Real-DFT Based DCO-OFDM and ACO-OFDM for Optical Communications Systems

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ABSTRACT
In this paper, a new solution is proposed for optical OFDM systems that eliminates the need for applying the Hermitian symmetry in the generation of optical OFDM signals. Our technique can completely reconstruct the OFDM signal from its pure real part without the need to transmit the imaginary data. Applying such technique has improved the performance on many aspects, where the power requirements has been reduced by 50% when compared to conventional ACO and DCO OFDM systems, the computational complexity has improved by 17%, 28% for radix-2 and radix-4 based FFT systems respectively whilst having same BER as traditional optical OFDM systems.

Keywords: Hermitian symmetry, optical communications, optical OFDMA.

1. INTRODUCTION
The huge demand for data services and the increasing need for broadband services has been exponentially growing during the last decade with new application being born such as Internet of Things (IoT). This has shifted the focus towards optical communications systems either by replacing copper wires with optical fiber, e.g., Fiber to the Home (FTTH), or by applying Visible Light Communications (VLC) to enhance the performance of wireless communications systems. Required data throughput for next generation PON Stage 2 (NG-PON2) have been determined by the full-service access network (FSAN) and the international telecommunications union (ITU-T) to be 40 Gb/s and 10 Gb/s for the downlink and uplink directions respectively [1]. Wide range of solutions have been suggested to meet the required performance for NG-PON2 standard. Wavelength-division multiplexing (WDM) [2] has been the main focus of research efforts as it can provide high rates. However, the exact tuning of wavelengths and the increasing costs limit the system scalability. Optical code-division multiple-access (OCDMA) [3] is another candidate for NG-PON2 which theoretically offers high flexibility in the system capacity and user access, but it is limited by the very high implementation complexity of the system. Frequency-division multiplexing (FDM) [4] is a low-cost solution for PONs but with a limitation of spectral efficiency due to the non-overlapping spectra. Orthogonal Frequency-Division Multiplexing (OFDM) is now a primary candidate to fulfil the requirements of modern standards for PONs due to its high spectral efficiency and reduced complexity of implementation [5].

Most optical communication systems rely on intensity modulation of the laser diodes or LEDs and direct detection using photodiodes (IM/DD) to avoid the need to use expensive coherent modulators and detectors. Two main IM/DD techniques for optical OFDM are widely used, namely, DC biased Optical-OFDM (DCO-OFDM) [6] and Asymmetrically Clipped Optical-OFDM (ACO-OFDM) [7]. For both techniques, the OFDM signal must be positive and real. Hermitian symmetry is the most popular technique used to generate a real OFDM signal. As for the elimination of the negative part of the signal, each technique has its own method. DCO-OFDM adds a DC bias to ensure the positive polarity of the output OFDM signal, while ACO modulates the odd subcarriers only to ensure time symmetry which prevents loss of data when clipping the negative samples.

In this paper, we propose a solution that does not rely on the use of Hermitian symmetry, instead it uses a Real DFT based algorithm which enhances the power efficiency while minimizing the number of complex operations needed. This technique is referred to as Real-DFT based O-OFDM which is applicable in bot PONs and VLC systems.

2. SYSTEM DESCRIPTION
The Real-DFT based algorithm has been suggested in [8]. The algorithm enables the recovery of the complex modulation symbols from the real part of the IDFT of the data. The idea behind the algorithm is to replace the complex conjugates of the subcarriers with zeros which allows the symmetry of the data for periodic signals.
That is, we can recover a signal $\tilde{x}(n)$ by applying DFT to the real samples of $x(n)$, defined as:

$$\tilde{x}(n) = \begin{cases} 
0 & \text{for } n = 0, \\
x(n-1) & \text{for } n \in \{1, 2, \ldots, N/2\}, \\
0 & \text{for } n \in \{N/2 + 1, \ldots, N-1\}.
\end{cases} \quad (1)$$

The output signal recovered at the receiver is $\tilde{y}(n)$ as given by:

$$\tilde{y}(n) = DFT(real(IDFT(\tilde{x}(n)))) = \begin{cases} 
0; & \text{for } n = 0, \\
x(n-1); & \text{for } n \in \{1, 2, \ldots, N/2\}, \\
x(N/2 - 1 - n); & \text{for } n \in \{N/2 + 1, \ldots, N-1\}.
\end{cases} \quad (2)$$

The schematic of the proposed system is shown in Fig. 1 and Fig. 2. In both DCO-OFDM and ACO-OFDM, Hermitian symmetry is replaced by the real DFT based alternative. For DCO-OFDM, DC bias is added to the resulting DFT signal for the signal to become unipolar, while in ACO-OFDM, the odd subcarriers are set to zero so that the output time domain signal is symmetrical.

