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Hybrid Power/Overlap Allocation Scheme for a Multirate Overlapped Optical CDMA System

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Abstract– **This paper addresses the problem of resource allocation in a multiservice optical network based on an Overlapped-CDMA system. A joint transmission power and overlapping coefficient (transmission rate) allocation strategy is provided via the solution of a constrained convex quadratic optimization problem. The solution to this problem maximizes the aggregate throughput subject to peak lasers transmission power constraints. The optimization problem is solved in a closed form, and the resource allocation strategy is simple to implement in an optical network. Results are presented showing a total agreement between the derived analytical solution and the one obtained using a numerical search method. In addition, analytical and numerical results show that the proposed resource allocation strategy can offer substantial improvement in the system throughput.**

Keywords– Overlapped CDMA, fiber-Bragg grating, capacity , throughput, multirate, power control, rate control, overlapping coefficient, quadratic function.

I. INTRODUCTION

Optical code division multiple access (OCDMA) has received considerable attention as a multiple access scheme for optical local area networks (LAN) [1][2]. In addition, heterogeneous services, entailing multirate transmission, are now feasible due to the rapid evolution of fiber optic technology that offers ultra-wide optical bandwidth capable of handling these multirate transmissions and fulfilling good quality of service (QoS) requirements [3].

The first work toward this target, a novel coding technique that leads to the generation of a new family of Optical Orthogonal Codes (OOC) called the *Strict* OOC was presented in [4][5].

Most of the analyses conducted on CDMA communication systems agree that optimal selection of the system's parameters such as the transmitted power and the bit rate would improve their performances [6]-[8]. This, in turn, gives rise to optimization problems which are rarely discussed in the literature of OCDMA. For instance, a non linear programming power control algorithm has been proposed in [9] to maximize the capacity of the multirate optical fast frequency hopping code division multiple access (OFFH-CDMA) system constrained by a predefined QoS. The power of multirate users is optimally regulated with variable optical attenuators. In [10], a power control algorithm, based on optical power selector consisting of a set of optical hardlimiters and couplers, has been inspected for a multirate optical DS-CDMA system using one signature for each user with time hopping. In this work, the transmission rate and the bit error rate are controlled by the hopping rate and the optical power, respectively, to improve the system performance. In addition, an adaptive overlapped pulse-position modulator, employed to create multirate and multiquality transmission schemes, has been investigated in [11] for OCDMA networks where the power control mechanism is done by means of an optical attenuators. Moreover, the power control problem is also addressed in [12] for temporal prime coded OCDMA systems taking into consideration the effect of the near-far problem caused by different fiber lengths connecting the users to the star network.

In this work, and for the first time, we propose a novel hybrid power/rate control algorithm for overlapped optical fast frequency hopping CDMA (OOFFH-CDMA) system [13] in which multirate transmission is achieved by overlapping consecutive bits while coded using fiber Bragg grating (FBG). Our purpose in this work is to find the optimal overlapping coefficient with which we can achieve maximum transmission rate with minimum transmitted optical power directly from a laser source according to a predefined QoS required at the optical receivers for each class of users.

Following the introduction, the paper is structured as follows. Section II introduces the system model and the optimization problem formulation. The resource allocation problem is obtained in Section III. Section IV presents the solution for a two-class system. Numerical results are covered in Section V. Finally, the conclusion is presented in Section VI.

II. SYSTEM MODEL AND PROBLEM FORMULATION

An OOFFH-CDMA system that supports M users in S classes, sharing the same optical medium in a star architecture, has been proposed in [13]. We will consider that all users transmitting their data at the same QoS are clustered in the same class. All classes have the same processing gain G. The encoding-decoding is achieved passively using a sequence of fiber Bragg gratings (FBG). The gratings spectrally and temporary slice the incoming broadband pulse into several components, equally spaced at chip interval T_c. The chip duration and the number of grating G determine the nominal bit duration to be $T_n = GT_c$. The corresponding nominal transmission rate is $R_n = 1/T_n$. Increasing the transmission rate beyond the nominal rate R_n without decreasing G introduces an overlapping of coefficient ε_j among the transmitted bits during the same period T_n, as revealed in Fig. 1.

