

Performance Analysis of Fractional Frequency Reuse Based on Worst Case Signal to Interference Ratio in OFDMA Downlink Systems

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Abstract— Fractional Frequency Reuse (FFR) is an efficient method to mitigate Inter Cell Interference in multicellular Orthogonal Frequency Division Multiple Access (OFDMA) systems. In this paper, we analyze the downlink worst case Signal to Interference Ratio for FFR schemes. A closed form expression is derived analytically for the worst SIR, outage probability, and Spectral Efficiency (SE). The proposed analytical technique is used to configure a FFR solution for the downlink of OFDMA cellular system. The analysis is performed using two-tiers cellular network with uniform user density and for three different cases of FFR, namely, Frequency Reuse Factor (FRF) = 3, FRF=4 and sectored FFR. The inner radius configuration depends on equalizing the worst SIR for both inner and outer edges of the cell. Numerical results show that sectored FFR yields the highest SE and low outage probability. Sectored FFR highly balances the needs of interference reduction and resource efficiency.

Keywords: Inter-cell interference cancellation, fractional frequency reuse

I. INTRODUCTION

In future mobile communication systems, the growing demand on wireless data traffic is the main motivation of the industry and academia investment in OFDMA based 4G networks. Several wireless cellular standards such as the Third General Partnership Project (3GPP) Long Term Evolution (LTE), 3GPP2 Ultra Mobile Broadband (UMB), and mobile Worldwide Interoperability for Microwave Access (WIMAX) are all OFDMA cellular systems [1]. The main interference in OFDMA system is due to inter-cell interference (ICI) which deteriorates cell edge user's performance. Cell-edge users suffer from strong interference when universal frequency or reuse one is used; leading to poor performance at cell edges.

ICI mitigation techniques can be grouped into three different approaches, namely, randomization, cancelation, and coordination [2, 3]. FFR belongs to coordination strategy where a significant capacity

improvement can be gained by using two different reuse factors instead of one [2]. Discussion and evaluation of different interference management techniques can be found in [4], [5].

Frequency reuse is the most commonly used interference coordination technique, where neighbouring cells do not use the same frequency band. In particular, let N_c be the number of cells per cluster, then clusters of size N_c share the whole bandwidth according to the value of FRF. In particular, each cell uses only $\frac{1}{N_c}$ of the BW; thus neighbouring cells inside the cluster would not interfere with each other. An obvious disadvantage of frequency reuse is the reduction of SE due to the use of only $\frac{1}{N_c}$ of the available BW [3, 4, and 15]. High frequency reuse improves cell edge performance at the expense of reducing the cell average throughput.

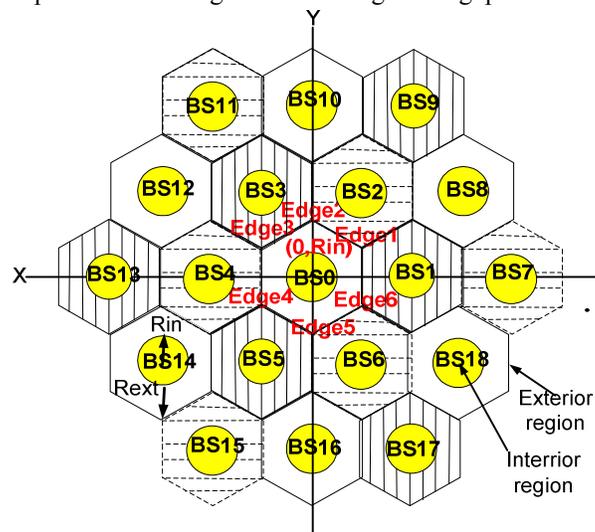


Figure 1. Two-tier network with FFR three

FFR with reuse three was first proposed in [7] to balance the link throughput across the coverage area where Cell Center Users (CCUs) use universal

frequency reuse and Cell Edge Users (CEUs) use reuse three; it was first proposed for GSM networks in [8]. Figure 1 illustrates the idea of FFR with reuse three, where R_{in} defines the interior region area illustrated by yellow circles. $R_{ext} - R_{in}$ defines the exterior region. Here, the bandwidth is divided into two main parts; the first part is assigned reuse one and the second part is divided equally into three parts for reuse three.

