Code for spectral amplitude coding optical CDMA systems

Xiang Zhou, H.H.M. Shalaby, Chao Lu and Teichiang Cheng

A new code structure for spectral amplitude coding optical code division multiple access (CDMA) is proposed and analysed. It is shown that such codes can effectively suppress the intensity noise and in turn increase the number of active users and improve the bit error rate performance.

Introduction: Optical CDMA systems employing the spectral amplitude coding of broadband sources have recently received much attention [1, 2]. Compared with some early optical CDMA systems based on coded sequences of incoherent pulse or coherent pulse, they both enable the limitations of unipolar codes to be avoided and do not entail the high complexity of coherent systems. Unfortunately, their performance is limited by the intensity noise mainly originating from the interference between incoherent sources [3]. In this Letter, we introduce a new code structure for spectral amplitude coding optical CDMA systems. Our theoretical analysis shows that such a code structure can effectively suppress intensity noise and hence improve the bit error rate (BER) performance.

Code design: In spectral amplitude coding optical CDMA systems, the frequency slots of different users will always be in use and multiuser interference can be completely cancelled out as long as the used codes (0,1) sequences satisfy the following conditions: (i) all the codewords have the same weight (defined as the number of 1s in it); (ii) for every two different codewords \( x_1, x_2, ..., x_N \) and \( y_1, y_2, ..., y_N \), we have \( \Theta = x_1 y_1 = y_1, where \( \lambda \) is a constant. Indeed any receiver that computes \( \Theta = x_1 y_1 \) will then reject the interference from any user having sequence \( y_i \), where \( \Theta = x_i y_i = y_i \). For convenience, we define \( N, w, \lambda \) code as a family of \( (0,1) \) sequences of length \( N \), weight \( w \) and \( \lambda \). In [2, 3], (0,1) m-sequences and Hadamard codes were proposed for spectral amplitude coding optical CDMA systems. They can be expressed as a \((N, (N + 1)/2, (N + 1)/4) \) code of size \( N \) and a \((N, N/2, N/4) \) code of size \( N - 1 \), respectively. We can see that the two codes have the same ratio of \( N \) to \( \lambda \), i.e., 2.

Based on the theorem of block designs [4], we can obtain a symmetric \((v, b, k, g) \) block design by using the points and hyperplanes of the projective geometry \( PG(m, q) \), where \( v = b = (g^v - 1)/(g - 1) \). It is immediately clear that such a block design just constructs a \((g^{v-1}-1, g^{v-1}-1, g^{v-1}-1, g^{v-1}-1) \) code of size \((g^{v-1}-1)/(g - 1) \). For such a code, we can see that the ratio of \( w \) to \( \lambda \) is tunable by choosing different values of \( q \) and \( m \) (by choosing \( q = 3, m = 4 \), we can construct a \((121, 40, 13) \) code; if choosing \( q = 4, m = 4 \), we can construct another code \((341, 85, 21) \)). In the following we show that, by taking a relatively large ratio of \( w \) to \( \lambda \), the intensity noise can be effectively suppressed.

Performance analysis: The ideal spectral amplitude coding optical CDMA system using an \((N, w, \lambda) \) code is shown in Fig. 1. The useful optical spectrum \( \Delta \nu \) is divided into \( N \) spectral spacings that are encoded according to the \( N \) elements of the used code. At the receiver, two masks that transmit the complementary frequency band and two photodetectors are used to perform the computation of \( \Theta = (\lambda w - \lambda) \Theta \). Assuming that: (i) the source spectra are ideally flat over a bandwidth of \( \nu_0 \); (ii) the power spectrum of each mask is identical (apart from the frequency translation); (iii) unpolarised signals are used; (iv) each user has equal power at the receiver. These are all best-case assumptions since we are interested in the upper bound on the system performance. Based on a similar method used in [3], we can obtain the following signal to noise ratio (SNR) limit due to intensity noise of

\[
\rho = \frac{\Delta \nu}{\Delta \nu + \frac{1}{\Delta \nu} + \frac{1}{\Delta \nu} + \frac{1}{\Delta \nu}}
\]

\[\text{where } q_1 = N/w, q_2 = w/\lambda, B = \text{the noise equivalent electrical bandwidth of the receiver}, K = \text{the number of active users} \text{ and } e = 1/4 \text{ for bit synchronism and } 1/6 \text{ ideally for bit asynchrony. For the } (g^{v-1}-1, g^{v-1}-1, g^{v-1}-1, g^{v-1}-1) \text{ code, if } m \text{ is large enough } (> 2), \text{ we have } q_1 = (g^{v-1}-1)/(g - 1) \text{ and } q_2 = (g^{v-1}-1)/(g - 1) \text{; we can then easily see that } \rho \text{ increases as } q \text{ increases. Thus the BER performance can be improved by choosing a relatively large value of } q. \text{ Such a result is clearly shown in Fig. 2, where we take } \Delta \nu = 2 \text{ and } B = 100 \text{MHz (signal bit rate is } 155 \text{Mb/s) and use a Gaussian noise approximation. It can be observed that, for a BER of } 10^{-9} \text{, while } q = 2, 4 \text{ and 6, the allowable maximum number of active users is 25, 44 and 55, respectively. Apparently, the result for } q = 2 \text{ is just the same as that for a conventional Hadamard code and m-sequence code.}

