Effect of Thermal Noise and APD Noise on the Performance of OPPM-CDMA Receivers

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***Abstract*—The effect of thermal noise and Avalanche-photodiode (APD) noise on the performance of optical overlapping pulse-position modulation/code-division multiple-access (OPPM-CDMA) systems with and without double optical hardlimiters is examined. Comparisons with the corresponding optical on–off keying/CDMA (OOK-CDMA) systems are presented as well. The maximum data rate that can be achieved under laser pulsewidth and bit error rate constraints is determined. It is shown that about 10 dB increase in the average power, with respect to the Poisson shot-noise-limited system, is required to compensate for the performance degradation due to thermal noise and APD noise. Moreover, it is indicated that for a bit error rate not exceeding 10^{-9}, a laser pulsewidth of 0.03 ns, and an average received optical power of -55 dBm, a data rate of more than 3 Gb/s can be achieved per channel when using OPPM-CDMA systems with double optical hardlimiters.**

***Index Terms*—Avalanche-photodiode noise (APD), code division multiple access (CDMA), direct detection optical channel, on–off keying (OOK), optical CDMA, optical correlator, optical hardlimiter, overlapping pulse-position modulation (OPPM), pulse-position modulation, thermal noise.**

I. Introduction

THE AVAILABLE bandwidth in a standard single-mode optical fiber is about 25 THz within the low-attenuation passband. This bandwidth decreases as the bit rate increases due to limitations in other components. For example a maximum of 10 000 channels can be loaded into a single fiber if the data rate per channel is about 1 Gb/s. Further, this bandwidth would be decreased down to about 5 THz in the case of long distance communications, where erbium-doped optical amplifiers are required. Time-division multiple-access (TDMA) technique cannot make full use of this bandwidth due to the few Gb/s speed limitation of the electronic devices. Wavelength-division multiple-access (WDMA) scheme with multiple wavelength channels is the best candidate to get around the limitations in TDMA. However today's available tunable optical receiver technology cannot resolve more than about 100 wavelengths. Thus it will not be efficient to allocate one wavelength to one channel with a data rate of few hundreds megabits per second, for example (which is sufficient to support image-based and multimedia applications). This means that WDMA cannot be used alone to make efficient use of the available bandwidth. TDMA can be used together with WDMA to mine the tera hertz bandwidth of the optical fiber. In this case several low-rate channels (few hundreds Mb/s each) are time-division multiplexed together to form a high-rate (1 Gb/s) channel. All high-rate channels are then wave-division multiplexed subject to the aforementioned limitations.

Optical code-division multiple-access (CDMA) technique stands as an attractive alternative to TDMA in future optical networks for the following reasons. CDMA is an asynchronous system that does not require time synchronization as TDMA. Idle users in CDMA do not bother the channel, whereas each node in TDMA might spend much time looking at bits that have nothing to do with it. Only simple communication protocols are required for CDMA and each subscriber makes full utilization of the entire time-frequency domain. CDMA provides flexibility in the network design and security against interception.

Both on–off keying CDMA (OOK-CDMA) and pulse-position modulation CDMA (PPM-CDMA) have appeared in literature [1]–[12]. Unfortunately, in conventional optical OOK- and PPM-CDMA systems, the laser pulsewidth must be stringently shortened in order to achieve the requirements on the very high data rates. Laser sources with very short pulsewidth are hardly realizable and the available ones are too expensive. Recently [14], we have suggested using overlapping pulse-position modulation (OPPM) rather than OOK or PPM in optical CDMA systems to come across this problem. This modulation technique can offer higher data rates without the need to decrease the pulsewidth [16]–[21]. Further, it retains some of the advantages of PPM where the transmitter involves only time delaying of the optical pulses and the receiver does not involve any threshold comparison. A special prototype of the OPPM-CDMA system was first introduced by Kwon [13] and was known as multibits/sequence-period optical orthogonal CDMA system. Although the OPPM-CDMA system can be considered as a generalization to the multibits/sequence-period CDMA system, the hardware implementation of the latter is more complex than the former.

Several trials to reduce the interference effect on the performance of the optical CDMA systems have appeared in literature [4], [8]–[12]. An optical hardlimiter, placed before the optical correlator at the receiver side, has been studied by Salehi and Brackett [4]. Although it was shown that this hardlimiter was able to remove some of the interference patterns in the case of ideal photodiodes, Kwon [6] has demonstrated that its effect is insignificant when considering more realistic systems, that is, if the effect of avalanche-photodiode (APD) noise and thermal noise is taken into account. On the other hand, Ohtsuki *et al.* [10] have proposed a synchronous optical CDMA system with double optical hardlimiters placed before and after the optical correlator at the receiver of an OOK-CDMA system. It has been

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shown that this system introduces an improvement in the performance over the system without hardlimiters even when considering Poisson shot-noise-limited (or PIN) photodiodes. In fact their receiver can remove more interference patterns than that removed by a single hardlimiter receiver. Although the improvement in [10] has been shown to be valid only when the number of users is not so large, Ohtsuki has shown that, in the case of asynchronous optical OOK-CDMA systems, this improvement continues for all possible number of users even with PIN photodiodes [9] or APD photodiodes [12]. In [15] we have added double optical hardlimiters before and after the optical correlator at the receiver side of the optical OPPM-CDMA receiver proposed in [14]. We were able to show that the performance of this system is *asymptotically* close to the optimum OPPM-CDMA system and its capacity is about 5.3 times greater than that of the optimum OOK-CDMA system.

In our previous studies of optical OPPM-CDMA systems, [14], [15], we have taken into account the effect of the multiple-user interference and assumed a Poisson shot-noise-limited receiver with a PIN photodetector. The effect of the receiver dark current, avalanche-photodiode (APD) noise, and thermal noise was, however, neglected in order to have some insight on the problem. In more realistic receivers the thermal noise cannot be neglected and the performance will be degraded with respect to the shot-noise-limited ones. One way to improve the receiver sensitivity, so as to overcome the degradation due to thermal noise, is to use APD's rather than PIN photodetectors. The random gain nature of the APD's, on the other hand, increases the noise term in the detection process.

Our aim in this paper is to study the effect of both thermal noise and APD noise on the performance of the optical OPPM-CDMA systems with and without double optical hardlimiters, which was not analyzed in both [14] and [15]. We also aim at determining the maximum data rate that can be achieved under constraints on both the laser pulsewidth and the bit error rate. Further, we compare our results to the corresponding OOK-CDMA systems. We are able to show that for a bit error rate not exceeding 10^{-9} and a laser pulsewidth of 0.03 ns, a total data rate of more than 3 Gb/s can be achieved per wavelength when using OPPM-CDMA systems with double optical hardlimiters.

