

Performance Comparison of Number- and Coherent-State Optical CDMA in Lossy Direct-Detection Photon Channels

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Abstract—The performance of optical code-division multiple-access (CDMA) communication systems utilizing number-state light field is evaluated. Lossy direct detection optical channels are assumed. Both on-off keying (OOK) and pulse-position modulation (PPM) schemes are investigated. For OOK, the exact bit error rate is evaluated taking into account the effect of both multiple-user interference and transmission loss. Upper and lower bounds on the bit error probability for PPM-CDMA systems are derived under the above considerations. The effect of both background and thermal noise is neglected in our analysis. A comparison between the performance of the number- and coherent-state OOK/PPM-CDMA networks is also presented. Our results demonstrate that the number-state system requires less than half the energy consumed by the coherent-state one in order to attain the same performance.

I. INTRODUCTION

The quantum fluctuations of laser photons generated in an optical *coherent state* lead to an uncertainty in estimating the number of photons contained in a coherent laser pulse. This number can be modeled as a Poisson random variable whose statistical expectation equals the average photon count per pulse [6]. The coherent-state random fluctuations, thus, form a major source of noise in optical communication systems. On the other hand, the photon count contained in a laser pulse generated in an optical *number state* is a nonrandom unique value [1]. In other words, in lossless channels, every transmitted photon will appear as is at the receiving end. If the optical channel is, however, lossy, some of the transmitted photons may disappear before reaching the photodetector. Assuming that $\eta \in [0, 1]$ denotes the transmittance coefficient of the lossy channel, the probability of detecting exactly n photons given that m photons have been transmitted can be written as

$$\Pr\{n|m\} = \binom{m}{n} \eta^n (1 - \eta)^{m-n}, \quad n \in \{0, 1, \dots, m\}.$$

The above equation demonstrates that, in lossy channels, number state optical pulses also yield random photon counting processes at the receiving end. Therefore performance degradation is expected as η decreases even in the absence of the background noise.

Recently, an increasing interest has been given to optical code-division multiple-access (CDMA) techniques because of their ultrafast signal-processing speeds [2–5]. Several models for optical CDMA communication systems have been suggested in literature. In a typical system model there are N simultaneous sources of information

(users) which produce continuous and asynchronous data. The data of each user modulates a laser source using either on-off keying (OOK) or M -ary pulse-position modulation (PPM) schemes. Each modulated signal is then multiplied by a periodic signature (code) sequence of length L and weight w . Assuming that the bit rate is denoted by R_0 bits/s, the chip time T_c of the sequence can be shown to be given by

$$T_c = \begin{cases} \frac{1}{LR_0} & \text{for OOK,} \\ \frac{\log_2 M}{MLR_0} & \text{for PPM,} \end{cases} \quad (1)$$

where M denotes the number of possible pulse positions (slots) within the PPM frame. The multiplication process can equivalently be performed using an optical splitter, optical tapped delay lines, and an optical combiner. The output of each signature forming device undergoes transmission loss in the channel before reaching the receiver. The received waveform is composed of the sum of N delayed and attenuated signals from each user in addition to the background noise. This waveform is passed to an optical correlator matched to the specific sequence [4]. The correlation process is equivalent in operation to multiplying the received waveform by the underlying code sequence. The output of the correlator is finally directed to an OOK/PPM demodulator which decides on the true data.

In this paper we aim at comparing between the bit error rate performance of coherent and number state optical CDMA systems utilizing either OOK or PPM modulation techniques. In our analysis we consider the effect of the transmission loss due to the attenuation in the optical channel. We neglect, however, the effect of both the background and thermal noise. In order to have some insight on the results obtained we assume chip synchronous uniformly-distributed relative delays among the transmitters and perfect photon counting processes at the receivers.

In the numerical analysis, we consider optical orthogonal codes (OOC's) [5] as the signature code sequences. To have minimal interference between the users we adopt OOC's with periodic cross-correlations and out-of-phase periodic auto-correlations that are bounded by 1 [5].

