Effect of Both Shot and Beat Noises on the Performance of a 2 D optical CDMA Correlation Receiver

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Abstract The effect of both shot beat noises on the performance of 2-D wavelength hopping/time spreading CDMA system is analyzed. Noise variance and probability of error are calculated for a general system employing asymmetric prime-hop code (ASPHC). Comparison with a noise-free system reveals the effect of noise on performance.

Index Terms—Code division multiple access optical CDMA interchannel interference multiuser channels optical communication photodetectors.

I. Introduction

Two-dimensional (2-D) wavelength hopping/time spreading code division multiple access (CDMA) systems is used in optical local area networks (LANs) [1]. Using 2-D coding has a number of advantages which are: the cross-correlation is reduced, the auto-correlation sidelobes vanishes, and the cardinality of the code family is greatly increased, which increases security. There are different ways to perform the coding in 2-D; each producing unique 2-D code sequences [1]. Our analysis is based on asymmetric prime-hop sequences where the algorithms for constricting 2-D codes are based on linear congruencies [1]. In the time domain, the code sequence has a weight of ω pulses (ω being a prime number); the position of each pulse is determined by the linear congruence operator. Hence, there are ω pulses in a code sequence of length ω^2. Wavelength domain coding is used to give pulses different colours, again based on the linear congruence operator; the available number of wavelengths is M (where M is also a prime number with $M \ \omega$) [1]. Since the number of wavelengths exceeds the pulses to be colored a column selection function has to be deployed. Each code family consist of $\omega \ M-1$ codes, each code having length ω^2, and Hamming weight ω; the cross-correlation is at most one with no auto-correlation sidelobes.

The system when implemented practically needs tunable laser sources and tunable fiber delay lines to select the pulse wavelength and time slot. The received signal consists of the data pulses plus pulses due to multiple-access interference. Due to the random arrival of photons the shot noise appears and due to the nonlinearity of the system there will be beating between pulses at the same wavelength giving rise to beat noise. The purpose of this paper is to assess the strength of this process and its impact on the probability of error.

II. Anal sis of idealized 2 D scheme

In order to calculate the bit-error rate, we begin by determining the conditional probability that ℓ users are sending a '1' bit given that interfering users are active. This yield

$$P(\ell|K-1) = \binom{K-1}{\ell}\left(\frac{1}{2}\right)^{\ell}\left(\frac{1}{2}\right)^{K-1-\ell} \tag{1}$$

where we have assumed that '0'- and '1'-bits are equiprobable. We assume also that the O-CDMA system is chip-synchronous, which yields an upper bound on the MAI [2]. On a single wavelength, the probability that a transmitted pulse is coincident with the lit chip in the code of the receiver is then $1/\omega^2$, with an average of wavelengths in common [1]

$$\mu_s = \frac{1}{\binom{M}{\omega}} \binom{M-1}{\omega-1} \frac{(\omega-1)(\omega-2)\ (M-2)}{M-2} \quad \binom{M-1}{\omega} \frac{\omega(\omega-1)}{M-2} \tag{2}$$

The total probability of an overlap is μ_s/ω^2. The probability of m interference pulses given that ℓ interfering users are present is therefore

$$P(m|\ell) = \binom{\ell}{m}\left(\frac{\mu_s}{\omega^2}\right)^{m}\left(1-\frac{\mu_s}{\omega^2}\right)^{\ell-m} \tag{3}$$

where we have used the fact that the peak cross-correlation between APHC signature sequences in the same set is unity.

Neglecting all physical noises, a bit-error can only occur when a '0' bit was transmitted (with probability 1/2) and ω or more interference pulses fall on the detector which makes the receiver to erroneously decide that a '1' bit had been sent. The probability of error is thus

$$P(\ell,m) = \frac{1}{2}\sum_{\ell=\omega}^{K-1} P(\ell|K-1) \sum_{m=\omega}^{\ell} P(m|\ell)$$

$$= \frac{1}{2}\sum_{\ell=\omega}^{K-1} \binom{K-1}{\ell} 2^{K-1} \sum_{m=\omega}^{\ell} \binom{\ell}{m}\left(\frac{\mu_s}{\omega^2}\right)^{m}\left(1-\frac{\mu_s}{\omega^2}\right)^{\ell-m} \tag{4}$$

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where we have set the threshold equal to the weight of the code $\omega = 5$, the available number of wavelengths is M = 31 and the system consists of $K = 15$ subscribers. Fig.1. shows the above BER versus the number of active users.

