# Proposal of an Optical CDMA Random Access Protocol

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**ABSTRACT**

A new random access protocol for optical code division multiple access (CDMA) communication systems is proposed. Stop & wait automatic repeat request (ARQ) is implemented at the optical link layer. A detailed state diagram and a mathematical model based on the equilibrium point analysis (EPA) technique are presented. Several performance measures are evaluated under different network parameters. We also compare the performance of the proposed protocol to that of the round robin receiver/transmitter ($R^3T$) protocol which is based on a go back-*n* technique. Finally, the proposed protocol is analyzed when a queuing subsystem is added. We prove by numerical analysis that the proposed protocol is less complex and significantly outperforms the $R^3T$ protocol. Our results also reveal that the performance of the proposed protocol with correlation receivers is competitive to that of the $R^3T$ protocol with chip-level receivers, which reduces the overall system cost. We finally show that a slight improvement in the protocol's performance can be achieved by only adding a single buffer to the system. The blocking probability is also reduced and the queuing delays that added to the total network latency are still acceptable.

**Keywords:** Chip-level receivers, code-division multiple-access, correlation receivers, on-off keying, optical CDMA protocols, optical link layer, optical networks, queuing.

## 1. INTRODUCTION

The success of modern communication systems has shifted the focus towards optical fiber networks. Especially, optical code division multiple access (CDMA) systems have been shown to be competitive candidates to support a large number of simultaneous users [1]-[12]. Most of researches in optical CDMA networks were concentrated in the physical layer. There are, however, a few authors that have examined the network or link layer of optical CDMA communication systems, [6]-[10]. In [6], [7] Hsu and Li have studied the slotted and unslotted optical CDMA networks. In [8] Shalaby has proposed two media access control (MAC) protocols for optical CDMA networks. However the effect of multi-packet messages, connection establishment and corrupted packets haven't been taken into account. Recently, Shalaby [9] has developed a new protocol called round robin receiver/transmitter ($R^3T$) protocol that has solved some of the above problems. The $R^3T$ protocol is based on a go-back $n$ automatic repeat request (ARQ), that is when a packet gets corrupted, the transmitter retransmits it and all sub-sequent packets. This scenario gives good performance for low population networks, while the performance is still low for larger population networks. Considering only the retransmission of corrupted packets, a selective reject ARQ has been applied in [10], which yields better results in case of higher population networks.

Our goal in this paper is to develop a new optical random access CDMA protocol which is based on a stop & wait ARQ in order to reduce the complexity of the previously proposed protocols. At the same time we aim at improving the system performance compared to the $R^3T$ protocol. Moreover, the proposed protocol is examined for the case of both chip-level and correlation receivers. Finally, the performance of this model is studied when introducing a queuing subsystem, namely increasing the number of available buffers.

The rest of this paper is organized as follows. In Section 2 we discuss the system and hardware architecture for our optical CDMA network. Section 3 is devoted for a description of the proposed protocol. The optical link layer is investigated and a complete state diagram of this protocol is presented. Section 4 is maintained for the mathematical model, where derivations of the steady state system throughput, the average packet delay, and the blocking probability are given. In Section 5 the proposed protocol is examined after adding a queuing subsystem, the same performance metrics are also considered. In Section 6 we discuss some of the numerical results obtained. A comparison between the proposed protocol and the $R^3T$ protocol, [9] is also considered. Finally we give our conclusions in Section 7.

## 2. SYSYTEM AND HARDWARE ARCHITECTURE

The network we use in our analysis is a broadcast star coupler based system connecting $N$ users. Optical orthogonal codes (OOCs) are used because they have both a peak cross-correlation and a shifted autocorrelation equal to one. The choice of code weight $w$ and code length $L$ for OOCs is arbitrary but these quantities determine the cardinality [1]. Due to the bursty nature of the traffic, we allow the number of users to exceed the number of available codes.

