

PERFORMANCE ANALYSIS OF SUCCESSIVE INTERFERENCE CANCELLATION WITH MODIFIED QUADRATIC CONGRUENCE AND HADAMARD CODES FOR OPTICAL CDMA

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Abstract— One of the techniques used to mitigate the problem of cross correlation is successive interference cancellation (SIC). This technique has been inspired of multi user detection technique (MUD), and can increase the capacity of optical CDMA system. A detailed analysis of this system is theoretically analyzed, taking into account multiple-access interference (MAI). In addition, the system performance is analyzed with both modified quadratic congruence (MQC) code and Hadamard codes. We have also compared the performance of this system (in terms of bit error rate) with conventional scheme. It has been shown that our proposed scheme performs better than the conventional scheme, and it has been found that signature code sets with higher weight are preferred in our SIC scheme.

Keywords: *Optical code-division multiple access (OCDMA), multiple-access interference (MAI), successive interference cancellation (SIC), Hadamard code, Modified quadratic congruence code (MQC).*

1. Introduction

Up to this time, optical CDMA methods have been studied and developed for optical systems. The main methods are used optical orthogonal code (OOC) [1, 2], which can be generated using one of the following devices: optical delay lines or optical switches [1, 2]; fiber Bragg grating (FBG) or arrayed waveguide gratings (AWGs) [3]. OCDMA using OOCs has many problems, such as a limitation on the number of distinct code sequences and low optical power because there are fewer 1s than 0s.

One of the key issues for OCDMA is to reduce the multiple access interference (MAI) among the users, and to come across this type of the problems many techniques have been studied on interference cancellation, such as Salehi and Brackett [4] have used an optical hardlimiter.

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Double optical hardlimiters placed before and after the optical correlator have been proposed in [5]. Lin and Wu in [6] have proposed a synchronous OCDMA system with an adaptive optical hardlimiter placed after the correlator receiver, they show the performance can be improved, compared with system with double hardlimiter. Recently one technique inspired by radio frequency communications, which is called multi-user detection (MUD) technique which typically employed in optical CDMA [7], to improve the capacity and overall throughput of the system, and it is known that optimum multi-user detection (MUD) has a much better theoretical performance than conventional detection [8]. Inspired of MUD we have been introducing the successive interference cancellation (SIC) [9]. In [10, 11] we have analyzed SIC scheme using optical orthogonal code (OOC) for optical CDMA, where the system is very efficient in elimination the effect of the multiple access interference.

Moreover, we have analyzed this technique in [12] using modified prime sequence code, and it is shown that the bit error rate performance of the system significantly improved at each stage of cancellation process using both OOC code and prime code and the system can suppress the MAI and increase the system capacity.

2. Basic Principle of SIC Scheme

The basic idea of this scheme is simple, it detect and demodulates the strongest user signal currently present in the overall received signal. Figure 1 shows the block diagram of a SIC receiver. In the next stage after this user has been detected and demodulated; the original signal is regenerated and subtracted from the overall received signal to get a new received signal [9-12]. In general algorithm the successive cancellations are carried as follows: i) Recognize the strongest signal (one with maximum correlation value), ii) Decode the strongest user, iii) Regenerate the strongest users' signal using its chip sequence, iv) Cancel the

strongest user, v) Repeat (until all users are decoded or a permissible number of cancellations are achieved).

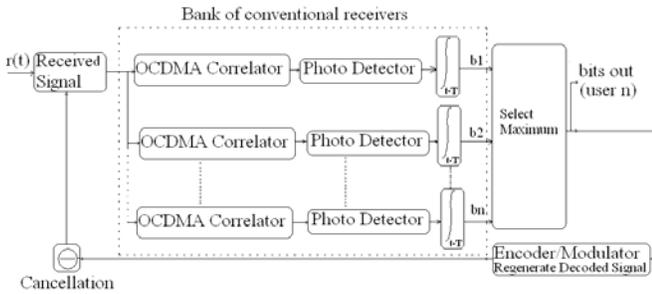


Figure 1. SIC receiver block diagram.

Several signature code sets have been proposed for optical CDMA, such as Hadamard, modified prime code, and modified quadratic congruence (MQC) code [13-18]. Each of these signature code sets can be represented by its length, and weight. The length prime code families have various length sequence code, weight as well. According to [13,14] it is known that an $(N \times N)$ Hadamard matrix of 1's and 0's has the property that any row differs from any other row in exactly $N/2$ positions, therefore let $c_n(i)$ denote the i th element of the n th code sequence, the code properties can be expressed as:

$$\sum_{i=1}^N c_n(i)c_l(i) = \begin{cases} \frac{N}{2}; & n = l \\ \frac{N}{4}; & n \neq l \end{cases} \quad n, l \in (2, 3, \dots, N). \quad (1)$$

In Hadamard code $(N-1)$ code sequence can be constructed each one has weight equal to $N/2$ and a length N .