The remaining of our paper is organized as follows. Section II is devoted for the derivation of the bit error rate for optical OOK-CDMA through both coherent and number state lossy channels. Upper and lower bounds on the bit error probability for PPM-CDMA systems are derived in Section III. Performance comparisons between the coherent and number state channels are illustrated at the end of the above two sections. Finally extensions and concluding remarks are given in Section IV.

II. BIT ERROR RATE FOR OOK-CDMA

In OOK a signature sequence is transmitted (of w laser pulses) to represent data bit "1". Data bit "0" is represented, however, by zero pulses. We denote by κ the number of pulses, from the other users, that cause interference to the desired user. In OOC's with cross-correlations bounded by one (since we assume chip synchronous) each undesired user may contribute only one pulse to this number or contribute no pulses at all. Hence κ is a binomial random variable with parameters $\frac{w^2}{2L}$ and $N-1$ [3]:

$$\Pr\{\kappa = l\} = \binom{N-1}{l} \left(\frac{w^2}{2L}\right)^l \left(1 - \frac{w^2}{2L}\right)^{N-1-l}, \quad l \in \{0, 1, \dots, N-1\}. \quad (2)$$

The Decision Rule

As usual, a threshold θ is set. If the received photon count is less than this threshold, "0" is declared, otherwise "1" is declared to be sent. The probability of bit error is thus given by

$$P_b(\theta) = \frac{1}{2} (P[E|0] + P[E|1]) \\ = \frac{1}{2} \sum_{l=0}^{N-1} (P[E|0, \kappa = l] + P[E|1, \kappa = l]) \Pr\{\kappa = l\} \quad (3)$$

where $P[E|i, \kappa = l]$ is the probability of error given that $i \in \{0, 1\}$ was sent and there are l interfering pulses with the desired user. To evaluate this probability of error we consider the following two cases (A and B).

A. Number State

We assume that exactly m photons are contained in each transmitted pulse (i.e., a total of mw photons are transmitted in the bit time of data bit "1"). A decoding error can thus occur, given that "0" was sent and l pulses have interfered with the desired user, if the number of received photons is at least equal to θ :

$$P[E|0, \kappa = l] = \begin{cases} \sum_{n=\theta}^{ml} \binom{ml}{n} \eta^n (1-\eta)^{ml-n}; & \text{if } \theta \leq ml, \\ 0; & \text{otherwise.} \end{cases} \quad (4-a)$$

Similarly,

$$P[E|1, \kappa = l] = \begin{cases} \sum_{n=\theta}^{\theta-1} \binom{m(w+l)}{n} \eta^n (1-\eta)^{m(w+l)-n}; & \text{if } \theta < m(w+l)+1, \\ 1; & \text{otherwise.} \end{cases} \quad (4-b)$$

B. Coherent State

Assuming that the average transmitted photons per pulse is equal to m , then the average received photons per pulse (due to channel loss) becomes ηm . Hence for a PIN photodetector which output is a Poisson random variable [6],

$$P[E|0, \kappa = l] = \sum_{n=\theta}^{\infty} \exp[-\eta ml] \frac{(\eta ml)^n}{n!}, \\ P[E|1, \kappa = l] = \sum_{n=\theta}^{\theta-1} \exp[-\eta m(w+l)] \frac{(\eta m(w+l))^n}{n!}. \quad (5)$$

Numerical Results

The optimal threshold which minimizes the bit error rate in (3) has been evaluated numerically for $w = 5$, $L = 500$, and different values of N and m . The minimum bit error rate $P_b = \min_{\theta} P_b(\theta)$ is plotted in Fig. 1 for both number state and coherent state. The superiority of the number state system is obvious from the figures. For example if $N = 5$, $\eta = 0.7$, and $P_b \leq 10^{-5}$ we need at least $m = 15$ for the number state whereas $m = 40$ for the coherent state. This indicates that more than 60% save in energy is gained when using the number state OOK. It is also noticed that for $\eta = 0.7$ the performance of the number state system with 10 simultaneous users is almost similar to the coherent state system with only 5 simultaneous users.