In our model, we assume that all codes are always available in a pool. In order to avoid receiver tunability, codes are assigned to users apriori. That is, when a user subscribes to the network, it is given a code (possibly used) randomly. Further, a code is randomly cyclic shifted around itself once assigned. In this way there is no need for pretransmission coordination. It has been shown that for fixed data rate and chip duration, there is no advantage in using pulse position modulation (PPM) in place of on-off keying (OOK). That's why each user is able to generate an optical OOK-CDMA signal according to its signature code that represents its data. Considering a message that is composed of $\ell > 0$ packets, each having $K > 0$ bits and taking only the effect of MAI into account, we can write the packet success probability $P_S(r_o)$ given $r_o \in \{1, 2, \ldots, N\}$ active users, for both chip-level and correlation receivers as in [8].

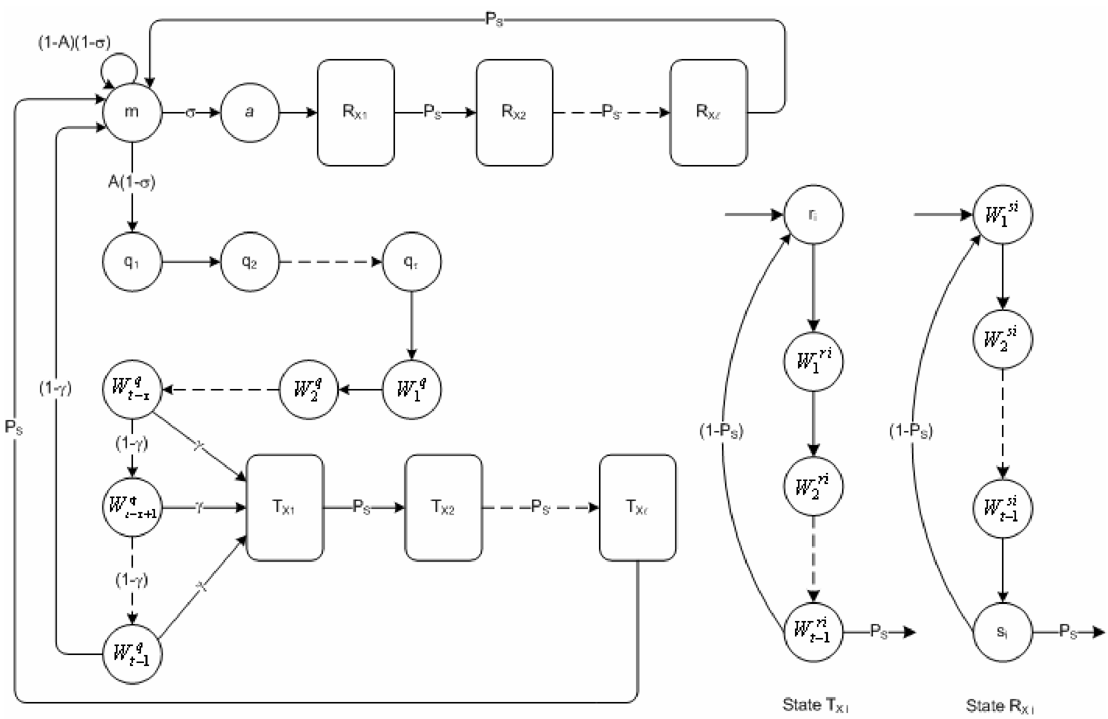


Fig. 1. Complete state diagram of the proposed Stop & Wait optical CDMA protocol.