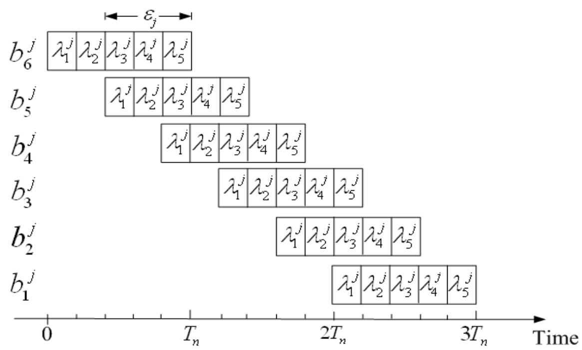

Fig. 1: The concept of overlapping among the bits of *class-j* users, showing the effect of the overlapping coefficient ε_j on their transmission rate.

In this case, the concept of overlapping is illustrated among six bits of $G=5$ and the overlapping coefficient of *class-j* is $\varepsilon_j = 3$, which means that there are three chips in each OCDMA-coded bit that overlap with three chips of the other bits in the same class. This, in turn, increases the overall transmission rate of the users involved in this class from three bits after $3T_n$ to six bits. In general, the overlapping coefficient represents the number of overlapped chips among consecutive bits of *class-j*. Accordingly, the new transmission rate of *class-j* is given by

$$R_j = \frac{G}{G-\varepsilon_j} R_n \quad (1)$$

where $0 \le \varepsilon_j \le G-1$ for $j \in \{0,\ldots,S-1\}$. This implies that $R^{(\ell)} \le R_j \le R^{(u)}$ where $R^{(\ell)} = R_n$ and $R^{(u)} = GR_n$ are the lower and the upper data rate common to all classes, respectively. Also, we assume that the system is chip-synchronous and of discrete rate variation. Furthermore, all users of the same class transmit with the equal power and have the same overlapping coefficient. Hence, each class is characterized by its own QoS. Thus, let P_j and β_j be the transmitted power and QoS of *class-j*, respectively.

A. The Signal-to-interference ratio as QoS measure:

In many cases, it is reasonable to take the QoS requirements as meeting the SIR constraints [7]-[9]. It was shown in [13] that the SIR for *class-j* using an OOFFH-CDMA system is given by

$$SIR_j = \frac{P_j G^2}{\frac{(M-1)G^2}{2F} \sum_{i=0}^{S-1} \frac{p^{(i)} P_i}{G-\varepsilon_i} + \sigma_n^2}, \quad j \in \{0,\ldots,S-1\} \quad (2)$$

where F is the total number of available frequencies used in the code construction [14], and σ_n^2 is the variance of AWGN. On the other hand, $p^{(i)}$ is the multimedia probability density function and it represents the probability that a user selects class-*i*, where $\sum_{i=0}^{S-1} p^{(i)} = 1$. We can easily simplify (2) into the following

$$SIR_j = \frac{P_j}{\sum_{i=0}^{S-1} M_i P_i R_i + \frac{\sigma_n^2}{G^2}}, \quad j \in \{0,\ldots,S-1\} \quad (3)$$

where R_i is given in (1) and M_i represents the weight factor of *class-i* and it is given by

$$M_i = \frac{M-1}{2FGR_n} p^{(i)} \quad (4)$$

Note that by increasing $p^{(i)}$ of *class-I* we are increasing its weight for fixed system parameters $F, G, M,$ and R_n.

B. System Throughput:

In this work, we aim at finding an appropriate resource allocation strategy that maximizes the transmission rates as well as minimizes the transmitted powers of the multirate users in an overlapped CDMA environment in order to maximize the system capacity. The criterion to achieve this optimality is to consider the aggregate throughput $\Omega^M : \mathbb{R}^S \times \mathbb{R}^S \to \mathbb{R}_+$ as the weighted sum of the ratios of the transmission rates over transmitted powers for the S classes and it is given by

$$\Omega^M(\mathbf{R},\mathbf{P}) = \sum_{j=0}^{S-1} M_j \frac{R_j}{P_j} \quad (5)$$

where $\mathbf{R} = (R_0, R_1, \ldots, R_{S-1})^T$ is the data rate vector, $\mathbf{P} = (P_0, P_1, \ldots, P_{S-1})^T$ is the power vector and M_j is the weight factor of *class-j*, as defined in (4) with $0 \le M_j \le 1$. This function of merit represents the system throughput, as the average number of bits per second per unit of power.

