A New Interference Cancellation Technique For Synchronous CDMA Communication Systems Using Modified Prime Codes

Hossam H. Shalaby[#], Mohab A. Mangoud[*] and Said E. El-Khamy[#]

Department of Electrical Engineering, Faculty of Engineering,
Alexandria University, Alexandria 21544, Egypt. E-Mail: elkhamy@alex.eun.eg

* Department of Electronics and Computer, Arab Academy for Science and Technology,
P.O.1029, Alexandria, Egypt. E-Mail: Mohab@alex.eun.eg

Abstract

A new technique of interference cancellation for synchronous optical CDMA communication systems is proposed. We utilized the properties of modified prime sequences under the assumption of Poisson shot noise model for the PIN photodiode and synchronization between users' symbols. The interference cancellation technique is based on estimating the multiuser interference (MUI) through the special correlation properties of the used codes. The estimated MUI is then used to adapt a threshold level to optimally detect the concerned user's information and to suppress MUI. Detailed derivations of the closed forms for the bit error rate are presented for both cases of with and without the suggested MUI suppression. Numerical results clearly show the advantages of the proposed technique and proves that it outperforms the system without interference cancellation.

1. Introduction

Synchronous accessing optical systems are preferred over asynchronous systems because it offers larger possible number of simultaneous users and easier interference cancellation techniques. From the previous efforts in the area of interference cancellation of CDMA, Aazhang has proposed a multistage detector [1] whose computational complexity is linear in the number of users. In this detector all users' symbols are estimated to reject the MUI. The disadvantage of this detector is that if the users are not allocated, then the total complexity of the system will be SK^2, where S is the number of stages and, K is the number of detectors per stage. In [2] Shalaby and sorour have proposed two different systems for interference cancellation. They have neglected both shot and background noises in their analysis. In this paper we investigate the performance of synchronous Optical OOK-CDMA system using modified prime sequences under the assumption of Poisson shot noise for the receiver photodetector and propose a new simple interference cancellation detector.

The paper unfolds as follows. **Section 2** describes CDMA system model without cancellation along with investigations of the system performance. In **Section 3**, a detector based on interference cancellation using estimates of MUI is described and the performance analysis of this "canceled- interference" detector is presented. Numerical analysis and comparisons between the two systems are provided in **Section 4**. Finally we give a conclusion in **Section 5**.

2. Optical CDMA Communication System Without Cancellation

We consider a system in which data bits are encoded (multiplied) by a user modified prime sequence $c_n(t)$ with p optical laser pulses where p is a prime number represent the weight of this code. The modified prime sequence codes [3] fit a synchronous transmission systems and have the following characteristics: There are p^2 code sequences that can be generated are divided into p groups in which each code has a length of p^2 chips. The correlation function C_{vq} between codes v and q at the location of synchronized chip is given by:

$$C_{vq} = \begin{cases} p & \text{if } v = q \\ 0 & \text{if } v \,\&\, q \text{ share the same group and } v \neq q \\ 1 & \text{if } v \,\&\, q \text{ are from different group and } v \neq q \end{cases}$$

The receiver of the optical synchronous OOK-CDMA direct detection communication system is shown in Fig. 1.

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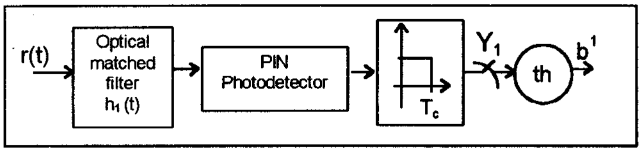

Fig.1 OOK-OCDMA user 1 Decoder without interference cancellation

This was presented in [2] except that in each group the last user is set idle (to be used in our proposed interference cancellation as will be shown later). Thus the total number of subscribers is equal to p^2-p. The number of simultaneous users is equal to N, so the remaining p^2-p-N codes are not used. Let the *jth* user data bits is assigned the data sequence b^k_j , where $b^k_j \in \{0,1\}$. We define a random variable x_j , $j \in \{1,2,\ldots,p^2\}$, as follows

$$x_j = \begin{cases} 1\ ; & \text{if user } j \text{ is active} \\ 0\ ; & \text{if user } j \text{ is idle} \end{cases}, \text{ thus: } \sum_{j=1}^{p^2} x_j = N$$