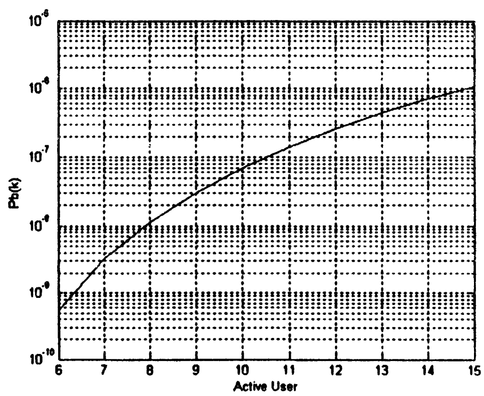

Fig.1. BER for idealized 2-D system. Codes have M = 31 wavelengths and code length $\omega^2 = 25$

III. Anal sis of the shot noise effect on 2 D scheme

The bit error probability of ASPHC correlation receiver when taking into account the effect of the shot noise [4] can be found in Eq (8). We derive here the corresponding BER as:

$$P_b\ K = \frac{1}{2}\ P\ E|0 \quad P\ E|1$$

$$= \frac{1}{2} \sum_{\ell=0}^{K-1} \sum_{m=0}^{\ell} P E|0,\ \ell, \kappa = m \quad P E|1,\ \ell, \kappa = m\ \ P\ \ell, m \tag{5}$$

$$P\ E|0,\ \ell, \kappa = m \ = \frac{1}{2} erfc \ \frac{\theta - \quad m}{\sqrt{2 \quad m}} \tag{6}$$

$$P\ E|1,\ \ell, \kappa = m \ = \frac{1}{2} erfc \ \frac{m \quad \omega \quad - \theta}{\sqrt{2 \quad m \quad \omega}} \tag{7}$$

where $= 2\mu/\omega^2$, is the average number of photons/pulse and μ the average number of photons/bit

The probability of error when taking into account the effect of the shot noise is

$$P_b K = \frac{1}{2} \sum_{\ell=0}^{K-1} \binom{K-1}{\ell} 2^{-K-1} \sum_{m=0}^{\ell} \binom{\ell}{m} \frac{\mu_s}{\omega^2}^{m} \ 1 - \frac{\mu_s}{\omega^2}^{\ell - m}$$

$$\frac{1}{2}\ erfc\ \frac{\theta - \quad m}{\sqrt{2 \quad m}} \quad erfc\ \frac{m \quad \omega \quad - \theta}{\sqrt{2 \quad m \quad \omega}} \tag{8}$$

where the optimum threshold is set through our software program, the available number of wavelengths is M = 31 and the system consists of $K = 15$ simultaneous users as shown in Fig.2.

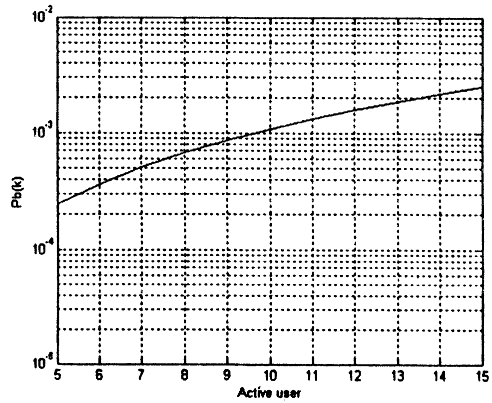

Fig.2. BER for shot noise effect on 2-D system. Codes have M=31 wavelengths, ω^2 code length =25 time chips

I . Anal sis of the beat noise effect on 2 D scheme

The bit error probability of ASPHC correlation receiver when taking into account the effect of the beat noise can be found in [3].

$$P_b = \frac{1}{2} \sum_{\ell=0}^{K-1} \binom{K-1}{\ell} 2^{-K-1} \sum_{m=0}^{\ell} \binom{\ell}{m} \frac{\mu_s}{\omega^2}^{m} \ 1 - \frac{\mu_s}{\omega^2}^{\ell - m}$$

$$\frac{1}{2}\ erfc\ \frac{\theta - m}{2\sqrt{\ ^2 \frac{m}{2}\frac{1}{\omega}}} \quad erfc\ \frac{m \quad \omega \quad - \theta}{2\sqrt{\ ^2\ m \quad ^2 \frac{m}{2}\frac{1}{\omega}}} \tag{9}$$

where we have set the Optimum threshold, the available number of wavelengths is M = 31 and the system consists of $K = 15$ simultaneous users as shown in Fig.3.