Accordingly, we are interested in computing the jointly optimal power and rate allocation for users in each class that maximizes the aggregate throughput, subject to predetermined QoS constraints in terms of the SIR of each class. The optimal allocation policy is obtained by solving the following optimization problem

$$(\Pi_1) \qquad (\mathbf{R}^*, \mathbf{P}^*) = \arg \max_{(\mathbf{R},\mathbf{P}) \in \Im} \{\Omega^M(\mathbf{R},\mathbf{P})\}$$

where the feasible set is given by

$$\Im = \left\{ \begin{matrix} (\mathbf{R},\mathbf{P}): & SIR_j = \beta_j, \ 0 < P_j \le P_{max}, \\ R_n \le R_j \le GR_n & \forall\ j \in \{0,\ldots,S-1\} \end{matrix} \right\} \quad (6)$$

where $P_{max} < \infty$ is the maximum permissible power of the laser source and β_j is the QoS of *class-j*.

To solve (Π_1), the problem is decoupled into two resource allocation scenarios: the power allocation scenario and the rate allocation scenario as will be shown in the next section.

III. Jointly Optimal Power and Rate Allocations

In this scenario, we consider that the intensity of the transmitted optical signal is directly adjusted from the laser source with respect to the transmission data rate of users of the *S* classes. Thus, each class is allocated the minimum optical power capable of handling the traffic rate of its users while observing the transmission rate of all other classes and at the same time maintaining a low level of interference at the desired receiver. To do so, we fix the transmission rate of all the classes and we find out the optimal transmitted power corresponding to the desired class for a given QoS. Therefore, by taking $SIR_j = \beta_j$ and rearranging terms in (3), we get a set of linear equality constraints in terms of P_j. That is,

$$\sum_{i=0}^{S-1} P_i M_i R_i - \frac{P_j}{\beta_j} + \frac{1}{SNR_n} = 0, \qquad \forall j \in \{0,\ldots,S-1\} \qquad (7)$$

where $SNR_n = \dfrac{G^2}{\sigma_n^2}$ is the nominal signal-to-noise ratio common to all classes. Then, by solving the linear system in (7) for P_j we get

$$P_j = \frac{1}{SNR_n} \times \frac{\beta_j}{1 - \sum_{i=0}^{S-1} \beta_i M_i R_i}, \qquad \forall j \in \{0,\ldots,S-1\} \qquad (8)$$

The power is defined when the denominator is strictly greater than zero. That is,

$$\sum_{i=0}^{S-1} M_i \beta_i R_i < 1 \qquad (9)$$

Consequently, the optimal *class-j* transmission power P_j^* is obtained by solving the rate allocation problem and finding the optimal rate R_j^*. Note that the thermal noise, dark current, surface leakage current of the system are taken into consideration through the presence of the factor SNR_n in the power allocation strategy.

A. Optimal Rate Allocation:

We will compute the optimal rate of the system classes that corresponds to the minimum power obtained in the previous section by substituting P_j in (8) into (5). We obtain

$$\Omega^M(\mathbf{R}) = SNR_n\left(-\mathbf{R}^T\mathbf{Q}\mathbf{R} + \mathbf{C}^T\mathbf{R}\right), \mathbf{Q} = \mathbf{Q}^T,\ \mathbf{Q} > 0,\ \mathbf{C} > 0 \quad (10)$$

where $\mathbf{Q}_{S\times S} = \left[\frac{1}{2} M_i M_j \left(\frac{\beta_i}{\beta_j} + \frac{\beta_j}{\beta_i}\right)\right]_{i,j=0,1\ldots,S-1}$ and $\mathbf{C} = \left[\frac{M_0}{\beta_0} \ \frac{M_1}{\beta_1} \ \cdots \ \frac{M_{S-1}}{\beta_{S-1}}\right]^T$. Notice that the throughput function is a quadratic function of the rate vector $\mathbf{R}$.

Thus, the optimization problem (Π_1) under the optimal power allocation becomes

$$(\Pi_2) \qquad \mathbf{R}^* = \arg\max_{\mathbf{R}\in\Im} \left\{\Omega^M(\mathbf{R})\right\} \qquad (11)$$

where the feasible set $\Im$ is given by

$$\Im = \left\{ \begin{array}{l} \mathbf{R}: \ \sum_{j=0}^{S-1} M_j \beta_j R_j \le 1 - \dfrac{\max(\beta_j)_{j=0,1,\ldots,S-1}}{P_{max} SNR_n}, \\ R_n \le R_j \le G R_n \quad \forall\ j \in \{0,\ldots,S-1\} \end{array} \right\} \qquad (12)$$