The authors in [9] have derived the theoretical capacity and outage rate of an OFDMA system using FFR. Numerical results showed that FFR achieves higher capacity compared to non-FFR systems besides lower outage rate. An analytical evaluation of FFR and Soft Frequency Reuse (SFR) is discussed in [10]; they have provided a detailed picture of the trade-offs of FFR systems. FFR provides larger overall network throughput and highest CEU SINR, while SFR balances the requirements of interference mitigation and resource efficiency. In [6] the authors used two main types of FFR deployments: strict FFR and SFR to model analytically BS locations based on homogenous Poisson point process. Reference [11] extends the work in [6], where the authors have studied the coverage probability for FFR and SFR with different connectivity models. They concluded that the nearest BS connectivity model provides better coverage than the maximum SIR connectivity with FFR.

In [12] a new dynamic FFR mechanism has been proposed. It selects the optimal frequency allocation based on the cell total throughput and user satisfaction. In [3] the author has provided not only the optimal frequency reuse factor of the exterior users but also the optimal bandwidth assigned to both interior and exterior regions. The optimal interior radius has been found to be about two-thirds of the overall cell radius. Combined FFR with Multiple Input Multiple Output (MIMO) systems provides superior performance; in [13] the authors provided combined three-cell MIMO network with rearranged tri-sector frequency partitioning strategy that can outperform the seven-cell network MIMO with omnidirectional antenna.

In this paper, a closed form solution for not only the worst case SIR but also for the optimum inner radius in OFDMA cellular system is provided. We propose a new method to compute the inner region radius based on worst SIR value. Paper organization is as follow: derivations of the worst SIR are provided in section two. Section three and four provide the corresponding results for FFR three and

four, respectively. Simulation results are provided in Section five.

II. SYSTEM MODEL FOR SECTORED FFR

Consider the FFR-OFDMA cellular network with 57 sectors shown in Fig 2. It is assumed that OFDMA system bandwidth is divided into four equal sub-bands; which correspond to f_1 , f_2 , f_3 , and f_4 in Fig. 2. Each BS uses tri-sector directional antenna for the exterior region and omnidirectional one for the interior region. Let where R be the cell radius, the distance between any two adjacent BSs is $2d$ where $d = \frac{\sqrt{3}}{2}R$. Users are assumed to be uniformly distributed across cells. Each BS transmits with a constant Power Spectral Density (PDF) $p = \frac{P_{tot}}{B_{tot}}$, where P_{tot} , is the total transmitted power at each base station and B_{tot} is the total Bandwidth of the system. The channel gain is given by $G = G_0 r^{-\alpha}$, where α is the path loss exponent, r is the distance between the BS and mobile User Equipment (UE), G_0 is given by $G_0 = (\frac{c}{4\pi f})^2$ where f is the center frequency and c is the speed of light.

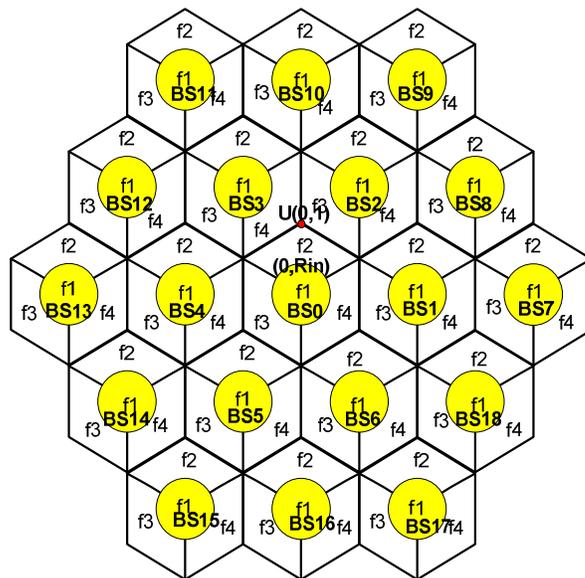


Figure 2. A 57-sector two-tier network (Sectorized FFR). The UE is located at the edge of home cell (0, 1)

