A Simplified Performance Analysis of Optical Burst-Switched Networks

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***Abstract*—A simplified mathematical model for evaluating the performance of optical burst switching networks is proposed. This model is described using a detailed state diagram. Two performance measures, namely, steady-state system throughput and average blocking probability, are derived based on the equilibrium point analysis technique. The effects of several design parameters on the above performance measures have been examined with the aid of a set of numerical examples.**

***Index Terms*—Bursty traffic, equilibrium point analysis (EPA), just-in-time (JIT) protocol, optical burst switching (OBS), optical networks, wavelength conversion.**

I. Introduction

OPTICAL BURST switching (OBS) is a new wavelength division multiplexing technology that retains some of the advantages of optical packet switching, yet is being more practical and realizable in the near future. This technology was first proposed by Qiao and Yoo [1], and then, many authors have studied it thoroughly, e.g., in [2]–[12]. The performance of OBS has appeared in literature by several authors, e.g., in [3], [7], [10], and [11]. This paper introduces a new simplified analytical model that can easily measure the performance of the OBS network and confer the alternatives to various design constraints.

A. System Architecture

The basic architecture of an OBS network is composed of a set of N interconnected nodes and a set of available wavelengths of cardinality w. An ingress node assembles the Internet protocol packets, that are coming from the local access networks and destined to the same egress node, into large bursts. A core node is composed of an optical cross-connect (OXC) fabric and a set of wavelength converters of cardinality $u \in \{0, 1, \ldots, w\}$. Each ingress node sends a control packet before the transmission of the optical burst starts. This control packet contains an information about the sender, receiver, and transmission wavelength of the corresponding burst. Its main function is to configure all the core nodes along the path to destination so that the burst travels smoothly in the optical domain without the need to be converted into the electrical domain.

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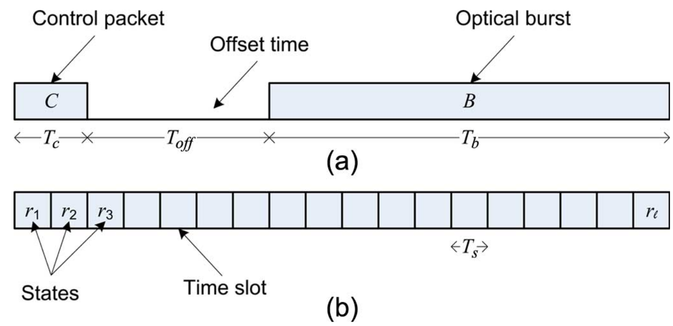

Fig. 1. (a) Transmission of an optical burst. (b) Slotted timing diagram.

B. OBS Protocol

In this paper, we focus on a one-way protocol, which is a modification of the just-in-time (JIT) protocol, for our OBS network [4], [5]. In this protocol, a wavelength is reserved (in a core node) for a burst immediately after the arrival of the corresponding control packet; if a wavelength cannot be reserved at that time, then the control packet is rejected and the corresponding burst is said to be blocked and dropped. Fig. 1(a) illustrates the operation of JIT-OBS protocol. Let a control packet C arrives at some OBS core node along the path to the destination. Once the processing of C is complete, a wavelength is immediately reserved for the upcoming burst, and the operation to configure the OXC fabric to switch the burst is initiated. It should be noticed that the optical burst arrives at the OBS node under consideration after an offset time T_{off} from the arrival of the control packet [Fig. 1(a)], which takes care of the processing and configuration times of the control packet and OXC fabric, respectively. Thus, the total time T spent from the transmission of the control packet until the end of the optical burst is

$$T = T_c + T_{\text{off}} + T_b$$

where T_c and T_b are the control packet and optical burst time durations, respectively.

C. Aim of the Paper

Obviously, if two control packets are to reserve the same wavelength at a given core node for two different bursts, then only one burst will be offered to this wavelength. The other will be blocked and lost (unless there is an available wavelength converter). Wavelength conversion can be used at some core nodes to relax the above contention problem and reduce the loss probability. Unfortunately, optical wavelength converters are too expensive and make the optical burst switches less competitive than electronic routers [8]. A simple question may be raised: Can we simply increase the number of available wavelengths rather than installing more wavelength converters and get the same performance?

In this paper, we aim at answering the above question and exploring the tradeoff between installing more wavelength converters and increasing the number of available wavelengths. To achieve our aims, we first describe a simplified state diagram of an OBS protocol. When trying to examine the network performance, we face a prohibitively large number of states, which makes the problem analytically intractable. Fortunately, the equilibrium point analysis (EPA) technique significantly simplifies the problem and makes it more tractable. This technique has been used successfully in studying the optical code division multiple access protocols [13] and motivates us to apply it here. In this technique, the system is always assumed to be operating at an equilibrium point. That is, at any time slot, the expected number of users entering any state is always equal to that departing from the state.

D. Paper Organization

The remainder of this paper is organized as follows. In Section II, we introduce a mathematical model and a basic description of the state diagram of the investigated protocol. Section III is devoted for a theoretical study for the performance of an OBS network with no wavelength conversions, where derivations of both the steady-state system throughput and average blocking probability are given. Considerations of the OBS networks with wavelength conversion capabilities are studied in Section IV. Section V is maintained for a numerical study of the derived performance measures by taking into account the effect of changing several design parameters. Finally, we give our conclusion in Section VI.

II. Mathematical Model

Our mathematical model depends on the construction of a state diagram that describes the status of an OBS node. We start by some definitions and preliminaries.

A. Slotted Timing Model

In our analysis, we use a slotted timing model [Fig. 1(b)] in which we divide the entire period T into small time slots, each of duration T_s, called slot time. The total number of slots ℓ is calculated as

$$\ell = \frac{T}{T_s}$$

where we assume, without loss of generality, that ℓ is an integer. In addition, we can assume that T_s is a multiple of the bit duration and will be held fixed throughout this paper. It should be emphasized that during a time slot T_s, the node would get enough information about the selected wavelength.