Notice that the gradient of $\Omega^M(\mathbf{R})$ can be computed as

$$\nabla\Omega^M = SNR_n(-2\mathbf{QR} + \mathbf{C}) \qquad (13)$$

and the Hessian matrix [15][16] is

$$\mathbf{H} = \nabla^2\Omega^M = -2SNR_n\mathbf{Q} \qquad (14)$$

Because the Hessian matrix is negative, the throughput function is a concave function in $\mathbf{R}$, and therefore, the Kuhn-Tucker (KT) condition [16] is sufficient for an optimal point to be a maximum. To solve (Π_2), we use the method of Lagrange multiplier. Consequently, the Lagrangian function is defined as

$$L(\mathbf{R},\mathbf{\Lambda}) = \Omega^M(\mathbf{R}) + \sum_{m=0}^{2S} \lambda_m g_m \qquad (15)$$

where g_m is the m^{th} constraint and λ_m is the corresponding Lagrangian multiplier and $\mathbf{\Lambda}$ is the vector of Lagrangian multipliers. Applying the KT condition on (Π_2) we obtain

$$\frac{\partial L(\mathbf{R},\mathbf{\Lambda})}{\partial R_j} = 0, \ \forall j \in \{0,\ldots,S-1\} \qquad (16)$$

$$\lambda_m g_m = 0, \ \forall m \in \{0,\ldots,2S\} \qquad (17)$$

$$g_m \ge 0 \qquad (18)$$

$$\lambda_m \ge 0 \qquad (19)$$

The nature of the stationary points is governed by the second order derivative of the Lagrangian function [15]. Notice that the second order derivative is strictly negative and independent of R_j i.e.

$$\frac{\partial^2 L(\mathbf{R},\mathbf{\Lambda})}{\partial R_j^2} = -2SNR_n M_j^2 \qquad (20)$$

and

$$\frac{\partial^2 L(\mathbf{R},\mathbf{\Lambda})}{\partial R_i \partial R_j} = -SNR_n M_i M_j \left(\frac{\beta_i}{\beta_j} + \frac{\beta_j}{\beta_i}\right), \ i \ne j \qquad (21)$$

This implies that (20) and (21) are sufficient conditions for the stationary points to be maxima [16]. The following two propositions show that the global maximum of $\Omega^M(\mathbf{R})$ is not the solution of (Π_2).

Proposition 1: *Given an $S\times S$ positive symmetric matrix $\mathbf{Q}$ of the form*

$$\mathbf{Q}_{S\times S} = \left[\frac{1}{2} M_i M_j \left(\frac{\beta_i}{\beta_j} + \frac{\beta_j}{\beta_i}\right)\right]_{\forall i,j\in\{0,1,\ldots,S-1\}}$$

where all $\beta_i \neq \beta_j$, $\mathbf{Q}$ is non-singular for $S=2$, and it is singular for $S \geq 3$.

Proof: ***Omitted due to the limited permissible space (can be provided upon request).*** ■

Proposition 2: *The global maximum of the optimization problem (Π_2) is not feasible.*

Proof: The throughput function has a global maximum only when the gradient is null, $\nabla\Omega^M = 0$. This implies

$$\mathbf{R}^* = \mathbf{Q}^{-1}\mathbf{C}/2 \quad (22)$$

The global maximum in (22) exists if and only if the matrix $\mathbf{Q}$ is invertible. In addition, $\mathbf{R}^*$ is feasible if it is in the feasible set $\Im$ of (Π_2). By proposition 1, $\mathbf{Q}$ is non singular for $S=2$. Thus, for any two-class system, say *class-i* and *class-j*, the global maximum is found when the gradient is null. That is $\nabla\Omega^M = 0$, which yields

$$R_i^* = \frac{\beta_i}{M_i\left(\beta_i^2 - \beta_j^2\right)}$$

$$R_j^* = \frac{\beta_j}{M_j\left(\beta_j^2 - \beta_i^2\right)}$$

By assumption, $\beta_i \neq \beta_j$, so both rates are finite. Also, notice that, $\beta_i M_i R_i + \beta_j M_j R_j = 1$, which violates the condition in (9). This means that this solution is not feasible and hence the global maximum. For $S \geq 3$, $\mathbf{Q}$ is singular by proposition 1 and therefore the global maximum does not exist. ■

B. Problem Solution:

The solution of (Π_2) is obtained via the following lemmas, the first of which shows that the feasible set $\Im$ can be reduced to its boundaries. It also shows that there is at most one class of users that has an overlapping coefficient between zero and full overlap, and the remaining classes either transmit with full overlap or with no overlap.