the received signal at any user is

$$r(t) = \sum_{k=1}^{N} \lambda_s \sum_{j=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} c_i^k\, b_j^k\, \Pi_{T_c}(t - iT_c)\,\Pi_{T_b}(t - jT_b) + n(t) \qquad (1)$$

where T_b = bit time for on off keying (OOK) = 1/(bit rate) $\Pi_{T_b}(t)$ is unit rectangular pulse on [0,T] ,$T_C=T_b/p^2$ is the chip time, $\lambda_s = \eta p_s/hf$ is the signal photon rate (which is assumed to be constant for all the transmitters), η is the quantum efficiency of the photodetector, P_S is the optical source power, hf is the photon energy, $\{b_j^k\ \Pi_T(t-jT_b)\}$ is the k *th* data bit defined on the bit interval $[jT_b , (j+1)T_b]$, T_b was defined previously, $\{c_i^k\ \Pi_{T_c}(t-jT_c)\}$ is the sequence of binary optical pulses with length $p^2 = T_b/Tc$ where $c_i^k \in \{0,1\}$, the number of i's with $c_i = 1$ equal to weight of the code p and n(t) is the noise due to optical background noise. Assume that user one is the desired user (x_1 = 1). Each active subscriber at the receiving end correlates the received signal with a stored replica to its signature used at the transmitter. So at receiver 1 the output of the PIN photodetector will be given by :

$$y(t) = [\,c_1(t)]^2\, b_1(t) + c_1(t)n(t) + \sum_{k=2}^{p^2} c_1(t) x_k c_k(t) b_k(t) \qquad (2)$$

The decision variables for the 1*st* user are $Y_1=Z_1+W_1+I_1$, where Z_1 and I_1 are conditionally independent Poisson photon counts given $\{b_k\}$ and $\{x_k\}$ due to the desired signal and MUI respectively and W_1 is the Poisson photon count due to the back ground noise and we can neglect this noise compared to the signal and MUI [4].The conditional means of these random variables are

$$E[Z_1 \mid b_0^1] = b_0^1\, p\, \lambda_s T_c\,, \qquad (3)$$

$$E[I_1 \mid \{b_0^k\}, x_k] = \sum_{k=p+1}^{p^2} x_k b_0^k\, \lambda_s T_c = \Omega \lambda_s T_c. \qquad (4)$$

Where Ω is a random variable represents the estimated interference and defined as, $\Omega = \sum_{k=p+1}^{p^2} x_k b_0^k$, Defined the following random variable,

$F = \sum_{j=1}^{p} x_j$, is the active users in group 1, thus $p-1 \geq f \geq 1$ and $p^2-p \geq N \geq f$. The probability distribution of the random variable " F " given that user 1 is active and the last user in each group is idle can be written as

$$P_F(f) = \frac{\binom{p^2-2p+1}{N-f}\binom{p-2}{f-1}}{\binom{p^2-p-1}{N-1}}, \ f \in \{f_1, f_1+1, \ldots\ldots\ldots, f_2\} \quad (5)$$

where $f_1 = \mathrm{Max}\{N+2p-p^2-1,\ 1\}$ and $f_2 = \mathrm{Min}\{N, p-1\}$.

the probability distribution of the Ω random variable given F = f is given by:

$$P_{\Omega \mid F}(\omega \mid f) = \binom{N-f}{\omega}\frac{1}{2^{N-f}},\ \omega \in \{0,1,2,\ldots\ldots\ldots,N\text{-}f) \qquad (6)$$

<u>*The decision rule*:</u> an optimal static threshold *th* , which depends on the number of active simultaneous users, is set. If the received photon count Y_1 is less than this threshold , " 0 " is declared, otherwise " 1 " is declared to be sent. We obtain the optimal value of *th* given N, p, λ by numerical calculations, where $\lambda = \lambda_s T_c$ is the photon count per pulse. The probability of bit error is given by,

$$P_b = \sum_{f=f_1}^{f_2} P_b^f\, P_F(f)\ ,\ P_b^f = \sum_{\omega=0}^{N-f} P_{e|\Omega}(\omega) P_{\Omega|F}(\omega|f)\ ,$$

$$P_{e|\Omega}(\omega) = \frac{1}{2}\Big[\sum_{y=th}^{\infty} e^{-\lambda\omega}\frac{(\lambda\omega)^y}{y!} + \sum_{y=0}^{th-1} e^{-\lambda(p+\omega)}\frac{(\lambda(p+\omega))^y}{y!}\Big] \quad (7)$$

