Control Systems And Their Components (EE391)

Lec. 12: Combined state feedback and state estimator

Thu. May 12th, 2016

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Lecture Outline

- Combining regulator with state estimator
- Combining state feedback + state estimator + pre-scaling reference input
Combining regulator with state estimator

- Now we will study the performance when combining a regulator designed as $u = -Kx$ but implemented as $u = -K\hat{x}$

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \quad \iff \text{plant state equation} \\
&= Ax(t) - BK\hat{x}(t) \quad \iff \text{apply state feedback} \quad u = -K\hat{x} \\
&= Ax(t) - BK(x(t) - e(t)) \\
&= (A - BK)x(t) + BKe(t)
\end{align*}
\]

\[
\begin{align*}
\dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + LC(x(t) - \hat{x}(t)) \\
\dot{e}(t) &= (A - LC)e(t)
\end{align*}
\]

\[
\begin{bmatrix}
\dot{x}(t) \\
\dot{e}(t)
\end{bmatrix} =
\begin{bmatrix}
A - BK & BK \\
0 & A - LC
\end{bmatrix}
\begin{bmatrix}
x(t) \\
e(t)
\end{bmatrix}
\quad \iff \text{augmented equation}
\]

\[
y(t) =
\begin{bmatrix}
C & 0
\end{bmatrix}
\begin{bmatrix}
x(t) \\
e(t)
\end{bmatrix}
\]
Combining regulator with state estimator

\[
\begin{bmatrix}
\dot{x}(t) \\
\dot{e}(t)
\end{bmatrix}
= \begin{bmatrix}
A - BK & BK \\
0 & A - LC
\end{bmatrix}
\begin{bmatrix}
x(t) \\
e(t)
\end{bmatrix}
\]

\[A_{cl}\]  

This says that the dynamics of both the state vector of the closed loop systems after introducing feedback as well as the estimation error are determined by the eigenvalues of \(A_{cl}\).

Since this is a block upper diagonal matrix, its eigenvalues are given by

\[
|sI - A_{cl}| = |sI - (A - BK)| \cdot |sI - (A - LC)|
\]

This means that the poles of the closed loop system are the union of the regulator and estimator poles.
Combining regulator with state estimator

This also means that you can design the compensator and estimator separately just as we did before and then combine them (separation principle).

As a design rule, you should place the estimator poles at >2 the real part of the regulator poles (found from transient specs required) to ensure the estimator converges fast and hence the estimated values used for feedback are good.
Combining regulator with state estimator

- The whole system (regulator + estimator) looks like

\[ u = -K\hat{x} \]

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}
\]

\[
\dot{\hat{x}} = (A - LC - BK)\hat{x} + Ly
\]

The full compensator (Regulator + Estimator)

- Compensator accepts the sensor outputs as its inputs and provides at its output the actuator input
- Sometimes called **dynamic output feedback compensator (DOFC)**
Full compensator equations

\[ u = -K \hat{x} \]

Plant
\[ \dot{x} = Ax + Bu \]
\[ y = Cx \]

Estimator
\[ \dot{\hat{x}} = (A - LC - BK)\hat{x} + Ly \]

The full compensator (Regulator + Estimator)
\[ \dot{\hat{x}} = (A - LC - BK)\hat{x} + Ly \]
\[ u = -K\hat{x} \]

\[ A_c \equiv A - LC - BK, \quad B_c \equiv L, \quad C_c \equiv -K \]

TF of compensator
\[ -K \left( sI - (A - LC - BK) \right)^{-1} L \]

SS Matrices of compensator
Guidelines of designing full compensator

- Applying the separation principle, we simply design the controller matrix \( K \) and the observer matrix \( L \) separately each to satisfy a desired set of poles
- \( K \) is found from the desired poles of closes loop system which can be obtained from the required specifications of the closed loop system (Max overshoot, settling time,…) similar to Lecture 9
- \( L \) is found from the desired poles estimator which are either given or chosen to be 2-10 times larger than the system poles on the real axis
- Do not expect the feedback of the estimated state vector \( u = -K\hat{x} \) to operate just as good as \( u = -Kx \) especially at the initial transient period where the estimation error hasn’t yet decayed from its initial nonzero value (refer to the MATLAB code of Lec.12 for more insight)
Pre-Scaling the full compensator (reference input)

- See section 11.6 page 857 of Dorf’s book for more details
Pre-Scaling the full compensator (reference input)

\[
\dot{x}(t) = Ax(t) + Bu(t) \quad \Leftarrow \quad \text{plant state equation}
\]

\[
= Ax(t) - BK\hat{x}(t) + BNr \quad \Leftarrow \quad \text{apply state feedback with pre-scaled reference i/p } u = Nr - K\hat{x}
\]

\[
= Ax(t) - BK(x(t) - e(t)) + BNr
\]

\[
= (A - BK)x(t) + BKe(t) + BNr
\]

\[
\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + LC(x(t) - \hat{x}(t))
\]

\[
= (A - LC)\hat{x}(t) + Bu(t) + Ly(t) \quad \Leftarrow \quad \text{estimator state equation}
\]

\[
\dot{e}(t) = (A - LC)e(t)
\]

\[
\begin{bmatrix}
\dot{x}(t) \\
\dot{e}(t)
\end{bmatrix} =
\begin{bmatrix}
A - BK & BK \\
0 & A - LC
\end{bmatrix}
\begin{bmatrix}
x(t) \\
e(t)
\end{bmatrix} +
\begin{bmatrix}
BN \\
0
\end{bmatrix}r \quad \Leftarrow \quad \text{augmented equation}
\]

\[
y(t) =
\begin{bmatrix}
C & 0
\end{bmatrix}
\begin{bmatrix}
x(t) \\
e(t)
\end{bmatrix}
\]