Lemma 1: *Assume that $\mathbf{R}^* = \left(R_0^*, R_1^*, \ldots, R_{S-1}^*\right)^T$ solves the optimization problem (Π_2). If $P_{Max} < \infty$, there exists at most one R_j^* such that $R^{(\ell)} \leq R_j^* \leq R^{(u)}$, and $R_i^* = R^{(u)}$ or $R_i^* = R^{(\ell)}$ $\forall i \neq j = 0,1,\ldots,S-1$*

Proof: Omitted due to the limited permissible space (can be provided upon request). ■

This means that there exists at most one class of users that transmits with rate between R_n and GR_n, and the remaining classes either transmit with the maximum rate GR_n or with the minimum rate R_n.

Lemma 2: *Consider that $\beta_i > \beta_j$ $\forall i < j$. If $\mathbf{R}^* = \left(R_0^*, R_1^*, \ldots, R_{S-1}^*\right)^T$ solves the optimization problem (Π_2), then $R_i^* \leq R_j^*$ if and only if $0 \leq i < j \leq S-1$.*

Proof: Omitted due to the limited permissible space (can be provided upon request). ■

IV. TWO CLASS SYSTEM

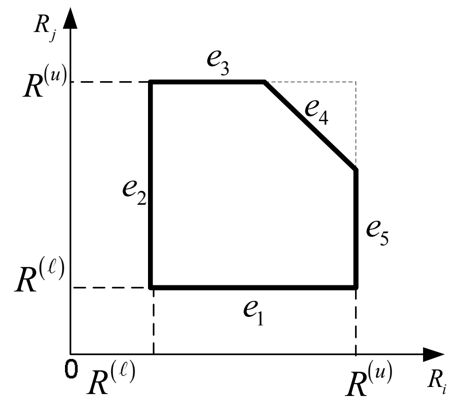

Fig. 2: The feasible region for a 2-class system

Due to the complexity of the problem under consideration, and without loss of generality, we present the case of a two-class system. Consider a two-class system, the *class-i* and the *class-j* with $\mathbf{R} = \left(R_i, R_j\right)^T$. The boundaries of the feasible region are illustrated in Fig. 2. To solve (Π_2), we should obtain the optimal solution which is defined in predetermined intervals of β_i and β_j. Let the set of edges of the feasible region be $\mathbf{E} = \{e_1, e_2, e_3, e_4, e_5\}$ to be the locus of our optimal solution. By Lemma 1, we know that at most one class of users transmits with transmission rate between $R^{(\ell)}$ and $R^{(u)}$, and the remaining classes either transmit with $R^{(u)}$ or with $R^{(\ell)}$. Thus, the search space of the optimal solution is $\mathbf{E}' = \{e_1, e_2, e_3, e_5\}$. In addition, without loss of generality, consider that $\beta_i > \beta_j$. By Lemma 2, we know that if $\beta_i > \beta_j$ then $R_i \leq R_j$. This means that the locus of the optimal solution has been reduced to $\mathbf{E}'' = \{e_2, e_3\}$.

V. NUMERICAL RESULTS

First of all, we consider the two-class system for which we assume that the processing gain of the user's signature is $G = 61$, the total number of available wavelengths is $F = 62$, the nominal signal-to-noise ratio is $SNR_n = 35.7$ dB, the upper bound on the laser power is $P_{\max} = 5$ dBm, the QoS of *class-0* is fixed to $\beta_0 = 8$ dB, and the nominal transmission rate is

$R_n = 1$ Mbps. Besides, in order to assess the validity of our results, we make use of a numerical method, the BFGS method (suggested by Broyden, Fletcher, Goldfarb, and Shannon) [16]. In addition, the performance of the proposed resource allocation strategy is compared with that of a classical power control algorithm with fixed transmission rates. We assume that *class-0* users transmit at rate $R_0 \cong 2R_1$ when $\beta_0 < \beta_1$, and *class-1* users transmit at rate $R_1 \cong 2R_0$ when $\beta_1 < \beta_0$, then the classical power control strategy allocates the best transmission laser power to each user in either classes in order to guarantee the QoS requirements.