B. Time Slots and Burst Arrivals

We assume that the optical bursts (or control packets) arrive to any OBS node with rate R_b bursts/s. In addition, the arrival process follows a Poisson distribution. Thus, the probability that n bursts arrive to an OBS node during time slot $i \in \{1, 2, \dots, \ell\}$ is given by

$$P_b(n) = e^{-R_b T_s} \frac{(R_b T_s)^n}{n!}, \qquad n \in \{0, 1, \dots\}.$$

Furthermore, we assume that the time slot duration T_s is small enough so that $P_b(n) \simeq 0$ for every $n \geq 2$. Thus, the last equation can be simplified to

$$P_b(n) = \begin{cases} e^{-R_b T_s} \simeq 1 - A, & \text{if } n = 0 \\ e^{-R_b T_s} R_b T_s \simeq A, & \text{if } n = 1 \\ e^{-R_b T_s} \frac{(R_b T_s)^n}{n!} \simeq 0, & \text{else} \end{cases}$$

where A denotes the probability of a burst arrival within a slot time, also called the user activity. It should be noted that, under fixed bit rate and transmission bandwidth, the probability of a burst arrival A decreases as we increase the burst length ℓ so that their product is fixed. This product $A\ell$, which is a measure to the data traffic, will be called the network traffic κ:

$$\kappa \stackrel{\text{def}}{=} A\ell \quad \text{bursts/burst time.}$$

C. Initial State and Transition Probabilities

We assume that initially, an OBS node is in state m, called the initial state (Fig. 2). If there is an arrival (with probability A), the OBS node will enter the following state r_1 and starts processing the control packet (cf. Fig. 2). On the other hand, if there is no arrivals (an event that occurs with probability $1 - A$), the OBS node will remain as is.

D. State Diagram

In the following two sections, we describe the state diagram of our OBS network models. We study here two models depending on the availability of the wavelength converters. In one model, we assume that there is no wavelength conversion in any OBS node. In the other model, however, we assume that there are wavelength conversion capabilities in all OBS nodes. In order to simplify the analysis and have some insight on the problem under consideration, we start by a simplified model and generalize it in a later stage. In any model, we always consider an OBS network with w wavelengths available for transmission of bursts. In addition, we assume that all bursts that arrive to an OBS node are of fixed lengths, as shown in Fig. 1. That is, an OBS node needs ℓ time slots to serve any accepted burst arrival. Although this assumption is not easy to achieve [5], we adopt it here in order to simplify the analysis. In a more realistic scenarios, ℓ can be considered as the average number of time slots.

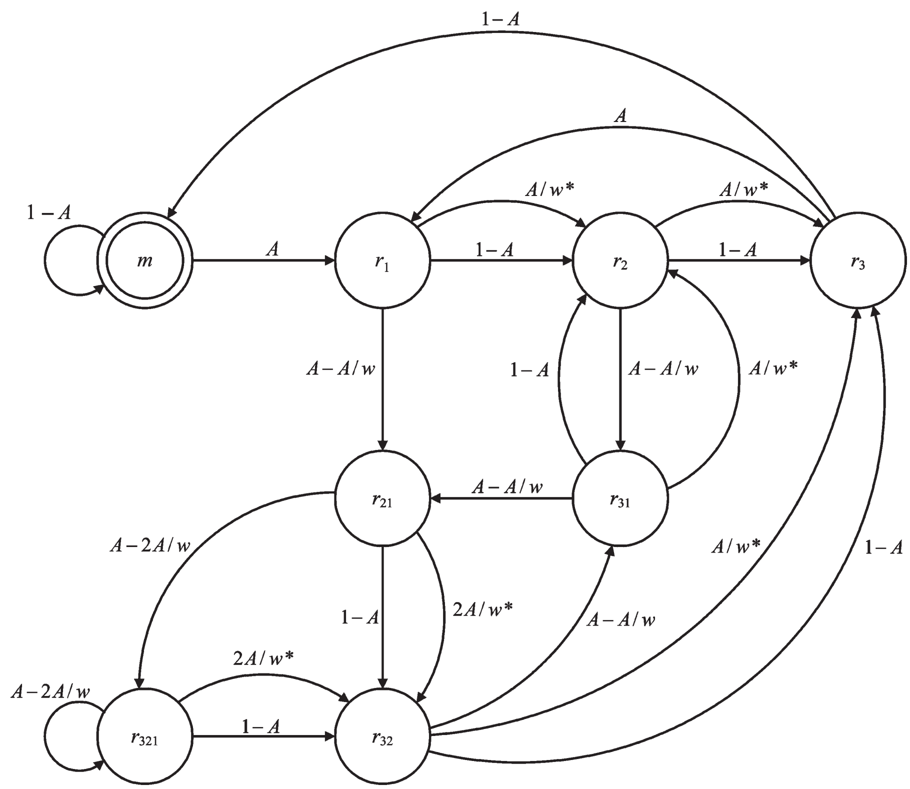

Fig. 2. State diagram for an OBS network with $\ell = 3$ and $w \geq \ell$. The stars denote blocking probabilities.

III. OBS Networks With No Wavelength Conversions

In this section, we focus on the case where there is no wavelength conversion in any OBS node. We start our analysis with a simplified model, namely with $\ell = 3$, and generalize it in a later stage.

A. State Diagram for an OBS Network With $\ell = 3$ and $w \geq \ell$

In this section, we consider an OBS network with $\ell = 3$ and $w \geq \ell$. The state diagram can be constructed as shown in Fig. 2. There are four types of states, each labeled by the probability of an OBS node being in the state.