Consider a user, $U(X, Y)$, that is uniformly located at a distance $r = \sqrt{X^2 + Y^2}$ from BS0, see Fig.2. According to his location, the user can be considered either an inner or an exterior user. Inner users suffer from 18 interfering BSs, due to universal frequency reuse in all BSs, namely, BS_1 to BS_{18} . An exterior user located at the corner of BS_0 , sector f_2 , is exposed to interference from sectors f_2 , from BS_5 , BS_6 , BS_{14} , BS_{18} , BS_{15} , BS_{16} , and BS_{17} respectively.

Thus, a general SIR expression for CCU and CEU at point (X, Y) can be written as

$$SIR_{CCU}(X, Y) = \frac{pG_0r^{-\alpha}}{\sum_{k=1}^{18} pG_0r_k^{-\alpha}} \quad (1)$$

$$SIR_{CEU}(X, Y) = \frac{pG_i r^{-\alpha}}{\sum_{z_e \in z_e} pG_i r_{z_e}^{-\alpha}} \quad (2)$$

where, p is the transmitted power from BS, z_e is the set of interfering BSs due to reuse factor in exterior region, r_k is the distance between U and the 18 interfering base stations, BS_k. r_{z_e} is the distance between exterior user and six interfering BSs that belong to set z_e . Let (x, y) be the normalized user coordinates, i.e. $(x, y) = (X/R, Y/R)$ shown in Table1; the normalized distance will equal $r^*R = \sqrt{x^2 + y^2}$. It follows from (1), and (2) that the SIR expression in normalized coordinates (x, y) becomes

$$SIR_{CCU}(x, y) = \frac{(x^2 + y^2)^{-\frac{\alpha}{2}}}{S_1(x, y)} \quad (3)$$

$$SIR_{CEU}(x, y) = \frac{(x^2 + y^2)^{-\frac{\alpha}{2}}}{S_{FRF}(x, y)} \quad (4)$$

where $S_1(x, y)$ is the sum of path loss distances of 18 interfering BSs with interior user and $S_{FRF}(x, y)$ is the sum of path loss distances of six interfering BSs with exterior user. $S_1(x, y)$ can be written as

$$S_1(x, y) = [(x \pm \sqrt{3})^2 + y^2]^{-\frac{\alpha}{2}} + [x^2 + (y \pm 3)^2]^{-\frac{\alpha}{2}} \\ + [(x \pm 2\sqrt{3})^2 + y^2]^{-\frac{\alpha}{2}} + [(x \pm \sqrt{3})^2 + (y \pm 3)^2]^{-\frac{\alpha}{2}} \\ + [(x \pm \frac{\sqrt{3}}{2})^2 + (y \pm \frac{3}{2})^2]^{-\frac{\alpha}{2}} + [(x \pm \frac{3\sqrt{3}}{2})^2 + (y \pm \frac{3}{2})^2]^{-\frac{\alpha}{2}} \quad (5)$$

The parameter S_{FRF} is the interference factor due to FFR value. In case of sectored FFR it is given by:-

$$S_{FRFsectored}(x, y) = [x^2 + (y + 3)^2]^{-\frac{\alpha}{2}} \\ + [(x \pm \frac{\sqrt{3}}{2})^2 + (y + \frac{3}{2})^2]^{-\frac{\alpha}{2}} \\ + [(x \pm \frac{3\sqrt{3}}{2})^2 + (y + \frac{3}{2})^2]^{-\frac{\alpha}{2}} \\ + [(x \pm \sqrt{3})^2 + (y + 3)^2]^{-\frac{\alpha}{2}} \quad (6)$$

