Transmission Analysis of M-ary QAM Based Wireless Services over Exponentiated Weibull Turbulent RoFSO Channels

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Abstract: Exact and approximate expressions for average BER of M-ary QAM-based wireless services, transmitted over radio-on-FSO systems, are derived. The effects of FSO turbulence exponentiated Weibull channels, fog, beam divergence, and pointing errors are considered.

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1. Introduction

Radio-on-free space optical (RoFSO) communication systems provide efficient and cost effective links to transfer high-data-rate radio signals. However, atmospheric conditions have strong effect on the performance of RoFSO systems. In this paper, a mathematical analysis for performance of transmission M-ary QAM based wireless services on FSO systems is carried out. Both exact and approximate expressions for average BER of M-ary QAM signals, transmitted over RoFSO systems, are derived. The effects of FSO turbulence exponentiated Weibull channels, fog, beam divergence, and pointing errors are considered The derived expressions are used to investigate system performance under both different weather conditions and pointing errors.

2. System and Channel Model

In our proposed radio-on-FSO system, an M-ary QAM signal, $S_{QAM}(t)$, is used to modulate the optical intensity of a laser diode. The output optical power is $P(t) = p\left[1+MS_{QAM}(t)\right]$, where p is the average transmitted optical power, M is the modulation index, $S_{QAM}(t) = A_I\cos(2\pi f_c t) - A_Q\sin(2\pi f_c t)$, A_I and A_Q are the signal amplitudes of in-phase and quadrature components, respectively, and f_c is the electrical carrier frequency. At receiver side, the output current of the photodetector PD is given by $y(t) = I_{ph}(t)MS_{QAM}(t) + n(t)$, where $I_{ph}(t) = \mathscr{R}pG(t)$, $\mathscr{R}$ is the responsivity of the photodiode, $n(t)$ is a Gaussian noise with variance σ_n^2, and G is the channel gain. Channel gain G includes effects of different parameters like turbulence gain h, fog attenuation, beam divergence attenuation, and pointing errors. Turbulence gain h is well described by exponentiated Weibull EW distribution [1].
In order to consider effects of fog and beam divergence, channel gain is given as $G = \xi h$, where ξ is a normalized path loss coefficient. The value of ξ is evaluated based on receiver and transmitter aperture diameters (D_R and D_T, respectively), weather dependent attenuation coefficient U, and optical beam divergence angle θ_T [2]. The pdf and cdf of instantaneous electrical signal to noise ration, $\gamma = \left(\frac{\mathscr{R}pG}{\sigma_n}\right)^2 = \overline{\gamma}G^2$, can be derived to be as follows.

$$f_{EW}(\gamma;\beta,\eta,\alpha) = \frac{\alpha\beta}{2\gamma}\left(\frac{\gamma}{\overline{\gamma}\eta^2\xi^2}\right)^{(\beta/2)}\exp\left(-\left(\frac{\gamma}{\overline{\gamma}\eta^2\xi^2}\right)^{\beta/2}\right)\left\{1-\exp\left(-\left(\frac{\gamma}{\overline{\gamma}\eta^2\xi^2}\right)^{\beta/2}\right)\right\}^{(\alpha-1)} \quad (1)$$

$$F_{EW}(\gamma;\beta,\eta,\alpha) = \left\{1-\exp\left(-\left(\frac{\gamma}{\overline{\gamma}\eta^2\xi^2}\right)^{\beta/2}\right)\right\}^{\alpha} \quad (2)$$

where $\overline{\gamma}$ is the average electrical SNR, $\beta > 0$ is a shape parameter related to the scintillation index, $\eta > 0$ is a scale parameter, and $\alpha > 0$ is an extra shape parameter that strongly dependent on the receiver aperture size as given in [1].

