Performance of Multipulse PPM Techniques in Free-Space Optics with Gamma-Gamma Channels

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Abstract: We derive both exact and approximate expressions for the symbol-error rate of free-space optics systems adopting multipulse PPM technique in gamma-gamma channels. Our expressions are then verified using traditional lognormal and exponential channel expressions.
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1. Introduction

Free-space optics (FSO) has several advantages over radio-frequency (RF) technology. Indeed, FSO offers extremely high bandwidth, license-free, and interference immunity [1]. This makes FSO a suitable candidate for last mile connectivity and optical-fiber backup [2]. One important phenomenon that degrades the performance of FSO is atmospheric turbulence or scintillation [1]. Scintillation results in random fluctuations in both the amplitude and phase of the received signal. Multi-pulse PPM (MPPM) has been recently proposed as a modulation technique in FSO because it is more bandwidth efficient than PPM, yet slightly more complex [1].

The studies that have been made to get an expression for the symbol-error rate (SER) of MPPM in the case of FSO under gamma-gamma optical scintillation are rare. Up till now, no one has got an exact expression for the SER of MPPM in the case of FSO under gamma-gamma distribution. In this paper, we derive both exact and approximate (based on Gauss-Laguerre quadrature) expressions for the SER of this system and verify our mathematical expressions by comparing them to traditional lognormal and exponential channel expressions.

2. MPPM SER Analysis in Atmospheric Turbulence Channel

A. MPPM in Poisson Discrete Memoryless Channels

At the receiver side, we assume that the received photon count per MPPM slot follows a Poisson distribution. In that case the SER is given by [3] after slight modifications:

$$P_e = \sum_{K_{min}=0}^{\infty}\sum_{l=1}^{M-n}\sum_{m=1}^{n}\binom{n}{m}\binom{M-n}{l} p_1(K_{min})^m\big(1-P_1(K_{min})\big)^{n-m}\Bigg[P_0(K_{min})^{M-n-l}\big(1-P_0(K_{min})\big)^l + p_0(K_{min})^l P_0(K_{min}-1)^{M-n-l}\left(1-\frac{1}{\binom{l+m}{m}}\right)\Bigg] \quad (1)$$

where M is the number of slots per frame, n is the number of signal slots, K_{min} is the minimum photon count over all signal slots,

$p_0(K_{min}) = \frac{{K_b}^{K_{min}}}{K_{min}!}e^{-K_b}$, $p_1(K_{min}) = \frac{(K_S+K_b)^{K_{min}}}{K_{min}!}e^{-(K_S+K_b)}$, $P_1(K_{min}) = \sum_{j=0}^{K_{min}}\frac{(K_S+K_b)^j}{j!}e^{-(K_S+K_b)}$, $P_0(K_{min}) = \sum_{j=0}^{K_{min}}\frac{K_b^j}{j!}e^{-K_b}$, and $P_0(K_{min}-1) = \sum_{j=0}^{K_{min}-1}\frac{K_b^j}{j!}e^{-K_b}$.

Here, K_b and $K_b + K_s$ denote the mean values in non-signal and signal slots, respectively.

B. Exact Mathematical Analysis of MPPM SER with Gamma-Gamma distribution

Several channel models have been assumed in literature, however, the most commonly used statistical models are lognormal distribution which is suitable in weak turbulence, exponential distribution which is suitable in strong turbulence and gamma-gamma distribution which is suitable in both strong and weak turbulence [4].

The average SER can be obtained by averaging P_e with respect to K_s using a gamma-gamma model. After rigorous mathematical analysis, we are able to show that the SER for MPPM under gamma-gamma distribution is obtained by replacing $p_1(K_{min})^m\big(1-P_1(K_{min})\big)^{n-m}$ in (1) by the value of $P_2(K_{min})$,

$$P_2(K_{min}) = \begin{cases} \sum_{i=0}^{mK_{min}} \frac{\left(\frac{\alpha\beta}{\lambda}\right)^{\frac{(\alpha+\beta-1)}{2}}}{\Gamma(\alpha)\Gamma(\beta)} \binom{K_{min}}{i} e^{-nK_b} \; {K_b}^{mK_{min}-i} \; \frac{\Gamma(i+\alpha)\,\Gamma(i+\beta)}{K_{min}!^m} e^{\left(\frac{\alpha\beta}{2n\lambda}\right)} n^{-\left(i+\frac{\alpha+\beta}{2}-\frac{1}{2}\right)} \; W_{-\left(i+\frac{\alpha+\beta}{2}-\frac{1}{2}\right),\frac{1}{2}(\alpha-\beta)}\left(\frac{\alpha\beta}{n\lambda}\right) & , n = m \\ \sum_{j=(n-m)(k_{min}+1)}^{\infty} \sum_{i=0}^{j+mK_{min}} \frac{\left(\frac{\alpha\beta}{\lambda}\right)^{\frac{(\alpha+\beta-1)}{2}}}{\Gamma(\alpha)\Gamma(\beta)} \binom{j+mK_{min}}{i} e^{-nK_b} \; {K_b}^{j+mK_{min}-i} \; \frac{\Gamma(i+\alpha)\,\Gamma(i+\beta)}{K_{min}!^m} \\ \qquad * f(j) e^{\left(\frac{\alpha\beta}{2n\lambda}\right)} n^{-\left(i+\frac{\alpha+\beta}{2}-\frac{1}{2}\right)} \; W_{-\left(i+\frac{\alpha+\beta}{2}-\frac{1}{2}\right),\frac{1}{2}(\alpha-\beta)}\left(\frac{\alpha\beta}{n\lambda}\right) & , n \neq m \end{cases} \tag{2}$$