The users experience the worst SIR in exterior region when they lie at the edge corners of the center cell. For interior users case it is at the edges of the inner circle, e.g., $(0, R_{in})$. The normalized coordinates of the corners of hexagon are $(\frac{\sqrt{3}}{2}, \frac{1}{2})$, $(0, 1)$, $(\frac{-\sqrt{3}}{2}, \frac{1}{2})$, $(-\frac{\sqrt{3}}{2}, -\frac{1}{2})$, $(0, -1)$, and $(\frac{\sqrt{3}}{2}, -\frac{1}{2})$. Consider a user located at the first corner with coordinates $(0, 1)$ by substituting his coordinates in (4), and (6). The worst SIR can be written as

$$SIR_{worst FFRsectored} = \frac{1}{2[7^{-\frac{\alpha}{2}} + 13^{-\frac{\alpha}{2}} + 19^{-\frac{\alpha}{2}}] + 16^{-\frac{\alpha}{2}}} \quad (7)$$

Table I

BSs coordinates and assigned frequencies for FFR

BS	x	y	FFR=3 Edge frequencies	FFR=4 Edge frequencies
0	0	0	f2	f2
1	$\sqrt{3}$	0	f3	f3
2	$\sqrt{3}/2$	3/2	f4	f4
3	$-\sqrt{3}/2$	3/2	f3	f5
4	$-\sqrt{3}$	0	f4	f3
5	$-\sqrt{3}/2$	-3/2	f3	f4
6	$\sqrt{3}/2$	-3/2	f4	f5
7	$2\sqrt{3}$	0	f4	f2
8	$3\sqrt{3}/2$	3/2	f2	f5
9	$\sqrt{3}$	3	f3	f2
10	0	3	f2	f3
11	$-\sqrt{3}$	3	f4	f2
12	$-3\sqrt{3}/2$	3/2	f2	f4
13	$-2\sqrt{3}$	0	f3	f2
14	$-3\sqrt{3}/2$	-3/2	f2	f5
15	$-\sqrt{3}$	-3	f4	f2
16	0	-3	f2	f3
17	$\sqrt{3}$	-3	f3	f2
18	$3\sqrt{3}/2$	-3/2	f2	f4

From (7), the worst SIR depends basically on the path loss exponent, α . In order to find the best FFR inner radius, the proposed technique calculates the worst SIR for both inner and exterior users. The worst SIR for exterior user that utilizes sectored FFR is given in (7). For reuse-one case, the worst SIR is given in terms of the inner cell radius, R_{in} , is found by substituting $(0, R_{in})$ in (3) and (5). By equalizing the worst SIR in the two cases, the result is an equation of one unknown which is the inner radius R_{in} in (9).

$$SIR_{worst FFR sectored} = SIR_{edge of reuse one} \quad (8)$$

$$\frac{1}{S_{FRFsectored}(0, 1)} = \frac{R_{in}^{-\alpha}}{S_1(0, R_{in})} \quad (9)$$

It should be noted that the SIR is an important parameter which reflects user's throughput and Quality of Service (QOS) in wireless systems. The throughput for user i in bits/sec is given by

$$R_i = \left(\frac{1}{T}\right) \log_2(1 + \Omega SIR_i) \quad (10)$$

where $\Omega = \frac{-1.5}{\ln(SBER)}$, SIR_i is the i -th user signal to interference power ratio, and T is the symbol duration.

Cell throughput is the aggregate data rate for all users throughputs inside the cell and given by: [10, 15]

$$R_{cell} = \sum_{i=1}^N R_i = \left(\frac{1}{T}\right) \log_2 \prod_i (1 + \Omega SIR_i) \quad (11)$$

High SE is obtained by maintaining high SIR in the system. SE for user i is shown in (12), where μ is the inverse of the reuse factor.

$$SE_i = \mu \log_2 (1 + SIR_i) \quad (12)$$

A user's QOS can be measured by his outage probability, β , which is defined as the probability that a user's SIR falls below certain threshold value SIR^{th} , and given by [10,15]

$$\beta_i = prob(SIR_i \leq SIR^{th}) = 1 - \prod_{k \in Z} \frac{1}{1 + SIR_i^{th} \frac{r_k^{-\alpha}}{r^{-\alpha}}} \quad (13)$$

