Evaluation of Nonlinear Interference in Few-Mode Fiber Using the Gaussian Noise Model

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Abstract: The nonlinear propagation in few-mode fibers (FMFs) is modeled using a Gaussian approach, where a closed-form formula for the nonlinear interference is derived. The impacts of different nonlinearity penalties are investigated using this model.

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1. Introduction

Space-division multiplexing (SDM) is a promised degree of freedom to increase the transmission capacity, which is rapidly approaching its fundamental limit in single mode fibers [1]. However, the nonlinear interaction between different propagation modes in FMFs (as the channel for SDM system) is a major source of performance limitation, which must be addressed for enabling its mitigation. Few analytical efforts have been developed to model the nonlinear propagation in multi-mode fibers [2,3]. In this paper, we extend the Gaussian noise (GN) model developed for single mode fibers [4] to address the different nonlinearities impact in FMFs. In [3], a general integral formula for the cross-modal nonlinear interaction has been proposed in multi-mode fibers. However, in this work, we formulate a simple closed-form expression (with less computational complexity) for the nonlinear interference power in the case of weak linear coupling regime among different spatial modes. In addition, a proposed formula for the nonlinear capacity of FMFs is obtained, which estimates the effect of different nonlinearity penalties for various constellation orders.

2. Proposed GN-Model for Few-Mode Fibers

The signal propagation of mode p in a FMF is described by Eq. (26) in [5], which is divided into a linear part (dispersion + attenuation) and a nonlinear part, given by $N_p = j\gamma\left(\frac{8}{9}f_{pppp}\left|\bar{\mathbf{A}}_p\right|^2 + \frac{4}{3}\sum_{h\neq p} f_{hhpp}\left|\bar{\mathbf{A}}_h\right|^2\right)$. Here $\bar{\mathbf{A}}_p$ is field envelope of mode p, γ is the fiber nonlinearity coefficient, f_{pppp} is the intra-modal nonlinear coefficient tensor of mode p, and f_{pphh} is the inter-modal nonlinear coefficient tensor between p and h spatial modes. The calculated values of these tensors have been reported in [1].

The GN-model for single mode fibers assumes that the nonlinearity source can be modeled as an additive Gaussian noise which is statistically independent from both the amplifier noise and the transmitted signal [4]. Also, it assumes the transmitted signal as a wavelength-division multiplexed (WDM) comb signal with N_{ch} channels. These assumptions can be extended for FMFs based on the fact that the interaction between any two orthogonal polarization modes is equivalent to that between two spatial modes. Therefore, the performance of a FMF link per mode can be determined by the optical signal-to-noise ratio as $OSNR_p = P_{tx,p}/(P_{ASE}+P_{NL_p})$, where $P_{tx,p}$ is the launch power per mode, P_{ASE} is amplified-spontaneous-emission (ASE) noise power, and $P_{NL,p}$ is the nonlinear interference power. After a rigorous mathematical analysis, we derive the nonlinear interference power formula through integrating its power spectral density (PSD) over the WDM bandwidth B_w. Furthermore, this PSD is obtained by statistical averaging the squaring absolute-value of the nonlinear optical field, which can be obtained from the solution of Eq. (26) in [5] by splitting it into linear and nonlinear parts. Next, by assuming a rectangular shaped WDM channel spectrum with bandwidth $B_{ch} = R_s$ (Nyquist case), where R_s is the baud rate, a closed-form expression for the nonlinear interference power per mode can be obtained at the center channel frequency as:

$$P_{NL,p} = \frac{4\gamma^2}{3M^3}\left(\frac{4}{9}f_{pppp}^2 + \sum_{h\neq p} f_{hhpp}^2\right) N_s\, L_{eff,p} B_n \frac{\log(\pi^2 B_w^2\left|\beta_{2,p}\right| L_{eff,p})}{\pi^2 B_w^3\left|\beta_{2,p}\right|} P_{tx}^3, \tag{1}$$

where M is the number of spatial modes, $L_{eff,p}$=$(1-e^{-\alpha_p L})/\alpha_p$ is the span effective length of a fiber with length L, and a mode fiber loss coefficient α_p, P_{tx} is the total lunch power, $\beta_{2,p}$ is the mode group-velocity dispersion (GVD), and N_s is the number of fiber spans. From Shannon's relation for the unconstrained additive-white Gaussian noise channel of single mode fibers and knowing that the electrical signal-to-noise ratio is given by $SNR_p = (B_n/R_s)\cdot OSNR_p$, we can formulate an extended overall FMF nonlinear capacity (bits/symbol) formula for dual-polarized signal as:

$$C = 2\frac{R_s}{B_{ch}}\sum_P \log_2\left(1 + \frac{P_{tx,p}}{B_n N_s F (G-1) h\nu + P_{NL,p}}\right), \tag{2}$$

where B_n is the noise bandwidth of 12.48 GHz (0.1 *nm* is the reference resolution for OSNR calculation), F is the amplifier noise factor, h is Plank's constant, ν is the center channel frequency, and G is the amplifier gain.