where σ_R^2 is unitless Rytov variance, λ is the average of K_S and α and β are the scintillation parameters that are dependent on σ_R^2[4].To define $f(j)$, let $S(j)$ be a set of vectors, each has a dimension of $(n-m)$. The vector elements are integer and are between $(K_{min}+1)$ and $j-(n-m-1)(K_{min}+1)$. In addition, the summation of all elements of any vector is equal to j:

$$f(j) = \sum_{i=1}^{number\ of\ vectors} 1/\prod_{j=1}^{n-m}(element\ number\ j\ in\ vector\ number\ i)! \tag{3}$$

C. Approximate Expression of MPPM SER with Gamma-Gamma distribution

As the computation of the exact expression is time consuming so we get approximate expression based on Gauss-Laguerre quadrature which its computation is faster than the exact expression and both expressions produce almost the same results. Using Gauss-Laguerre quadrature, we are able to show that.

$$P_2(K_{min}) = \sum_{i=1}^{c} w_i \frac{e^{-nK_b}}{K_{min}!^m} (x_i + K_b)^{mK_{min}} \left[e^{(x_i+K_b)} - \sum_{j=0}^{K_{min}} \frac{(x_i+K_b)^j}{j!}\right]^{n-m} f(x_i) \tag{4}$$

where w_i denotes the weighting coefficient, x_i is a root of the Laguerre polynomial with degree c and $f(x_i)$ is the atmospheric turbulence distribution [4].

3. Numerical Results

We use both exact and approximate (numerical) expression to get MPPM average SER under gamma-gamma distribution for $\sigma_R = 5$ and as shown in figure 1 below they are nearly identical so we will produce our results using approximate expression as its computation is faster as we said before. Figure 2 illustrates the change of SER with λ under gamma-gamma model verified by the curves for both exponential and lognormal distributions with $M = 8$, $n = 4$, $K_b = 1$, Laguerre polynomial degree $c = 50$ and $\sigma_R = \{0.25, 0.75, 5\}$. All of the distributions have the same mean value for K_s which equals λ, with the variance of the lognormal distribution governed by the same scintillation index as gamma-gamma model. We conclude that at strong turbulences with $\sigma_R = 5$, the gamma-gamma model approaches the exponential model. On the other hand, at weak turbulences gamma-gamma approaches lognormal models as σ_R decreases. So at $\sigma_R = 0.25$ gamma-gamma model reflect more lognormal model than at $\sigma_R = 0.75$.

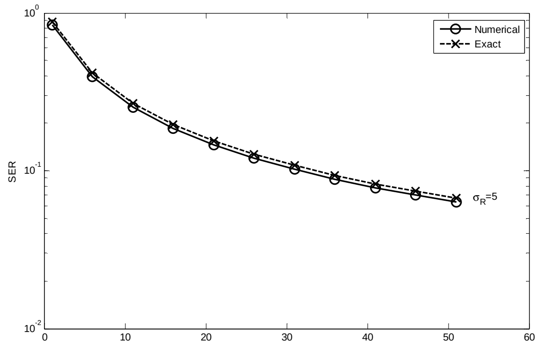

Fig. 1. Average SER versus average of K_S using exact and approximated method under gamma-gamma distribution with $M = 8$, $n = 4$, and $K_b = 1$.

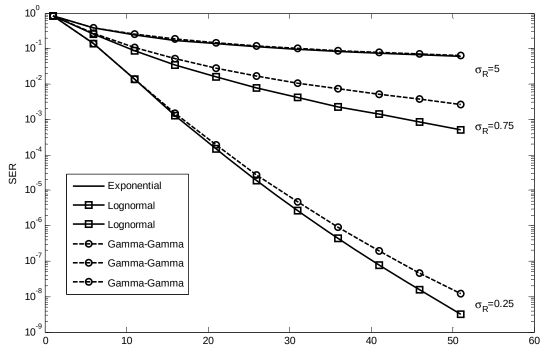

Fig. 2. Average SER versus average of K_S using the approximate method for different channel models.

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