3. Interference Cancellation in Optical OOK-CDMA

In this section we propose a detector in which the MUI can be canceled, each active user *i* performs the correlation and photodetection processes as in the previous section. At the same time the received signal is passed to a matched filter which is matched to the last idle code in the i *th* user group then passed to a photodetector. A decision, representing the MUI, is made and used to decide the data of the i *th* user by getting an optimal adaptive threshold θ_c that depends on the estimated MUI. By this way we decrease the effect of MUI in detection. As shown in Fig. 2, each active user in this canceller performs two primary decisions Y_i and Y_{i-p} after correlating with $a_i(t)$ and a_{i-p} (t) code waveformes respectively (where $a_{i\sim p}(t)$ is the last code in the i th group). The desired user then uses the $Y_{i\sim p}$ decision to estimate the interference $\hat{I}=\Omega$ and use it to set an adaptive threshold θ_c to recover the data bits. For receiver 1 the means of two primary photon counts equal to: $E[Y_1] = \lambda p b^1 + \lambda\Omega$, $E[Y_{1\sim p}] = \lambda\Omega$. We assume for simplicity $Y_{1\sim p} = Y_p$.

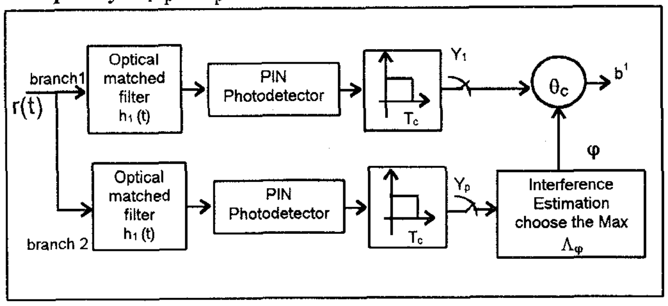

Fig 2. User 1 OOK-OCDMA decoder with interference cancellation

The probability distribution of the random variable f given that user 1 is active and the last user in each group is idle and the probability distribution of the Ω random variable given F = f are as before.

3.1. Estimation for The Multiple-User Interference "Ω"

The mean count $E[Y_p] \in \{0, \lambda, \ldots\ldots, (N-f)\lambda\}$ given $Y_p = K$, where K the 2 *nd* branch photon count for the 1*st* user receiver. We decide that $\Omega = \omega$ if,

$$P(\Omega = \omega \mid Y_p = K) > P(\Omega = \alpha \mid Y_p = K) \qquad \forall\, \alpha \neq \omega ,$$

$\alpha \in \{0, 1, \ldots., N\text{-}f\}$ this means that ,

$$\frac{P(Yp = K|\ \Omega = \omega)P(\Omega = \omega)}{P(Yp = K)} > \frac{P(Yp = K|\ \Omega = \alpha)P(\Omega = \alpha)}{P(Yp = K)} \tag{8}$$

Substituting by the Poisson probability distribution ,

$$e^{-\lambda\omega}\frac{(\lambda\omega)^K}{K!}\binom{N-f}{\omega}\frac{1}{2^{N-f}} > e^{-\lambda\alpha}\frac{(\lambda\alpha)^K}{K!}\binom{N-f}{\alpha}\frac{1}{2^{N-f}} \tag{9}$$

$$\left(\frac{\omega}{\alpha}\right)^K > e^{\lambda(\omega-\alpha)}\frac{\binom{N-f}{\alpha}}{\binom{N-f}{\omega}} \tag{10}$$

$$K\log\left(\frac{\omega}{\alpha}\right) > \lambda(\omega-\alpha) + \log\binom{N-f}{\alpha} - \log\binom{N-f}{\omega} \tag{11}$$

$$K\log(\omega) + \log\binom{N-f}{\omega} - \lambda\omega > K\log(\alpha) + \log\binom{N-f}{\alpha} - \lambda\alpha \tag{12}$$

In our interference estimation stage we consider the decision variable,

$\Lambda_\omega = K\log(\omega) + \log\binom{N-f}{\omega} - \lambda\omega$, and choose maximum Λ_ω to estimate the interference "$\Omega = \omega$" and then using it to adapt the threshold θ_c of the "canceled-interference" detector.