III. FFR WITH REUSE THREE CASE

For the FRF=3 case, the exterior region uses reuse three as the total bandwidth equals $BW_{tot} = B_1 + 3B_2$. Where B_1 is the BW allocated to reuse one region and the rest of the BW is divided equally to three equal values as shown in Fig 1. For the inner user case equations (1) and (3) will be the same. For the exterior user, interfering base stations will be different. The interfering BSs will be $BS_8, BS_{10}, BS_{12}, BS_{14}, BS_{16},$ and BS_{18} . Hence, S_{FRF} in (6) will be

$$S_{FRF3}(x, y) = \frac{[(x \pm \frac{3\sqrt{3}}{2})^2 + (y \pm \frac{3}{2})^2]^{-\frac{\alpha}{2}}}{[x^2 + (y \pm 3)^2]^{-\frac{\alpha}{2}}} \quad (14)$$

The worst SIR in this case will change to

$$SIR_{worstFRF3} = \frac{1}{(2^{-\alpha} + 2 * 13^{-\frac{\alpha}{2}} + 2 * 7^{-\frac{\alpha}{2}} + 4^{-\alpha})} \quad (15)$$

For optimum inner radius in this case (9) will change to:

$$\frac{1}{S_{FRF3}(0,1)} = \frac{R_{in}^{-\alpha}}{S1(0, R_{in})} \quad (16)$$

IV. FFR WITH REUSE FOUR CASE

The same analysis is repeated for FRF=4 case. Six interfering BSs is the result of a user lie at the edges of home cell and uses FRF=4. They are $BS_7, BS_9, BS_{11}, BS_{13}, BS_{15},$ and BS_{17} . The sum of interference distances will change to (17). By substituting on $(x=0, y=1)$ the worst SIR will result as shown in (18). The optimal inner radius is driven as the previous two cases method by applying (19).

$$S_{FRF4}(x, y) = \frac{[(x \pm 2\sqrt{3})^2 + y^2]^{-\frac{\alpha}{2}}}{[(x \pm \sqrt{3})^2 + (y \pm 3)^2]^{-\frac{\alpha}{2}}} \quad (17)$$

$$SIR_{worstFRF4} = \frac{1}{2 * [13^{-\frac{\alpha}{2}} + 7^{-\frac{\alpha}{2}} + 19^{-\frac{\alpha}{2}}]} \quad (18)$$

$$\frac{1}{S_{FRF4}(0,1)} = \frac{R_{in}^{-\alpha}}{S1(0, R_{in})} \quad (19)$$

V. NUMERICAL RESULTS

Figure 3 shows the relation between SIR and α for different possible values of α using equations (7), (15), and (17) of the three different FFR cases. Practically, the SIR at different locations is greater than worst SIR at the cell corner. It should be noted that as α increases, the worst SIR also increases. This is because as α increases (attenuation increases) both the received signal and interfering signals powers decrease. However the decrease in the interfering signal is more than that of received signal resulting in an increase of SIR. The worst SIR arising from sectored FFR is nearly close to that of FFR4. It is obvious from the curve that reuse one has lowest values of SIR due to large interference at the edges. For $\alpha=3.6$, the worst SIR=7.7, 11, and 10dB for FRF=3, FRF=4, and sectored FFR respectively.

Figure 4 shows the relation between the optimum inner radius and α for $R=1$ km. The curves are drawn using equations (9), (16), and (19). For $\alpha = 3.6$ $R_{opt} = 560m$ for FRF=3 which is close to $630m$ obtained in [3] for maximum throughput. Generally for the same value of α , optimum inner radius of FRF=3 is the largest one. This is due to the small worst SIR that FRF=3 have. The inner radius of sectored FFR is in the middle between FFR3, FFR4. The larger the inner radius means the more reuse one area in the cell center hence the better overall SE of the system. Bandwidth resources can be distributed in proportion to the cell areas or to maximize the system capacity. Figure 5 previews the SE of the three FFR cases. It is obvious that there is a trade off between SIR (QOS) and SE (capacity). SE is calculated using (12). Sectored FFR has the best SE due to the efficient use of all frequencies in the cell. Figure 6 shows the outage probability α using (13) at $SIR^{th} = 0$ dB. It is obvious that reuse one has higher outage probability while both FFR4 and sectored FFR have the lowest one.