1) Initial state $\{m\}$: An OBS node is in this state (with probability m) if it is not serving any burst. After staying in the initial state for T_ss, one of the two events happens. Either there is an arrival to the node (an event that occurs with probability A) or there are no arrivals. In the first case, the OBS node will enter state r_1 (defined below), whereas in the second case, the node will remain as is.
2) 1-λ states $\{r_1, r_2, \ldots, r_\ell\}$: An OBS node is in these states if it is using one of the available wavelengths (as identified in the control packet). That is, for $\ell = 3$, if an OBS node is in state m and there is an arrival, it will reserve one of the available wavelengths and enter state r_1. If it is in state r_1 and there are no arrivals after T_ss, it enters state r_2 (corresponding to time slot 2). On the other hand, if it is in state r_1 and there is an arrival that needs to use the same wavelength (an event that occurs with probability A/w), the burst will be blocked and the node will again enter state r_2. However, if the node is in state r_1 and there is an arrival that needs to use another wavelength (an event that occurs with probability $A(w-1)/w$), the node will serve both bursts and enters state r_{21} (defined below). The process on these states is the same until being in the last state $r_\ell = r_3$, where after staying for T_ss in this state, the node returns back to the initial state if there are no arrivals (where the current burst has already served) or returns to state r_1 if there is a new arrival.
3) 2-λ states $\{r_{21}, r_{31}, r_{32}\}$: An OBS node is in these states if it is using two of the available wavelengths and is serving two different bursts. That is, the node is in state r_{ij}, $i, j \in \{1, 2, 3\}$ and $i > j$, if it is serving slots i and j of the two bursts. For example, if the node is in state r_{21}, then it is serving slot 2 of the first burst and slot 1 of the second burst. After T_ss, if there is an arrival that needs to use one of the reserved wavelengths (an event that occurs with probability $2A/w$), it will be blocked, and the node enters state r_{32} to serve slot 3 of the first burst and slot 2 of the second burst. If the arrival, however, needs to use another wavelength, it will be served, and the node enters state r_{321} to serve slot 3 of the first burst, slot 2 of the second burst, and slot 1 of the new burst.
4) 3-λ state $\{r_{321}\}$: An OBS node is in this state if it is using three of the available wavelengths and is serving three different bursts (as mentioned above). For example, if the node is in state r_{321}, it is serving slot 3 of the first burst, slot 2 of the second burst, and slot 1 of the third burst. If after T_ss there are no arrivals (an event that occurs with probability $1 - A$), then the node enters state r_{32} to serve

slot 3 of the second burst and slot 2 of the third burst, where the first burst has already been done.

The rest of the state diagram can be easily followed in a similar way.

B. Theoretical Analysis

We start by writing the flow equations of the above state diagram. We should emphasize that each state is labeled by its probability.

$$\begin{aligned} r_1 &= A(m + r_3) \\ r_2 &= \left(1 - A + \frac{A}{w}\right)(r_1 + r_{31}) \\ r_3 &= \left(1 - A + \frac{A}{w}\right)(r_2 + r_{32}) \\ r_{21} &= \left(A - \frac{A}{w}\right)(r_1 + r_{31}) \\ r_{31} &= \left(A - \frac{A}{w}\right)(r_2 + r_{32}) \\ r_{32} &= \left(1 - A + \frac{2A}{w}\right)(r_{21} + r_{321}) \\ r_{321} &= \left(A - \frac{2A}{w}\right)(r_{321} + r_{21}). \end{aligned} \quad (1)$$

It can be shown (using some algebraic manipulations) that the above equations reduce to

$$\begin{aligned} r_1 = r_2 = r_3 &= \frac{A}{1-A} \cdot m = \frac{w}{w \cdot \frac{1-A}{A}} \cdot m \\ r_{21} = r_{31} = r_{32} &= \frac{(w-1)A^2}{[w - (w-1)A](1-A)} \cdot m \\ &= \frac{w}{w \cdot \frac{1-A}{A}} \cdot \frac{w-1}{w \cdot \frac{1-A}{A} + 1} \cdot m \\ r_{321} &= \frac{(w-1)(w-2)A^3}{[w-(w-2)A][w-(w-1)A](1-A)} \cdot m \\ &= \frac{w}{w \cdot \frac{1-A}{A}} \cdot \frac{w-1}{w \cdot \frac{1-A}{A} + 1} \cdot \frac{w-2}{w \cdot \frac{1-A}{A} + 2} \cdot m. \end{aligned} \quad (2)$$

Imposing the condition that the sum of all probabilities equals to 1

$$m + 3r_1 + 3r_{21} + r_{321} = 1 \quad (3)$$

we can obtain the probability that an OBS node is in the initial state m:

$$m = \left[1 + 3 \cdot \frac{w}{w \cdot \frac{1-A}{A}} + 3 \cdot \frac{w}{w \cdot \frac{1-A}{A}} \cdot \frac{w-1}{w \cdot \frac{1-A}{A} + 1} + \frac{w}{w \cdot \frac{1-A}{A}} \cdot \frac{w-1}{w \cdot \frac{1-A}{A} + 1} \cdot \frac{w-2}{w \cdot \frac{1-A}{A} + 2}\right]^{-1}. \quad (4)$$

C. State Diagram for an OBS Network With $\ell = 3$ and $w < \ell$

In this section, we consider an OBS network with $\ell = 3$ and $w < \ell$.

Case 1—$w = 2$: In this case, state $r_{321} = 0$ in (1)–(3), and the state diagram is the same as that in Fig. 2 but when removing state r_{321} and all arrows to or from it.

Case 2—$w = 1$: In this case, states $r_{21} = r_{31} = r_{32} = r_{321} = 0$ in (1)–(3), and the state diagram is the same as that in Fig. 2 but when removing states r_{21}, r_{31}, r_{32}, r_{321} and all arrows to or from them. Of course, (4) reduces to

$$m = \begin{cases} \left[1 + 3 \cdot \frac{w}{w \cdot \frac{1-A}{A}} + 3 \cdot \frac{w}{w \cdot \frac{1-A}{A}} \cdot \frac{w-1}{w \cdot \frac{1-A}{A} + 1}\right]^{-1}, & \text{if } w = 2 \\ \left[1 + 3 \cdot \frac{w}{w \cdot \frac{1-A}{A}}\right]^{-1}, & \text{if } w = 1. \end{cases}$$

D. Steady-State Throughput and Blocking Probability

In this section, we calculate both the throughput and the blocking probability in our OBS network. The steady-state throughput $\beta(A, \ell, w)$ is defined as the average number of successfully received bursts/burst time:

$$\begin{aligned} \beta(A, \ell, w) = &\sum_{i=1}^{\ell} r_i + \sum_{i=j+1}^{\ell} \sum_{j=1}^{\ell-1} 2r_{ij} \\ &+ \sum_{i=j+1}^{\ell} \sum_{j=k+1}^{\ell-1} \sum_{k=1}^{\ell-2} 3r_{ijk} + \cdots \\ &+ \sum_{i_\ell = i_{\ell-1}+1}^{\ell} \sum_{i_{\ell-1} = i_{\ell-2}+1}^{\ell-1} \\ &\cdots \sum_{i_2=1}^{2} (\ell-1) r_{i_\ell i_{\ell-1} \cdots i_2} + \ell r_{\ell(\ell-1)\cdots 1}. \end{aligned} \quad (5)$$