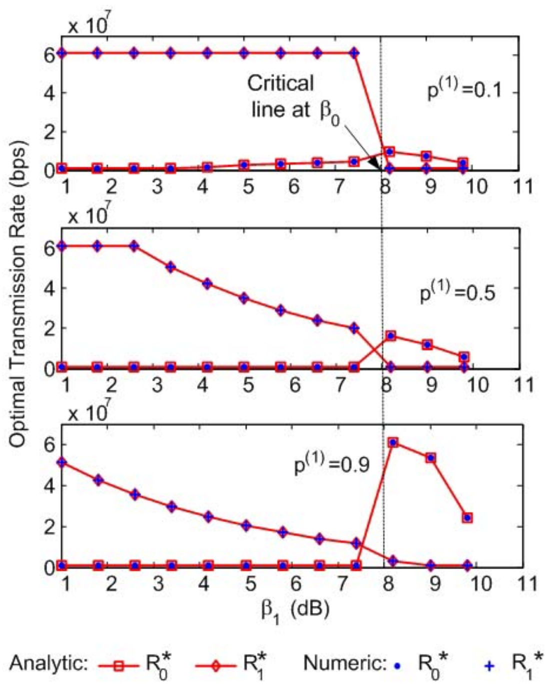

Fig. 3: The transmission rates versus QoS of *class-1* users for different multimedia distribution.

In Fig. 3, the transmission rates are evaluated as a function of β_1, and they are plotted for different multimedia distributions. For $p^{(1)} = 0.1$, the multimedia traffic is more dense in *class-0* rather than in *class-1*. Since a small number of users are choosing *class-1*, the minority *class-1* users transmit at rate $R_1^* = R^{(u)}$ for $\beta_1 < 8\,\text{dB}$. On the other hand, the majority *class-0* users transmit at rate $R_0^* = R^{(\ell)}$ for $\beta_1 < 4\,\text{dB}$, and $R^{(\ell)} < R_0^* < R^{(u)}$ for $4\,\text{dB} < \beta_1 < 8\,\text{dB}$. In addition, for $\beta_1 > 8\,\text{dB}$, *class-1* users now transmit at rate $R_1^* = R^{(\ell)}$, while *class-0* users transmit at rate $R^{(\ell)} < R_0^* < R^{(u)}$. For $p^{(1)} = 0.9$, *class-1* users are allowed to transmit only at $R^{(\ell)} < R_1^* < R^{(u)}$ for $\beta_1 < 8$ dB, while *class-0* users transmit at $R^{(\ell)}$. Also, notice that for $8\,\text{dB} < \beta_1 < 9\,\text{dB}$, $R^{(\ell)} < R_1^* < R^{(u)}$ and $R_0^* = R^{(u)}$. This, in turn, shows a total agreement with the hypotheses proposed in the two lemmas, proposed in previous sections.

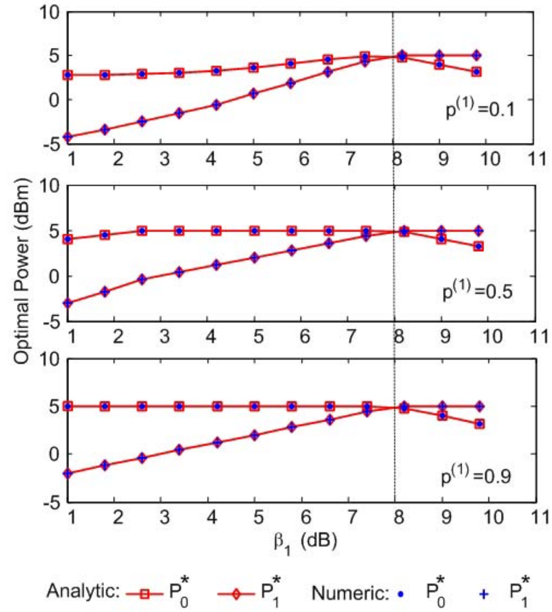

Fig. 4: The power consumption of *class-0* and *class-1* for different multimedia distributions.

The optimal transmission power for the corresponding multimedia distributions is illustrated in Fig. 4. Note that as the number of *class-0* users decreases, their allowable transmission power increases. So that when $p^{(1)}$ increases from 0.1 to 0.9, the MAI effect of *class-0* on *class-1* decreases, and the system allows *class-0* to transmit at the upper-bound laser power to improve the service requirement. In addition, *class-1* power is proportional to the QoS. Therefore, it is monotonically increasing as β_1 until it reaches a constant level at the maximum attainable laser power for $\beta_1 > \beta_0$. The constant power in this interval of β_1 is necessary to assure the data transmission at such QoS. Further, we remark that an additional augmentation of β_1 above 10 dB is no longer supportable because the laser power of the source becomes inadequate.

