Direct and Indirect Semiconductors

Chapter 3  Carrier Action

**E-k Diagrams**

- **GaAs, GaN** (direct semiconductors)
  - Little change in momentum is required for recombination
  - Momentum is conserved by photon (light) emission

- **Si, Ge** (indirect semiconductors)
  - Large change in momentum is required for recombination
  - Momentum is conserved by mainly phonon (vibration) emission + photon emission
Excess Carrier Concentrations

Deviation from equilibrium values

Values under arbitrary conditions

Equilibrium values

\[ \Delta n \equiv n - n_0 \]
\[ \Delta p \equiv p - p_0 \]

Positive deviation \( \Delta n, \Delta p > 0 \) corresponds to a carrier excess, while negative deviation \( \Delta n, \Delta p < 0 \) corresponds to a carrier deficit.

Charge neutrality condition:

\[ \Delta n = \Delta p \]
Often, the disturbance from equilibrium is small, such that the majority carrier concentration is not affected significantly.

However, the minority carrier concentration can be significantly affected.

- For an $n$-type material $\Delta p \ll n_0$, $n \approx n_0$ 
  $\Delta p >> p_0$

- For a $p$-type material $\Delta n \ll p_0$, $p \approx p_0$ 
  $\Delta n >> n_0$

This condition is called “low-level injection condition”.

The *workhorse* of the diffusion in low-level injection condition is the minority carrier (which number increases significantly) while the majority carrier is practically undisturbed.
Photoconductivity is an optical and electrical phenomenon in which a material becomes more electrically conductive due to the absorption of electro-magnetic radiation such as visible light, ultraviolet light, infrared light, or gamma radiation.

When light is absorbed by a material like semiconductor, the number of free electrons and holes changes and raises the electrical conductivity of the semiconductor.

To cause excitation, the light that strikes the semiconductor must have enough energy to raise electrons across the band gap.
Example: Photoconductor

Consider a sample of Si at 300 K doped with $10^{16} \text{ cm}^{-3}$ Boron, with recombination lifetime 1 $\mu$s. It is exposed continuously to light, such that electron-hole pairs are generated throughout the sample at the rate of $10^{20} \text{ per cm}^3 \text{ per second}$, \textit{i.e.} the \textbf{generation rate} $G_L = 10^{20}/\text{cm}^3/\text{s}$.

\begin{enumerate}
  \item \textbf{What are $p_0$ and $n_0$?}

    \[ p_0 = 10^{16} \text{ cm}^{-3} \]

    \[ n_0 = \frac{n_i^2}{p_0} = \frac{(10^{10})^2}{10^{16}} = 10^4 \text{ cm}^{-3} \]

  \item \textbf{What are $\Delta n$ and $\Delta p$?}

    \[ \Delta p = \Delta n = G_L \cdot \tau = 10^{20} \times 10^{-6} = 10^{14} \text{ cm}^{-3} \]

  \begin{itemize}
    \item \textbf{Hint: In steady-state (equilibrium), generation rate equals recombination rate}
  \end{itemize}
\end{enumerate}
Consider a sample of Si at 300 K doped with $10^{16}$ cm$^{-3}$ Boron, with recombination lifetime 1 $\mu$s. It is exposed continuously to light, such that electron-hole pairs are generated throughout the sample at the rate of $10^{20}$ per cm$^3$ per second, i.e. the generation rate $G_L = 10^{20}$/cm$^3$/s.

c) What are $p$ and $n$?

$$p = p_0 + \Delta p = 10^{16} + 10^{14} \approx 10^{16} \text{ cm}^{-3}$$
$$n = n_0 + \Delta n = 10^4 + 10^{14} \approx 10^{14} \text{ cm}^{-3}$$

d) What are $np$ product?

