

# Maximum Achievable Constrained Power Efficiencies of MPPM- $L$ QAM Techniques

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**Abstract**—The maximum achievable power efficiency of a multipulse pulse-position modulation- $L$ -ary quadrature-amplitude modulation (MPPM- $L$ QAM) technique under a constraint on its spectral efficiency is obtained. Variations to this technique that include polarization multiplexing and switching are considered as well. A simple characterization to the maximum achievable power efficiency is derived under the above constraint. Our results are compared with that of traditional QAM techniques under same constraints. The results reveal that at spectral efficiency constraints of 1.5, 2.5, and 3.5 bit/sym/pol, the power efficiency of MPPM- $L$ QAM technique is better than that of traditional QAM technique by  $\sim 3.9$ , 2.2, and 1.4 dB, respectively.

**Index Terms**—Multipulse pulse-position modulation, polarization multiplexing, polarization switching, power efficiency, quadrature-amplitude modulation, spectral efficiency.

## I. INTRODUCTION

TWO main design goals in modern optical communications systems are spectral and power efficiencies. Increasing the spectral efficiency corresponds to increasing the data transmission rate at a given bandwidth, while increasing the power efficiency corresponds to asymptotically reducing the bit-error rate (or improving the receiver sensitivity) [1].

Spectral efficiencies can be increased by adopting multilevel signaling and/or polarization-division multiplexing (PDM) techniques [2]. However, the price paid here is the reduction of power efficiencies (or receiver sensitivities). Coherent light-wave systems with optically pre-amplified receivers are often used for high spectral-efficient optical transmission [2], [3]. Specifically, an optical binary differential phase-shift keying (DPSK) technique has been demonstrated in [4]. Aided with Erbium-doped fiber amplifiers (EDFAs), a 100 Gb/s coherent non-return-to-zero (NRZ) polarization-division-multiplexing quadrature phase-shift keying (PDM-QPSK) signal has been transmitted over  $16 \times 100$  km of standard single mode fiber [5].

Recently, hybrid modulation formats have been proposed to increase the power efficiency of multilevel signals in optical links, e.g., polarization-switched QPSK (PS-QPSK) and PDM-QPSK superimposed on pulse-position-modulation (PPM) signals have been shown to provide higher power efficiency than PDM-QPSK at the expense of reduced spectral efficiency [6]–[8]. By combining multi-pulse pulse-position modulation (MPPM) with BPSK, QPSK,

PS-QPSK, or polarization-multiplexed QPSK (PM-QPSK) one can increase both the spectral and power efficiencies over pure traditional QPSK [9]–[12]. The maximum achievable spectral efficiency using these techniques has been shown to be 2.32 bit/sym/pol [12].

In order to increase the spectral efficiency above 2.32 bit/sym/pol, one has to use higher level  $L$ -ary quadrature-amplitude modulation ( $L$ QAM) signals rather than QPSK signals. In this letter we aim at characterizing the best power efficiency that can be achieved under a constraint on the spectral efficiency of a hybrid MPPM- $L$ QAM technique. We compare our results to that of traditional QAM technique under same spectral efficiency constraint. In addition, variations to the above system are studied as well, namely, polarization-multiplexed MPPM- $L$ QAM (PM-MPPM- $L$ QAM), MPPM-PM- $L$ QAM, and MPPM-PS- $L$ QAM techniques. The importance of these techniques is that they can be used in multimode or multicore fibers, where a mode or a core replaces a time slot of an MPPM frame to obtain what is called mode-selection or core-selection modulation [12], [13]. The simplicity of the obtained power efficiency expressions gives more insight about the sensitivity gains of the systems studied. However, the obtained results in this letter are only valid for asymptotically high signal-to-noise ratios (SNRs) or small bit-error rates (BERs). The BER of an MPPM- $L$ QAM system over a wide range of transmitted power has been analyzed in [14].

