Throughput and Efficiency Capacities of Optical OOK-CDMA Communication Systems

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Abstract—**A comparison between the performance of traditional optical code-division multiple-access correlation receivers and that of recently proposed chip-level receivers is presented. The performance is measured in terms of both throughput and efficiency capacities. Signature code correlations bounded by either one or two are employed. The bit error probabilities for chip-level receivers, with code correlations bounded by two, are derived. That with code correlations bounded by one is recited from previous literature. Our results reveal that chip-level systems are much more efficient and their throughput capacities are much higher than that of traditional correlation systems. Further, the throughput capacity of chip-level systems can be increased by almost a factor of 3.4 when increasing the code-correlation constraint from one to two.**

Index Terms—**Optical communications, optical networks, code division multiple access, optical CDMA, channel capacity.**

I. Introduction

Optical code-division multiple-access (CDMA) techniques can be utilized in fiber-optic local area networks because of the great advantages resulting from employing high-bandwidth optical components [1]–[11]. When compared to time-division multiple-access (TDMA), optical CDMA techniques do not require time synchronization and provides flexibility in the network design and security against interception. Optical CDMA systems, on the other hand, suffer from the multiple-user interference, which degrades their bit error probabilities and reduces their bit rates as the number of users increases. Further, they exhibit error probability floors, which cannot be reduced without the addition of interference cancellation subsystems [10].

The traditional method to recover the data at the receiving end of an optical CDMA system is to use an optical correlator followed by a photodetector and a decision device [2]. To enhance the performance of the correlation receiver, Salehi and Brackett have added an optical hardlimiter before the correlator at the receiver side [2]. Although many authors have adopted the optical hardlimiters in their systems' design, the problem with these devices is that their technology is not yet mature and their ideal characteristics are very difficult to be implemented [12]. Fortunately, the recently proposed chip-level receiver [8] does not require the optical hardlimiters or the correlators in its implementation. So it is much more practical than the correlation receiver with hardlimiters [11].

One way to come across the throughput (bit rate) limitation of optical CDMA systems is to use error control coding [5],[6]. Our goal in this paper is to evaluate the throughput and efficiency capacities of both optical OOK-CDMA chip-level and correlation systems when using two different code-correlation constraints, namely, $\lambda \in \{1,2\}$. We employ the optical orthogonal codes (OOCs) [1], with periodic cross-correlations and out-of-phase periodic auto-correlations that are bounded by either one or two ($\lambda \in \{1,2\}$), as the users' signature code sequences in our theoretical analysis.

The remainder of this paper is organized as follows. The optical OOK-CDMA receiver models are described in Section II. The bit error probabilities for the optical OOK-CDMA systems, under code correlations bounded by one, are recited in Section III. Section IV is devoted for the development of the bit error probabilities of the optical OOK-CDMA chip-level receivers under code-correlation constraint equal to two. The effect of both the receiver shot noise and the multiple-user interference is taken into account in our derivation. In Section V, we compare between the throughput and efficiency capacities of OOK-CDMA systems with different code-correlation constraints. Finally the conclusion is given in Section VI.

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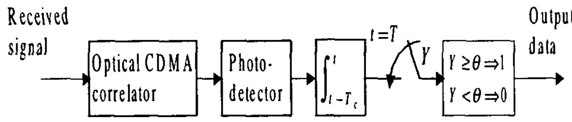

Fig. 1. An optical OOK-CDMA correlation receiver.

II. Optical OOK-CDMA Receiver Models

The block diagram for the correlation receiver is shown in Fig. 1, where T and T_c denote the bit time and the chip time durations, respectively. The optical CDMA correlator usually splits the received optical signal into a number of branches, which is equal to the code weight w, and then combines these branches after properly delaying the split optical pulses in accordance to the signature code. This splitting process wastes most of the received optical signal. The electronic switch in Fig. 1 samples at a rate that is equal to the data bit rate $R_b = 1/T$. This rate is much less than the optical processing rate $R_c = 1/T_c$. In fact $R_b = R_c/L$, where L is the code length. This is an advantage of correlation receivers and is very useful in practice.