3. Performance Analysis

In this section, we have analyzed and presented the SIC scheme more in details with modified quadratic congruence (MQC) code. More details of MQC can be found in [15], where MQC code shows that is much better than Hadamard codes for SAC optical CDMA systems. Therefore under the worst case i.e., $\tau_n = 0$, for all n , synchronized condition [1],

the cross correlation function I_{xy} between any pair of code sequences x and y is given by [15]:

$$I_{x,y} = \begin{cases} p+1; & x = y \\ 0; & x \text{ and } y \text{ are in the same group} \\ 1; & x \text{ and } y \text{ are in different group} \end{cases} \quad (2)$$

Assuming that each user is assigned a unique prime sequence code of length $p^2 + p$ with weight $(p+1)$ and considering the total number of users $N=p^2$. Then the receiver signal $r(t)$ at the front end of the receiver is modeled as follows:

$$r(t) = \sum_{n=1}^{N=p^2} A_n b_n(t - \tau_n) \sum_{i=1}^{p^2+p} c_n^i(t - \tau_n) + n(t) \quad (3)$$

Where; N is the total number of users; A_n is the signal strength of the n th user; $b_n(t)$ is the bit sequence of n th user; $c_n(t)$ is the spreading chip sequence of the n th user; $n(t)$ is the noise signal (thermal noise); and τ_n is the time delay of the n th user.

Detailed analysis of the SIC scheme for incoherent DSOCDMA system can be found in [9-12]. After j th cancellation, the decision variable for the $(j+1)$ th user taking in account the code properties indicated in Eq. (1), is given by:

$$Z_{j+1} = \zeta_{j+1} + l_{j+1} \quad (4)$$

Where

$$\zeta_{j+1} = \frac{A_{j+1} b_{(j+1)}}{p} \quad (5)$$

$$l_{j+1} = \frac{1}{p^2} \left[\sum_{n=j+2}^{N=p^2} A_n b_n I_{n,j+1}(\tau_{n,j+1}) - \right. \quad (6)$$

$$\left. \sum_{i=1}^j l_i I_{i,i+1}(\tau_{i,i+1}) \right] + n_{j+1}(t)$$

In the above expression, the first term is MAI of the uncanceled users; and the second term is duo to cumulative noise from imperfect cancellation. The original prime code is analyzed by using a Gaussian approximation. The negative effects of shot noise, effect of the receiver's dark current, and other sources of noise are neglecting in order to focus only on the interference (i.e., MAI), created by other simultaneous users, furthermore the thermal noise.

When N represented the users are transmitting simultaneously, the total interference at a given receiver is

the superposition of $(N - 1)$ different cross correlation functions. If the $(N - 1)$ interference is uncorrelated, then the variance of the total interference is equal to the sum of the variance of the $(N - 1)$ cross correlation functions. We can define the variance of I_{j+1} conditioned on A_n as follows:

$$\Gamma^2_{j+1}|_{-MAI} = \frac{1}{p^2} \left[\sum_{n=j+2}^{N=p^2} A_n^2 b_n^2 \cdot \text{var}[I_{n,j+1}(\tau_{n,j+1})] + \sum_{i=1}^j \Gamma^2_i \cdot \text{var}[I_{i,i+1}(\tau_{i,i+1})] \right] \quad (7)$$

And the variance of cross correlation in (2) is found as [11]:

$$\text{var}(I_{n,j+1}) \approx \frac{1}{p^2 + p} \quad (8)$$

Substituting (8) into (7) we get the variance of the noise in the decision variable:

$$\Gamma^2_{j+1}|_{-MAI} = \frac{1}{p^2(p^2 + p)} \left[\sum_{n=j+2}^{N=p^2} A_n^2 + \sum_{i=1}^j \Gamma^2_i \right] \quad (9)$$

Furthermore the thermal noise has a Gaussian distribution effect has been considered:

$$\sigma_{th} = 4K_b T_n B / R_L \quad (10)$$

Where B is the noise-equivalent electrical bandwidth of the receiver in hertz; K_b is the Boltzmann's constant in joules per Kelvin = $1.38 \times 10^{-23} \text{ JK}^{-1}$; T_n is the absolute receiver noise temperature in Kelvin; and R_L is the receiver load resistor in ohm.

The responsivity of the PDs is given by $\mathfrak{R} = \eta e / h \nu_c$. Here, η is the quantum efficiency, e is the electron's charge, h is the Plank's constant, and ν_c is the central frequency of the original broadband optical pulse.

The signal to noise ratio function (SNR) is represented as the ratio of the squared of the peak of the autocorrelation function to the variance of the amplitude of the interference. Then from (4), (9) and (10), we can get the signal to noise ratio function of A_n

$$SNR_{j+1} = \frac{\mathfrak{R}^2 A_{j+1}^2 / p^2}{\frac{\mathfrak{R}^2}{p^2(p^2 + p)} \left[\sum_{n=j+2}^{N=p^2} A_n^2 + \sum_{i=1}^j \Gamma^2_i \right] + 4K_b T_n B / R_L} \quad (11)$$