## 3. MATHEMATICAL MODEL

Because of its simplicity over other ARQs, stop & wait is implemented at the data link layer of our optical CDMA network. After a packet is sent, a user enters a waiting mode to get a feedback of that packet. If a positive acknowledgement is received the user will send the next packet, otherwise he will retransmit another version of the corrupted packet. We impose the following assumptions in our model for optical CDMA protocol:

- Time is slotted with slot size $T_s$, a two way propagation delay time is assumed to be equal to $t$ time slots, and a timeout duration of $\tau$ time slots is selected such that $1 \le \tau \le t$.
- A message is composed of $\ell > t$ packets each having $K > 0$ bits. One packet should fit in a time slot $T_s = K\,\ell\,T_c$, where $T_c$ is the chip duration.
- A maximum of 1 message can arrive at each time slot to a station with probability $A$ (also called user activity). This message is stored in a buffer till its successful transmission.
- Any arrival to a non empty buffer will be blocked.
- Connection requests and acknowledgements are exchanged between stations.
- Transmission times for connection requests and acknowledgements are neglected.
- A priority is given for the reception mode than for the transmission mode.
- Receivers use a cyclic redundancy check (CRC) to determine whether a received packet is correctly detected or not.

The complete state diagram of the proposed optical CDMA random access protocol with stop & wait ARQ is illustrated in Fig. 1. Each state is labeled by its number of users. Transition between states is on a slot basis; that is the duration of each state equals to one time slot. At any time slot, any user in the network will be in one of the following states or modes:

- Initial state, $\{m\}$. Users in the initial state scan across all codes in a round-robin manner. If a connection request (event happening with probability $\sigma$) is found, a station will proceed to the acknowledgement mode. If there is a message arrival and there is no connection request, it will go to the requesting mode. If there is neither message arrival nor connection request, the station will remain in the initial state.
- Requesting mode, $\{q_1, q_2, \ldots, q_\tau\}$. Stations in this mode send repeated requests $\{q_1, q_2, \ldots, q_\tau\}$ for $\tau$ time slots. Then the station should wait for a feed back and thus enters a waiting mode $\{W_1^{\,q}, W_2^{\,q}, \ldots, W_{t-1}^{\,q}\}$, for $t-1$ time slots, as depicted in Fig. 1. Whenever a waiting station gets a positive acknowledgement (event occurring with probability $\gamma$) from the destination, it starts sending its message and enters the transmission mode, otherwise it remains in the waiting mode. In the last waiting state, if an acknowledgement is not received, the station is timed out and returns to the initial state.
- Acknowledgement state, $\{a\}$. In this state, the station sends an acknowledgement to a requesting station and then enters a waiting mode $\{W_1^{\,s1}, W_2^{\,s1}, \ldots, W_{t-1}^{\,s1}\}$ for $t$ time slots till the reception of the first packet.
- Reception mode, $\{Rx_1, Rx_2, \ldots, Rx_\ell\}$. A user in the reception mode receives either new packets or retransmitted ones. Fig. 1 illustrates the structure of the states $R_{X\,i}$ , $i \in \{1, 2, \ldots, \ell\}$. In states $s_i$, a user receives packet $i$, if it is successfully received, the user will move to states $\{W_1^{si+1}, W_2^{si+1}, \ldots, W_{t-1}^{si+1}\}$ waiting for the next packet, otherwise he will ask for retransmission and enters a waiting mode $\{W_1^{si}, W_2^{si}, \ldots, W_{t-1}^{si}\}$. If the station receives the last packet successfully it goes back to the initial state.
- Transmission mode, $\{Tx_1, Tx_2, \ldots, Tx_\ell\}$. This mode

involves transmission states $r_i$, $i \in \{1, 2, \ldots, \ell\}$ and waiting states $W_j^{ri}$, $j \in \{1, 2, \ldots, t-1\}$, Fig. 1. A user in state $r_i$ is transmitting packet $i$, then he enters a waiting mode $\{W_1^{ri}, W_2^{ri}, \ldots, W_{t-1}^{ri}\}$ to get the acknowledgement of that packet. If a positive feedback is received the user will proceed to state $r_{i+1}$ otherwise he will return to state $r_i$ for retransmission. After successful transmission of the last packet the user will return to the initial state.

## 4. PERFORMANCE ANALYSIS

Because of the complexity of the above model and the prohibitively large number of states, the problem will be analytically intractable if we try to calculate the transition probabilities between states and the stationary probabilities using Markov chains [12]. Fortunately, the equilibrium point analysis (EPA) technique significantly simplifies the problem. In this technique, the system is always assumed to be operating at an equilibrium point [11]; that is the number of users entering a state is equal to the number of users departing from the same state. By writing down the flow equations for each state, the performance of the proposed optical CDMA protocol can be evaluated.