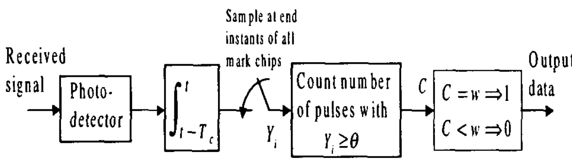

Fig. 2. An optical OOK-CDMA chip-level receiver.

The block diagram for the chip-level receiver is shown in Fig. 2, where we can see that this receiver does not require the optical correlator and hence it does not waste the received optical power as in the correlation receivers. The information about the signature code is provided in the electronic switch, which samples only at the end instants of the mark chips. The average sampling rate of this electronic switch is still very low compared to the optical processing rate. In fact it samples at an average rate of wR_b.

III. Bit Error Probabilities for OOK-CDMA Systems with $\lambda = 1$

The bit error probability for the chip-level receiver can be found in [8]. It is recited here for convenience

$$P_b = \frac{1}{2}(P[E|0] + P[E|1]) \ , \tag{1}$$

where

$$P[E|0] = \sum_{i=0}^{w}(-1)^i \binom{w}{i} \left[1 - i\frac{w}{2L} + i\frac{w}{2L}e^{-Q}\right]^{N-1} \tag{2}$$

and

$$P[E|1] = -\sum_{i=1}^{w}(-1)^i \binom{w}{i} e^{-Qi} \left[1 - i\frac{w}{2L} + i\frac{w}{2L}e^{-Q}\right]^{N-1} \tag{3}$$

Here N denotes the number of simultaneous users and Q denotes the average photons per chip pulse. It is related to the average photons/bit μ as follows

$$Q = \begin{cases} \frac{2\mu}{w} \ ; & \text{for chip-level receivers,} \\ \frac{2\mu}{w^2} \ ; & \text{for correlation receivers.} \end{cases} \tag{4}$$

In the limiting case, when $Q \to \infty$, the last two probabilities reduce to

$$P_\infty[E|0] = \sum_{i=0}^{w}(-1)^i \binom{w}{i} \left[1 - i\frac{w}{2L}\right]^{N-1} \text{ and } P_\infty[E|1] = 0 \ , \tag{5}$$

respectively. It should be emphasized that the error probability for the chip-level receiver from [8] was derived under the assumption of a constant threshold $\theta = 1$ (cf. Fig. 2). This threshold is independent of the number of users and the average optical power, which adds to the advantages of chip-level receivers.

IV. Bit Error Probabilities for OOK-CDMA Systems with $\lambda = 2$

A. The Interference Probability

Let p_t, $t \in \{1, 2\}$, denote the probability that a single user interferes with the desired user at t pulse positions. It was shown in [3] that

$$p_1 + 2p_2 = \frac{w^2}{2L} \ , \tag{6}$$

In this section, we only study the worst case, which occurs when $p_1 = 0$ in the last equation, and hence

$$p_2 = \frac{w^2}{4L} \ . \tag{7}$$

That is, if a single user interferes with the desired user, it will cause interference to exactly two mark positions of the desired user. The system performance in this case provides an upper bound to the more general one with $p_1 \geq 0$ [3].