III. BIT ERROR RATE FOR PPM-CDMA

In M -ary PPM a time frame of duration T is divided into M disjoint slots each having a width $\tau = T/M$. Symbol $i \in \{0, 1, \dots, M-1\}$ is represented by transmitting a signature sequence within slot number i . We denote by κ_i , $i \in \{0, 1, \dots, M-1\}$ the number of pulses, from other users, that cause interference to slot i of the desired user. As in the case of OOK, κ_i is a binomial random variable with probability distribution given by (2). The joint distribution of any two random variables κ_i and κ_j , $i \neq j$ is given by

$$\Pr\{\kappa_i = l_i, \kappa_j = l_j\} = \sum_{r=0}^{l_i \wedge l_j} \frac{(N-1)!}{r!(l_i-r)!(l_j-r)!(N-1-l_i-l_j+r)!} \\ \times P_{11}^r(i, j) P_{10}^{l_i-r}(i, j) P_{01}^{l_j-r}(i, j) P_{00}^{N-1-l_i-l_j+r}(i, j), \\ l_i, l_j \in \{0, 1, \dots, N-1\}, \quad (6)$$

where

$$P_{11}(i, j) = \frac{|i-j|}{M^2} \cdot \frac{w^2}{ML}, \\ P_{10}(i, j) = \left(1 - \frac{|i-j|}{M^2}\right) \frac{w^2}{ML}, \\ P_{01}(i, j) = P_{10}(i, j), \\ P_{00}(i, j) = 1 - P_{11}(i, j) - P_{10}(i, j) - P_{01}(i, j). \quad (7)$$

The derivation of this distribution can be found in Appendix A.

The Decision Rule

We denote the photon count collected in slot i , $i \in \{0, 1, \dots, M-1\}$ by Y_i . Symbol "r" is declared to be transmitted if $Y_i > Y_j$ for every $j \neq i$. We now provide a union bound on the probability of word error P_E . The bit error rate P_b is related to P_E by the well known formula $P_b = \frac{M/2}{M-1} P_E$.

$$P_E = \sum_{i=0}^{M-1} P[E|i] \Pr\{i\},$$

where $\Pr\{i\} = 1/M$ in the case of equally likely data and

$$\begin{aligned} P\{E|i\} &= \Pr\{Y_j \geq Y_i, \text{ some } j \neq i|i\} \\ &\leq \sum_{j=0, j \neq i}^{M-1} \Pr\{Y_j \geq Y_i|i\}. \end{aligned}$$

Denoting the union bound by P_E^U and noticing that $\Pr\{Y_j \geq Y_i|i\}$ depends only on the difference $|i - j|$, we get

$$\begin{aligned} P_E^U &= \frac{1}{M} \sum_{i=0}^{M-1} \sum_{j=0, j \neq i}^{M-1} \Pr\{Y_j \geq Y_i|i\} \\ &= \frac{2}{M} \sum_{d=1}^{M-1} (M-d) \Pr\{Y_j \geq Y_i|i, |i-j|=d\} \\ &= \frac{2}{M} \sum_{d=1}^{M-1} (M-d) \Pr\{Y_d \geq Y_0|0\}. \end{aligned} \quad (8)$$

The probability under the summation can be evaluated as follows

$$\Pr\{Y_d \geq Y_0|0\} = \sum_{l_0=0}^{N-1} \sum_{l_d=0}^{N-1} \Pr\{Y_d \geq Y_0|0, \kappa_0 = l_0, \kappa_d = l_d\} \times \Pr\{\kappa_0 = l_0, \kappa_d = l_d\}. \quad (9)$$

This union bound is still too complex. We thus provide tight upper and lower bounds on this union bound.