VI. CONCLUSIONS

FFR is one of the most important ICI mitigation techniques. A closed form expression for the worst SIR was derived. The problem of calculating the universal reuse inner distance is solved by equalizing worst SIR of both inner and outer regions. The approach was applied to three different FFR cases FRF=3, FRF=4 and sectored FFR. It was found that the worst SIR depends basically on the path loss exponent as FRF has the largest one. Sectored FFR prove its efficiency as it has high SIR value, low outage probability and the largest SE. Sectored FFR performance is near FFR4 performance but with larger SE. Inner radius of sectored FFR lies between FFR3, and FFR4 for the same α .

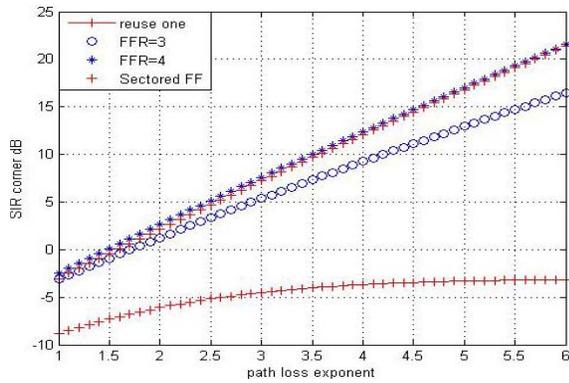


Figure 3. SIR worst vs. path loss exponent for FFR=3, FFR=4, and sectored FFR

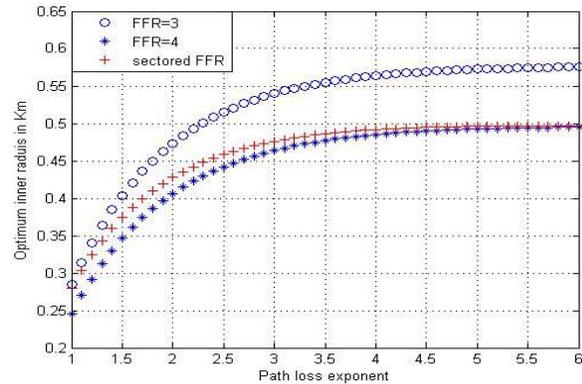


Figure 4. Optimum inner radius vs. path loss exponent for FFR=3, FFR=4, and sectored FFR

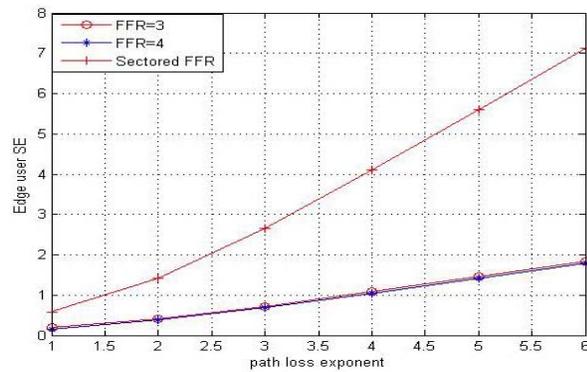


Figure 5. Worst case edge SE vs. path loss exponent for FFR=3, FFR=4, and sectored FFR

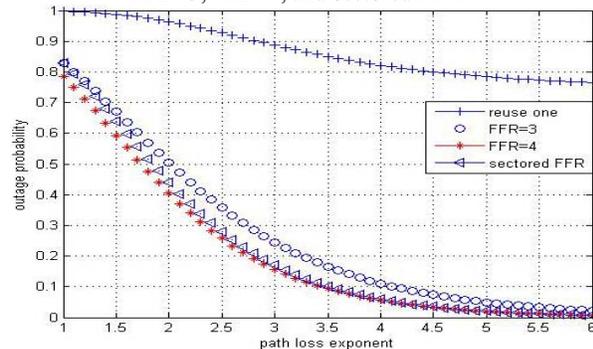


Figure 6. Outage probability vs. path loss exponent for FFR=3, FFR=4, and sectored FFR at 0dB threshold

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