The remaining of this letter is organized as follows. Preliminaries and problem statement are presented in Section II. Both spectral and power efficiencies are derived in Section III. Section IV is devoted for the problem formulation and solution of an optimization problem of the best achievable power efficiency under a constraint on the spectral efficiency of the MPPM- $L$ QAM technique. Variations to this technique with polarization multiplexing and switching are considered in this section as well. In Section V, we present some numerical results, where we give comparisons between maximum power efficiencies of the above hybrid techniques and traditional QAM technique under various spectral efficiency constraints. Finally, we give our conclusions in Section VI.

## II. PRELIMINARIES AND PROBLEM STATEMENT

In this section we cite the definitions of both power and spectral efficiencies. We also introduce the problem statement aimed to be characterized in the letter.

### A. Spectral Efficiency

The spectral efficiency  $\eta$  is defined as [1], [6]:

$$\eta \stackrel{\text{def}}{=} \frac{\log_2 M}{N/2} \quad \text{bit/sym/pol}, \quad (1)$$

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where  $M$  is the number of constellation vectors (or symbols) and  $N$  is the dimension of a constellation vector.

### B. Asymptotic Power Efficiency

The asymptotic power efficiency  $\gamma$  is defined as [1], [6]:

$$\gamma \stackrel{\text{def}}{=} \frac{d_{\min}^2 \log_2 M}{4\mathcal{E}_s}, \quad (2)$$

where  $\mathcal{E}_s$  is the average symbol energy and  $d_{\min}$  is the minimum Euclidean distance between two symbols in the constellation space, given by:

$$d_{\min} \stackrel{\text{def}}{=} \min_{\substack{\mathbf{x}, \mathbf{z} \in \mathcal{S}(M): \\ \mathbf{x} \neq \mathbf{z}}} \|\mathbf{x} - \mathbf{z}\|, \quad (3)$$

where  $\|\mathbf{x}\|^2 \stackrel{\text{def}}{=} \sum_{i=1}^N |x_i|^2$  and  $\mathcal{S}(M) \stackrel{\text{def}}{=} \{\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_M\}$  is the set of all possible vectors (of dimension  $N$ ) in the constellation space. Here  $\mathbf{c}_k = (c_{k1}, c_{k2}, \dots, c_{kN})$ ,  $k \in \{1, 2, \dots, M\}$ , is an  $N$ -dimensional vector with  $c_{ki} \in \mathbb{Z}$  for any  $i \in \{1, 2, \dots, N\}$ .  $\mathbb{Z}$  denotes the set of integers.

### C. Problem Statement

In this letter we aim at finding the maximum power efficiency of an MPPM-LQAM technique under a constraint on its spectral efficiency. That is, we maximize the power efficiency subject to the constraint that the spectral efficiency is not less than a certain threshold  $S_e$ .

## III. EFFICIENCIES OF MPPM-LQAM TECHNIQUE

In this section we evaluate both the spectral and power efficiencies of an MPPM-LQAM technique.

### A. MPPM Constellation

For any integer  $K \geq 1$  and any  $n \in \{1, 2, \dots, K\}$ , an MPPM symbol is represented by a vector  $\mathbf{d} = (d_1, d_2, \dots, d_K)$  selected from the set:

$$\mathcal{S}_{MPPM} \left( \binom{K}{n} \right) \stackrel{\text{def}}{=} \left\{ \mathbf{d} \in \{0, 1\}^K : \sum_{i=1}^K d_i = n \right\}. \quad (4)$$

It is clear that the cardinality of this set is  $\binom{K}{n}$ .