In order to extend analysis to include the impact of pointing errors under exponentiated Weibull turbulence, let channel gain $G = hh_p$ be composed by two independent random processes, neglecting path loss, where the random term h_p is the misalignment, pointing error, fading. The pointing errors h_p has been modeled as the result of considering independent identical Gaussian distributions, with variance σ_s^2, for the elevation and horizontal displacement [3]. The model for statistics of the pointing error fading is given in [4]. By applying the standard statistical procedures, the distribution of the overall channel gain G is given as a conditional random process given a turbulence state h [3] and its cdf can be evaluated to be as follows [5].

$$F_G(G;\beta,\eta,\alpha) = \sum_{j=0}^{\infty} \frac{\delta^2 \alpha \Gamma(\alpha)(-1)^j (1+j)^{\frac{\delta^2}{\beta}-1} G^{\delta^2}}{\beta A_o^{\delta^2} \eta^{\delta^2} \Gamma(\alpha-j) j!} G_{2,3}^{2,1}\left((1+j)\left(\frac{G}{\eta A_o}\right)^{\beta} \middle| \begin{matrix} 1-\frac{\delta^2}{\beta},1 \\ 0,1-\frac{\delta^2}{\beta},-\frac{\delta^2}{\beta} \end{matrix} \right) \tag{3}$$

where $\delta = \frac{W_{Leq}}{2\sigma_s}$ is the ratio between equivalent beam radius and pointing error jitter at the receiver, $A_o = (erf(v))^2$, $v^2 = \frac{\pi a^2}{2W_L^2}$, a is the radius of the receiver, W_L is the beam radius at the receiver plane, and $W_{L_{eq}}^2 = \frac{W_L^2 \sqrt{\pi} erf(v)}{2v\exp(-v^2)}$.

3. BER Performance Analysis

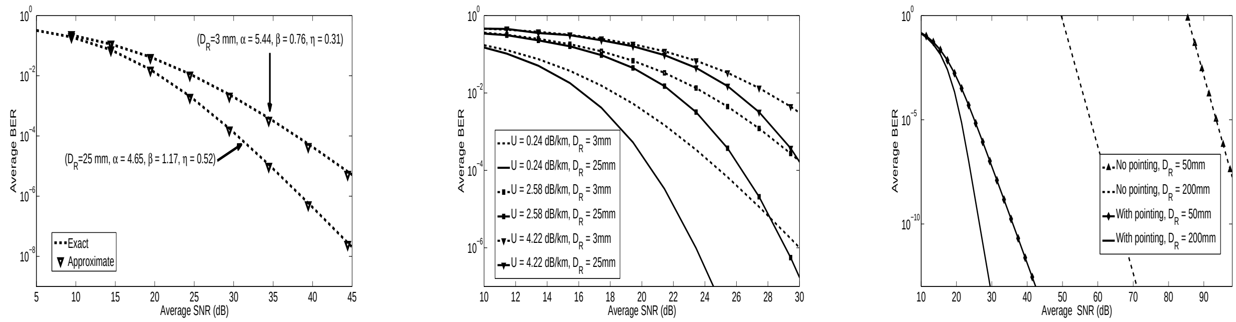
After rigorous mathematical analysis, we are able to get exact and approximate expressions for the average of BER_{QAM} with respect to γ as given in (4) and (5), respectively. Where the effects of both fog and beam divergence have been considered. Derivation of approximate expression is based on generalized Gauss-Laguerre quadrature rule [6]. An approximate expression for the average BER of RoFSO system over EW turbulent channel considering pointing errors can be evaluated using generalized Gauss-Laguerre quadrature rule as given in (6).