For the example above ($\ell = 3$), (5) reduces to

$$\beta(A, 3, w) = \begin{cases} 3(r_1 + 2r_{21} + r_{321}), & \text{if } w \geq 3 \\ 3(r_1 + 2r_{21}), & \text{if } w = 2 \\ 3r_1, & \text{if } w = 1. \end{cases}$$

The steady-state blocking probability $P_B(A, \ell, w)$ is defined as the probability that an arrival is being blocked. Referring to Fig. 2, it is given by

$$\begin{aligned} P_B(A, 3, w) &= r_1 \cdot \frac{A}{w} + r_2 \cdot \frac{A}{w} + r_{31} \cdot \frac{A}{w} + r_{32} \cdot \frac{A}{w} \\ &\quad + r_{21} \cdot \frac{2A}{w} + r_{321} \cdot \frac{2A}{w} \\ &= \frac{A}{w} \cdot 2(r_1 + 2r_{21} + r_{321}) \\ &= \frac{2A}{3w} \beta(A, 3, w). \end{aligned}$$

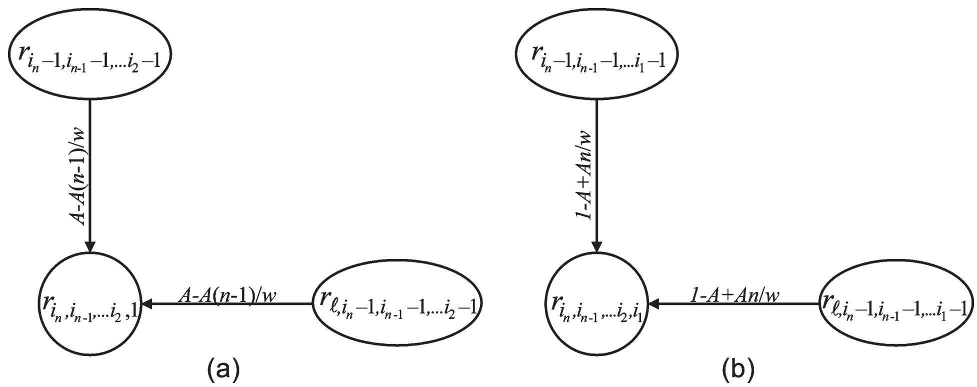

Fig. 3. Generation of an n-λ state: (a) $i_1 = 1$. (b) $i_1 \neq 1$.

E. General Model

We consider an OBS network with w wavelengths and fixed-length bursts (each of length $\ell \geq 1$ time slots). In addition, we assume that the user activity is A and there are no wavelength converters per any node. Fig. 3 shows an n-λ state $r_{i_n,i_{n-1},\ldots,i_1}$ (where $n \in \{1,2,\ldots,\ell \wedge w\}$ and $i_1, i_2, \ldots, i_n \in \{1,2,\ldots,\ell\}$ with $i_n > i_{n-1} > \cdots > i_1$. Here, $\ell \wedge w \stackrel{\text{def}}{=} \min\{\ell, w\}$). The node in this state is serving slot i_n of the first burst, slot i_{n-1} of the second burst, and so on. Two different scenarios may generate this state.

1) $i_1 = 1$: That is, the above node is serving first slot of a new arrival. The previous states are, thus, either an $(n-1)$-λ state $r_{i_n-1,i_{n-1}-1,\ldots,i_2-1}$ or an n-λ state $r_{\ell,i_n-1,i_{n-1}-1,\ldots,i_2-1}$ [Fig. 3(a)]. The transition probability is given by

$$\begin{aligned} P_{n1} &= \Pr\{\text{a new arrival}\} \\ &\quad \cdot \Pr\{\text{the arrival selects an unused wavelength}\} \\ &= A\left[1 - \frac{(n-1)}{w}\right] \\ &= A - \frac{(n-1)A}{w}. \end{aligned}$$

The corresponding flow equation is thus

$$r_{i_n,i_{n-1},\ldots,i_2,1} = \left[A - \frac{(n-1)A}{w}\right] \times \left(r_{i_n-1,i_{n-1}-1,\ldots,i_2-1} + r_{\ell,i_n-1,i_{n-1}-1,\ldots,i_2-1}\right). \quad (6)$$

2) $i_1 \neq 1$: That is, there is either no new arrival or the new arrival is blocked. The previous states are either an n-λ state $r_{i_n-1,i_{n-1}-1,\ldots,i_1-1}$ or an $(n+1)$-λ state $r_{\ell,i_n-1,i_{n-1}-1,\ldots,i_1-1}$ [Fig. 3(b)]. The transition probability in this case is given by

$$\begin{aligned} P_{n2} &= \Pr\{\text{no arrivals}\} + \Pr\{\text{a new arrival}\} \\ &\quad \cdot \Pr\{\text{the arrival selects a used wavelength}\} \\ &= 1 - A + \frac{nA}{w}. \end{aligned}$$

The corresponding flow equation is thus

$$r_{i_n,i_{n-1},\ldots,i_2,i_1} = \left[1 - A + \frac{nA}{w}\right] \times \left(r_{i_n-1,i_{n-1}-1,\ldots,i_1-1} + r_{\ell,i_n-1,i_{n-1}-1,\ldots,i_1-1}\right). \quad (7)$$

Solution of the State Equations: The complete set of state equations is described by (6) and (7) for any $n \in \{1,2,\ldots,\ell \wedge w\}$. Similar to (2), the unique solution occurs when all n-λ states are equal. That is, for any $i_1, i_2, \ldots, i_n \in \{1,2,\ldots,\ell\}$ with $i_n > i_{n-1} > \cdots > i_1$:

$$r_{i_n,i_{n-1},\ldots,i_2,i_1} = e_n$$

where e_n can be determined by substitution in (6) and (7):

$$\begin{aligned} e_n &= \left[A - \frac{(n-1)A}{w}\right](e_{n-1} + e_n) \\ &\Rightarrow e_n = \frac{A - \frac{A}{w}\cdot(n-1)}{1 - A + \frac{A}{w}\cdot(n-1)} e_{n-1} \\ e_n &= \left[1 - A + \frac{nA}{w}\right](e_n + e_{n+1}) \\ &\Rightarrow e_{n+1} = \frac{A - \frac{A}{w}\cdot n}{1 - A + \frac{A}{w}\cdot n} e_n. \end{aligned}$$