### B. LQAM Constellation

For any positive integer  $L$  satisfying  $L = 2^\ell$  with  $\ell \in \{1, 2, \dots\}$ , a symbol  $\mathbf{a} = (a_1, a_2)$  on an arbitrary rectangular LQAM constellation is selected from the

set [15]:

$$\mathcal{S}_{QAM}(L) \stackrel{\text{def}}{=} \begin{cases} \left\{ -[2^{\ell/2} - 1], \dots, -3, -1, \right. \\ \left. 1, 3, \dots, [2^{\ell/2} - 1] \right\}^2; & \text{if } \ell \text{ is even,} \\ \left\{ -[2^{(\ell+1)/2} - 1], \dots, -3, -1, \right. \\ \left. 1, 3, \dots, [2^{(\ell+1)/2} - 1] \right\} \\ \times \left\{ -[2^{(\ell-1)/2} - 1], \dots, -3, -1, \right. \\ \left. 1, 3, \dots, [2^{(\ell-1)/2} - 1] \right\}; & \text{if } \ell \text{ is odd and } \ell \neq 1, \\ \left\{ (-1, -1), (1, 1) \right\}; & \text{if } \ell = 1. \end{cases} \quad (5)$$

### C. MPPM-LQAM Constellation

Combining the above, an MPPM-LQAM symbol  $\mathbf{c} = (c_1, c_2, \dots, c_{2K})$  is selected from the set given in Eq. (6), as shown at the bottom of this page. The cardinality of this set is  $M = L^n \binom{K}{n}$  and the symbol dimension is  $N = 2K$ .

### D. Spectral and Power Efficiencies

It is easy to evaluate spectral efficiency for the MPPM-LQAM system using (1):

$$\eta = \frac{n \log_2 L + \log_2 \binom{K}{n}}{K} \quad \text{bit/sym/pol.} \quad (7)$$

The corresponding average symbol energy  $\mathcal{E}_s$  is given by:

$$\mathcal{E}_s = E \left\{ \|\mathbf{c}\|^2 \right\} = nE \left\{ \|\mathbf{a}\|^2 \right\}, \quad (8)$$

where  $E\{X\}$  denotes the expected value of  $X$  and  $\mathbf{a} \in \mathcal{S}_{QAM}(L)$ . From [15] and [16], we get that:

$$\mathcal{E}_s = nE \left\{ \|\mathbf{a}\|^2 \right\} = \frac{2}{3} n (\kappa_\ell L - 1), \quad (9)$$

where

$$\kappa_\ell \stackrel{\text{def}}{=} \begin{cases} 1; & \text{if } \ell \text{ is even,} \\ \frac{5}{4}; & \text{if } \ell \text{ is odd and } \ell \neq 1, \\ 2; & \text{if } \ell = 1, \end{cases} \quad (10)$$

with  $\ell = \log_2 L$  as mentioned earlier. Using (2) and noticing that  $d_{\min} = 2$ , the corresponding power efficiency is given by:

$$\gamma = \frac{3}{2} \cdot \frac{n \log_2 L + \log_2 \binom{K}{n}}{n(\kappa_\ell L - 1)}. \quad (11)$$

### E. Observations and Motivations

To show the efficiency advantages of the MPPM-LQAM technique, we plot in Fig. 1 both the power and spectral efficiencies of this technique versus the ratio  $n/K$ . In our plots we have arbitrary fixed  $K = 32$  and used different values of QAM levels  $L$ . It can be seen from the figure that the spectral efficiency follows a concave function with the ratio  $n/K$  and increases with the increase of QAM level  $L$ . On the

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$$\mathcal{S}(M) \stackrel{\text{def}}{=} \left\{ \mathbf{c} \in \mathbb{Z}^{2K} : \forall (d_1, d_2, \dots, d_K) \in \mathcal{S}_{MPPM} \text{ and } \forall i \in \{1, 2, \dots, K\} (c_{2i-1}, c_{2i}) \in \begin{cases} \mathcal{S}_{QAM}(L); & \text{if } d_i = 1, \\ \{(0, 0)\}; & \text{if } d_i = 0. \end{cases} \right\}, \quad (6)$$