$$BER_{QAM} = \begin{cases} \sum_{j=0}^{\infty} \sum_{i=1}^{\sqrt{M_q}/2} \frac{(-1)^j (1-1/\sqrt{M_q}) k^{0.5} l^{(\beta/2-1)} \alpha\beta\Gamma(\alpha)(4(M_q-1))^{\beta/2}}{\sqrt{\pi} log_2(M_q)\Gamma(\alpha-j) j! (2\pi)^{0.5(l+k)-1} (3M^2(2i-1)^2)^{\beta/2} (\overline{\gamma}\eta^2\xi^2)^{\beta/2}} G_{2l,k+l}^{k,2l}\left(\frac{(1+j)^k (4l(M_q-1)/3M^2)^l}{k^k (2i-1)^{2l} (\overline{\gamma}\eta^2\xi^2)^{k\beta/2}} \middle| \begin{matrix} \Delta(l,1-\beta/2),\Delta(l,0.5-\beta/2) \\ \Delta(k,0),\Delta(l,-\beta/2) \end{matrix} \right) & \text{if } m \text{ even}, \\ \sum_{j=0}^{\infty} \frac{(-1)^j k^{0.5} l^{(\beta/2-1)} \alpha\beta\Gamma(\alpha)(4(M_q-1))^{\beta/2}}{\sqrt{\pi} log_2(M_q)\Gamma(\alpha-j) j! (2\pi)^{0.5(l+k)-1} (3M^2)^{\beta/2} (\overline{\gamma}\eta^2\xi^2)^{\beta/2}} G_{2l,k+l}^{k,2l}\left(\frac{(1+j)^k (4l(M_q-1)/3M^2)^l}{k^k (\overline{\gamma}\eta^2\xi^2)^{k\beta/2}} \middle| \begin{matrix} \Delta(l,1-\beta/2),\Delta(l,0.5-\beta/2) \\ \Delta(k,0),\Delta(l,-\beta/2) \end{matrix} \right) & \text{if } m \text{ odd}, \end{cases} \tag{4}$$

$$BER_{QAM} = \begin{cases} \sum_{j=1}^{c} \sum_{i=1}^{\sqrt{M_q}/2} \frac{2}{log_2(M_q)\sqrt{\pi V_j}} \left(1-\frac{1}{\sqrt{M_q}}\right) \Lambda_j \left\{ 1-\exp\left(-\left(\frac{4(M_q-1)V_j}{3M^2(2i-1)^2\overline{\gamma}\eta^2\xi^2}\right)^{\beta/2}\right)\right\}^{\alpha}, & \text{if } m \text{ is even}, \\ \sum_{j=1}^{c} \frac{2}{log_2(M_q)\sqrt{\pi V_j}} \Lambda_j \left\{ 1-\exp\left(-\left(\frac{4(M_q-1)V_j}{3M^2\overline{\gamma}\eta^2\xi^2}\right)^{\beta/2}\right)\right\}^{\alpha}, & \text{if } m \text{ is odd}, \end{cases} \tag{5}$$

where M_q is the number of modulation levels, $m = log_2(M_q)$, $\Delta(k,a) = \frac{a}{k}, \frac{a+1}{k}, \cdots, \frac{a+k-1}{k}$, $l/k = \beta/2$, both l and k are integers, $c > 1$ denotes the number of terms; and for any $i \in \{1,2,\ldots,c\}$, V_i is the ith root of Laguerre polynomial $L_c(X)$ with degree c, and Λ_i is the corresponding weighting coefficient.