We denote by κ_i, $i \in \mathcal{X} \stackrel{\text{def}}{=} \{1, 2, \dots, w\}$, the number of pulses (from other users) that interfere to chip

i of the mark positions of the signature code of the desired user. Further, we denote by n_{ij}, $j > i$ and $i, j \in \mathcal{X}$, the number of users that interfere with both chips i and j of the mark positions of the desired user. Thus we have the following set of linear equations:

$$\begin{aligned}\kappa_1 &= n_{12} + n_{13} + \ldots + n_{1w} \\ \kappa_2 &= n_{12} + n_{23} + \ldots + n_{2w} \\ &\vdots \\ \kappa_w &= n_{1w} + n_{2w} + \ldots + n_{w-1,w} \; . \end{aligned} \tag{8}$$

It is easy to check that the random variables $\{n_{ij}: j > i \text{ and } i, j \in \mathcal{X}\}$ admit a multinomial distribution with parameters $N-1$ and $\rho = w/2(w-1)L$. That is

$$\begin{aligned}\Pr\{\mathbf{n}_1 = \mathbf{l}_1, \mathbf{n}_2 = \mathbf{l}_2, \ldots, \mathbf{n}_{w-1} = \mathbf{l}_{w-1}\} &= \frac{(N-1)!}{(N-1-S)! \cdot \prod_{i=1}^{w-1} \prod_{j=i+1}^{w} l_{ij}!} \\ &\times \rho^S \left(1 - \frac{w^2}{4L}\right)^{N-1-S} , \end{aligned} \tag{9}$$

where, for every $i \in \{1, 2, \ldots, w-1\}$, $\mathbf{n}_i$ and $\mathbf{l}_i$ denote the vectors $(n_{i,i+1}, n_{i,i+2}, \ldots, n_{i,w})^T$ and $(l_{i,i+1}, l_{i,i+2}, \ldots, l_{i,w})^T$, respectively, and

$$S = \sum_{i=1}^{w-1} \sum_{j=i+1}^{w} l_{ij} \; . \tag{10}$$

Let Y_i, $i \in \mathcal{X}$, be the photon count collected from chip number i of the mark positions. In order to simplify the analysis and have some insights on the problem under consideration, we focus here on the Poisson shot-noise-limited case only.

B. The Decision Rule

If the collected photon count from each mark chip of the underlying code is positive, "1" is declared, otherwise "0" is declared to be sent. That is

$$\text{Decide} \begin{cases} 1 \; ; & \text{if } Y_i \geq 1 \quad \forall i \in \mathcal{X}, \\ 0 \; ; & \text{otherwise.} \end{cases} \tag{11}$$

C. The Error Probability for Chip-Level Receivers

The probability of bit error is thus given by (1) with

$$\begin{aligned} P[E|1] &= \Pr\{Y_i = 0, \text{ some } i \in \mathcal{X} | D = 1\} \\ &= -\sum_{i=1}^{w} (-1)^i \binom{w}{i} \\ &\quad \times \Pr\{Y_1 = Y_2 = \ldots = Y_i = 0 | D = 1\} \end{aligned} \tag{12}$$

and

$$\begin{aligned} P[E|0] &= \Pr\{Y_i \geq 1 \; \forall i \in \mathcal{X} | D = 0\} \\ &= 1 - \Pr\{Y_i = 0, \text{ some } i \in \mathcal{X} | D = 0\} \\ &= 1 + \sum_{i=1}^{w} (-1)^i \binom{w}{i} \\ &\quad \times \Pr\{Y_1 = Y_2 = \ldots = Y_i = 0 | D = 0\} \end{aligned} \tag{13}$$

Thus, for $b \in \{0, 1\}$, we can evaluate the probabilities under the last two summations as follows

$$\begin{aligned} &\Pr\{Y_1 = Y_2 = \ldots = Y_i = 0 | D = b\} \\ &= \sum_{l_1, \ldots, l_i} \Pr\{Y_1 = Y_2 = \ldots = Y_i = 0 | D = b, \kappa_1 = l_1, \ldots, \kappa_i = l_i\} \\ &\quad \times \Pr\{\kappa_1 = l_1, \ldots, \kappa_i = l_i\} \\ &= \sum_{l_1, \ldots, l_i} \Pr\{\kappa_1 = l_1, \ldots, \kappa_i = l_i\} \\ &\quad \times \prod_{j=1}^{i} \Pr\{Y_j = 0 | D = b, \kappa_j = l_j\} \\ &= \sum_{l_1, \ldots, l_i} \Pr\{\kappa_1 = l_1, \ldots, \kappa_i = l_i\} \\ &\quad \times \prod_{j=1}^{i} \exp[-Q(b + l_j)] \\ &= \exp[-Qbi] \cdot E\Big\{ \exp\Big[-Q \sum_{j=1}^{i} \kappa_j\Big]\Big\} \; . \end{aligned} \tag{14}$$