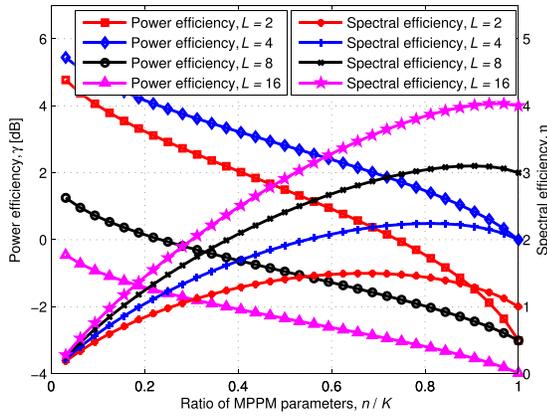


Fig. 1. Power and spectral efficiencies for MPPM-LQAM technique, with  $K = 32$  and different values of  $L$ .

other hand, the figure shows that the power efficiency is a decreasing function with both the ratio  $n/K$  and the QAM level  $L$  (for  $L \geq 4$ ). It is interesting to notice that the spectral efficiency can easily reach values well above 2.32 bit/sym/pol by proper choices of  $L$  and ratio  $n/K$ . This motivates us to determine the best we can get from MPPM-LQAM technique under a constraint on the spectral efficiency.

#### IV. PROBLEM FORMULATION AND CHARACTERIZATION OF A CONSTRAINED POWER EFFICIENCY

##### A. Optimization Problem and Constrained Power Efficiency

As mentioned earlier, our aim in this letter is to characterize the maximum power efficiency of MPPM-LQAM technique under a constraint on its spectral efficiency. We notice from the last section that both  $\eta$  and  $\gamma$  are functions of  $K$ ,  $n$ , and  $L$ . In view of this, we restate the main problem in Subsection II-C as:

$$\theta(S_e) \stackrel{\text{def}}{=} \max_{\substack{(K,n,L) \in \mathcal{Y}: \\ \eta \geq S_e}} \gamma(K, n, L), \quad (12)$$

where the set  $\mathcal{Y}$  is defined as:

$$\mathcal{Y} \stackrel{\text{def}}{=} \{(K, n, L) \in \mathbb{N}^3 : n \leq K, \log_2 L \in \mathbb{N}\}. \quad (13)$$

Here  $\mathbb{N}$  denotes the set of positive integers (natural numbers). We call the last maximum  $\theta(S_e)$  as the *constrained power efficiency*.

##### B. Characterization of the Constrained Power Efficiency

*Theorem 1: In MPPM-LQAM technique, if the spectral efficiency threshold is  $S_e$ , then the constrained power efficiency as defined in (12) can be characterized as:*

$$\theta_{\text{MPPM-LQAM}}(S_e) = \frac{3}{2(\kappa_{\ell_0} 2^{\ell_0} - 1)} \left[ \ell_0 + \frac{h(p_0)}{p_0} \right], \quad (14)$$

where  $\ell_0 = \max\{\lceil \log_2(2^{S_e} - 1) \rceil, 2\}$ ,  $p_0 \in [0, 1]$  is the smaller root of the equation  $p_0 \ell_0 + h(p_0) = S_e$ , and  $h(q) \stackrel{\text{def}}{=}} -q \log_2 q - (1-q) \log_2(1-q)$  is the binary entropy function. Here  $\lceil x \rceil$  denotes the minimum integer not less than  $x$ .

*Proof:* Using the method of types [17], we can write:

$$2^{Kh(\frac{n}{K})} \geq \binom{K}{n} \geq \frac{1}{K+1} 2^{Kh(\frac{n}{K})}. \quad (15)$$