Lower bound on P_E^U

We can write

$$\Pr\{Y_d \geq Y_0|0\} \geq \sum_{l_d=0}^{N-1} \Pr\{Y_d \geq Y_0|0, \kappa_0 = 0, \kappa_d = l_d\} \times \Pr\{\kappa_0 = 0, \kappa_d = l_d\}.$$

Using (6) we have

$$\Pr\{\kappa_0 = 0, \kappa_d = l_d\} = \binom{N-1}{l_d} P_{01}^{l_d}(0, d) P_{00}^{N-1-l_d}(0, d),$$

where

$$\begin{aligned} P_{01}(0, d) &= \left(1 - \frac{d}{M^2}\right) \frac{w^2}{ML}, \\ P_{00}(0, d) &= 1 - \frac{w^2}{ML} - P_{01}(0, d). \end{aligned}$$

Hence

$$P_E^U \geq \frac{2}{M} \sum_{l_d=0}^{N-1} \Pr\{Y_d \geq Y_0|0, \kappa_0 = 0, \kappa_d = l_d\} \times \sum_{d=1}^{M-1} (M-d) \binom{N-1}{l_d} P_{01}^{l_d}(0, d) P_{00}^{N-1-l_d}(0, d) \quad (10)$$

Upper bound on P_E^U

This bound is provided by noticing that

$$\Pr\{Y_d \geq Y_0|0, \kappa_0 = l_0, \kappa_d = l_d\} \leq \Pr\{Y_d \geq Y_0|0, \kappa_0 = 0, \kappa_d = l_d\}.$$

Hence by substitution in (9)

$$\Pr\{Y_d \geq Y_0|0\} \leq \sum_{l_d=0}^{N-1} \Pr\{Y_d \geq Y_0|0, \kappa_0 = 0, \kappa_d = l_d\} \times \Pr\{\kappa_d = l_d\}.$$

From (8) and (2), we obtain

$$\begin{aligned} P_E^U &\leq \frac{2}{M} \sum_{l_d=0}^{N-1} \Pr\{Y_d \geq Y_0|0, \kappa_0 = 0, \kappa_d = l_d\} \times \\ &\quad \sum_{d=1}^{M-1} (M-d) \binom{N-1}{l_d} \left(\frac{w^2}{ML}\right)^{l_d} \left(1 - \frac{w^2}{ML}\right)^{N-1-l_d} \\ &= (M-1) \sum_{l_d=0}^{N-1} \binom{N-1}{l_d} \left(\frac{w^2}{ML}\right)^{l_d} \left(1 - \frac{w^2}{ML}\right)^{N-1-l_d} \times \\ &\quad \Pr\{Y_d \geq Y_0|0, \kappa_0 = 0, \kappa_d = l_d\}. \end{aligned} \quad (11)$$

What remains to complete the evaluation of the error bounds is to get expressions on $\Pr\{Y_d \geq Y_0|0, \kappa_0 = 0, \kappa_d = l_d\}$ under both number state (A) and coherent state (B).

A. Number State

Assuming that exactly m photons are transmitted per pulse, we can write

$$\begin{aligned} \Pr\{Y_d \geq Y_0|0, \kappa_0 = 0, \kappa_d = l_d\} &= \\ &\sum_{n_d=0}^{m l_d} \binom{m l_d}{n_d} \eta^{n_d} (1-\eta)^{m l_d - n_d} \sum_{n_0=0}^{n_d \wedge m w} \binom{m w}{n_0} \eta^{n_0} (1-\eta)^{m w - n_0}. \end{aligned} \quad (12)$$

B. Coherent State

Assuming that the average transmitted photons per pulse is equal to m , we have

$$\Pr\{Y_d \geq Y_0|0, \kappa_0 = 0, \kappa_d = l_d\} = \sum_{n_d=0}^{\infty} \exp[\eta m l_d] \frac{(\eta m l_d)^{n_d}}{n_d!} \sum_{n_0=0}^{n_d} \exp[\eta m w] \frac{(\eta m w)^{n_0}}{n_0!}. \quad (13)$$