The remainder of this paper is organized as follows. The optical OPPM-CDMA system model is described in Section II. Section III is devoted for the development of the bit error probabilities for the optical OPPM-CDMA system (with and without optical hardlimiters) taking into account the effect of both the APD noise and the thermal noise in addition to the multiple-user interference. Our numerical results are presented in Section IV, where the variations of the performance of the above systems with different design parameters are illustrated and compared to that of the traditional OOK-CDMA systems and to that of the corresponding systems when neglecting the effect of the APD noise and/or the thermal noise. Our conclusion is finally given in Section V.

II. Optical OPPM-CDMA System Model

A. Optical OPPM-CDMA Signal Formats

In an OPPM-CDMA communication system, the transmitter is composed of N simultaneous sources of information (users). Each user produces continuous and asynchronous data symbols, which take values from the discrete finite set $\{0, 1, \dots, M-1\}$. Each symbol is then converted to an optical OPPM-CDMA waveform as was described in [15]. An example of optical OPPM-CDMA signal format with $M = 5$ is shown in Fig. 1. An optical CDMA waveform (standing for the signature code sequence) is placed in one of M time slots (or spreading intervals) to represent a data symbol. Each time slot has a duration $\tau = LT_c$, where L denotes the CDMA code length and T_c denotes the chip time duration. Any two adjacent time slots are allowed to overlap with a depth of $(1 - 1/\gamma)\tau$, where γ denotes the index of overlap. Thus, all the M time slots constitute an optical OPPM time frame with duration T

$$T = (M - 1 + \gamma)\frac{\tau}{\gamma} = (M - 1 + \gamma)\frac{L}{\gamma} T_c. \tag{1}$$

For the optical CDMA waveform to fit properly within a time slot, the following condition must be satisfied:

$$\frac{\tau}{\gamma} = \text{integer} \times T_c \qquad \text{or} \qquad \frac{L}{\gamma} = \text{integer}. \tag{2}$$

It should be recited that an optical CDMA waveform can be generated by splitting an input optical pulse, of duration T_c, into w shorter pulses within w different branches, where w denotes the CDMA code weight, then delaying each pulse in accordance to the mark positions of the signature code, and finally combining all the branches back into one [1], [7].

B. Optical OPPM-CDMA Receiver Model

The composite optical signal from all the N transmitters is then broadcast to all receivers. The block diagram of an optical OPPM-CDMA receiver with double optical hardlimiters is shown in Fig. 2. The output optical powers of the ideal optical hardlimiters are defined as follows. For every $i \in \{1, 2\}$

$$g_i(x_i) = \begin{cases} u_i, & \text{if } x_i \geq v_i \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

where x_i, g_i, and v_i denote the input optical power, the output optical power, and the threshold power level of the ith hardlimiter, respectively, and u_i is a constant. Thus the ith optical hardlimiter clips its input optical power to u_i whenever it exceeds the threshold level v_i . The CDMA correlator is similar to the CDMA encoder (at the transmitter) but with reverse time delays [1], [7]. The output of the second optical hardlimiter is photodetected using an APD and passed to the OPPM decoder where chip integrations are made at the end of each slot inside an OPPM time frame. The values of these integrations within the time frame are then compared for the maximum, and the corresponding slot will identify the transmitted symbol for that frame. In our analysis we choose

$$(\forall\, i \in \{1, 2\}) \qquad u_i = v_i = P_p, \tag{4}$$

where P_p denotes the received peak laser power of a single user. This can be justified as follows. Assume that the desired user is sending a 1 and there is no multiple-user interference, then the input to the first hardlimiter takes only two values from the set $\{0, P_p\}$, whereas that to the second hardlimiter takes three

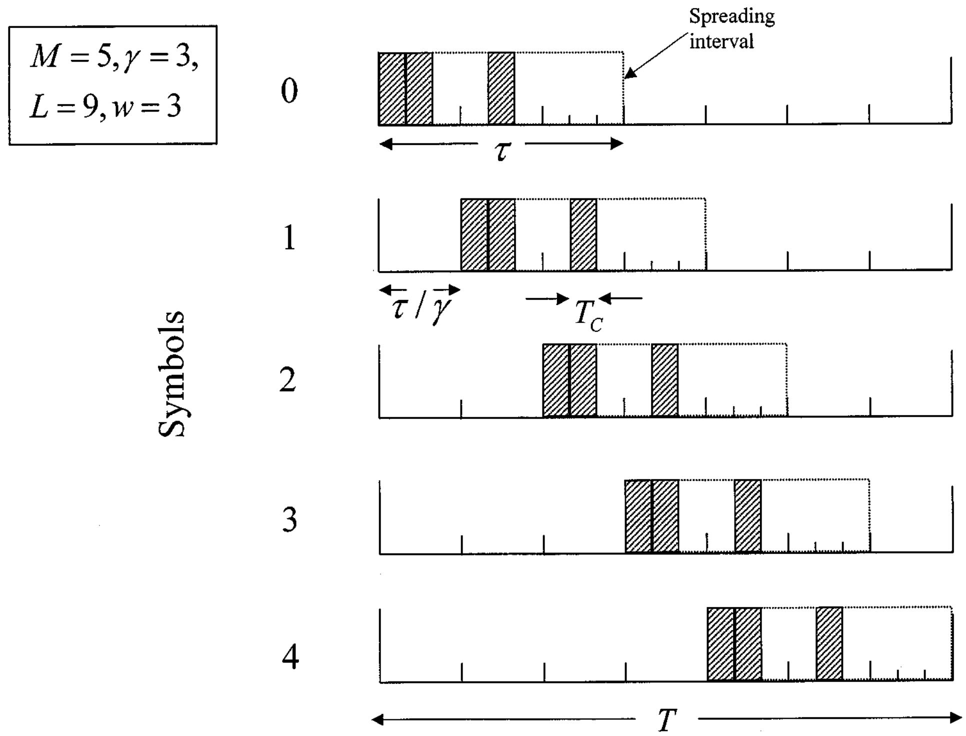

Fig. 1. Optical OPPM-CDMA signal formats.