Noticing that  $\ell = \log_2 L \in \mathbb{N}$  and substituting in (7) and (11), we get:

$$p\ell + h(p) \geq \eta \geq p\ell + h(p) - \frac{\log_2(K+1)}{K} \quad (16)$$

and

$$\begin{aligned} \frac{3}{2(\kappa_{\ell} 2^{\ell} - 1)} \left[ \ell + \frac{h(p)}{p} \right] &\geq \gamma \\ &\geq \frac{3}{2(\kappa_{\ell} 2^{\ell} - 1)} \left[ \ell + \frac{h(p)}{p} - \frac{\log_2(K+1)}{Kp} \right], \end{aligned} \quad (17)$$

respectively, where  $p \stackrel{\text{def}}{=} n/K$ . It is remarkable to notice that the last two bounds are tight for large values of  $K$  and it is easy to check that:

$$\theta_{\text{MPPM-LQAM}}(S_e) = \max_{\substack{p \in (0,1), \ell \in \mathbb{N}: \\ p\ell + h(p) \geq S_e}} \frac{3}{2(\kappa_{\ell} 2^{\ell} - 1)} \left[ \ell + \frac{h(p)}{p} \right]. \quad (18)$$

The constraint function is concave with maximum of  $p\ell + h(p) \leq \log_2(1+2^{\ell})$ , achieved at  $p = 2^{\ell}/(1+2^{\ell})$ . This means that  $\ell \geq \log_2(2^{S_e} - 1)$ . Since the power efficiency (the objective function) decreases with  $\ell$  (for  $\ell > 2$ ),  $\theta_{\text{MPPM-LQAM}}(S_e)$  is maximized at the smallest value of  $\ell$  that satisfies the constraint, i.e., at  $\ell = \max\{\lceil \log_2(2^{S_e} - 1) \rceil, 2\}$ . Finally, from the concavity of the constraint function with  $p$  and the decrease of the objective function with  $p$ , it follows that the maximum is achieved at the smaller root (of two roots) of  $p\ell + h(p) = S_e$ . ■

##### C. Other Related Cases

1) *LQAM Constrained Power Efficiency:* This is immediate by setting  $p = 1$  in (18):

$$\theta_{\text{LQAM}}(S_e) = \frac{3}{2} \cdot \frac{\lceil S_e \rceil}{\kappa_{\lceil S_e \rceil} \cdot 2^{\lceil S_e \rceil} - 1}. \quad (19)$$

It is clear that  $\theta_{\text{LQAM}}(S_e) \leq 1$ .

2) *PM-MPPM-LQAM Constrained Power Efficiency:* The same constrained power efficiency as given in Theorem 1 can be achieved for the case of polarization-multiplexed MPPM-LQAM. In this case  $M = \binom{K}{n}^2$  and  $N = 4K$ .

3) *MPPM-PM-LQAM Constrained Power Efficiency:* The constrained power efficiency for the case of polarization-multiplexed LQAM is evaluated as follows.  $M = L^{2n} \binom{K}{n}$  and  $N = 4K$ . Following a similar argument as given above, we get:

$$\theta_{\text{MPPM-PM-LQAM}}(S_e) = \frac{3}{2(\kappa_{\ell_1} 2^{\ell_1} - 1)} \left[ \ell_1 + \frac{h(p_1)}{2p_1} \right], \quad (20)$$

where  $\ell_1 = \max\{\lceil 0.5 \log_2(2^{2S_e} - 1) \rceil, 2\}$ ,  $p_1$  is the smaller root of the equation  $p_1 \ell_1 + h(p_1)/2 = S_e$ .

4) *MPPM-PS-LQAM Constrained Power Efficiency:* The constrained power efficiency for the case of polarization-switched LQAM is evaluated as follows.  $M = (2L)^n \binom{K}{n}$  and  $N = 4K$ . Following a similar argument as given above, we get:

$$\theta_{\text{MPPM-PS-LQAM}}(S_e) = \frac{3}{2(\kappa_{\ell_2} 2^{\ell_2} - 1)} \left[ \ell_2 + 1 + \frac{h(p_2)}{p_2} \right], \quad (21)$$