Numerical Results

Upper and lower bounds on P_E^U have been evaluated numerically for the case of number state with $w = 5$, $L = 500$, $N = 20$, and different values of M and m . These bounds (scaled to the bit error probability) are shown in Fig. 2. It is clear that the upper bound on P_E^U is so close (same order of magnitude) to the true union bound especially for large M . Because of its simplicity we use

the upper bound on P_E^U in the following numerical analysis. A comparison between number- and coherent-state bit error rate is shown in Fig. 3 under the above parameter values. The superiority of the number state system over the coherent state one is clear from the figures. As an example if $N = 20$, $\eta = 0.7$, and $P_b \leq 10^{-7}$, at least $m = 9$ photons/pulse are required for the number state whereas $m = 28$ for the coherent state if $M = 32$. When $M = 16$, m becomes 16 in the case of the number state and becomes 48 for the coherent state. The above numbers indicate that more than 66% save in energy is gained when using the number state PPM. Another remark on the curves is that the performance improves as M increases. From Fig. 3 with $P_b \leq 10^{-7}$ there is about 44% save in energy per pulse when switching the number state system from $M = 16$ to $M = 32$. This percentage is, however, misleading; a fair comparison should be based on the transmitted photons per bit not per pulse. Hence for $M = 32$, $m * w / \log_2 M = 9 * 5/5 = 9$ photons/bit is consumed versus $16 * 5/4 = 20$ photons/bit for $M = 16$ to attain the above bit error rate. That is, the true save in energy is about 55% (not 44%). A serious problem in realization may arise as M increases above 32 because the chip time must be decreased in order to hold the bit rate fixed. The resulting laser pulsewidth might be difficult to generate with the current optical technology. A quick look at the curves suggests a crucial solution to the above problem by using number state systems instead of coherent state. The performance of the number state with $M = 16$ (8) is almost competitive to the coherent state system with $M = 32$ (16) for m exceeding 30.

IV. EXTENSIONS AND CONCLUDING REMARKS

Bit error rates for optical chip-synchronous CDMA communication systems utilizing both number and coherent state light fields have been derived for lossy direct-detection photon channels. Exact expressions have been obtained for the case of an OOK modulation scheme. Tight upper and lower bounds on the union bound have been provided when PPM is used. The effect of the multiple-user interference and transmission loss has been considered in the numerical analysis. Our results suggest using the number state system instead of the coherent state one in optical CDMA because of its superiority over the latter. Namely, the number state system requires less than half the energy consumed by the coherent state one to attain the same performance.

In our analysis of PPM-CDMA we have used an upper bound on the bit error rate. In order to figure out the uncertainty on the exact P_b , we have the following lower bound.

$$P_E = \sum_{i=0}^{M-1} P[E|i] \Pr\{i\} \\ \geq \sum_{i=0}^{M-2} \Pr\{i\} \Pr\{Y_{i+1} \geq Y_i|i\} +$$

$$\Pr\{M-1\} \Pr\{Y_{M-2} \geq Y_{M-1}|M-1\} \\ = \sum_{l_0=0}^{N-1} \sum_{l_1=0}^{N-1} \Pr\{Y_1 \geq Y_0|0, \kappa_0 = l_0, \kappa_1 = l_1\} \times \\ \Pr\{\kappa_0 = l_0, \kappa_1 = l_1\} \times \\ \geq \sum_{l_1=0}^{N-1} \Pr\{Y_1 \geq Y_0|0, \kappa_0 = 0, \kappa_1 = l_1\} \times \\ \Pr\{\kappa_0 = 0, \kappa_1 = l_1\}.$$

The upper and lower bounds on P_b have been evaluated numerically for the case of number state with $w = 5$, $L = 500$, $N = 20$, $\eta = 0.7$, and different values of M , m . These bounds are shown in Fig. 4. It is clear that the upper bound determines the exact bit error rate within 1.5 orders of magnitude.