$$BER_{QAM} = \begin{cases} \sum_{j=0}^{\infty} \sum_{i=1}^{\sqrt{M_q}/2} \sum_{a=0}^{c} \frac{2}{\sqrt{\pi}\log_2 M_q} \left(1-\frac{1}{\sqrt{M_q}}\right) \frac{\delta^2\alpha\Gamma(\alpha)(-1)^j(1+j)^{\frac{\delta^2}{\beta}-1}}{\beta A_o^{\delta^2}\eta^{\delta^2}\Gamma(\alpha-j)j!} \left(\frac{4(M_q-1)}{3M^2\overline{\gamma}((2i-1)^2}\right)^{\delta^2/2+0.5} \Lambda_a V_a^{\delta^2/2-0.5} \\ \times G_{2,3}^{2,1}\left((1+j)\left(\frac{4(M_q-1)V_i}{3M^2\overline{\gamma}((2i-1)^2(\eta A_o)^2}\right)^{\beta/2} \middle| \begin{matrix} 1-\frac{\delta^2}{\beta},1 \\ 0,1-\frac{\delta^2}{\beta},-\frac{\delta^2}{\beta} \end{matrix} \right) \text{ if } m \text{ is even}, \\ \sum_{j=0}^{\infty} \sum_{a=0}^{c} \frac{2\delta^2\alpha\Gamma(\alpha)(-1)^j(1+j)^{\frac{\delta^2}{\beta}-1}}{\sqrt{\pi}\log_2 M_q \beta A_o^{\delta^2}\eta^{\delta^2}\Gamma(\alpha-j)j!} \left(\frac{4(M_q-1)}{3M^2\overline{\gamma}}\right)^{\delta^2/2+0.5} \Lambda_a V_a^{\delta^2/2-0.5} G_{2,3}^{2,1}\left((1+j)\left(\frac{4(M_q-1)V_i}{3M^2\overline{\gamma}(\eta A_o)^2}\right)^{\beta/2} \middle| \begin{matrix} 1-\frac{\delta^2}{\beta},1 \\ 0,1-\frac{\delta^2}{\beta},-\frac{\delta^2}{\beta} \end{matrix} \right); \\ \text{if } m \text{ is odd}. \end{cases} \tag{6}$$



(a) (b) (c)

Fig. 1: Average bit-error rates versus average electrical SNR in dB: (a) Considering moderate fading, clear weather: $U = 0.19$ dB/km and $\theta_T = 2$ mrad. (b) Considering weak fading, $\theta_T = 2$ mrad and different weather conditions. (c) Considering weak turbulent channel and effect of pointing error with $\sigma_s = 0.3m$.

4. Discussion and Numerical Results

Figure 1(a) shows average BER performance of 8-QAM based wireless signal over FSO system under moderate turbulent channels, clear weather ($U = 0.19$ dB/km) and beam divergence $\theta_T = 2$ mrad. Two receiver diameters considered are: $D_R = 3$mm with (α, β, η) = (5.44, 0.76, 0.31) and $D_R = 25$mm with (α, β, η) = (4.65, 1.17, 0.52). Modulation index is $M = 0.8$ and operating wavelength is $\lambda = 780$ nm. Both exact and approximate expressions with $c = 100$ are used in evaluating the average BER in that figure. It is clear that there is a high degree of agreement between exact and approximate expressions curves. Also we can conclude from the figure that increasing receiver diameter leads to significant improvement in FSO system BER performance. Specifically, using $D_R = 25$mm improves performance by 7 dB at BER 10^{-4} when compared with that using $D_R = 3$mm. Indeed, increasing receiver aperture helps in reducing fading effect through averaging it over the aperture.

It is clear from Fig. 1(b) that increasing aperture size will help in improving system performance in case of bad weather conditions because using larger size means collecting more optical power and decreasing path loss. Specifically, system using aperture size $D_R = 25$mm outperforms that using $D_R = 3$mm by 6 dB at BER 10^{-6} in case of clear weather, 4 dB at BER 10^{-4} in case of haze, and by 2 dB at BER 10^{-4} in case of thing fog.

Figure 1(c) shows average BER performance under weak turbulent channel ($\sigma_R^2 = 0.32$) for two aperture sizes: ($D_R = 50$ mm, $\alpha = 2.81$, $\beta = 3.31$, $\eta = 0.88$) and ($D_R = 200$ mm, $\alpha = 0.99$, $\beta = 20$, $\eta = 1.03$), and for the two cases of considering effect of pointing error with $\sigma_s = 0.3m$ and neglecting it. It is clear from the figure that, pointing error has high effect on system performance. For $D_R = 50$ mm, system performance is degraded by 60 dB at BER = 10^{-5}, while for $D_R = 200$ mm it is degraded by 35 dB at BER = 10^{-10}.

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