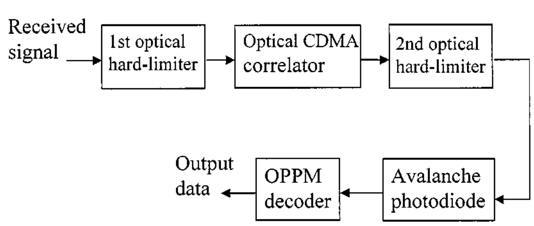

Fig. 2. Optical OPPM-CDMA receiver model with double optical hardlimiters.

values from the set $\{0, P_p/w, P_p\}$. Since the OPPM decoder integrates over last chips of all slots, it can only see the two values 0 and P_p and the choices in (4) would ensure the elimination of some of the *extra* power due to the interference.

III. Theoretical Analysis

We assume that $Y_i, i \in \mathcal{M} \stackrel{\text{def}}{=} \{0, 1, 2, \cdots, M-1\}$, denotes the photon count collected from slot i of an OPPM time frame. From the above discussion, we have the following decision rule. Decide that symbol i was transmitted if $Y_i > Y_j$ for every $j \in \mathcal{M}$ and $j \neq i$. An incorrect decision is otherwise declared. The probability of word error for equally probable data symbols is thus given by

$$P_E = \frac{1}{M} \sum_{i=0}^{M-1} \Pr\{Y_j \geq Y_i, \text{ some } j \neq i | D = i\} \tag{5}$$

where $D \in \mathcal{M}$ denotes the transmitted data symbol. The bit error probability P_b can be obtained from P_E using the formula $P_b = [M/2(M-1)]P_E$ [22].

Furthermore, we assume that $\kappa_{ij}, i \in \mathcal{X} \stackrel{\text{def}}{=} \{1, 2, \cdots, w\}$, $j \in \mathcal{M}$, denotes the number of pulses (from other users) that interfere with chip i of the mark positions (of the desired user's code) in slot j . We also denote the vector $(\kappa_{1j}, \kappa_{2j}, \cdots, \kappa_{nj})^T$, $n \in \mathcal{X}$, by $\boldsymbol{K}_j^n$. We have shown in [15] that the vector $\boldsymbol{K}_j^n$ obeys *approximately* a multinomial distribution with parameters $p_1 = \gamma w/(M-1+\gamma)L$ and $N-1$. That is if $\boldsymbol{L}_j^n = (l_{1j}, l_{2j}, \ldots, l_{nj})^T$ is a realization vector of $\boldsymbol{K}_j^n$, then

$$\Pr\{\boldsymbol{K}_j^n = \boldsymbol{L}_j^n\} = \frac{(N-1)!}{l_{1j}!l_{2j}!\cdots l_{nj}!s_{nj}!} p_1^{N-1-s_{nj}}(1-np_1)^{s_{nj}} \tag{6}$$

where

$$s_{nj} = N-1-\sum_{i=1}^{n} l_{ij}, \qquad n \in \mathcal{X}, j \in \mathcal{M}. \tag{7}$$

A. Optical OPPM-CDMA Receiver with Double Optical Hardlimiters

We derive here an upper bound to the word error probability in (5) for the OPPM-CDMA system with double optical hardlimiters: Consider first any of the probabilities in the right hand side of (5). We use a union bound and keep in mind that the code pulses in slot i of the desired user may hit (self-interfere) with at most $\gamma - 1$ adjacent slots

$$\begin{aligned} &\Pr\{Y_j \geq Y_i, \text{ some } j \neq i | D = i\} \\ &\quad \leq (M-\xi)\Pr\{Y_1 \geq Y_0 | D = 0, \nu_1 = 0\} \\ &\qquad + \sum_{j=1}^{\xi-1} \Pr\{Y_j \geq Y_0 | D = 0\} \end{aligned} \tag{8}$$

where, $\xi \overset{\text{def}}{=} \min\{M, \gamma\}$ and $\nu_j \in \{0, 1\}, j \in \{1, 2, \ldots, M-1\}$, denotes the number of pulses (from the desired user) that cause a self-interference to slot j due to signature code pulses sent in slot 0. The first term in the right hand side of the last inequality is due to the $M-1-(\xi-1)$ slots that do not have self-interference with slot 0. That is, $\nu_j = 0$ with probability 1 for these slots. On the other hand, the second term is due to the remaining $\xi - 1$ slots which interfere with slot 0 by a positive probability. If the code marks are uniformly distributed within the code sequence, then $\Pr\{\nu_j = 1\} \le w(w-1)/(L-1)$, and consequently

$$\begin{aligned}\Pr\{Y_j \ge Y_0,\ \text{some } j \ne i | D = i\} \\ \le (M-\xi)\Pr\{Y_1 \ge Y_0 | D = 0,\ \nu_1 = 0\} \\ + r(\xi-1)\Pr\{Y_1 \ge Y_0 | D = 0,\ \nu_1 = 1\} \\ + (1-r)(\xi-1)\Pr\{Y_1 \ge Y_0 | D = 0,\ \nu_1 = 0\} \end{aligned} \tag{9}$$

where $r \overset{\text{def}}{=} w(w-1)/(L-1)$. By substituting (9) in (8) and (5), we get the following upper bound on the word error probability:

$$P_E \le [M - 1 - r(\xi-1)]P_0 + r(\xi-1)P_1 \tag{10}$$

where

$$P_0 = \Pr\{Y_1 \ge Y_0 | D = 0,\ \nu_1 = 0\}$$

and

$$P_1 = \Pr\{Y_1 \ge Y_0 | D = 0,\ \nu_1 = 1\}. \tag{11}$$

We can evaluate P_0 and P_1 as follows:

$$\begin{aligned}P_0 &= \Pr\{Y_1 \ge Y_0 | D = 0,\ \nu_1 = 0\} \\ &= \Pr\{Y_1 \ge Y_0 | D = 0,\ Z_1 = w,\ \nu_1 = 0\}\Pr\{Z_1 = w\} \\ &\quad + \Pr\{Y_1 \ge Y_0 | D = 0,\ Z_1 \ne w,\ \nu_1 = 0\}\Pr\{Z_1 \ne w\}.\end{aligned} \tag{12}$$