APPENDIX A

The probability of exactly one interference-pulse hit in slot i is given by $P_i(1) = \frac{w^2}{ML}$ [2]. The probability of exactly one interference-pulse hit in slot j given that a hit has occurred in slot i , $j \neq i$, is given by $P_{j|i}(1|1) = \frac{|i-j|}{M^2}$. Hence

$$\Pr\{\kappa_j = l_j | \kappa_i = l_i\} = \\ \sum_{r=0}^{l_i \wedge l_j} \sum_{l_j - r}^{l_i \wedge l_j - (N-1-l_i)} \binom{l_i}{r} P_{j|i}^r(1|1) (1 - P_{j|i}(1|1))^{l_i - r} \times \\ \binom{N-1-l_i}{l_j - r} P_{j|i}^{l_j - r}(1|0) (1 - P_{j|i}(1|0))^{N-1-l_i - (l_j - r)},$$

where

$$P_{j|i}(1|0) = \frac{P_{i,j}(0,1)}{P_i(0)} = \frac{P_j(1) - P_{i,j}(1,1)}{P_i(0)} \\ = \frac{w^2/ML}{1 - w^2/ML} \left(1 - \frac{|i-j|}{M^2}\right). \quad \square$$

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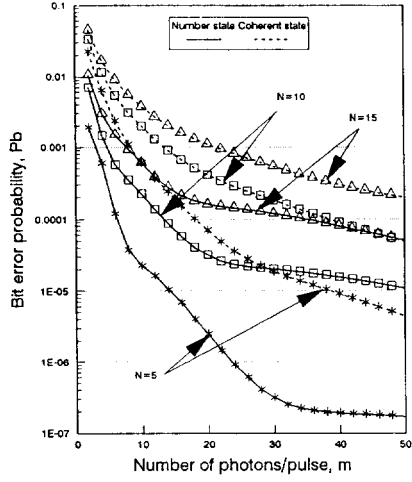


Fig. 1. Bit error probability versus the number of photons/pulse for OOK scheme with $n=0.7$, $w=5$, and $L=500$.

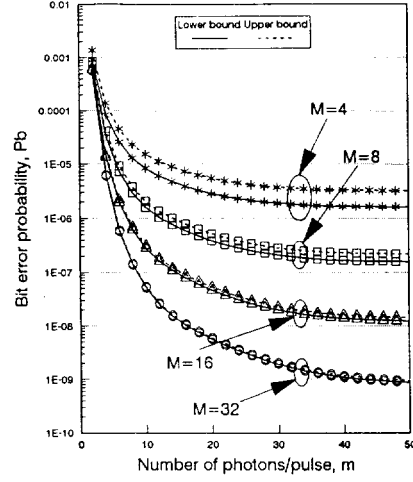


Fig. 2. Upper and lower bounds on the bit error probability union bound for PPM scheme with $n=0.7$, $w=5$, $L=500$, and $N=20$.

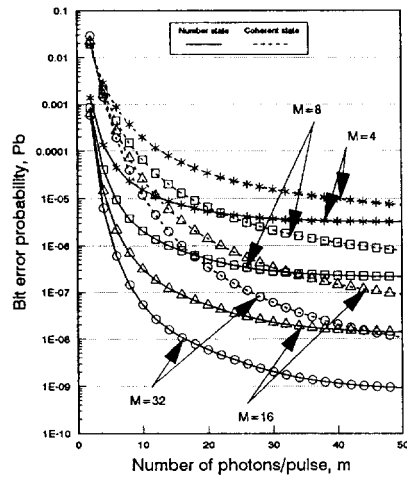


Fig. 3. Bit error probability versus the number of photons/pulse for PPM scheme with $n=0.7$, $w=5$, $L=500$, and $N=20$.

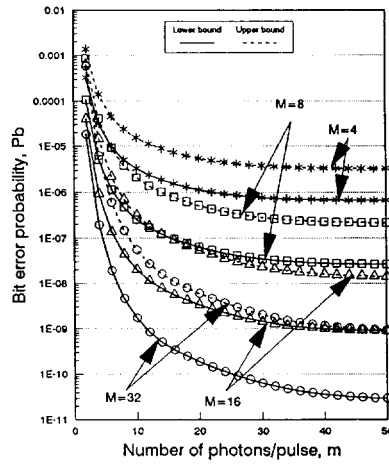


Fig. 4. Upper and lower bounds on the bit error probability for PPM scheme with $n=0.7$, $w=5$, $L=500$, and $N=20$.