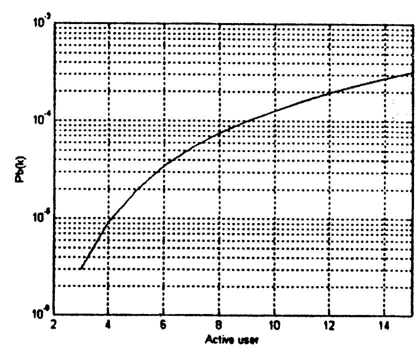

Fig.3. BER for the beat noise effect on 2-D system. Codes have M=31 wavelengths, ω^2 code length = 25 time chips

. Anal sis of the beat noise and the shot noise effect on 2 D scheme

The bit error probability of ASPHC correlation receiver when taking into account the effects of the beat noise shot noise can be found in Eq (13). We derive here the corresponding BER as:

$$P_b \; K = \frac{1}{2} \; P \; E|0 \quad P \; E|1$$
$$= \frac{1}{2} \sum_{\ell=0}^{K-1} \sum_{m=0}^{\ell} \; P E|0, \ell, \kappa = m \quad P E|1, \ell, \kappa = m \; P \; \ell, m$$

(10)

$$P \; E|0, \ell, \kappa = m = \frac{1}{2} erfc \frac{\theta - \quad m}{\sqrt{2 \quad m \; 4 \; ^2 \; \frac{m}{2} \; \frac{1}{\omega}}}$$

(11)

$$P \; E|1, \ell, \kappa = m =$$

$$\frac{1}{2} erfc \frac{m \; \omega \; - \theta}{\sqrt{2 \quad m \; \omega \quad 4 \; m \; ^2 \; 4 \; ^2 \; \frac{m}{2} \; \frac{1}{\omega}}}$$

(12)

The probability of error when taking into account the effects of the beat noise and the shot noise is:

$$P_b \; K = \frac{1}{2} \sum_{\ell=0}^{K-1} \text{K-1} \; \ell \; 2^{-K-1} \sum_{m=0}^{\ell} \; \ell \; m \; \frac{\mu_s}{\omega^2}^{m} \; 1 - \frac{\mu_s}{\omega^2}^{\ell-m} \quad 0.5$$

$$erfc \frac{\theta - \quad m}{\sqrt{2 \quad m \; 4 \; ^2 \; \frac{m}{2} \; \frac{1}{\omega}}}$$

$$erfc \frac{m \; \omega \; - \theta}{\sqrt{2 \quad m \; \omega \quad 4 \; m \; ^2 \; 4 \; ^2 \; \frac{m}{2} \; \frac{1}{\omega}}}$$

(13)

where we have set the Optimum threshold, the available number of wavelengths is M = 31 and the system consists of $K = 15$ simultaneous users as shown in Fig.4.

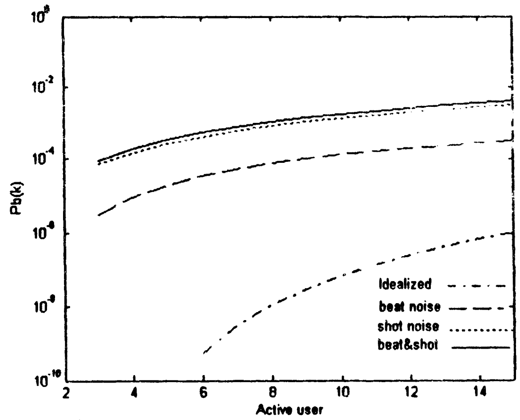

Fig.4. BER for the beat noise and the shot noise effects on 2-D system. Codes have M = 31 wavelengths, ω^2 code length =25 time chips

I. Conclusion

The effect of both the beat noise and shot noise on the system performance of 2-D Optical CDMA systems employing coherent laser sources has been investigated. Calculations have revealed that the system performance is seriously degraded in presence of one or both noises. However, the effect of shot noise is predominant.

II. References

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