Here the random variable $Z_1 \in \{0, 1, \ldots, w\}$ denotes the number of interfered mark positions in slot 1 of the desired user immediately after the first optical hardlimiter. But the error probability would increase if we enforced the condition that a zero interference always occurs to slot 0 (the signal slot). Thus

$$\begin{aligned}P_0 \le \Pr\{Y_1 \ge Y_0 | D = 0,\ \kappa_0 = 0,\ Z_1 = w,\ \nu_1 = 0\} \\ \times \Pr\{Z_1 = w\} \\ + \Pr\{Y_1 \ge Y_0 | D = 0,\ \kappa_0 = 0,\ Z_1 \ne w,\ \nu_1 = 0\} \\ \times \Pr\{Z_1 \ne w\}\end{aligned} \tag{13}$$

where $\kappa_0 = \sum_{i=1}^{w} \kappa_{i0}$ denotes the interference from other users to slot 0 of the desired user. Similarly

$$\begin{aligned}P_1 \le \Pr\{Y_1 \ge Y_0 | D = 0,\ \kappa_0 = 0,\ Z_1' = w-1,\ \nu_1 = 1\} \\ \times \Pr\{Z_1' = w-1\} \\ + \Pr\{Y_1 \ge Y_0 | D = 0,\ \kappa_0 = 0,\ Z_1' \ne w-1,\ \nu_1 = 1\} \\ \times \Pr\{Z_1' \ne w-1\}\end{aligned} \tag{14}$$

where the random variable $Z_1' \in \{0, 1, \ldots, w-1\}$ is similar to Z_1 but for nonself-interfered mark positions. The probability distributions for both Z_1 and Z_1' can be evaluated as comes next.

$$\begin{aligned}\Pr\{Z_1 = w\} \\ &= \Pr\{\kappa_{i1} \ge 1\, \forall\, i \in \mathcal{X}\} \\ &= 1 - \Pr\{\kappa_{i1} = 0,\ \text{some } i \in \mathcal{X}\} \\ &= 1 + \sum_{n=1}^{w} (-1)^n \binom{w}{n} \Pr\{\kappa_{11} = \kappa_{21} = \cdots = \kappa_{n1} = 0\}\end{aligned} \tag{15}$$

where the last equality is justified by expanding the probability of a union of events. Using (6) in (15), we get

$$\Pr\{Z_1 = w\} = \sum_{i=0}^{w} (-1)^i \binom{w}{i} \left[1 - i\frac{\gamma w}{(M-1+\gamma)L}\right]^{N-1}. \tag{16}$$

In a similar way, we can have

$$\Pr\{Z_1' = w-1\} = \sum_{i=0}^{w-1} (-1)^i \binom{w-1}{i} \left[1 - i\frac{\gamma w}{(M-1+\gamma)L}\right]^{N-1}. \tag{17}$$

For sufficiently large average signal and noise power, the discrete APD counting probabilities for both Y_0 and Y_1 can be approximated by continuous Gaussian densities [22]. Thus the probabilities P_0 and P_1 in (13) and (14), respectively, can be approximated as follows:

$$\begin{aligned}P_0 &\le \Pr\{Y_1 - Y_0 \ge 0 | D = 0,\ \kappa_0 = 0,\ Z_1 = w,\ \nu_1 = 0\} \\ &\quad \times \Pr\{Z_1 = w\} \\ &\quad + \Pr\{Y_1 - Y_0 \ge 0 | D = 0,\ \kappa_0 = 0,\ Z_1 \ne w,\ \nu_1 = 0\} \\ &\quad \times \Pr\{Z_1 \ne w\} \\ &= Q\left(-\frac{m_{eq}}{\sigma_{eq}}\right)\Pr\{Z_1 = w\} \\ &\quad + Q\left(-\frac{m_{\text{neq}}}{\sigma_{\text{neq}}}\right)(1 - \Pr\{Z_1 = w\}) \\ &= \frac{1}{2}\Pr\{Z_1 = w\} + Q\left(-\frac{m_{\text{neq}}}{\sigma_{\text{neq}}}\right)(1 - \Pr\{Z_1 = w\})\end{aligned} \tag{18}$$

where the function $Q(x)$ is the normalized Gaussian tail probability, given by

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-s^2/2}\, ds \tag{19}$$

$$\begin{aligned}m_{eq} &= E\{Y_1 - Y_0 | D = 0,\ \kappa_0 = 0,\ Z_1 = w,\ \nu_1 = 0\} \\ &= G(qw + q_d) - G(qw + q_d) = 0\end{aligned} \tag{20}$$

$$\begin{aligned}\sigma_{eq}^2 &= E\{Y_1^2 - (EY_1)^2 | D = 0, \kappa_0 = 0 \\ &\qquad Z_1 = w,\ \nu_1 = 0\} + \sigma_n^2 \\ &\quad + E\{Y_0^2 - (EY_0)^2 | D = 0, \kappa_0 = 0 \\ &\qquad Z_1 = w,\ \nu_1 = 0\} + \sigma_n^2 \\ &= [G^2F(qw + q_d) + \sigma_n^2] + [G^2F(qw + q_d) + \sigma_n^2] \\ &= 2G^2F(qw + q_d) + 2\sigma_n^2,\end{aligned} \tag{21}$$

$$\begin{aligned}m_{\rm neq} &= E\{Y_1 - Y_0 | D=0,\, \kappa_0 = 0,\, Z_1 \neq w,\, \nu_1 = 0\}\\ &= Gq_d - G(qw+q_d) = -Gqw \end{aligned} \tag{22}$$

$$\begin{aligned}\sigma_{\rm neq}^2 &= E\{Y_1^2 - (EY_1)^2 | D=0,\, \kappa_0 = 0\\ &\qquad Z_1 \neq w,\, \nu_1 = 0\} + \sigma_n^2\\ &\quad + E\{Y_0^2 - (EY_0)^2 | D=0,\, \kappa_0 = 0\\ &\qquad Z_1 \neq w,\, \nu_1 = 0\} + \sigma_n^2\\ &= [G^2 F q_d + \sigma_n^2] + [G^2 F(qw+q_d) + \sigma_n^2]\\ &= G^2 F(qw + 2q_d) + 2\sigma_n^2. \end{aligned} \tag{23}$$