3. Model Results

In this section, we apply this proposed model for a system with the following FMF parameters [1] ($\alpha_p = 0.22$ dB/km, $\beta_{2,p} = -21.2$ ps^2/km, and $\gamma_p = 1.3$ W^{-1}km^{-1}) for three modes (LP_{01}, $LP_{11a,b}$). For WDM system, the specifications are assumed as: $R_s = 32$ GBaud (that is, a net baud rate of 25 GBaud + 20% for forward error correction (FEC) and network protocols overheads [4]) and $N_{ch} = 5$. The used amplifier is an erbium-doped fiber amplifier (EDFA) with a noise figure of 6 dB and a gain that compensates the fiber span loss: $G = e^{\alpha_p L}$. Fig. 1 shows the impacts of different nonlinearity penalties on both the maximum channel reach and OSNR. Fig. 1-a shows that the penalty effect is greater in the fundamental LP_{01} than the degenerated modes (LP_{11a}, LP_{11b}) in both the inter-modal and the intra-modal limits by $\approx 20\%$ at optimal launched power. Also, the inter-modal penalty is greater than those for intra-modal one by $\approx 7\%$ for all spatial modes. This penalty variation is related to the different spatial interactions and the fiber effective areas of different modes. Also, Fig. 1-b shows the same contributions for different nonlinearity penalties on OSNR at fixed maximum reach of 5000 km.

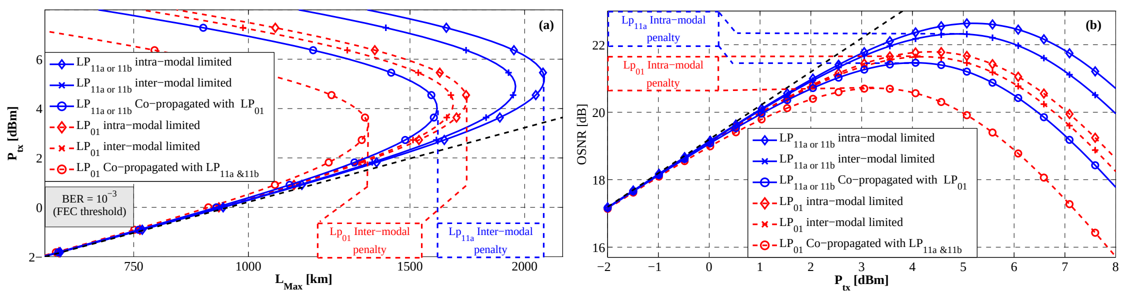

Fig. 1: a) Launch power versus maximum reach and b) OSNR versus lunch power, for a FMF with $L_s = 100$ km, PM-QPSK modulation format, and data rate = 100 Gb/s/mode.

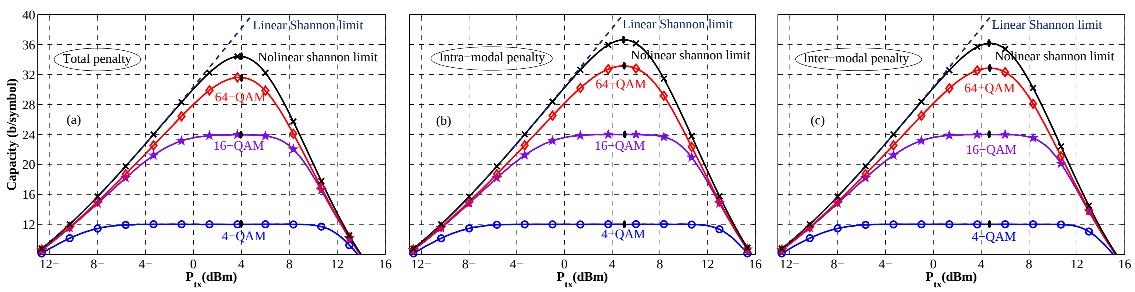

Fig. 2: Capacity versus channel lunch power at different nonlinear penalties for a FMF of $L_s = 100$ km and $N_s = 5$.

The capacity for different constellation levels (4, 16, 64)-QAM is compared to both the nonlinear and linear Shannon limit in Fig. 2. The nonlinearities impact does not appear at low constellation levels (4-QAM). However, at moderate levels (16-QAM), the different nonlinearity penalties become significant and limit the FMF capacity from reaching its maximum value (which is 24 b/symbol for a 3-mode dual-polarized signal). In addition, both the inter- and intra-modal impacts are approximately equal as shown in Figs. 2-b and 2-c. At high constellation levels, the nonlinearities impact become more significant for different penalties. Furthermore, the inter-modal impact becomes greater than the intra-modal one by $\approx 1.5\%$ at nonlinear Shannon limit. These nonlinearity penalties are clear in the nonlinear Shannon capacity curves for different nonlinearity limits. The optimal launched power (Top points on curves in Fig. 2) only depends on the penalty limit (nonlinear tensor's values) not on the constellation order.

4. Conclusions

The GN-model has been extended for FMFs in order to estimate the effects of different nonlinearity penalties. A closed-form formula for the nonlinear interference power has been derived. Furthermore, an expression for FMF capacity has been obtained. Using this model, it has been verified that the performance degradation due to the inter-modal penalty is greater than those for the intra-modal ones. In addition, the nonlinear impact on the fundamental mode is greater than that for the degenerated modes.

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