Figure 1. System block diagram for the proposed real DCO DFT

Figure 2. System block diagram for the proposed real ACO DFT

This algorithm is applicable for both PONs and VLC systems, where the optical channel in Fig. 1 and Fig. 2 is modelled as an attenuating low pass filter and additive white Gaussian noise (AWGN).

3. Performance Analysis

The proposed real-DFT based optical OFDM system is simulated using Matlab with $10^5$ frames, 128 subcarriers, and both binary phase shift keying (BPSK) and quadrature phase shift keying (QPSK) modulations. The performance of system is determined and compared to traditional Hermitian symmetry solutions. The main points of comparison are the average signal power, the computational complexity, and the bit-error rate.

3.1 Average Signal Power

In order to fairly compare the average power of optical OFDM symbol generated by both traditional and proposed real-DFT based methods by analytical calculations and system simulation. We analytically compare the formulas for power of the input modulation symbols to the systems as following:

$$P_{\text{Hermitian}} = \sum_{n=0}^{N-1} |X(n)|^2 = \sum_{n=0}^{N/2-1} |X(n)|^2 + \sum_{n=N/2}^{N-1} |X(n)|^2$$

$$P_{\text{Hermitian}} = 4 \times \sum_{n=0}^{N/2-1} |X(n)|^2$$  \quad (3)
\[ P_{\text{real}} = \sum_{n=0}^{N-1} |X(n)|^2 = \sum_{n=0}^{N/2-1} |X(n)|^2 + \sum_{n=N/2}^{N-1} |X(n)|^2 \]

(5)

\[ P_{\text{Hermitian}} = 4 \cdot P_{\text{real}} \]

(6)

where \( P_{\text{Hermitian}} \) is the power equation for both Hermitian DCO-OFDM and ACO-OFDM systems, \( P_{\text{real}} \) is the power for proposed real-DFT-based OFDM. It is clear from (4) and (6) that the Hermitian based symmetry systems require four-fold the power needed from proposed technique. This is verified by the Matlab simulation as well.

### 3.2 Computational Complexity

In order to assess the complexity requirement between the two mention techniques, the number of arithmetic operations for generating the O-OFDM is used as a metric. Real-DFT-based algorithm reduces the overall number of required operation due to the fact that half of the subcarriers is set to zero.

In Table 1, the number of required operations is compared for the two methods using various algorithms for calculation of DFT. For radix-2 based FFT the number of complex additions and multiplications is reduced by an average of 15% where the reduction is exactly 20% for 32 subcarriers, 13% for 256 subcarriers, and 9% for 2048 subcarriers. Further, complexity improvement can be achieved when applying radix-4 FFT algorithm where the average number of computations is dropped by 27%. The computational complexity reduction is exactly reduced by 40% for the case of 64 subcarriers, 25% for the 256 subcarriers and 18% for the 2048 subcarriers. In Fig. 3, the saving percentage for real-DFT against Hermitian OFDM is shown for radix-2 and radix-2.

**TABLE 1.** Complexity of Mathematical Operations for both Proposed Real-DFT Based and Traditional Techniques.

<table>
<thead>
<tr>
<th>OFDM Calculation Method</th>
<th>Traditional DCO-OFDM</th>
<th>Real-DFT Based DCO-OFDM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical Operation</td>
<td>Multiplication</td>
<td>Addition</td>
</tr>
<tr>
<td>DFT</td>
<td>( N^2 )</td>
<td>( N(N-1) )</td>
</tr>
<tr>
<td>FFT radix-2</td>
<td>( (N/2) \log_2 N )</td>
<td>( N \log_2 N )</td>
</tr>
<tr>
<td>FFT radix-4</td>
<td>( (3N/4 \log_4 N) )</td>
<td>( (12N/4 \log_4 N) )</td>
</tr>
</tbody>
</table>

**Figure 3.** Computational complexity comparison between Hermitian OFDM and real-DFT
3.3 BER PERFORMANCE

Using Monte-Carlo Simulation method, the BER is determined for both conventional and real-DFT based O-OFDMA solutions. Fig. 4 show that same BER is achieved for both techniques, indicating that the real-DFT based method does not impact the performance. It must be noted that the below simulation results hold for both VLC and NG-PONs systems.

![Figure 4](image)

*Figure 4. (a) BER performance comparison between proposed real-DFT and traditional DCO-OFDM techniques for QPSK modulation (b) BER performance comparison between proposed real-DFT and traditional ACO-OFDM techniques for QPSK modulation*

4. CONCLUSION

In this work, a novel approach for generation of O-OFDM has been suggested as an alternative to traditional Hermitian symmetry method. The solution is based on real-DFT generation algorithm which achieves 50% power savings and 15% to 27% savings in the number of required complex computations. This is achieved while maintaining the same BER as conventional solutions.

REFERENCES