**Transmission Mode**

This mode involves states $r_i$ and states $W_j^{ri}$, where $i \in \{1, 2, \ldots, \ell\}$ and $j \in \{1, 2, \ldots, t-1\}$. From Fig. 1, we have the following flow equations.

$$r_1 = r_2 = \ldots = r_\ell, \quad \text{and} \quad W_j^{ri} = r_1.$$

Let $r_o$ denotes the number of transmitting users in a given slot such that

$$r_o = \sum_{i=1}^{\ell} r_i = \ell\, r_1. \tag{1}$$

We define $W^r$ as the number of users waiting after transmission

$$W^r = \sum_{i=1}^{\ell}\sum_{j=1}^{t-1} W_j^{ri} = \ell(t-1)\cdot r_1. \tag{2}$$

**Reception Mode**

Assuming that the number of users transmitting packet $i$ is equal to that receiving the same packet, for $i = 1$ we can directly write $r_1 = s_1$. From Fig. 1, we can write the flow equations for states $s_i$ and $W_j^{si}$, for $i \in \{1, 2, \ldots, \ell\}$ and $j \in \{1, 2, \ldots, t-1\}$ as follows:

$$W_1^{s1} = a + (1 - P_S)\cdot s_1, \tag{3}$$

$$s_1 = s_2 = \ldots = s_\ell = r_1, \quad \text{and} \quad W_j^{si} = r_1.$$

We define the following two variables

$$s = \sum_{i=1}^{\ell} s_i = \ell\, r_1, \quad W^s = \sum_{i=1}^{\ell}\sum_{j=1}^{t-1} W_j^{si} = \ell(t-1)\cdot r_1. \tag{4}$$

**Acknowledgement Mode**

Again by writing the flow equations for the states in this mode, described in Fig. 1 and using equation (3) we get:

$$a = \sigma\, m = P_S\, r_1. \tag{5}$$

Therefore

$$m = \frac{P_S}{\sigma} r_1. \tag{6}$$

**Requesting Mode**

Fig. 1 illustrates the requesting states and waiting states in the requesting mode. By writing down the flow equations as in [9] we define:

$$q + W^q = \sum_{i=1}^{\tau} q_i + \sum_{i=1}^{t-1} W_i^q = \left[t - 1 + \frac{1}{\gamma}\left(1 - (1-\gamma)^\tau\right)\right] A \frac{1-\sigma}{\sigma} P_S\, r_1. \tag{7}$$

The probability that a request is found by a scanning user $\sigma$ and the probability that a station gets an acknowledgement $\gamma$ can be computed as follows [9]:

$$\sigma = \frac{1}{2}\left[\sqrt{\left(AP_S\tau\frac{r_1}{N}\right)^2 + 4\left(AP_S\tau\frac{r_1}{N}\right)} - AP_S\tau\frac{r_1}{N}\right].$$

$$\gamma = 1 - \left[1 - \frac{\sigma}{A(1-\sigma)}\right]^{1/\tau}$$

**Steady State System Throughput**

The steady state system throughput $\beta(N, A, t, \tau, \ell)$ is defined as the average number of successful received packets per slot. It can be calculated as follows:

$$\beta(N, A, t, \tau, \ell) = \sum_{i=1}^{\ell} s_i \cdot P_S = P_S(r_o)\,\ell\, r_1.$$

Substituting with equation (1) we get

$$\beta(N, A, t, \tau, \ell) = r_o\, P_S(r_o). \tag{8}$$

To compute $r_o$ we assume that the total number of users in all states is equal to $N$, yielding

$$\begin{aligned} N &= m + r + W^r + s + W^s + a + q + W^q \\ &= \frac{r_o}{\ell}\left[\frac{P_S}{\sigma} + 2\,\ell\, t + P_S + \left\{t - 1 + \frac{1}{\gamma}\left(1 - (1-\gamma)^\tau\right)\right\} A \frac{1-\sigma}{\sigma} P_S\right], \end{aligned} \tag{9}$$

where we have used equations (1) - (7).