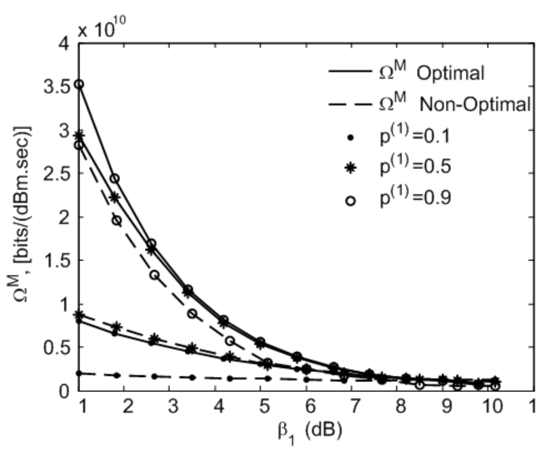

Fig. 5: The throughput versus QoS of *class-1* for different multimedia distributions.

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The optimal and non-optimal throughputs are plotted and compared in Fig. 5. The optimal throughput decays as β_1 increases because at high QoS the allocated resources are performed to preserve the QoS requirement rather than to increase the system capacity. In addition, we observe that an appropriate traffic distribution gradually enhances the system throughput according to allocated resources. For $p^{(1)} = 0.1$, both R_0^* and R_1^* are high, hence the interference. Alongside, the allowable transmission power is relatively low. This yields the lowest optimal throughput due to the fact that the allocated power is insufficient to satisfy the QoS requirement and to combat the MAI increase. As $p^{(1)}$ increases, the MAI relaxes and the average system throughput increases. Also, we can clearly observe that the system throughput of the proposed resource allocation strategy is superior to that of the non-optimal one for the different multimedia distributions.

Next, we study the effectiveness of the proposed resource allocation strategy with respect to the number of stations accessing the system for different multimedia distributions. We keep the same parameter settings as in the previous part but in this case we fix QoS of *class-1* to $\beta_1 = 5$ dB, which is 3 dB less than β_0. By this setting, the classical power control criteria with constant transmission rate turns out to be the equal energy criteria (EEC). We show that the EEC results are feasible but non optimal.

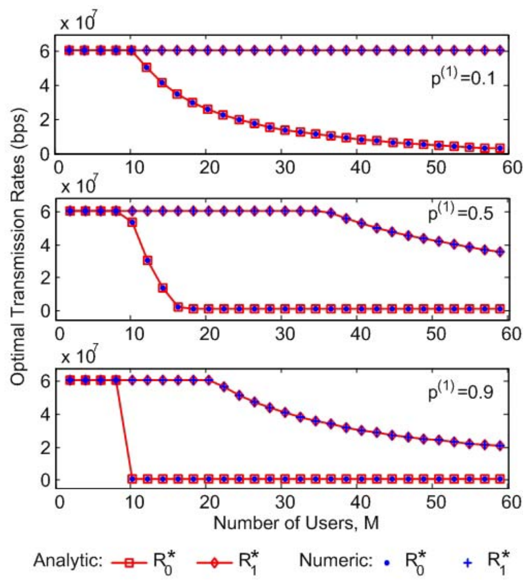

Fig. 6: The optimal transmission rates versus the number of users for a two-class system.

The optimal transmission rates in terms of *M* are examined in Fig. 6. Consequently, for small *M*, the system allows both classes to transmit at maximum rate $R^{(u)}$ because the MAI is sufficiently small. However, when $M \geq 10$ as the user population in a given class increases, its transmission rate decreases to keep the MAI at regular levels and to accommodate to the QoS requirement. Note that, when $R^{(\ell)} \leq R_0^* < R^{(u)}$, $R_1^* = R^{(u)}$, and when $R_0^* = R^{(\ell)}$, $R^{(\ell)} < R_1^* \leq R^{(u)}$ thus consistent with Lemma 1. Furthermore, because $\beta_0 > \beta_1$, it is clear that we always have $R_0^* \leq R_1^*$, which validates Lemma 2.

By obtaining the optimal rates R_0^* and R_1^* for both classes, the optimal overlapping coefficients ε_0^* and ε_1^* satisfying the system requirements are now computed using (1) and depicted in Fig. 7.

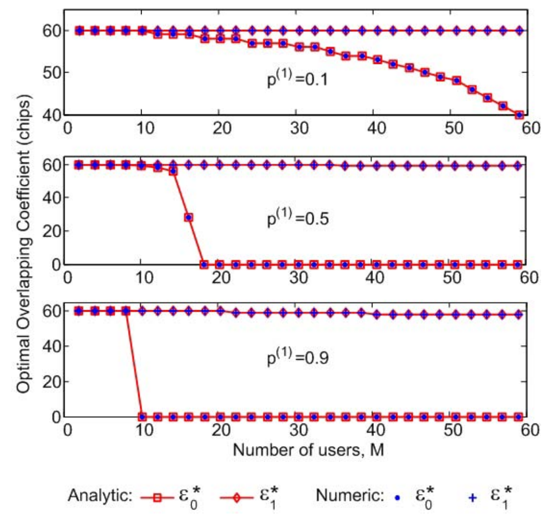

Fig. 7: The optimal overlapping coefficient versus the number of users for a two-class system.