3.2. Another Way to Estimate The Interference "Ω" and The Probability Distribution of Estimated Interference

We can define $a_{\omega\alpha}$ as ,

$$a_{\omega\alpha} = \frac{\lambda(\omega-\alpha) + \log\binom{N-f}{\alpha} - \log\binom{N-f}{\omega}}{\log(\frac{\omega}{\alpha})}, \tag{13}$$

from Section 3.1 note that, $a_{\omega\alpha} = a_{\alpha\omega}$, then we can make the "$\Omega=\omega$" by another way if, $Y_p > a_{\Omega\alpha}\ \forall\ \alpha < \omega$, or, $Y_p < a_{\omega\alpha}\ \forall\ \alpha > \omega$. this means that: $a_\omega < Y_p < b_\omega$, where $a_\omega = \mathrm{Max}_{\alpha:\alpha<\omega}\{a_{\omega\alpha}\}$ and $b_\omega = \mathrm{Min}_{\alpha:\alpha>\omega}\{a_{\omega\alpha}\}$.The decision rule in the second branch can be made by the following test:

"Decide $\Omega=\omega$ if : $a_\omega < Y_p < a_{\omega+1}$ "

where $a_\omega = \dfrac{\lambda + \log\dfrac{\omega}{N-f-\omega+1}}{\log\dfrac{\omega}{\omega-1}}$,

we also can note that, $a_0 = 0^-$, $a_{N-f-1} = \infty$.

3.3. Adaptive Threshold θ_c and Canceled Interference Detector Decision Rule

The decision rule is to choose $b^1=1$ if "$Y_1 = H \geq \theta_c$" given $\Omega = \omega$, where Y_1 is the photon count in the 1 *st* branch of the 1*st* user receiver and Ω is the estimate of interference from the 2 nd branch decision described in Section 3.1. This decision is true if:

$$P(b^1 = 1|\ Y_1 = H, \Omega = \omega) > P(b^1 = 0|\ Y_1 = H, \Omega = \omega) \ , \tag{14}$$

by applying bayes' rule, we note

$$\frac{P(Y_1 = H|b^1 = 1, \Omega = \omega)P(b^1 = 1)}{P(Y_1 = H|\Omega = \omega)} > \frac{P(Y_1 = H|b^1 = 0, \Omega = \omega)P(b^1 = 0)}{P(Y_1 = H|\Omega = \omega)} \tag{15}$$

this is equivalently to

$$e^{-\lambda(p+\omega)}\frac{(\lambda(p+\omega))^H}{H!} > e^{-\lambda\omega}\frac{(\lambda\omega)^H}{H!}, \tag{16}$$

then $$(1+\frac{p}{\omega})^H > e^{-\lambda p} \tag{17}$$

Hence we can make the data bit recovery decision by the test:

$$Y_1 \underset{<}{>} \frac{\lambda p}{\log(1+\frac{p}{\omega})} = \theta_c \tag{18}$$

We note that we can obtain our optimal adaptive threshold θ_c by knowing the values of λ , p , ω . the term adaptive is used because in a real systems, any change in the estimated interference Ω due to other data users variations, would change this optimal adaptive threshold.

3.4. Canceller Bit Error Probability

The bit error probability of "canceled-Interference" detector is defined as

$$P_b = \sum_{f=f_1}^{f_2} P_b^{\,f}\, P_F(f), \tag{19}$$

where,

$$P_b^{\,f} = \sum_{\omega=0}^{N-f} P_{e|\Omega,F}(\omega,f)P_{\Omega|F}(\omega|f), \tag{20}$$

$$P_{e|\Omega,F}(\omega) = \sum_{\beta=0}^{N-f} P_{e|\Omega,K,\hat{I}}(\omega,f,\beta)P_{\hat{I}|\Omega,F}(\beta|\omega,f) \quad . \tag{21}$$

where Ω and $\hat{I}$ are the real and estimated interference random variables and

$$P_{e|\Omega,\hat{I}}(\omega,\beta) = \frac{1}{2}(P_{e|\Omega,\hat{I},b}(\omega,\beta,0) + P_{e|\Omega,\hat{I},b}(\omega,\beta,1)),$$