**Blocking Probability**

The blocking probability is defined as the probability of an arrival being blocked. In this case the blocking probability is equal to the probability that the station is not in the initial state $m$ and there is a message arrival $A$ or the station is in the initial state $m$ but there is a request for connection and at the same time there is a message arrival $A$. Thus, we can write

$$P_B = \frac{m}{N}\cdot\sigma\cdot A + \left(1 - \frac{m}{N}\right)\cdot A.$$

Substituting with equations (1) and (6) we get

$$P_B = A\left[1 - \frac{P_S}{\ell N}\left(\frac{1-\sigma}{\sigma}\right) r_o\right]. \tag{10}$$

**Average Packet Delay**

The average packet delay $D$ can be calculated from Little's theorem:

$$D = \frac{NA\cdot(1-P_B)}{\beta(N, A, t, \tau, \ell)} \quad \text{slots}, \tag{11}$$

where $NA\cdot(1-P_B)$ denotes the total traffic in the network. Note that equations (10) and (11) are valid for both optical random access CDMA protocols; with Stop & Wait ARQ and with go-back $n$ ARQ [9].

## 5. QUEUING MODEL

As argued before, each node is equipped with only a single buffer that is able to store only the message being served, thus any arrival to a non empty buffer is dropped. This gives rise to a high blocking probability. In this section we introduce a queuing subsystem that is able to store one more message (message waiting to be served) if the main buffer is busy. We add the following assumptions in our model for optical CDMA protocol:

- A maximum of 1 message can arrive at each time slot to a station and is stored in the queue if the server is busy.