Here the second equality holds because the random variables $Y_1, \ldots, Y_i$ are independent given $\kappa_1, \ldots, \kappa_i$, and E denotes the expected value. To evaluate the last expectation we make use of (8) and (9). Thus

$$\begin{aligned} &E\Big\{ \exp\Big[-Q \sum_{j=1}^{i} \kappa_j\Big]\Big\} \\ &= E\Big\{ \exp\Big[- Q\Big(2 \sum_{r=1}^{i-1} \sum_{s=r+1}^{i} n_{rs} + \sum_{r=1}^{i} \sum_{s=i+1}^{w} n_{rs}\Big)\Big]\Big\} \\ &= \Big[1 - i\frac{i-1}{2}\rho - i(w-i)\rho \\ &\qquad + i\frac{i-1}{2}\rho e^{-2Q} + i(w-i)\rho e^{-Q}\Big]^{N-1} \\ &= \Big[1 - i\Big(w - 1 - \frac{i-1}{2}\Big)\rho \\ &\qquad + i\frac{i-1}{2}\rho e^{-2Q} + i(w-i)\rho e^{-Q}\Big]^{N-1} . \end{aligned} \tag{15}$$

By substitution in (14) and then in both (13) and (12), we obtain

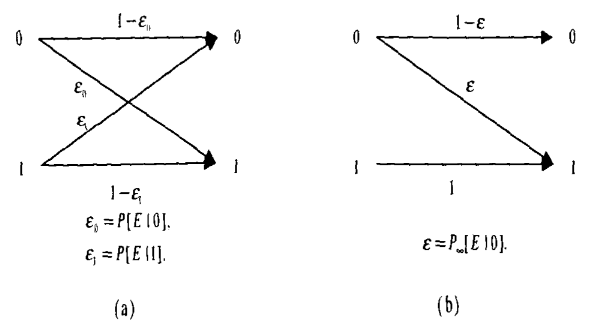

Fig. 3. Channel models of the optical OOK-CDMA systems. (a) Poisson shot-noise-limited case. (b) Noiseless case.

$$P[E|0] = \sum_{i=0}^{w}(-1)^i\binom{w}{i}\Big[1 - i\Big(w-1-\frac{i-1}{2}\Big)\rho + i\frac{i-1}{2}\rho e^{-2Q} + i(w-i)\rho e^{-Q}\Big]^{N-1} \quad (16)$$

and

$$P[E|1] = -\sum_{i=1}^{w}(-1)^i\binom{w}{i}e^{-Qi}\Big[1 - i\Big(w-1-\frac{i-1}{2}\Big)\rho + i\frac{i-1}{2}\rho e^{-2Q} + i(w-i)\rho e^{-Q}\Big]^{N-1}, \quad (17)$$

respectively. In the limiting case, when $Q \to \infty$, the last two probabilities reduce to

$$P_\infty[E|0] = \sum_{i=0}^{w}(-1)^i\binom{w}{i}\Big[1 - i\frac{w}{4L}\Big(2-\frac{i-1}{w-1}\Big)\Big]^{N-1} \quad \text{and } P_\infty[E|1] = 0\ , \quad (18)$$

respectively.

Equations (1), (16), and (17) provide the error probability calculations for chip-level receivers under a Poisson shot-noise-limited assumption. The corresponding error rates for the correlation receivers without optical hardlimiters are found in [3] but only for ideal photodetectors, i.e., when $Q \to \infty$

V. A Capacity Comparison

In this section we compare between the throughput capacities of the OOK-CDMA systems with $\lambda \in \{1, 2\}$. We consider both chip-level receivers and correlation receivers.