![Figure 1 - Ideal spectral amplitude coding optical CDMA system](image1)

![Figure 2 - Bit error rate against K for various q (intensity noise limit)](image2)

![Performance analysis: The ideal spectral amplitude coding optical CDMA system using an (N, w, λ) code is shown in Fig. 1. The useful optical spectrum Δν is divided into N spectral spacings that are encoded according to the N elements of the used code. At the receiver, two masks that transmit the complementary frequency band and two photodetectors are used to perform the computation of Θ = (λw - λ)Θ. Assuming that: (i) the source spectra are ideally flat over a bandwidth of ν0; (ii) the power spectrum of each mask is identical (apart from the frequency translation); (iii) unpolarised signals are used; (iv) each user has equal power at the receiver. These are all best-case assumptions since we are interested in the upper bound on the system performance. Based on a similar method used in [3], we can obtain the following signal to noise ratio (SNR) limit due to intensity noise of](image3)

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Improved performance analysis of spiral curve-phase precoding

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An efficient method for computing the bit error probability in spiral precoding for M-ary phase shift keying over an intersymbol interference channel is presented, consisting of a simplified single formula. The results show that this method is more effective than previously proposed methods.

Introduction: Precoding using the spiral curve technique has been devised as a pre-equalisation method for phase modulated signals in an intersymbol interference (ISI) channel for personal wireless communications [1]. An analysis of its performance, with a significant improvement for slow varying Rician and Rayleigh fading channels, was presented in [1] for quadrature phase shift keying (QPSK) modulation only and it has not been extended to higher levels of phase modulation. In this Letter, an M-PSK bit error rate (BER) computation method which considers the effect of both amplitude and phase noise is presented. This improved BER computation method can be used for a spiral curve-precoded signal in any M-ary PSK modulation scheme.

where \( r_i \) and \( \varphi_i \) are the \( i \)-th received signal amplitude and phase, respectively. Gaussian noise with amplitude \( n_i \) and phase \( \theta_i \) is represented by amplitude noise \( n_i \) and phase noise \( \theta_i \) on the right hand side of eqn. 1. These \( n_i \) and \( \theta_i \) are independent Gaussian noise components with zero mean, and variance \( \sigma^2 \) and \( \sigma^2(\sigma^2/4) \) at a given \( r_i \), respectively. Next, the real received signal in spiral curve precoding

\[
r_i = A \left( 1 + \frac{(\beta_i - \varphi_i)}{C} \right)
\]

[1] (where \( C \) is the spiral curve constant, and \( A \) and \( \beta_i \) are the amplitude and phase of the original signal, respectively) is substituted with the concerned amplitude and phase noise. As a result, the detected phase \( \tilde{\beta}_i \) is obtained with an error by

\[
\Delta \tilde{\beta}_i = \tilde{\beta}_i - \beta_i \text{ (modulo } 2\pi) \quad (2)
\]

The solution of eqn. 2 shows the relationship of Gaussian noise with a spiral curve-precoded signal. Hence, the spiral curve-Gaussian random variable is as follows:

\[
n_g = n_{gA} + n_{g\varphi}(C\pi)/A \quad (3)
\]

It has a zero mean and variance \( \sigma_g^2 = ((1/\pi^2) + (C\pi/A)^2)\sigma^2 \), and its probability density function (PDF) is

\[
p(s_g) = \frac{1}{\sigma_g\sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( s_g/\sigma_g \right)^2 \right] \quad (4)
\]

Note that signal amplitude \( A \) and received amplitude \( r_i \) for phase noise variance is simply normalised as one. At high SNRs, the effect of phase noise can be neglected; nevertheless, it still plays an important role if \( C \) is small.

**Fig. 1** Performance of QPSK-spiral curve precoding

- - - - - - - - AWGN (reference)
- - - - - - - - C = 1, simulation
- - - - - - - - C = 1, analysis
- - - - - - - - C = 5, simulation
- - - - - - - - C = 5, analysis
- - - - - - - - C = 10, simulation
- - - - - - - - C = 10, analysis
- - - - - - - - C = 10, new analysis
- - - - - - - - C = 10, Zhuang's analysis

Precoding analysis: First, consider an ISI-free signal perturbed by Gaussian noise:

\[
r_i e^{j\theta_i} = (r_i + n_{r_i})e^{j(\theta_i + n_{\theta_i})} \quad (1)
\]

BER computation and simulation: For an AWGN channel, an accurate method for computing the BER under M-PSK modulation has been developed based on the signal-space concept [2]. This method can be used to compute the error probability for any M-ary signal by the summation of Q-functions with knowledge of the noise spectral density. Consequently, the implication is that the modification is required for computing the BER for spiral curve precoding if the spectral noise density \( N_{\text{spiral}} \) (from the effect of Gaussian noise on the precoded signal) with bit energy \( E_b \) is determined under ISI-free conditions. As a result, the equation for the BER probability in [2] can be modified in the case of precoding with previous knowledge of the spiral curve-Gaussian noise as follows:

\[
P_e = \frac{2}{\log_2 M} \sum_{i=1}^{M/4} Q \left( \sqrt{2E_b \log_2 M} N_{\text{spiral}} \min \left( \frac{2i-1}{M} \pi \right) \right) \quad (5)
\]

where \( Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} f(t)dt \) and \( f(t) = \exp(-t^2/2)/\sqrt{2\pi} \).