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Maximum Achievable Number of Users in Optical PPM-CDMA

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Abstract — **An Optical code-division multiple-access communication network employing optical orthogonal codes is considered. The data symbols of each multiple-access user is encoded, before multiplexing, using pulse-position modulation technique with sufficiently large pulse position multiplicity. The concepts of both users rate and users exponent are introduced. Using these concepts the *best* number of simultaneous users that can be accommodated by the optical PPM-CDMA channel while maintaining the probability of error below some prescribed threshold $0 < \epsilon < 1$ is determined.**

I. Introduction

In optical code-division multiple-access (CDMA) systems N users transmit information simultaneously over a common optical channel. Each user is assigned a code (called the signature code) of length L and weight w. We focus on optical orthogonal codes (OOC's) with both off-peak auto-correlation and cross-correlation bounded by $\lambda \in \{1, 2, \ldots, w-1\}$. Methodologies in the design and analysis of such codes can be found in [1, 2]. An optical pulse (laser on) of duration T_c is transmitted whenever a 1 occurs in the signature code and a 0 is transmitted (laser off) for the same duration, otherwise.

Each user generates M-ary data symbols $D \in \{0, 1, \ldots, M-1\}$. These symbols are encoded with the aid of pulse-position modulation (PPM) schemes.

In our study of optical PPM-CDMA networks we assume equi-probable data symbols, i.e., $\Pr\{D = d\} = 1/M$, $d \in \{0, 1, \ldots, M-1\}$. The transmitted information in nats per channel use is thus equal to $\log M$. For the sake of convenience we shall denote the last pieces of information by m:

$$m \stackrel{\text{def}}{=} \log M \qquad \text{nats/channel use} .$$

Since both the maximum number of simultaneous users and the probability of error depend on M (or equivalently m), they may be denoted by N_m and $P_m[E]$, respectively.

Our aim in this paper is to determine the *best* number of users that can be accommodated simultaneously by the optical PPM-CDMA system while maintaining $P_m[E] < \epsilon$, any $0 < \epsilon < 1$. In our analysis, we let λ be any value in $\{1, 2 \ldots, w-1\}$ allowing a larger subscribers limit since in this case the number of possible codes is upper bounded by [1]

$$\frac{(L-1)(L-2)\cdots(L-\lambda)}{w(w-1)\cdots(w-\lambda)} .$$

Further, we focus on synchronous networks only and investigate two slightly different systems. In the first we neglect the Poisson shot noise of the photodetector and consider only the multiple-access interference. In the second system, however, we take into account the effect of both the Poisson shot noise and the multiple-users interference. The former system can be considered as the limit of the latter one with $\rho \to 0$, where ρ is the transmitted information per photon. Instead, for $\rho > 0$, it provides a lower bound to the error rate or an upper bound to the number of users of the second system.

Our results for the first system are complete in the sense that we develop both direct and converse proofs on the best users' size. For the second system, there is a tradeoff between the users' size and ρ. If ρ is less than some positive threshold, the users' exponent becomes independent of ρ and coincides with that of the first system. Of course within this region the converse part of system 1 is applicable as well for system 2. Outside this region, however, the users' exponent decreases as ρ increases. Only a direct result is demonstrated in this case. To avoid an enormous increase of the average interfering pulses per slot, we assume (in the converse part only) that $\lim_M \frac{N_m}{ML} = 0$. In the conclusion, however, we justify that this assumption does not lose the generality of the problem and can be removed without disturbing our results.

II. The Main Result

Theorem: The ϵ-users exponent, in a synchronous optical PPM-CDMA channel employing OOC's with weight $w > 1$, length $L \geq w^2$, and auto- and cross-correlation constraint $\lambda < w$, is given by

$$\theta(L, w, \lambda, \epsilon) \stackrel{\text{def}}{=} \lim_{\substack{m \to 0: \\ P_m[E] \leq \epsilon}} \frac{1}{m} \log \frac{N_m}{L} = 1 - \left\lceil \frac{w}{\lambda} \right\rceil^{-1} ,$$

where $\epsilon \in (0, 1)$.

Remark: This result holds for system 1 with any $\rho > 0$ and for system 2 with $\rho \leq \frac{\lambda \lceil \frac{w}{\lambda} \rceil}{\lambda \lceil \frac{w}{\lambda} \rceil - w + 1} (\log \frac{w}{w-1} - \frac{1}{w})$.

References

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[2] H. Chung and P. V. Kumar, "Optical orthogonal codes–new bounds and an optimal construction," *IEEE Trans. Inform. Theory*, vol. IT-36, pp. 866–873, July 1990.