It is clear from SNR equation the variance of the amplitude of the cross correlation function ($\Gamma^2_{j+1}|_{-MAI}$) (the denominator of the SNR) decreases with prime number p . To calculate the bit error rate, we shall assume the noise I_{j+1} is Gaussian with zero mean and variance Γ^2_{j+1} . Then the probability of error, conditioned on the A_n , is given below as follows:

$$BER_{j+1} = Q \left(\frac{|\zeta_{j+1}|}{\Gamma_{j+1}} \right) = Q \left(\sqrt{SNR_{j+1}} \right) \quad (12)$$

4. Numerical Results

In this section, numerical results are presented to highlight the advantages of this system (SIC). For the analysis, we have used modified quadratic congruence codes as a signature codes when the prime number set has a length 7, 11, and 13, Hadamard code also has been tested for this scheme where the number of users $N = 128$. The typical parameters used in our analysis given in table 1. Figure 2 shows the relation between the number of active users and the SNR using both sequence codes. It is shown that in this figure, using effective power of -20 dBm, the SNR of each curve ends at the point where the number of active users is equal to the code size. For a given code weight, the SNR decreases as the number of active users increases, a degradation in the SNR implies worsening in the error probability. That is the more simultaneous users, the poorer system performance becomes. Furthermore, the SNR is improved as the prime number increased; it is clear from the figure number of active users increase with prime number. Hence, the SNR increased by 5 dB, for constant number of active users (40) for both prime numbers ($p = 11, 13$), whereas the SNR increased more than (12 dB) between prime numbers ($p = 7, 11$). Moreover it is shown that the SNR of the system with MQC code is higher than Hadamard code especially when the number of prime code increases.

Therefore, SIC system can be mitigate MAI, and significantly increase the system capacity.

Figure 3 shows the system's BER versus the number of active users, where the comparison of BER performance between modified quadratic congruence codes and Hadamard codes has been established. Moreover, for different numbers of users, it is shown that the bit error rate improves as the number of prime code increases, and it is clearly shown from the figure MQC codes has much lower BER than the Hadamard code. (i.e., better performance).

Table 1. Typical parameters used for the calculation

Parameter	Value
Operation Wavelength	193.1 THz
PD quantum efficiency	0.6
Receiver noise temperature	300k
Receiver load resistor	1030Ω
Electrical equivalent bandwidth	80MHz

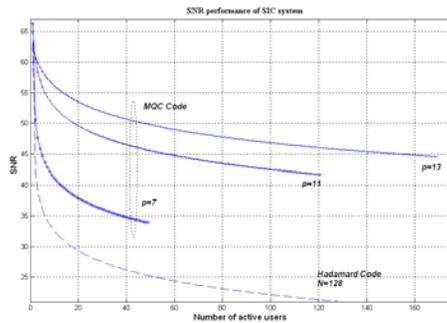


Figure 2. SNR versus number of active users under effective power (-20dBm)

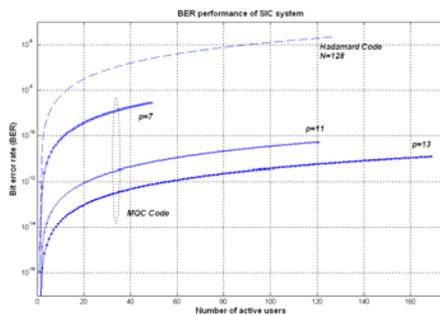


Figure 3. BER versus number of active users under effective power (-20dBm)

In figure 4 the BER function for the system with and without cancellation has been plotted for the sake of comparison. The bit error rate from the analysis obtains shows that the cancellation scheme has better performance than conventional scheme. Assuming that prime number set as a length 13, it is clearly for similar BER performance (10^{-12}), less than 20 users can be active with conventional scheme, which can be increased to 60 users with SIC cancellation scheme, giving substantial increase in capacity.

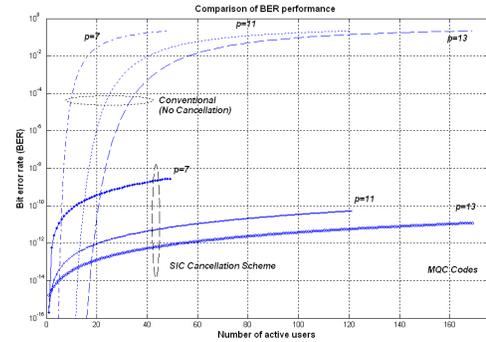


Figure 4. Comparison of BER performance

5. Conclusion

In this paper we have presented an analysis of the performance of SIC system using both modified quadratic congruence and Hadamard codes, taking in to account effects of thermal noise sources. The BER performance of both codes for this system have been compared and it is clear that SIC scheme gives better results of BER when the MQC code used, and the BER improves with increasing prime code p . Clearly the MQC code gets a good performance results compared to Hadamard code, which means using MQC code for this system can effectively suppress the effect of multiple access interference (MAI). When P_{er} is large, the Multiple access interference (MAI) acts as the main factor to limiting the system performance. However, the system shows much lower BER performance with SIC cancellation as compared to the one without cancellation. Hence, much more number of users can accommodate by our proposed system. This increase in the number of users results from the cancellation process. Indeed, at each stage of the cancellation, the number of interfering users decreases, the cross correlation between

users decreases, the SNR increases, and finally the BER decreases.

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