Here,

$$P_{e|\Omega,\hat{I},b}(\omega,\beta,0) = P(Y_1 \geq \theta_c | b^1 = 0) = \sum_{K=\theta_c}^{\infty} e^{-\lambda\omega}\frac{(\lambda\omega)^K}{K!}, \tag{22}$$

$$P_{e|\Omega,\hat{I},b}(\omega,\beta,1) = P(Y_1 < \theta_c | b^1 = 1) = \sum_{K=0}^{\theta_c - 1} e^{-\lambda(p+\omega)}\frac{(\lambda(p+\omega))^K}{K!} \tag{23}$$

where, θ_c is the optimal adaptive, $\theta_c = \dfrac{\lambda p}{\log(1+\frac{p}{\beta})}$

and the conditional probability distribution of estimated interference given real interference $\Omega = \omega$ as shown in Section 3.2 is defined as :

$$P_{\hat{I}|\Omega}(\beta|\omega) = P(a_\beta < Y_p < a_{\beta+1}|\Omega = \omega) = \sum_{K=a_\beta}^{a_{\beta+1}} e^{-\lambda\omega}\frac{(\lambda\omega)^K}{K!} \tag{24}$$

where: $a_\beta = \dfrac{\lambda + \log\frac{\beta}{N-f-\beta+1}}{\log\frac{\beta}{\beta-1}}$, $P_F(f)$ and $P_{\Omega|F}(\omega|f)$ are as before.

4. Numerical Results

Numerical calculations are done to find the optimal static threshold th which minimizes BER with all other parameters fixed as shown in Fig 3 for the OOK-CDMA system described in Section 2. We plot BER versus the number of simultaneous users N with $\lambda = 20$ for p=5,9 in Fig. 4 which shows a comparison between the performance of the system with and without cancellation, the program of calculating BER for the system with cancellation can be found in Appendix E. It is obvious that Pe improves with the proposed canceller all over the range of N and the improvement increase as p increases. Fig 5 shows BER versus the prime number p when N is multiple of p and λ=20, with N>2p system without MUI canceller is not reliable but when using canceller this system may be used. In Fig. 6 the performance of the two systems are compared when varying number of photon counts per pulse with p=5,7 and we can see that there is a remarkable increase in the improvement of the system performance with canceled interference detector.

5. Conclusion

Interference cancellation technique have been proposed for synchronous optical CDMA communication systems. In this technique, the desired user perform two primary decisions Y_i and Y_{i-p} after correlation with the user assigned code and the last code in its group. The desired user then employ Y_{i-p} decision to estimate the interference and adapt the threshold θ_c to recover the transmitted data bits. Bit error rate for this system have been derived and compared to the system without interference cancellation. Our results show significant improvement in performance when using this canceller.

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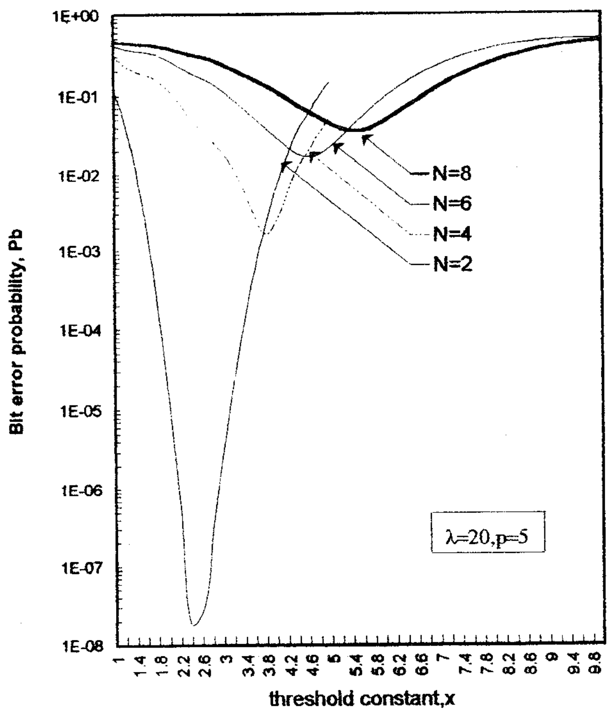

Fig. 3 Bit error rate versus threshold per pulse photons count, with p=5,λ=20.