Here G denotes the average APD gain and q denotes the average number of absorbed photons per received single-user pulse, given by

$$q = \frac{\eta P_p T_c}{hfw} = \frac{\eta P_{av} T}{hfw^2} = \frac{\eta \lambda P_{av} T}{hCw^2} \quad \text{photons/pulse} \tag{24}$$

where

$$P_{av} = P_p \frac{wT_c}{T} \tag{25}$$

is the average single-user received laser power, f is the laser frequency, λ its wavelength, η is the APD efficiency, $h = 6.626 \times 10^{-34}$ J·s is Plank's constant, and $C = 3 \times 10^8$ m/s is the speed of light. q_d denotes the photon count due to the APD dark current within a chip interval. It is given by

$$q_d = \frac{I_d}{e} T_c \tag{26}$$

where I_d is the APD dark current and $e = 1.6 \times 10^{-19}$ Cb is the electron charge. F denotes the excess noise factor, given by

$$F = k_{\rm eff} G + (2 - 1/G)(1 - k_{\rm eff}) \tag{27}$$

where $k_{\rm eff}$ is the APD effective ionization ratio. σ_n^2 denotes the variance of the thermal noise within a chip interval. It is given by

$$\sigma_n^2 = \frac{2k_B T^\circ}{e^2 R_L} T_c \tag{28}$$

where $k_B = 1.38 \times 10^{-23}$ J/°K is Boltzmann's constant, T° is the receiver noise temperature, and R_L is the receiver load resistor. In a similar way, we can obtain

$$\begin{aligned}P_1 &\leq \tfrac{1}{2} \Pr\{Z_1' = w-1\}\\ &\quad + Q\left(-\frac{m_{\rm neq}}{\sigma_{\rm neq}}\right)(1 - \Pr\{Z_1' = w-1\}). \end{aligned} \tag{29}$$

Finally, by substituting (18) and (29) in (10), we get

$$\begin{aligned}P_E &\leq [M-1-r(\xi-1)]P_0 + r(\xi-1)P_1\\ &\leq [M-1-r(\xi-1)]\left[\tfrac{1}{2}\Pr\{Z_1 = w\}\right.\\ &\qquad \left. + Q\left(-\frac{m_{\rm neq}}{\sigma_{\rm neq}}\right)(1 - \Pr\{Z_1 = w\})\right]\\ &\quad + r(\xi-1)\left[\tfrac{1}{2}\Pr\{Z_1' = w-1\}\right.\\ &\qquad \left. + Q\left(-\frac{m_{\rm neq}}{\sigma_{\rm neq}}\right)(1 - \Pr\{Z_1' = w-1\})\right]. \end{aligned} \tag{30}$$

B. Optical OPPM-CDMA Correlator Receiver Without Hardlimiters

An upper bound to the word error probability in (5) for this case can be found in [15]

$$P_E \leq \sum_{l=1}^{N-1} [\{M-1-r(\xi-1)\}R_0 + r(\xi-1)R_1] \Pr\{\kappa_1 = l\} \tag{31}$$

where

$$R_0 = \Pr\{Y_1 \geq Y_0 | D=0,\, \kappa_0 = 0,\, \kappa_1 = l,\, \nu_1 = 0\} \tag{32}$$

and

$$R_1 = \Pr\{Y_1 \geq Y_0 | D=0,\, \kappa_0 = 0,\, \kappa_1 = l,\, \nu_1 = 1\}. \tag{33}$$

Here $\kappa_1 = \sum_{i=1}^{w} \kappa_{i1}$ denotes the interference from other users to slot 1 of the desired user. From (6), it turns out that its probability distribution is binomial:

$$\Pr\{\kappa_1 = l\} = \binom{N-1}{l} p^l (1-p)^{N-1-l} \tag{34}$$

where

$$p = w \cdot p_1 = \frac{\gamma w^2}{(M-1+\gamma)L}. \tag{35}$$

Using the continuous Gaussian approximation for both Y_0 and Y_1, we can show that

$$R_0 = Q\left(-\frac{m_0}{\sigma_0}\right) \quad \text{and} \quad R_1 = Q\left(-\frac{m_1}{\sigma_1}\right) \tag{36}$$

or

$$\begin{aligned}P_E \leq \sum_{l=1}^{N-1} &\left[\{M-1-r(\xi-1)\} Q\left(-\frac{m_0}{\sigma_0}\right)\right.\\ &\left. + r(\xi-1) Q\left(-\frac{m_1}{\sigma_1}\right)\right] \Pr\{\kappa_1 = l\} \end{aligned} \tag{37}$$

where

$$\begin{aligned}m_0 &= E\{Y_1 - Y_0 | D=0,\, \kappa_0 = 0,\, \kappa_1 = l,\, \nu_1 = 0\}\\ &= G(ql + q_d) - G(qw + q_d) = Gq(l-w) \end{aligned} \tag{38}$$

$$\begin{aligned}\sigma_0^2 &= E\{Y_1^2 - (EY_1)^2 | D=0,\, \kappa_0 = 0\\ &\qquad \kappa_1 = l,\, \nu_1 = 0\} + \sigma_n^2\\ &\quad + E\{Y_0^2 - (EY_0)^2 | D=0,\, \kappa_0 = 0\\ &\qquad \kappa_1 = l,\, \nu_1 = 0\} + \sigma_n^2\\ &= [G^2 F(ql + q_d) + \sigma_n^2] + [G^2 F(qw + q_d) + \sigma_n^2]\\ &= G^2 F[q(l+w) + 2q_d] + 2\sigma_n^2 \end{aligned} \tag{39}$$

$$\begin{aligned}m_1 &= E\{Y_1 - Y_0 | D=0,\, \kappa_0 = 0,\, \kappa_1 = l,\, \nu_1 = 1\}\\ &= G[q(l+1) + q_d] - G(qw + q_d) = Gq(l+1-w) \end{aligned} \tag{40}$$

$$\begin{aligned}\sigma_1^2 &= E\{Y_1^2 - (EY_1)^2 | D=0,\, \kappa_0 = 0\\ &\qquad \kappa_1 = l,\, \nu_1 = 1\} + \sigma_n^2\\ &\quad + E\{Y_0^2 - (EY_0)^2 | D=0,\, \kappa_0 = 0\\ &\qquad \kappa_1 = l,\, \nu_1 = 1\} + \sigma_n^2\\ &= [G^2 F\{q(l+1) + q_d\} + \sigma_n^2] + [G^2 F(qw + q_d) + \sigma_n^2]\\ &= G^2 F[q(l+1+w) + 2q_d] + 2\sigma_n^2. \end{aligned} \tag{41}$$

TABLE I
TYPICAL LASER LINK PARAMETERS

Name	Symbol	Value
APD responsivity (at unity gain)	$\mathcal{R} = e\eta\lambda/hC$	0.84 A/W
APD gain	G	100
APD eff. ionization ratio	k_{eff}	0.02
APD dark current	I_d	1 nA
Receiver load resistor	R_L	50 Ω
Data bit rate	R_T	12 Mb/s
Laser pulsewidth	T_c	0.03 ns
Receiver noise temperature	T°	300, 1000 °K