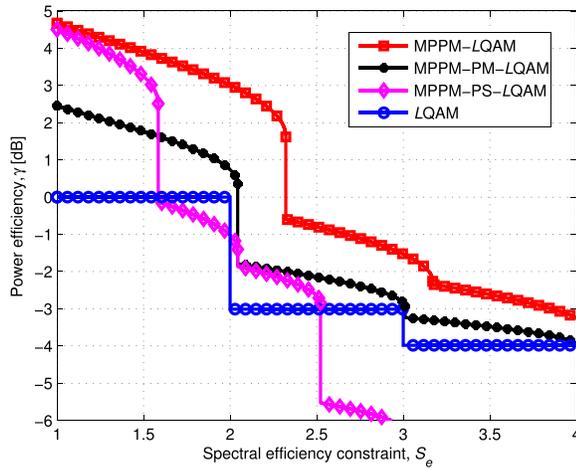


Fig. 2. Comparison between maximum power efficiencies of MPPM-LQAM, its variants, and traditional LQAM techniques versus spectral efficiency constraints.

where  $\ell_2 = \max \{ \lceil \log_2 (2^{2S_e} - 1) \rceil - 1, 2 \}$ ,  $p_2$  is the smaller root of the equation  $p_2(\ell_2 + 1) + h(p_2) = 2S_e$ .

## V. NUMERICAL RESULTS

In this section we give a comparison between maximum achievable power efficiencies of the MPPM-LQAM, its variants, and traditional LQAM techniques for various spectral efficiency constraints. Figure 2 shows such a comparison for the constrained power efficiencies of these techniques versus spectral efficiency constraint. The sharp jumps in the curves indicate increases in the QAM levels in order to satisfy the constraints on the spectral efficiencies.

It is clear from the figure that the most power efficient technique is MPPM-LQAM (or equivalently, PM-MPPM-LQAM). Specifically, at spectral efficiency constraints of 1.5, 2.5, and 3.5 bit/sym/pol, its constrained power efficiency is better than that of traditional QAM technique by about 3.9, 2.2, and 1.4 dB, respectively. That is, this technique can achieve spectral efficiencies higher than 2.32 bit/sym/pol (of traditional MPPM-QPSK) and still give good power efficiency gains above traditional QAM.

As expected, the constrained power efficiency of MPPM-PM-LQAM technique is slightly better than that of traditional QAM, but worse than that of MPPM-LQAM. Specifically, at spectral efficiency constraints of 1.5, 2.5, and 3.5 bit/sym/pol, the constrained power efficiency of MPPM-PM-LQAM technique is better than that of traditional QAM technique by about 1.76, 0.85, and 0.52 dB, respectively.

Interestingly, the MPPM-PS-LQAM technique is no longer power efficient for  $\ell > 2$  and it is even worse than traditional QAM (except for a couple of QAM levels). That is, MPPM-PS-LQAM technique is only suitable for spectral efficiencies not exceeding 1.585 bit/sym/pol. The reason is that when the spectral efficiency threshold  $S_e$  increases, the corresponding QAM level is achieved at high values of  $L$  and the power efficiency would drop significantly.

## VI. CONCLUSIONS

The maximum achievable power efficiency of a multipulse pulse-position modulation- $L$ -ary quadrature-amplitude modulation (MPPM-LQAM) technique under a constraint on

its spectral efficiency has been obtained and characterized. Variations to this technique that include polarization multiplexing and switching have been considered as well. Numerical comparisons with traditional QAM techniques have been worked out under same spectral efficiency constraints. Our results reveal that the most power efficient technique is MPPM-LQAM (or equivalently, PM-MPPM-LQAM). For example at spectral efficiencies of 1.5, 2.5, and 3.5 bit/sym/pol, the power efficiency of MPPM-LQAM technique is better than that of traditional QAM technique by about 3.9, 2.2, and 1.4 dB, respectively.