The similarity of the two equations ensures the consistency of our solution. Performing the induction method on one of the last equations yields

$$e_k = \prod_{i=0}^{k-1} \frac{w-i}{w \cdot \frac{1-A}{A} + i} \cdot e_0 = \prod_{i=0}^{k-1} \frac{w-i}{w \cdot \frac{1-A}{A} + i} \cdot m.$$

The value of the initial probability m can be determined by imposing the condition that the sum of all probabilities equals to 1:

$$\begin{aligned} m + \sum_{n=1}^{\ell \wedge w} \binom{\ell}{n} e_n = 1 \\ \Rightarrow m = \left[1 + \sum_{n=1}^{\ell \wedge w} \binom{\ell}{n} \prod_{i=0}^{n-1} \frac{w-i}{w \cdot \frac{1-A}{A} + i}\right]^{-1}. \end{aligned}$$

Hence, for any $k \in \{1, 2, \ldots, \ell \wedge w\}$

$$e_k \stackrel{\text{def}}{=} \frac{\prod_{i=0}^{k-1} \frac{w-i}{w \cdot \frac{1-A}{A} + i}}{1 + \sum_{n=1}^{\ell \wedge w} \binom{\ell}{n} \prod_{i=0}^{n-1} \frac{w-i}{w \cdot \frac{1-A}{A} + i}}. \tag{8}$$

Steady-State Throughput and Blocking Probability: The following theorem provides the expressions for both the steady-state throughput and blocking probability for the generalized model.

Theorem 1: In an OBS network with w wavelengths and fixed-length bursts (each of length $\ell \geq 1$ time slots), if the user activity is A and there are no wavelength converters per any node, then the steady-state throughput and blocking probability are given by

$$\beta(A, \ell, w) = \sum_{k=1}^{\ell \wedge w} \binom{\ell}{k} \cdot k e_k$$
$$P_B(A, \ell, w) = \frac{A(\ell - 1)}{w\ell} \cdot \beta(A, \ell, w)$$

respectively. Here, e_k, $k \in \{1, 2, \ldots, \ell \wedge w\}$ are given by (8).

Proof: The proof is a simple generalization for the case of $\ell = 3$. Indeed, for $w \geq \ell$, $\beta(A, \ell, w)$ can be reduced to

$$\begin{aligned}
\beta(A,\ell,w) &= \sum_{i=1}^{\ell} e_1 + \sum_{i=j+1}^{\ell} \sum_{j=1}^{\ell-1} 2e_2 + \sum_{i=j+1}^{\ell} \sum_{j=k+1}^{\ell-1} \sum_{k=1}^{\ell-2} 3e_3 + \cdots \\
&\quad + \sum_{i_\ell = i_{\ell-1}+1}^{\ell} \sum_{i_{\ell-1} = i_{\ell-2}+1}^{\ell-1} \cdots \sum_{i_{\ell-\ell\wedge w+1}=1}^{\ell-\ell\wedge w+1} (\ell \wedge w) \cdot e_{\ell\wedge w} \\
&= \ell \cdot e_1 + \binom{\ell}{2} \cdot 2e_2 + \binom{\ell}{3} \cdot 3e_3 + \cdots \\
&\quad + \binom{\ell}{\ell \wedge w} \cdot (\ell \wedge w) \cdot e_{\ell \wedge w} \\
&= \sum_{k=1}^{\ell \wedge w} \binom{\ell}{k} \cdot k e_k.
\end{aligned}$$

The proof of the blocking probability $P_B(A, \ell, w)$ can be performed as follows. Consider an n-λ state $r_{i_n, i_{n-1}, \ldots, i_1}$ (where $n \in \{1, 2, \ldots \ell \wedge w\}$ and $i_1, i_2, \ldots, i_n \in \{1, 2, \ldots, \ell\}$ with $i_n > i_{n-1} > \cdots > i_1$). The node in this state is serving slot i_n of the first burst, slot i_{n-1} of the second burst, and so on. After T_ss, if there is an arrival that needs to be served, then two blocking cases may arise.

1) If $i_n \neq \ell$, the arrival will be blocked with probability nA/w. The blocked node will enter state $r_{i_n+1, i_{n-1}+1, \ldots, i_1+1}$.
2) If $i_n = \ell$, the arrival will be blocked with probability $(n-1)A/w$. The blocked node will enter state $r_{i_{n-1}+1, \ldots, i_1+1}$.

The blocking probability $P_B(A, \ell, w, n)$ for this node is thus

$$\begin{aligned}
&P_B(A, \ell, w, n) \\
&= e_n \cdot \Big[n\frac{A}{w} \cdot \text{number of times } (i_n \neq \ell) \text{ occurs in } \\
&\qquad r_{i_n, i_{n-1}, \ldots, i_1} + (n-1)\frac{A}{w} \cdot \text{number of times} \\
&\qquad (i_n = \ell) \text{ occurs in } r_{i_n, i_{n-1}, \ldots, i_1} \Big] \\
&= e_n \cdot \frac{A}{w} \left[n\binom{\ell - 1}{n} + (n-1)\binom{\ell - 1}{n-1} \right] \\
&= e_n \cdot \frac{A}{w} \cdot \frac{\ell - 1}{\ell} \cdot n\binom{\ell}{n}.
\end{aligned}$$

Thus, the total blocking probability is

$$\begin{aligned}
P_B(A, \ell, w) &= \sum_{k=1}^{\ell \wedge w} P_B(A, \ell, w, k) \\
&= \frac{A}{w} \cdot \frac{\ell - 1}{\ell} \cdot \sum_{k=1}^{\ell \wedge w} \binom{\ell}{k} \cdot k e_k \\
&= \frac{A(\ell - 1)}{w\ell} \cdot \beta(A, \ell, w).
\end{aligned}$$

■

IV. OBS Networks With Wavelength Conversion Capabilities

In this section, we focus on the case where there are some wavelength conversion capabilities in all OBS nodes.