IV. NUMERICAL RESULTS

In our numerical calculations, we hold the data rate per user fixed at $R_T = 12$ Mb/s and assume that the laser pulsewidth equals $T_c = 0.03$ ns. This arbitrary (and practical) choice involves a constant throughput per chip time of $R_0 = R_T T_c = 2.5 \times 10^{-4}$ nats/chip time. Other parameters for our link are shown in Table I. For any given pulse-position multiplicity M and index of overlap γ, the signature code length L is determined as the maximum length that satisfy the above throughput constraint and the condition given in (2). The code weight w is determined as the maximum weight that satisfies the constraint on the number of users N as was determined in [2]

$$N \leq \frac{L-1}{w(w-1)}. \tag{42}$$

The bit error rates for the OPPM-CDMA systems with and without optical hardlimiters are shown in Figs. 3 and 4, versus the average received laser power, for $N = 100$ and 150, respectively, and for two different values of noise temperatures $T^\circ \in \{300, 1000\}$ K. The two parameter M and γ are changed as N changes so as to control the error rate below 10^{-9}. The corresponding Poisson shot-noise-limited error probabilities (obtained from [15]) are depicted in the same figures as well. It is evident that thermal noise degrades the decoding performance as the noise temperature increases. However, it can be seen that by increasing the average laser power by 10 dB, the receiver with double hardlimiters can tolerate the thermal noise effect and achieve same error probability as the Poisson shot-noise-limited case. This is because optical CDMA systems exhibit error probability floors on their performance. For the receivers without hardlimiters, the performance is far above the required threshold of 10^{-9}. Lower bounds on the bit error probabilities for the OPPM-CDMA system with optimum receiver (obtained from [15]) are included in the figures as well. It can be seen that the performance of the OPPM-CDMA systems with double optical hardlimiters can be very close to that of the OPPM-CDMA system with optimum receiver when the power levels exceed -60 dBm. The difference between the error probability floors of both the optimum and the proposed OPPM-CDMA receivers is in fact much smaller (probably zero) than that in Figs. 3 and 4. This is because here we are comparing a lower bound on the bit error rate of the optimum OPPM-CDMA receiver with upper bounds on that of the proposed receivers.

The corresponding curves for the optical OOK-CDMA systems with and without double optical hardlimiters (cf., the Appendix) are plotted in Fig. 5 for the sake of comparison. It is evident that these systems do not grant reliable communication under the aforementioned constraints. The throughput and/or the number of users should be reduced in this case to keep the error rate below the required threshold.

In Fig. 6 we plot the bit error rates for both OOK- and OPPM-CDMA systems with double optical hardlimiters versus the number of users. The parameters M and γ are selected such that the throughput becomes maximum (or close to maximum). It has been shown in [21] that for a given γ, the parameter M would achieve a very-close-to-maximum throughput if it equals $\lfloor 2\gamma/\log(1+\gamma) \rfloor$. From this figure it can be extracted that, under a constraint of 10^{-9} on the bit error rate, the OPPM-CDMA system with double hardlimiters can accommodate 261 users each transmitting at 12 Mb/s, whereas the OOK-CDMA system with double hardlimiters can only accommodate 50 users. Of course this large improvement is acquired at the expense of increasing the system complexity by increasing the code length. Thus for the case of OPPM-CDMA systems, there is a wide room for trading between the number of users and the hardware complexity. Careful examination of Fig. 6 shows that for small number of users ($N < 30$) the error rate of the OOK-CDMA system with double hardlimiters improves as N increases. This is because we always choose the code weight w as the maximum weight that satisfies the constraint in (42). Thus for small values of N and a fixed value of the code length, w is large and hence the hit probability among the users' codes is large as well leading to a large multiple-user interference. As N increases, both w and the interference decrease till the number of users reaches a certain value, above which the multiple-user interference starts to increase (even though w is small) due to the large value of N.

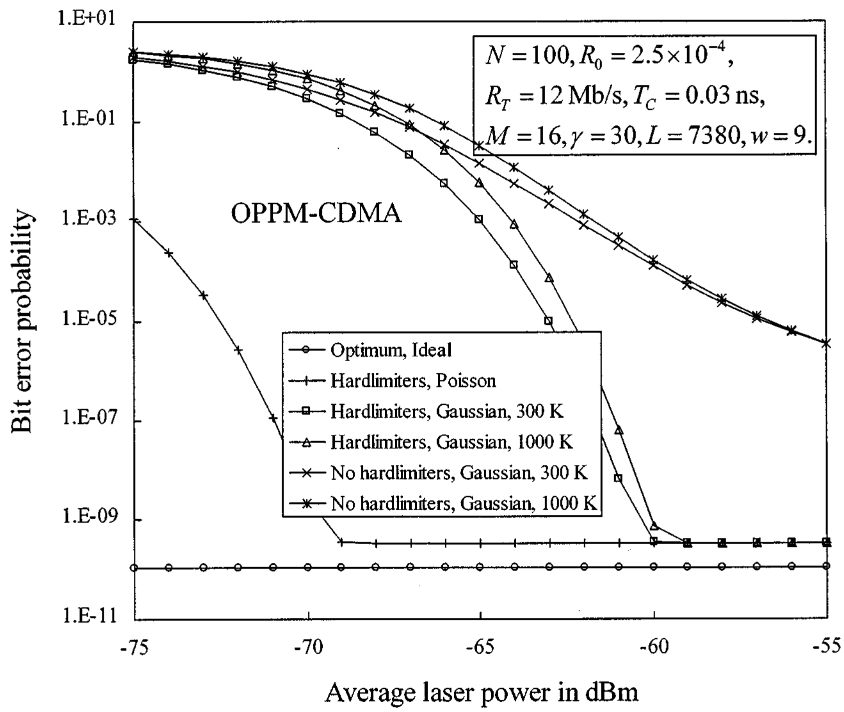

Fig. 3. Bit-error probabilities versus the average laser power for OPPM-CDMA systems with $N = 100$ and different receiver temperatures.