A. Channel Models

We assume, in our channel models, that the desired receiver is only interested in the message transmitted by the desired transmitter, i.e., no cooperation between users is permitted. Thus the multiple-access channel is reduced to N single-user channels [14], each is subject to multiple-user interference from the others. In our evaluation of the channel capacities we calculate the mutual information $I(X \wedge Y)$ between the binary input $X = \{0,1\}$ and the binary output $Y = \{0,1\}$, for equiprobable inputs. Of course this yields lower bounds to the actual channel capacities but is suited to most practical coding schemes [13]. The channel model is shown in Fig. 3(a), which is a binary asymmetric channel. For sufficiently large values of Q, this channel reduces to the Z-channel shown in Fig. 3(b).

B. Definitions

The channel capacity in nats/channel use, the *total* throughput capacity in nats/s, and the efficiency capacity in nats/photon are defined as

$$\begin{aligned} C &= I(X \wedge Y) \qquad \text{nats/cu} \\ C_T &= \frac{NC}{T} = \frac{NC}{LT_c} \qquad \text{nats/s} \\ C_{ph} &= \frac{C}{\mu} \qquad \text{nats/ph}\ . \end{aligned} \quad (19)$$

It is easy to check that the mutual information functions for the channels of Fig. 8 (in the case of equiprobable inputs) are given by

$$I(X \wedge Y) = \begin{cases} \log 2 - \frac{1-\epsilon_1+\epsilon_0}{2}h\big(\frac{\epsilon_0}{1-\epsilon_1+\epsilon_0}\big) \\ \quad -\frac{1-\epsilon_0+\epsilon_1}{2}h\big(\frac{\epsilon_1}{1-\epsilon_0+\epsilon_1}\big)\ ; & \text{Poisson,} \\ \log 2 - \frac{1+\epsilon}{2}h\big(\frac{\epsilon}{1+\epsilon}\big)\ ; & \text{ideal,} \end{cases} \quad (20)$$

where

$$\epsilon_0 = P[E|0], \qquad \epsilon_1 = P[E|1], \qquad \epsilon = P_\infty[E|0]\ , \quad (21)$$

and $h(p)$ is the binary entropy function, given by

$$h(p) = -p\log p - (1-p)\log(1-p)\ . \quad (22)$$

C. Numerical Results

The throughput capacities of both correlation and chip-level OOK-CDMA systems with $\lambda = 1$ are shown in Fig. 4 versus the code length when $N = 25$, $w = 4$, and $\mu = 10$. In view of the OOC constraint [1]

$$N \le \frac{(L-1)(L-2)\cdots(L-\lambda)}{w(w-1)\cdots(w-\lambda)}\ , \quad (23)$$

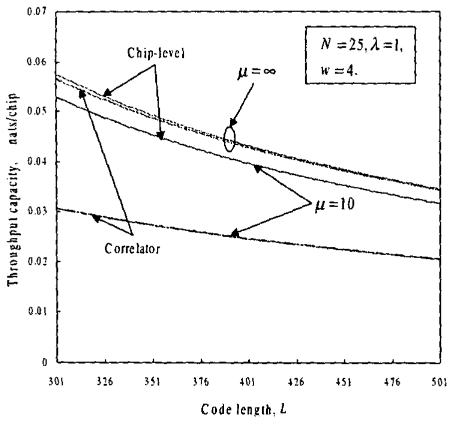

Fig. 4. Throughput capacities versus the code length for both chip-level and correlation OOK-CDMA systems when N=25, λ= 1, and w=4.

the code length should be greater than $1+w(w-1)N = 301$ when $\lambda = 1$. The advantage of chip-level receivers over that of correlation receiver is obvious from the figure. The limiting throughput capacity when $\mu \to \infty$ for chip-level systems is depicted in the same figure as well. It can be seen that the increase in the chip-level system's capacity is not that significant when μ increases above 10, which demonstrates that chip-level receivers are very efficient. The maximum throughput is achieved at the boundary of the feasible region, i.e., at minimum L.