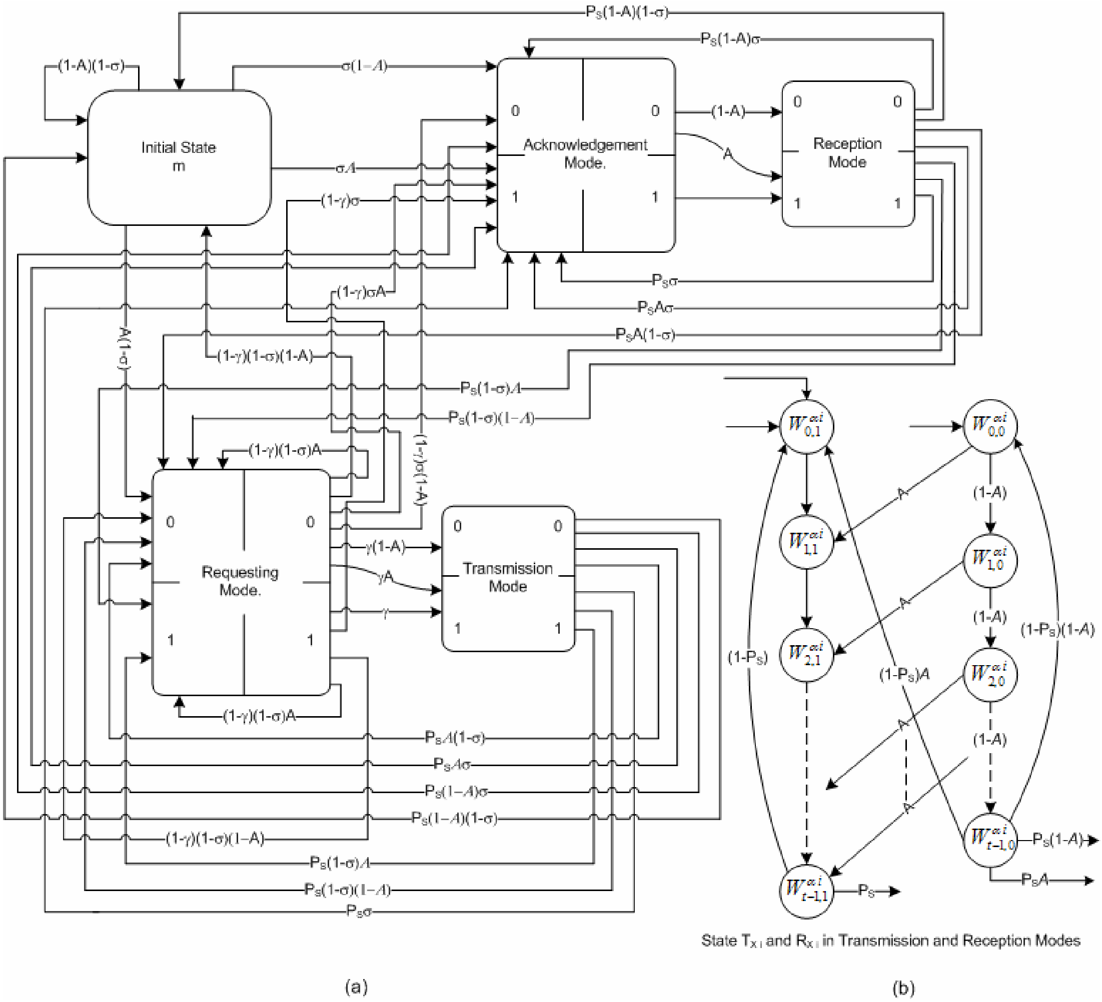


Fig. 2. State diagram of the proposed optical CDMA protocol with a single buffer in the queue.

- Any arrival to a non empty queue is blocked. The queue is freed once the stored message is moved to the server for being transmitted.
- A station scans for connection requests only after a successful transmission or reception or when it is idle.

The state diagram of the proposed optical CDMA protocol with a single buffer in the queue is illustrated in Fig. 2. Each mode is divided into two sets of states: states marked with '0' indicating that the buffer is empty and states marked with '1' indicating that the buffer is full. Transition between states is on a slot basis. Users move from states marked with '0' to states marked with '1' if there is a message arrival.

Similarly, by writing down the flow equations for all states and carrying out summations which involve mathematical series, we can compute the number of users in all states, and thus the problem becomes tractable. The steady state system throughput and the average packet delay in this case are also computed using equations (8) and (11), respectively. Whereas the blocking probability is equal to the probability that the station is not in the initial state, there is a message arrival, and the queue is full in addition to:

1- After successful transmission / reception: If there is a connection request and there is a message arrival, blocking will occur.
2- After request: A message is blocked, if the station is timed-out, there is a message arrival and a connection request is found, or if the station got a positive acknowledgement and there is a message arrival.

## 6. NUMERICAL RESULTS

In this section, we discuss some numerical results for the proposed optical random access CDMA protocol. The steady state system throughput, the blocking probability, and the average packet delay derived above have been evaluated and compared to the results in [9]. Our results are plotted in Figs. 3-7. In our simulations a user bit rate of $R_b = 127$ Mbps is held constant. Optical orthogonal codes with length $L = 31$, code weight $w = 3$ and unity correlation constraints are used. A message length of $\ell = 15$ packets each having $K = 127$ bits are selected. A two way propagation delay time $t \in \{2, 4, 6\}$ slots (or interstation distances of $z = vT_s t/2 \in \{200, 400, 600\}$ m, where $v \approx 2\times10^8$ m/s is the speed of light inside a fiber) and a timeout duration $\tau = 1$ slot are imposed in our simulations. The performance of the proposed protocol using both chip-level receivers and correlation receivers is also presented. Correlation receivers are only considered in Fig. 3.