$$np \approx 10^{16} \cdot 10^{14} = 10^{30} \text{ cm}^{-3} \gg n_i^2$$

• **Note:** The $np$ product can be very different from $n_i^2$ in case of perturbed / agitated semiconductor.
Consider carrier-flux into/out of an infinitesimal volume:

\[ Adx \cdot \left( \frac{\partial n}{\partial t} \right) = \frac{1}{q} \left[ J_N(x + dx) - J_N(x) \right] A \]
\[ J_N(x + dx) = J_N(x) + \frac{\partial J_N(x)}{\partial x} dx \]

\[ \frac{\partial n}{\partial t} = \frac{1}{q} \frac{\partial J_N(x)}{\partial x} + \frac{\partial n}{\partial t} \bigg|_{\text{thermal R–G}} + \frac{\partial n}{\partial t} \bigg|_{\text{other processes}} \]

\[ \frac{\partial p}{\partial t} = -\frac{1}{q} \frac{\partial J_P(x)}{\partial x} + \frac{\partial p}{\partial t} \bigg|_{\text{thermal R–G}} + \frac{\partial p}{\partial t} \bigg|_{\text{other processes}} \]
The minority carrier diffusion equations are derived from the general continuity equations, and are applicable only for minority carriers.

Simplifying assumptions:

- The electric field is small, such that:

\[
\mathbf{J}_N = q\mu_n n \mathbf{E} + qD_N \frac{\partial n}{\partial x} \approx qD_N \frac{\partial n}{\partial x}
\]

- For \( p \)-type material

\[
\mathbf{J}_P = q\mu_p p \mathbf{E} - qD_P \frac{\partial p}{\partial x} \approx -qD_P \frac{\partial p}{\partial x}
\]

- For \( n \)-type material

Equilibrium minority carrier concentration \( n_0 \) and \( p_0 \) are independent of \( x \) (uniform doping).

Low-level injection conditions prevail.
Starting with the continuity equation for electrons:
\[
\frac{\partial n}{\partial t} = \frac{1}{q} \frac{\partial J_N(x)}{\partial x} - \frac{\Delta n}{\tau_n} + G_L
\]

\[
\frac{\partial (n_0 + \Delta n)}{\partial t} = \frac{1}{q} \frac{\partial}{\partial x} \left[ q D_N \frac{\partial (n_0 + \Delta n)}{\partial x} \right] - \frac{\Delta n}{\tau_n} + G_L
\]

The **minority carrier lifetime** \( \tau \) is the average time for excess minority carriers to “survive” in a sea of majority carriers.

Therefore
\[
\frac{\partial \Delta n}{\partial t} = D_N \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_n} + G_L
\]

Similarly
\[
\frac{\partial \Delta p}{\partial t} = D_P \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau_p} + G_L
\]
Carrier Concentration Notation

- The subscript “n” or “p” is now used to explicitly denote n-type or p-type material.
  - \( p_n \) is the hole concentration in n-type material
  - \( n_p \) is the electron concentration in p-type material

Thus, the minority carrier diffusion equations are:

\[
\frac{\partial \Delta n_p}{\partial t} = D_N \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L
\]

\[
\frac{\partial \Delta p_n}{\partial t} = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L
\]

- Partial Differential Equation (PDE)!
- The so called “Heat Conduction Equation”
Simplifications (Special Cases)

■ Steady state:

\[ \frac{\partial \Delta n_p}{\partial t} = 0, \quad \frac{\partial \Delta p_n}{\partial t} = 0 \]

■ No diffusion current:

\[ D_N \frac{\partial^2 \Delta n_p}{\partial x^2} = 0, \quad D_p \frac{\partial^2 \Delta p_n}{\partial x^2} = 0 \]

■ No thermal R–G:

\[ \frac{\Delta n_p}{\tau_n} = 0, \quad \frac{\Delta p_n}{\tau_p} = 0 \]

■ No other processes:

\[ G_L = 0 \]

• Solutions for these common special-case diffusion equation are provided in the textbook
Consider the special case:

- Constant minority-carrier (hole) injection at \( x = 0 \)
- Steady state, no light absorption for \( x > 0 \)

\[
0 = D_P \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} \\
\Delta p_n(0) = \Delta p_{n0} \\
G_L = 0 \text{ for } x > 0
\]

\[
\frac{\partial^2 \Delta p_n}{\partial x^2} = \frac{\Delta p_n}{D_P \tau_p} = \frac{\Delta p_n}{L_P^2}
\]

The **hole diffusion length** \( L_P \) is defined to be: 
\[
L_P = \sqrt{D_P \tau_p}
\]

Similarly, 
\[
L_N = \sqrt{D_N \tau_n}
\]
The general solution to the equation \( \frac{\partial^2 \Delta p_n}{\partial x^2} = \frac{\Delta p_n}{L_p^2} \) is:

\[
\Delta p_n(x) = Ae^{-x/L_p} + Be^{x/L_p}
\]