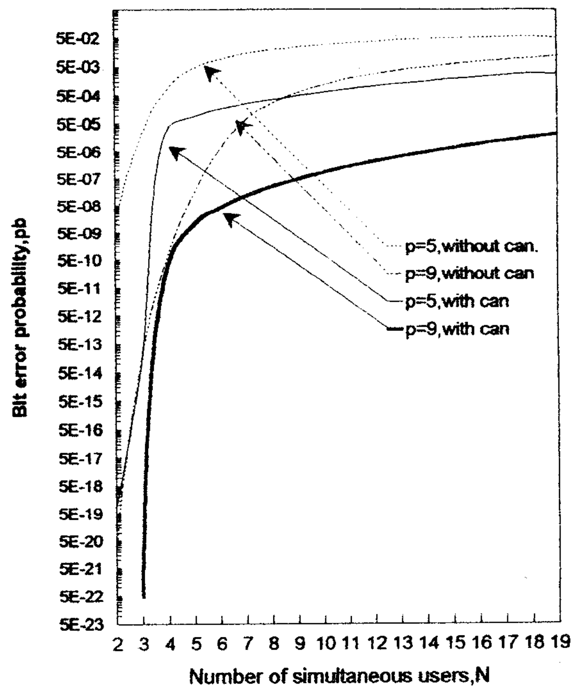

Fig. 4 Bit error rate versus the number of simulatneous users without interference cancellation with 20 photon/pulse

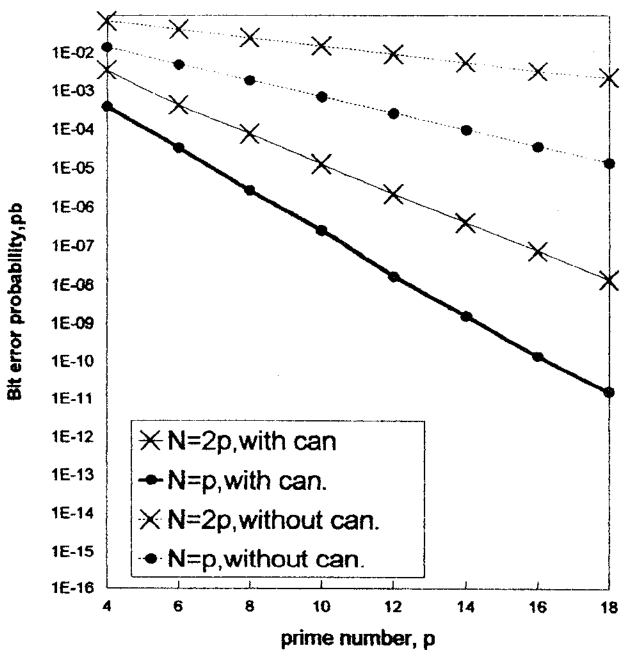

Fig. 5 Bit error probability versus the prime number with and without MUI cancelation with 20 photons/pulse

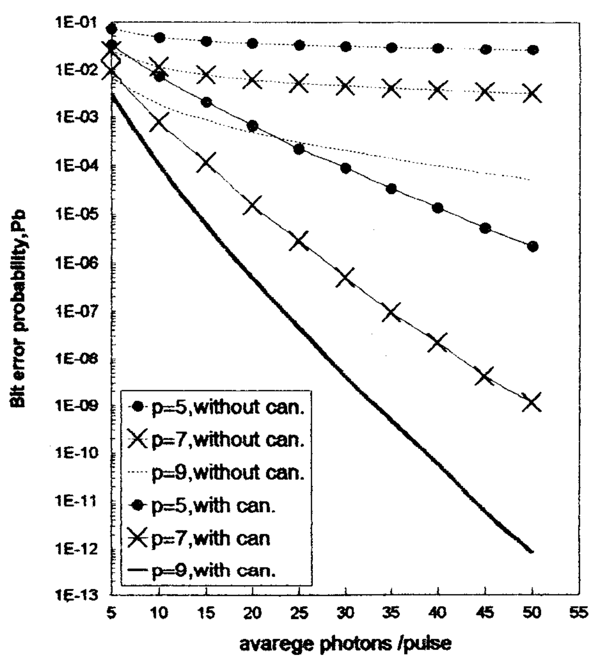

Fig. 6 Bit error probability versus the average photons count per pulse with n=8