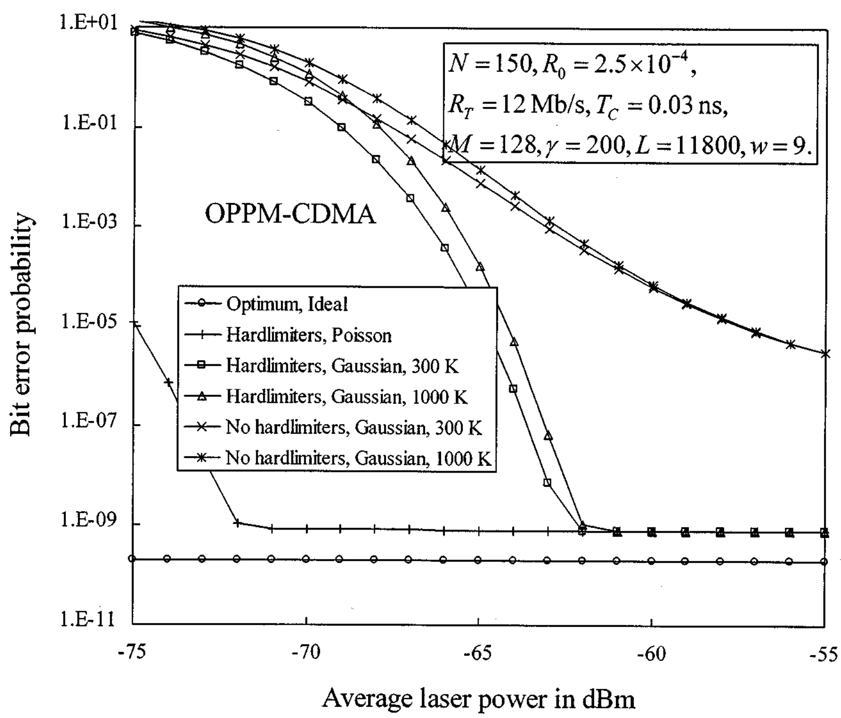

Fig. 4. Bit-error probabilities versus the average laser power for OPPM-CDMA systems with $N = 150$ and different receiver temperatures.

Finally the effect of the APD mean gain G for the ideal (shot-noise-limited with $I_d = 0$ and $T^\circ = 0$) and nonideal cases is shown in Fig. 7. The Poisson case is also included for convenience. When the thermal noise and the dark current are not factors, the results show that the performance is degraded by the added randomness of the gain more than it is improved by the increase in the signal power. On the other hand, the performance improves as the gain increases if the thermal noise and the dark current are significantly large.

V. Concluding Remarks

The effect of the thermal noise and the APD noise on the performance of direct-detection optical OPPM-CDMA systems

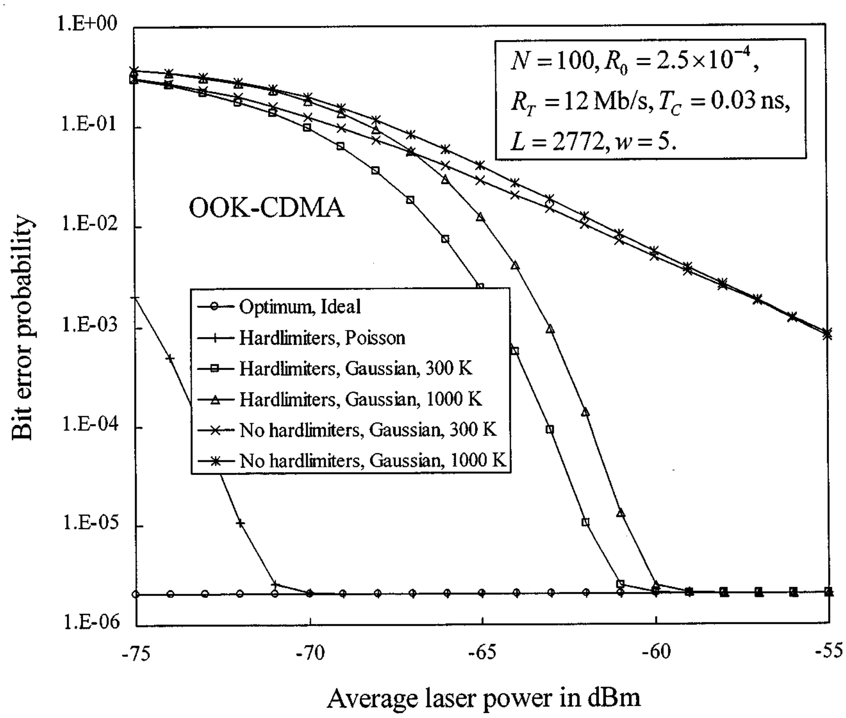

Fig. 5. Bit-error probabilities versus the average laser power for OOK-CDMA systems with $N = 100$ and different receiver temperatures.

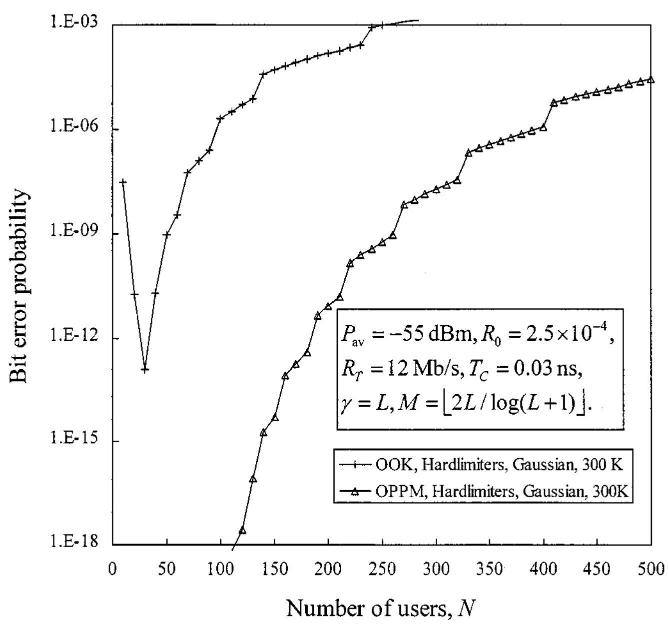

Fig. 6. Bit-error probabilities, for both OOK- and OPPM-CDMA systems with double optical hardlimiters, versus the number of users with close-to-optimum parameters and average power of -55 dBm.

with and without double optical hardlimiters has been studied. The Gaussian approximation has been employed in our derivations of the bit error rates. Our results have also been compared to the performance of the corresponding OOK-CDMA systems. We conclude with the following few remarks.

1) About 10 dB increase in the average power is required to compensate for the performance degradation due to the thermal noise and the APD noise (with respect to the Poisson shot-noise-limited system).
2) The required amount of average power, for the performance of optical OPPM-CDMA systems with double optical hardlimiters to be almost equal to that of the optimum receiver, is still very small (less than -55 dBm).
3) Given an average power of -55 dBm and a bit error rate of 10^{-9}, a data rate of 3 Gb/s can be achieved when using optical OPPM-CDMA systems with double optical hardlimiters, whereas only 600 Mb/s can be achieved with OOK-CDMA systems under same pulsewidth constraint of 0.03 ns.