A. General Model

We consider an OBS network with w wavelengths. The set of available wavelengths is denoted by $\Lambda \stackrel{\text{def}}{=} \{\lambda_1, \lambda_2, \ldots, \lambda_w\}$. In addition, each node in the network is equipped with u wavelength converters, $u \in \{0, 1, 2, \ldots, w\}$. Only u wavelengths of Λ can be converted to any other wavelength in the set; the rest $w - u$ wavelengths cannot be converted. The factor

$$\rho \stackrel{\text{def}}{=} \frac{u}{w}, \qquad 0 \leq \rho \leq 1$$

is called the network conversion capability. If $\rho = 0$, then the network has no conversion capability, whereas if $\rho = 1$, then the network has a full conversion capability. It can be assumed that, at any node, all wavelengths are available in a pool. When an arriving burst is to be served with a specific wavelength, this wavelength is removed from the pool until after the service is complete. If another arriving burst is to be served with a wavelength not available in the pool, it will be converted to another one from the pool. This latter wavelength is then removed, and u is decreased by one. Blocking occurs whenever the pool is empty, or a used wavelength is needed while $u = 0$. The state diagram in this case can be described as follows. Consider an n-λ state $r_{i_n,i_{n-1},\ldots,i_1}$ (where $n \in \{1, 2, \ldots \ell \wedge w\}$ and $i_1, i_2, \ldots, i_n \in \{1, 2, \ldots, \ell\}$ with $i_n > i_{n-1} > \cdots > i_1$). Three different scenarios may generate this state.

1) $i_1 = 1$: That is, the above node is serving a new arrival. The previous states are either an $(n-1)$-λ state $r_{i_n-1,i_{n-1}-1,\ldots,i_2-1}$ or an n-λ state $r_{\ell,i_n-1,i_{n-1}-1,\ldots,i_2-1}$. The transition probability is given by

$$\begin{aligned} P_{n1} &= \Pr\{\text{a new arrival}\} \\ &\quad \cdot \Pr\{\text{the arrival selects an unused wavelength} \\ &\qquad \text{or a convertible used wavelength}\} \\ &= A \cdot \left(1 - \frac{n-1}{w} + \rho\frac{n-1}{w}\right) \\ &= A - (1-\rho)\frac{(n-1)A}{w}. \end{aligned}$$

The corresponding flow equation is thus

$$\begin{aligned} r_{i_n,i_{n-1},\ldots,i_2,1} &= \left[A - (1-\rho)\frac{(n-1)A}{w}\right] \\ &\quad \times \left(r_{i_n-1,i_{n-1}-1,\ldots,i_2-1} + r_{\ell,i_n-1,i_{n-1}-1,\ldots,i_2-1}\right). \end{aligned} \tag{9}$$

2) $i_1 \neq 1$ and ($w \geq \ell$ or $n \neq w$): That is, there is either no new arrival or the new arrival is blocked. The previous states are either an n-λ state $r_{i_n-1,i_{n-1}-1,\ldots,i_1-1}$ or an $(n+1)$-λ state $r_{\ell,i_n-1,i_{n-1}-1,\ldots,i_1-1}$. The transition probability in this case is given by

$$\begin{aligned} P_{n2} &= \Pr\{\text{no arrivals}\} + \Pr\{\text{a new arrival}\} \\ &\quad \cdot \Pr\{\text{the arrival selects a} \\ &\qquad \text{nonconvertible used wavelength}\} \\ &= 1 - A + A \cdot (1-\rho)\frac{n}{w} \\ &= 1 - A + (1-\rho)\frac{nA}{w}. \end{aligned}$$

The corresponding flow equation is thus

$$\begin{aligned} r_{i_n,i_{n-1},\ldots,i_2,i_1} &= \left[1 - A + (1-\rho)\frac{nA}{w}\right] \\ &\quad \times \left(r_{i_n-1,i_{n-1}-1,\ldots,i_1-1} + r_{\ell,i_n-1,i_{n-1}-1,\ldots,i_1-1}\right). \end{aligned} \tag{10}$$

3) $i_1 \neq 1$, $w < \ell$, and $n = w$: That is, there is either no new arrival or the new arrival is blocked. Here, the previous state should be a w-λ state $r_{i_w-1,i_{w-1}-1,\ldots,i_1-1}$. The transition probability in this case is unity $P_{n3} = 1$, and the corresponding flow equation is thus

$$r_{i_w,i_{w-1},\ldots,i_2,i_1} = r_{i_w-1,i_{w-1}-1,\ldots,i_1-1}. \tag{11}$$

Solution of the State Equations: The complete set of state equations is described by (9)–(11) for any $n \in \{1, 2, \ldots, \ell \wedge w\}$. Again, the unique solution occurs when all n-λ states are equal. That is, for any $i_1, i_2, \ldots, i_n \in \{1, 2, \ldots, \ell\}$ with $i_n > i_{n-1} > \cdots > i_1$:

$$r_{i_n,i_{n-1},\ldots,i_2,i_1} = e_n$$

where e_n can be determined by a substitution in (9)–(11):

$$\begin{aligned} e_n &= \frac{A - (1-\rho)(n-1)A/w}{1 - A + (1-\rho)(n-1)A/w} e_{n-1} \\ &= \frac{w - (1-\rho)(n-1)}{w \cdot \frac{1-A}{A} + (1-\rho)(n-1)} \cdot e_{n-1}. \end{aligned}$$

After some algebraic manipulations as were done earlier, we get for any $k \in \{1, 2, \ldots, \ell \wedge w\}$:

$$e_k \overset{\text{def}}{=} \frac{\prod_{i=0}^{k-1} \frac{w - i(1-\rho)}{w \cdot \frac{1-A}{A} + i(1-\rho)}}{1 + \sum_{n=1}^{\ell \wedge w} \binom{\ell}{n} \prod_{i=0}^{n-1} \frac{w - i(1-\rho)}{w \cdot \frac{1-A}{A} + i(1-\rho)}}. \tag{12}$$

Steady-State Throughput and Blocking Probability: The following theorem provides expressions for both the steady-state throughput and blocking probability for the generalized model.