A and B are constants determined by boundary conditions:

\[
\Delta p_n(\infty) = 0 \quad \Rightarrow \quad B = 0
\]

\[
\Delta p_n(0) = \Delta p_{n0} \quad \Rightarrow \quad A = \Delta p_{n0}
\]

Therefore, the solution is:

\[
\Delta p_n(x) = \Delta p_{n0} e^{-x/L_p}
\]

Physically, \( L_p \) and \( L_N \) represent the average distance that a minority carrier can diffuse before it recombines with a majority carrier.
Example: Minority Carrier Diffusion Length

Given $N_D = 10^{16}$ cm$^{-3}$, $\tau_p = 10^{-6}$ s. Calculate $L_p$.

From the plot,

$\mu_p = 437 \text{ cm}^2/\text{V} \cdot \text{s}$

$D_p = \frac{kT}{q} \mu_p$

$$= 25.86 \text{ mV} \cdot 437 \text{ cm}^2/\text{V} \cdot \text{s}$$

$$= 11.3 \text{ cm}^2/\text{s}$$

$L_p = \sqrt{D_p \tau_p}$

$$= \sqrt{11.3 \text{ cm}^2/\text{s} \cdot 10^{-6} \text{ s}}$$

$$= 3.361 \times 10^{-3} \text{ cm}$$

$$= 33.61 \mu\text{m}$$
Whenever $\Delta n = \Delta p \neq 0$ then $np \neq n_i^2$ and we are at non-equilibrium conditions.

In this situation, now we would like to preserve and use the relations:

$$n = n_i e^{(E_F - E_i)/kT}, \quad p = n_i e^{(E_i - E_F)/kT}$$

On the other hand, both equations imply $np = n_i^2$, which does not apply anymore.

The solution is to introduce to quasi-Fermi levels $F_N$ and $F_P$ such that:

$$n = n_i e^{(F_N - E_i)/kT}, \quad p = n_i e^{(E_i - F_P)/kT}$$

$$F_N = E_i + kT \ln \left( \frac{n}{n_i} \right), \quad F_P = E_i - kT \ln \left( \frac{p}{n_i} \right)$$

- The quasi-Fermi levels is useful to describe the carrier concentrations under non-equilibrium conditions
Example: Quasi-Fermi Levels

Consider a Si sample at 300 K with $N_D = 10^{17} \text{ cm}^{-3}$ and $\Delta n = \Delta p = 10^{14} \text{ cm}^{-3}$.

a) What are $p$ and $n$? • The sample is an $n$-type

\[
n_0 = N_D = 10^{17} \text{ cm}^{-3}, \quad p_0 = \frac{n_i^2}{n_0} = 10^3 \text{ cm}^{-3}
\]

\[
n = n_0 + \Delta n = 10^{17} + 10^{14} \approx 10^{17} \text{ cm}^{-3}
\]

\[
p = p_0 + \Delta p = 10^3 + 10^{14} \approx 10^{14} \text{ cm}^{-3}
\]

b) What is the $np$ product?

\[
np \approx 10^{17} \cdot 10^{14} = 10^{31} \text{ cm}^{-3}
\]
Consider a Si sample at 300 K with $N_D = 10^{17} \text{ cm}^{-3}$ and $\Delta n = \Delta p = 10^{14} \text{ cm}^{-3}$.

c) Find $F_N$ and $F_P$?

$$F_N = E_i + kT \ln \left( \frac{n}{n_i} \right)$$

$$F_N - E_i = 8.62 \times 10^{-5} \cdot 300 \cdot \ln \left( \frac{10^{17}}{10^{10}} \right)$$

$$= 0.417 \text{ eV}$$

$$F_P = E_i - kT \ln \left( \frac{p}{n_i} \right)$$

$$E_i - F_P = 8.62 \times 10^{-5} \cdot 300 \cdot \ln \left( \frac{10^{14}}{10^{10}} \right)$$

$$= 0.238 \text{ eV}$$

$$np = n_i e^{(F_N - E_i)/kT} \cdot n_i e^{(E_i - F_P)/kT}$$

$$= 10^{10} e^{0.417 \cdot 0.02586} \cdot 10^{10} e^{0.238 \cdot 0.02586}$$

$$= 1.000257 \times 10^{31}$$

$$\approx 10^{31} \text{ cm}^{-3}$$