APPENDIX
OPTICAL OOK-CDMA RECEIVER WITH DOUBLE OPTICAL HARDLIMITERS

We derive here an expression for the bit error rate in the case of OOK-CDMA system with double optical hardlimiters.

A. The Decision Rule

A threshold θ is set. If the collected photon count in any bit interval is greater than this threshold, "1" is declared, otherwise "0" is declared to be sent. Denoting the photon count collected in one bit interval by Y, then the probability of bit error is given by

$$P_b = \frac{1}{2} \min_{\theta}(\Pr\{Y \leq \theta | D = 1\} + \Pr\{Y > \theta | D = 0\}) \quad (43)$$

where D denotes the transmitted data bit. Assuming the contin-

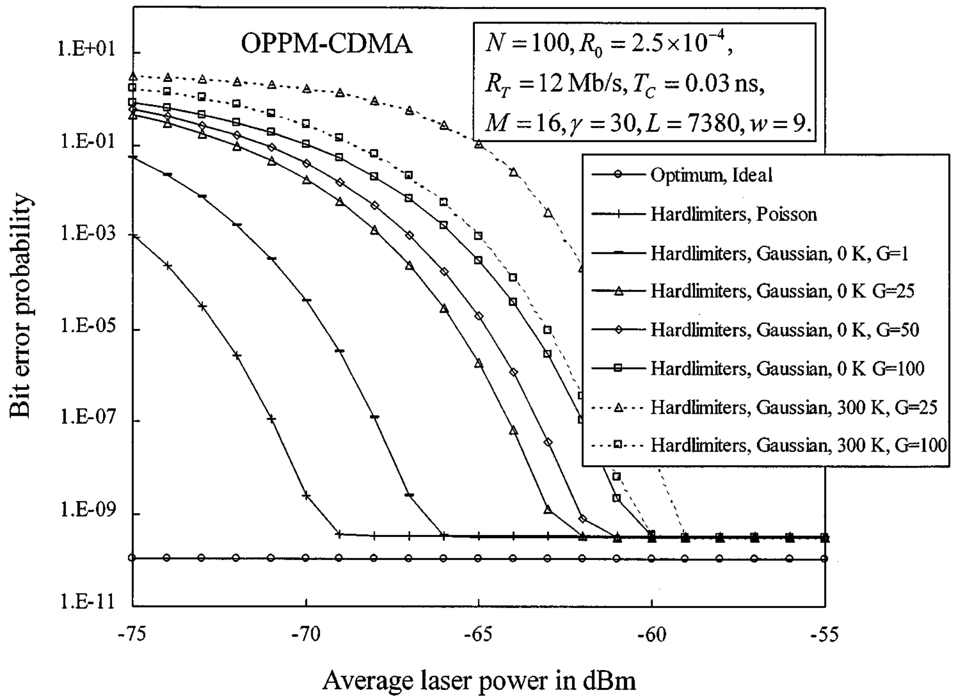

Fig. 7. Bit error probabilities versus the average laser power for OPPM-CDMA systems with $N = 100$ and various APD gain values.

uous Gaussian approximation for the photon count Y, we obtain for the first probability in the right-hand side of (43)

$$\Pr\{Y \le \theta | D = 1\} = Q\left(\frac{m_1 - \theta}{\sigma_1}\right) \tag{44}$$

where

$$m_1 = E\{Y|D=1\} = G\left(qw + \frac{I_d}{e} T_c\right) \tag{45}$$

$$\begin{aligned}\sigma_1^2 &= E\{Y^2 - (EY)^2 | D = 1\} + \sigma_n^2 \\ &= G^2 F\left(qw + \frac{I_d}{e} T_c\right) + \sigma_n^2.\end{aligned} \tag{46}$$

Here q denotes the average number of absorbed photons per received single-user pulse, given by

$$q = \frac{\eta P_p T_c}{hfw} = \frac{2\eta P_{av} L T_c}{hfw^2} = \frac{2\eta\lambda P_{av} L T_c}{hCw^2} \quad \text{photons/pulse.} \tag{47}$$

The rest of the parameters are defined as before. Next, consider the second probability in the right-hand side of (43):

$$\begin{aligned}&\Pr\{Y > \theta | D = 0\} \\ &\quad = \Pr\{Y > \theta | D = 0, Z = w\}\Pr\{Z = w\} \\ &\quad\quad + \Pr\{Y > \theta | D = 0, Z \ne w\}\Pr\{Z \ne w\}\end{aligned} \tag{48}$$

where the random variable $Z \in \{0, 1, \cdots, w\}$ denotes the number of interfered mark positions in the bit interval of the desired user immediately after the first optical hardlimiter. It can be evaluated in a similar way to (15). Hence

$$\Pr\{Z = w\} = \sum_{i=0}^{w} (-1)^i \binom{w}{i} \left[1 - i\frac{w}{2L}\right]^{N-1}. \tag{49}$$

The remaining probabilities in (48) are calculated as follows:

$$\Pr\{Y > \theta | D = 0, Z = w\} = Q\left(\frac{\theta - m_0'}{\sigma_0'}\right) \tag{50}$$

and

$$\Pr\{Y > \theta | D = 0, Z \ne w\} = Q\left(\frac{\theta - m_0}{\sigma_0}\right) \tag{51}$$

respectively, where

$$m_0' = E\{Y|D=0, Z=w\} = G\left(qw + \frac{I_d}{e} T_c\right) = m_1 \tag{52}$$

$$\begin{aligned}\sigma_0'^2 &= E\{Y^2 - (EY)^2 | D = 0, Z = w\} + \sigma_n^2 \\ &= G^2 F\left(qw + \frac{I_d}{e} T_c\right) + \sigma_n^2 = \sigma_1^2,\end{aligned} \tag{53}$$

$$m_0 = E\{Y|D=0, Z\ne w\} = G\left(\frac{I_d}{e} T_c\right) \tag{54}$$

and

$$\begin{aligned}\sigma_0^2 &= E\{Y^2 - (EY)^2 | D = 0, Z \ne w\} + \sigma_n^2 \\ &= G^2 F\left(\frac{I_d}{e} T_c\right) + \sigma_n^2.\end{aligned} \tag{55}$$

After some algebraic manipulations involving the optimization in (43), we obtain

$$P_b = \frac{1}{2}\Pr\{Z = w\} + Q\left(\frac{m_1 - m_0}{\sigma_1 + \sigma_0}\right)(1 - \Pr\{Z = w\}). \tag{56}$$

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