Theorem 2: In an OBS network with w wavelengths and fixed-length bursts (each of length $\ell \geq 1$ time slots), if the user activity is A and the conversion capability in any node is ρ, $\rho \in \{0, 1/w, 2/w, \ldots, 1\}$, then the steady-state throughput and blocking probability are given by

$$\begin{aligned} &\beta(A, \ell, w, \rho) \\ &\quad = \sum_{k=1}^{\ell \wedge w} \binom{\ell}{k} \cdot k e_k \\ &P_B(A, \ell, w, \rho) \\ &\quad = \begin{cases} \frac{A(\ell-1)}{w\ell}(1-\rho) \cdot \beta(A, \ell, w, \rho), & \text{if } w \geq \ell \\ \frac{A(\ell-1)}{w\ell}(1-\rho) \cdot \beta(A, \ell, w, \rho) + \binom{\ell-1}{w} A\rho \cdot e_w, & \text{if } w < \ell \end{cases} \end{aligned}$$

respectively. Here, e_k, $k \in \{1, 2, \ldots, \ell \wedge w\}$ are given by (12).

Proof: The proof of the throughput $\beta(A, \ell, w, \rho)$ is exactly the same as that of theorem 1. The proof of the first assertion of blocking probability $P_B(A, \ell, w, \rho)$ is also exactly the same as that of theorem 1. The proof of the second assertion of blocking probability can be performed as follows. Assume that $w < \ell$, and consider an n-λ state $r_{i_n,i_{n-1},\ldots,i_1}$ (where $n \in \{1, 2, \ldots w\}$ and $i_1, i_2, \ldots, i_n \in \{1, 2, \ldots, \ell\}$ with $i_n > i_{n-1} > \cdots > i_1$). After T_ss, if there is an arrival that needs to be served, then three blocking cases may arise.

1) If $i_n = \ell$, the arrival will be blocked with probability $(1-\rho)(n-1)A/w$. The blocked node will enter state $r_{i_{n-1}+1,\ldots,i_1+1}$.
2) If $i_n \neq \ell$ and $n \neq w$, the arrival will be blocked with probability $(1-\rho)nA/w$. The blocked node will enter state $r_{i_n+1,i_{n-1}+1,\ldots,i_1+1}$.
3) If $i_n \neq \ell$ and $n = w$, the arrival will be blocked with probability A. The blocked node will enter state $r_{i_n+1,i_{n-1}+1,\ldots,i_1+1}$.

Thus, if $n \neq w$, the blocking probability $P_B(A, \ell, w, \rho, n)$ for this node is given by

$$P_B(A, \ell, w, \rho, n) = \frac{A(\ell-1)}{w\ell}(1-\rho) \cdot n\binom{\ell}{n} e_n.$$

If $n = w$, however, it is given by

$$\begin{aligned}
&P_B(A, \ell, w, \rho, w) \\
&= e_w \cdot \Big[A \cdot \text{number of times } (i_w \neq \ell) \text{ occurs in} \\
&\qquad r_{i_w, i_{w-1}, \ldots, i_1} + (1-\rho)(w-1)\frac{A}{w} \\
&\qquad \cdot \text{number of times } (i_w = \ell) \\
&\qquad \text{occurs in } r_{i_w, i_{w-1}, \ldots, i_1} \Big] \\
&= \frac{A}{w}\left[w\binom{\ell-1}{w} + (1-\rho)(w-1)\binom{\ell-1}{w-1} \right] e_w \\
&= \frac{A}{w}\left[(1-\rho)w\binom{\ell-1}{w} \right. \\
&\qquad \left. + (1-\rho)(w-1)\binom{\ell-1}{w-1} + \rho w\binom{\ell-1}{w} \right] e_w \\
&= \frac{A(\ell-1)}{w\ell}(1-\rho) \cdot w\binom{\ell}{w} e_w + A\rho\binom{\ell-1}{w} e_w.
\end{aligned}$$

Thus, the total blocking probability in this case is

$$\begin{aligned}
P_B(A, \ell, w, \rho) &= \sum_{k=1}^{w} P_B(A, \ell, w, \rho, k) \\
&= \frac{A(\ell-1)}{w\ell}(1-\rho) \cdot \beta(A, \ell, w, \rho) \\
&\quad + \binom{\ell-1}{w} A\rho \cdot e_w. \qquad \blacksquare
\end{aligned}$$

V. Numerical Results

Both the steady-state system throughput and average blocking probability derived above have been evaluated under different network parameters. Our results are plotted in Figs. 4–7. A burst length of $\ell = 500$ slots is held fixed in all figures but Fig. 7. An average network traffic constraint of $\kappa \leq 50$ bursts/burst time is imposed in all figures. This keeps the user activity A below 0.1 bursts/slot time.

In Fig. 4, the average throughput has been plotted versus the average network traffic for a fixed number of wavelengths $w = 16$ and different conversion capabilities $\rho \in \{0, 0.5, 1\}$. General and expected trends of the curves can be noticed. Indeed, the throughput increases as the network traffic increases and as the conversion capability increases. In addition, it can be deduced that the normalized throughput decreases as κ increases. Of course, as ρ increases, the system complexity and cost increases as well. Indeed, the optical wavelength

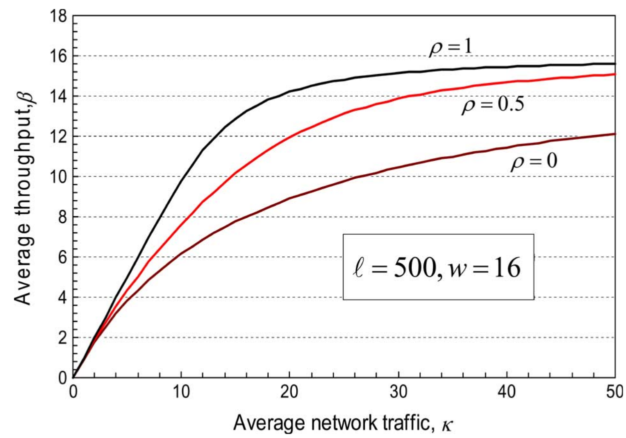

Fig. 4. Average throughput versus network traffic for different conversion capabilities and the same number of wavelengths.

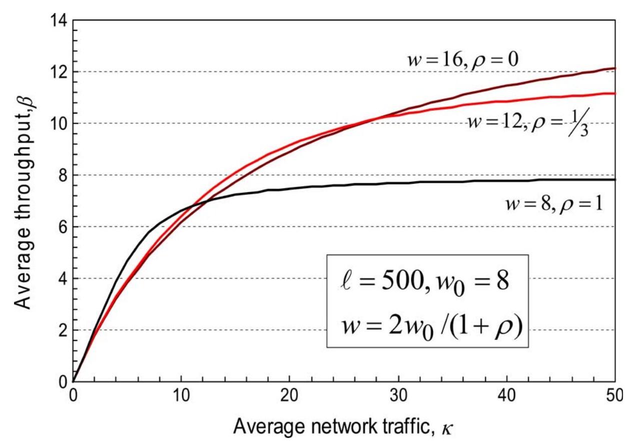

Fig. 5. Average throughput versus network traffic for different conversion capabilities and a constraint on the sum of the available wavelengths and wavelength converters.