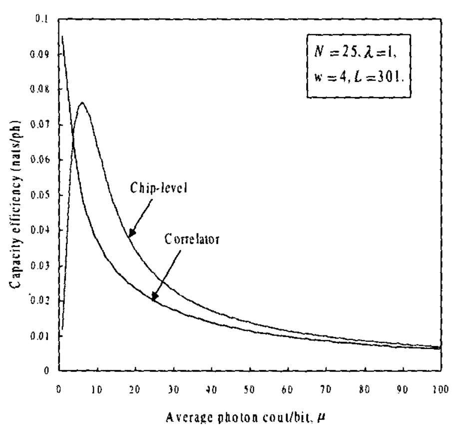

Fig. 5. Efficiency capacities versus the average photons per bit for both chip-level and correlation OOK-CDMA systems when N=25, λ=1, L=301, and w=4.

The capacity efficiencies are plotted in Fig. 5 for $\lambda = 1$ and minimum value of L. For the correlation receiver the efficiency decreases as the average number of photons μ increases, whereas there exists an optimum value of μ that maximizes the efficiency for the case of chip-level receiver. This is because we did not choose an optimum threshold for chip-level receivers, which makes the system a suboptimum one for very low values of μ. However, when μ increases a bit, the threshold becomes optimum and chip-level receivers become more efficient. It is also seen from the figure that for large values of μ, the capacities of both systems coincide and the advantages of the chip level receivers are lost.

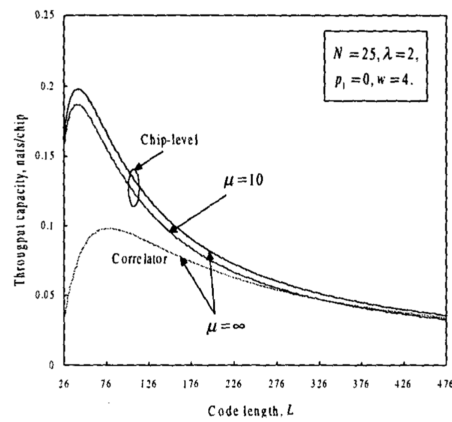

Fig. 6. Throughput capacities versus the code length for both chip-level and correlation OOK-CDMA systems when N=25, λ= 2, and w=4.

From Fig. 4, it can be noticed that the throughput can be increased by decreasing L below the boundary of the feasible region. By switching λ from 1 to 2, the possible values of the code length can be increased, that is, $L \geq 26$ for $N = 25$ and $w = 4$. The corresponding throughput capacities are plotted in Fig. 6 for $\mu \in \{10, \infty\}$. It is obvious that the optimum values of the code lengths occur inside the feasible region and the advantages of chip-level receivers are retained even for infinite energy. Comparing Fig. 4 to Fig. 6, we conclude that the capacity of chip-level receivers increases by a factor of 3.4 when using $\lambda = 2$ rather than $\lambda = 1$.

IV. Conclusion

The bit error probability for chip-level receivers, that are used in recovering optical CDMA signals, has been derived under code-correlation constraint equal to two. The effect of both the receiver shot noise and the multiple-user interference has been taken into account. Both the throughput capacity and the efficiency of chip-level systems and traditional correlation systems are derived and evaluated under code correlations bounded by one and two. We have the following concluding remarks.

i) Chip-level receivers are very efficient when compared to correlation receivers.

ii) The throughput capacity of chip-level systems is much higher than that of correlation systems whenever the optical power is finite.

iii) When the optical power increases without limit and the code-correlation constraint is equal to one, the above advantage will be lost.

iv) This advantage, however, is retained (even for infinite optical power) if the code-correlation constraint is equal to two rather than one.

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