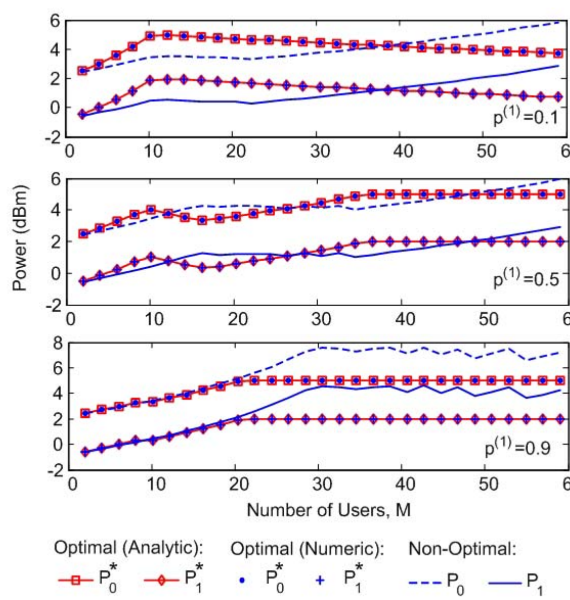

Fig. 8: The transmission power versus the number of users for a two-class system.

In Fig. 8, the optimal power is compared to the non optimal one as *M* varies. Notice that when $p^{(1)}$ is small, the EEC allocates less power for small *M* and more power for large *M* compared to our newly proposed algorithm. This criterion always makes the users susceptible to MAI. In contrast, our proposed strategy provides more power for small *M* to improve the optical signal, and less power for large *M* to reduce the MAI intensity. On the other hand, when $p^{(1)} = 0.5$, the EEC follows the optimal one. Finally, when $p^{(1)} = 0.9$, the EEC power exceeds the upper bound laser power for large *M*. On the other hand, our proposed strategy controls this excess of power by clamping it to the maximum allowable laser power.

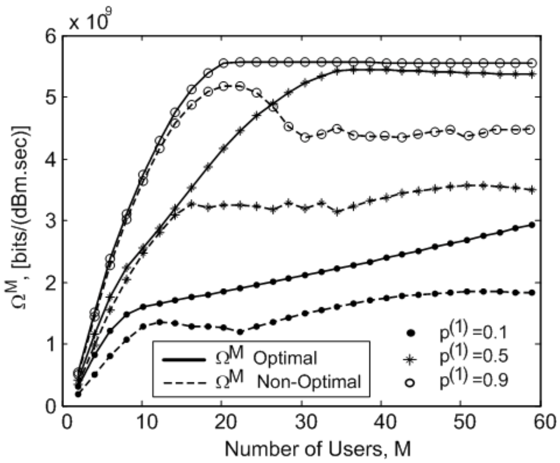

Fig. 9: The optimal throughput versus the total number of users for the two-class system.

The impact of the total number of users on the system throughput is shown in Fig. 9. For small *M*, the performance of the EEC approaches the optimal one especially when the probability of selecting *class-1* is high. As *M* increases, the system throughput of our proposed strategy outperforms that of the EEC. Note that as *M* becomes higher, the system throughput increases for increasing values of $p^{(1)}$. This happens because when $p^{(1)}$ is small, both classes transmit at rates higher than the nominal one as revealed in Fig. 3. This in turn requires high overlapping coefficients. It follows that the interference level in the optical channel increases. In addition, the transmission power level dedicated for such rate is also low as shown in Fig. 4. This creates degradation in the system throughput. When $p^{(1)}$ increases, the transmission rates decrease, whereas the power relatively increases. Consequently, the throughput is significantly improved.

VI. CONCLUSION

A new resource allocation strategy was proposed for overlapped OFFH-CDMA system. Two recourse allocation scenarios were derived for both power and rate to simplify the analysis. Then, the KT theorem was applied on the rate allocation scenario to find out the optimal transmission rates upon which the optical intensity of each class was optimally regulated directly form the laser source. It was proven that this system, in general, has no global maximum but local maximum for given QoS's. Simulation assessed that multirate transmission alternates among classes depending on which QoS region the users are adhering to, as well as on the total number of users exploiting the system and their distribution among those classes.

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