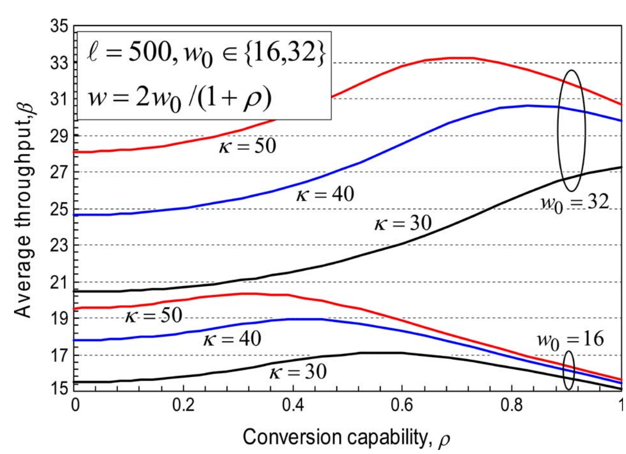

Fig. 6. Average throughput versus conversion capability for different network traffics and a constraint on the sum of the available wavelengths and wavelength converters.

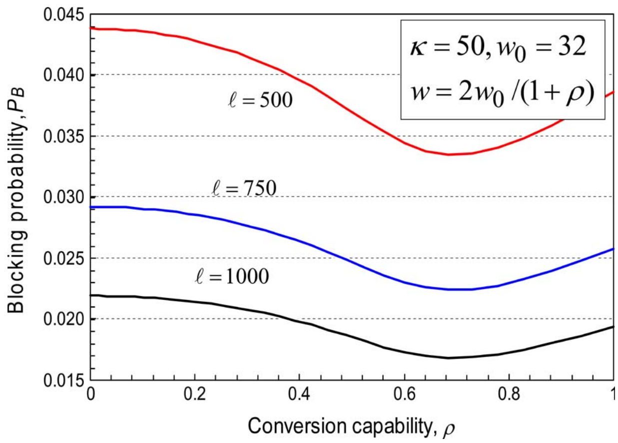

Fig. 7. Average blocking probability versus conversion capability for different burst lengths and a constraint on the sum of the available wavelengths and wavelength converters.

conversion remains expensive, and there are no realistic expectations that it will become cheap in the near future. This makes the comparison between the different curves unfair.

To make the comparison somewhat fair, we proceed as follows. Assume that the initial number of wavelengths in a given node is w_0. If we wish to increase the node resources, then we either increase the number available wavelengths by $v \in \{0, 1, \ldots, w_0\}$ and/or install wavelength converters $u \in \{0, 1, \ldots, w_0\}$ so that

$$u + v = w_0.$$

The total number of available wavelengths w is related to the conversion capability $\rho = u/w$ as follows:

$$\begin{aligned} w =& w_0 + v \\ =& w_0 + (w_0 - u) \\ =& 2w_0 - \rho w \\ \Rightarrow w =& \frac{2w_0}{1+\rho}. \end{aligned} \tag{13}$$

This relation provides a tradeoff between the number of available wavelengths and the conversion capability. For example, if a node has full conversion capability $\rho = 1$, then the total available wavelengths is only $w = w_0$. On the other hand, if a node has no conversion capability $\rho = 0$, then the total available wavelengths is $w = 2w_0$. Using the condition above with $w_0 = 8$, the average throughput has been plotted in Fig. 5 versus the average network traffic for different conversion capabilities $\rho \in \{0, 1/3, 1\}$. The curves are competitive to each other depending on the traffic. For example, if the traffic is low, then the systems with higher conversion capabilities perform better than that with lower conversion capabilities. This conclusion reverses if the traffic is high. The reason is that, for low traffic, most available wavelengths' pipes are free; therefore, if two bursts with the same wavelength arrive, one can use a wavelength converter to switch one of them to another pipe. On the other hand, for high traffic, most of the pipes are full, and the need of wavelength converter is not as important as installing more pipes to relax the high load.

A deeper study of the last conclusion has been performed, and its results are plotted in Fig. 6, with $w_0 \in \{16, 32\}$. It can be seen that for a fixed traffic and condition (13) satisfied, the throughput changes with the conversion capability, and an optimum value of ρ_{opt} that maximizes the throughput always exists. This ρ_{opt} decreases as the traffic increases and increases as w_0 increases, confirming our previous conclusion.

Finally, in Fig. 7, the average blocking probability has been plotted versus the conversion capability [with $w_0 = 32$ and condition (13) satisfied] for different burst lengths $\ell \in \{500, 750, 1000\}$ slots. It can be seen that the blocking probability decreases as ℓ increases. The reason is that, as ℓ increases (with slot duration fixed), more arriving packets are combined into one large burst, reducing the number of competing bursts and, hence, reducing the blocking probability. It should be noticed that the blocking probability improves by the same factor of the increase in length. It should be noted that the price to be paid by increasing ℓ is the increase of the packet latency.

VI. Concluding Remarks

Simplified mathematical model has been proposed for evaluating the performance of OBS networks. Two main performance measures have been derived based on the EPA technique. These measures are the steady-state system throughput and average blocking probability. The effects of several design parameters on the system performance measures have been investigated and presented numerically. In order to have some insight on the problem under consideration, focus has been given to the wavelength blocking only, and other sources of noise have been neglected. To have a somewhat fair comparison between different systems, a formula that compensates for the tradeoff between system complexity and total number of wavelengths has been derived.The following concluding remarks can be extracted from our results.

1) Lower conversion capabilities and higher number of wavelengths are more suitable if the network traffic is low. However, higher conversion capabilities and lower number of wavelengths are more suitable if the network traffic is high.
2) There always exist optimum values of both the conversion capability and the total number of available wavelengths that maximizes the throughput for a given network traffic.
3) The blocking probability improves as the burst length increases.

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