In Fig. 3 we have plotted the throughput versus the number of users for both chip-level receivers and correlation receivers. General trends of the curves can be noticed. As the number of users in the network increases more packets become available for transmission with low interference. Thus the throughput increases till it reaches its peak. As the number of users is further increased the effect of MAI becomes significant and the throughput starts to decay. In the case of stop & wait ARQ the

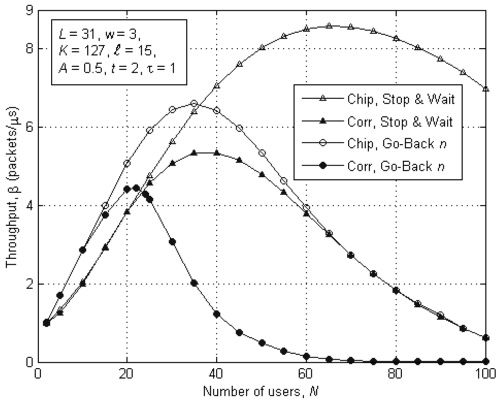


Fig. 3. Throughput vs. number of users for different receivers.

protocol supports a larger number of users and reaches higher values of throughput. This is because the channel is not busy all the time, as users enter a waiting mode after sending each packet. On the other hand for a protocol depending on a go-back $n$ ARQ, users continuously send their packets, which contributes to higher traffic loads, yielding lower packet success probabilities and lower throughput values. It can be inferred that the performance of both protocols is reduced when using correlation receivers and that the performance of the stop & wait with correlation receivers is nearly close to that of the $R^3T$ protocol with chip-level receivers.

In Fig. 4 we have plotted the system throughput and the average packet delay versus the average activity for different number of users $N \in \{30, 70\}$. It can be noticed that the $R^3T$ protocol exhibits higher throughput values and lower delays at $N = 30$. Whereas the proposed protocol outperforms the $R^3T$ protocol at $N = 70$, as argued in Fig. 3. For short interstation distances, as the user activity increases the throughput also increases till it reaches saturation whereas for longer distances the throughput falls after reaching its peak. In fact the initial increase of throughput is because as $A$ increases above zero, more packets become available with low interference. The throughput decay in the case of long propagation delays after reaching its peak because the number of active users increases while other users already in the transmission mode are still busy transmitting their messages over long distances. The interference would thus increase rapidly and packet failures become more probable. Finally, it is noticed that for longer interstation distances and an increase in user activity, the average packet delay increases.

The relation between the system throughput and the number of users for our proposed protocol is depicted in Fig. 5. It can be seen that for longer propagation delays, the proposed protocol accommodates a higher number of users. Also the quality of service (QOS) requirements is achieved in a large dynamic range. This is because as $t$ increases, users wait longer times after transmitting their packets, giving chance to other users to start transmission.

In Fig. 6 we have plotted the relation between the throughput and the average packet delay when varying the average user activity and for different number of users in the network. General trends of the curves can be noticed. As the user activity increases, both the throughput and the average packet delay .increase till the throughput reaches its saturation value. It can be shown that this value depends on both the code length and the interstation distance [9]. It is immediate to notice that for larger number of users and higher activity levels, the queuing delay will add to the total latency of the network.

Finally, the blocking probability has been plotted against both the number of users and the average activity in Fig. 7. It can be

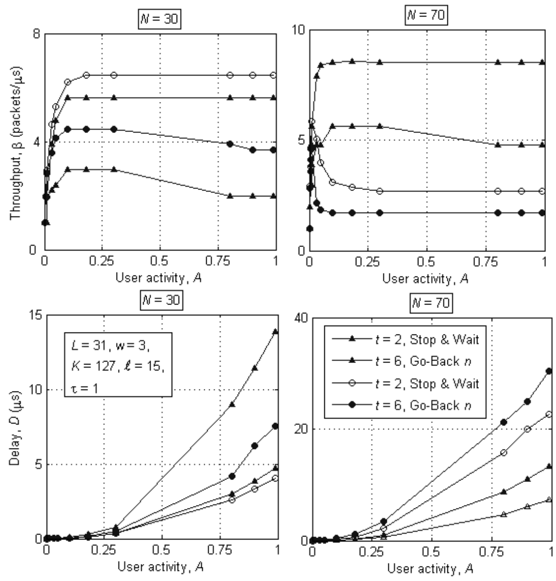


Fig. 4. Throughput and delay vs. activity.