## REFERENCES

- [1] S. Benedetto and E. Biglieri, *Principles of Digital Transmission: With Wireless Applications*. New York, NY, USA: Kluwer, 1999.
- [2] K.-P. Ho, *Phase-Modulated Optical Communication Systems*. New York, NY, USA: Springer-Verlag, 2005.
- [3] P. J. Winzer, "High-spectral-efficiency optical modulation formats," *J. Lightw. Technol.*, vol. 30, no. 24, pp. 3824–3835, Dec. 15, 2012.
- [4] N. W. Spellmeyer, J. C. Gottschalk, D. O. Caplan, and M. L. Stevens, "High-sensitivity 40-Gb/s RZ-DPSK with forward error correction," *IEEE Photon. Technol. Lett.*, vol. 16, no. 6, pp. 1579–1581, Jun. 2004.
- [5] J. Renaudier *et al.*, "Transmission of 100Gb/s coherent PDM-QPSK over  $16 \times 100$ km of standard fiber with all-erbium amplifiers," *Opt. Exp.*, vol. 17, no. 7, pp. 5112–5119, Mar. 2009.
- [6] E. Agrell and M. Karlsson, "Power-efficient modulation formats in coherent transmission systems," *J. Lightw. Technol.*, vol. 27, no. 22, pp. 5115–5126, Nov. 15, 2009.
- [7] M. Karlsson and E. Agrell, "Generalized pulse-position modulation for optical power-efficient communication," in *Proc. 37th Eur. Conf. Opt. Commun. (ECOC)*, Geneva, Switzerland, Sep. 2011, pp. 1–3, paper Tu.6.B.6.
- [8] X. Liu, T. H. Wood, R. W. Tkach, and S. Chandrasekhar, "Demonstration of record sensitivities in optically preamplified receivers by combining PDM-QPSK and M-ary pulse-position modulation," *J. Lightw. Technol.*, vol. 30, no. 4, pp. 406–413, Feb. 15, 2012.
- [9] H. Selmy, H. M. H. Shalaby, and Z.-I. Kawasaki, "Proposal and performance evaluation of a hybrid BPSK-modified MPPM technique for optical fiber communications systems," *J. Lightw. Technol.*, vol. 31, no. 22, pp. 3535–3545, Nov. 15, 2013.
- [10] H. Selmy, H. M. H. Shalaby, and Z. Kawasaki, "Enhancing optical multipulse pulse position modulation using hybrid QPSK-modified MPPM," in *Proc. IEEE Photon. Conf. (IPC)*, San Diego, CA, USA, Oct. 2014, pp. 617–618.
- [11] J. H. B. Nijhof, "Generalized L-out-of-K pulse position modulation for improved power efficiency and spectral efficiency," in *Proc. Opt. Fiber Commun. Conf. (OFC)*, Los Angeles, CA, USA, Mar. 2012, pp. 1–3, paper OW3H.7.
- [12] T. A. Eriksson, P. Johannisson, B. J. Puttnam, E. Agrell, P. A. Andrekson, and M. Karlsson, "K-over-L multidimensional position modulation," *J. Lightw. Technol.*, vol. 32, no. 12, pp. 2254–2262, Jun. 15, 2014.
- [13] P. J. Winzer, "Communication via a multimode constellation," U.S. Patent 8817895 B2, Aug. 26, 2014.
- [14] H. S. Khallaf and H. M. H. Shalaby, "Proposal of a hybrid QAM-MPPM technique for optical communications systems," in *Proc. 16th Int. Conf. Transparent Optic. Netw. (ICTON)*, Graz, Austria, Jul. 2014, pp. 1–4, paper Tu.B1.7.
- [15] K. Cho, D. Yoon, W. Jeong, and M. Kavehrad, "BER analysis of arbitrary rectangular QAM," in *Proc. 35th Asilomar Conf. Signals, Syst. Comput.*, Pacific Grove, CA, USA, Nov. 2001, pp. 1056–1059.
- [16] S. Haykin, *Communication Systems*, 4th ed. Hoboken, NJ, USA: Wiley, 2001.
- [17] I. Csiszár and J. Körner, *Information Theory: Coding Theorems for Discrete Memoryless Systems*. New York, NY, USA: Academic, 1982.