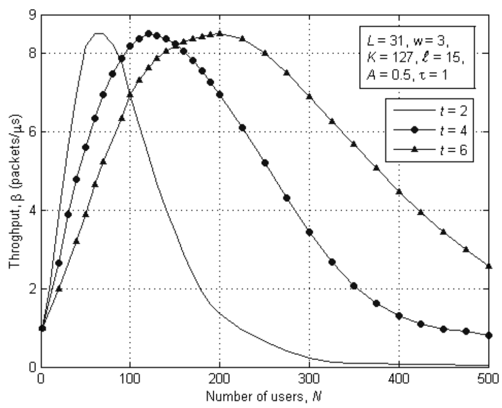


Fig. 5. Throughput vs. number of users.

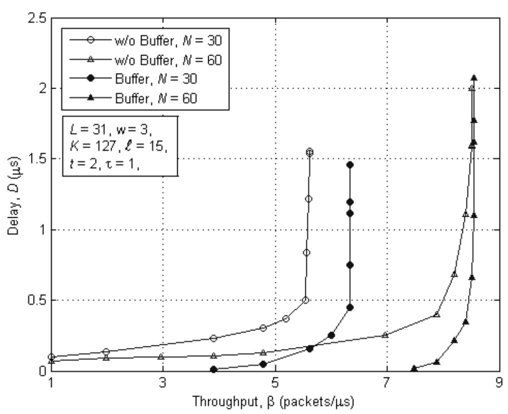


Fig. 6. Average packet delay vs. throughput.

noticed that by including only a single buffer to the system the blocking probability can be reduced.

## 7. CONCLUSIONS

In this paper we have proposed an optical random access CDMA protocol based on a stop & wait ARQ. A mathematical description of this protocol has been presented using a detailed

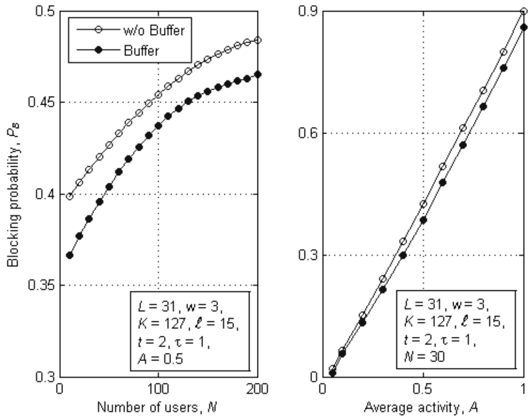


Fig. 7. Blocking probability vs. number of users and average activity.

state diagram. Several performance measures were considered, namely, the steady state system throughput, the blocking probability, and the average packet delay. In our numerical calculations, we have focused only on the effect of MAI. Both correlation and chip-level receivers were also examined. The following concluding remarks can be extracted from our results.

1- The performance of the proposed CDMA protocol using correlation receivers is almost the same as that of the $R^3T$ protocol with chip-level receivers.
2- The proposed protocol outperforms the $R^3T$ protocol (in terms of the system throughput and average packet delay) in high population networks, whereas for smaller size networks the $R^3T$ protocol slightly outperforms the proposed one.
3- Both protocols exhibit satisfactory blocking probability only for small traffic loads. Furthermore, the average packet delay is acceptable under different network parameters.
4- For larger interstation distances, the proposed protocol accommodates a higher number of users. Also the quality of service (QOS) requirements can be achieved in a larger dynamic range (wider range of users).
5- The complexity of the proposed protocol is significantly reduced compared to the $R^3T$ protocol. Also the cost is reduced when using correlation receivers.
6- The proposed protocol with queuing subsystem has proved a slight improvement in the performance for low propagation delays and small population networks. Also the blocking probability is reduced and the queuing delay is negligible.

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