Direct Detection Optical Overlapping PPM-CDMA Communication Systems with Double Optical Hardlimiters

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***Abstract*— Direct detection optical code-division multiple-access (CDMA) communication systems involving overlapping pulse-position modulation (OPPM) is considered. Double optical hardlimiters placed before and after the correlator at the receiver side is proposed for this system. The performance (in terms of the bit error probability) of this system is evaluated taking into account the effect of both the multiple-user interference and the photodetector shot noise. Both the receiver dark current and thermal noise are ignored in our analysis since their effect is very minor. The performance of the above receiver is compared to that of the OPPM-CDMA correlator receiver without hardlimiters, OPPM-CDMA optimum receiver, and OOK-CDMA optimum receiver. Our results reveal that, for given pulsewidth and throughput constraints, significant improvement in the performance is acquired when adding double optical hardlimiters to the correlator of the OPPM-CDMA receiver. Moreover the performance of this system is *asymptotically* close to the optimum OPPM-CDMA system and is considerably superior to the optimum OOK-CDMA system. It is also reported that the capacity of the proposed system is about 5.3 times greater than that of the optimum OOK-CDMA system.**

***Index Terms*— Code division multiple access (CDMA), direct detection optical channel, ON–OFF keying, optical CDMA, optical CDMA optimum receivers, optical hardlimiters, overlapping pulse-position modulation, pulse-position modulation.**

I. INTRODUCTION

OPTICAL code division multiple access (CDMA) stands as an attractive multiplexing candidate for future ultrafast all-optical data networks [1]–[10]. This is because CDMA is an asynchronous scheme which does not require time synchronization or frequency management as in the case of time division multiple access (TDMA) or wavelength division multiple access (WDMA), respectively.

Conventional optical ON–OFF keying (OOK) CDMA systems need very long code sequences with good auto- and cross-correlation properties in order to support many simultaneous users. The laser pulsewidth must in turn be stringently shortened so as to be able to achieve the requirements on the very high data rates. Recently, it has been shown that new modulation schemes, such as optical overlapping pulse-position modulation (OPPM) can offer large throughput without the need to decrease the laser pulsewidth [11]–[14]. In [10], we have suggested using synchronous OPPM technique in optical CDMA channels. Only correlator receivers have been used in that study and we were able to show that under the constraints of both throughput and chip time, OPPM-CDMA system superperforms both OOK- and PPM-CDMA systems. Ohtsuki [8] has studied the performance of OOK-CDMA when double optical hardlimiters are placed before and after the optical correlator at the receiving end. He was able to show that a significant improvement can be gained when using double hardlimiters' correlator in the detection of OOK-CDMA.

In this paper we aim at studying the performance of OPPM-CDMA when double optical hardlimiters are added to the correlator at the receiver side. We employ a slightly different model than the one described in [10]. Namely, *wrapped signals* (as defined in [10]) are not permitted here and focus is given to asynchronous system models rather than synchronous ones. This of course will reduce the complexity of both the transmitter and the receiver of the OPPM-CDMA system at the expense of reducing the maximum achievable throughput. The effect of both the multiple-user interference and the photodetector's Poisson shot noise is taken into account in our theoretical derivations. The effect of both the receiver dark current and thermal noise is, however, neglected since it is very minor. Although we are considering asynchronous CDMA system model, we derive our results under the assumption of chip-synchronous uniformly distributed relative delays among the receivers. This reduces the complexity of the analysis and gives more insights into the problem under consideration. In the more realistic situation of chip-asynchronous, the results will improve even further since the bit error rates in chip-synchronous systems provide upper bounds to that of chip-asynchronous ones [4]. Finally, we choose optical orthogonal codes (OCC's), with cross-correlation and out-of-phase autocorrelation not exceeding one, as the signature code sequences for the channel users [1].

The performance of the OPPM-CDMA receiver with double hardlimiters is then compared to that of the OPPM-CDMA receiver without hardlimiters. Further, it is compared to the performance of both OOK- and OPPM-CDMA optimum receivers. Our results reveal that OPPM-CDMA systems with correlator receivers and double optical hardlimiters considerably outperform both OOK-CDMA systems with optimum

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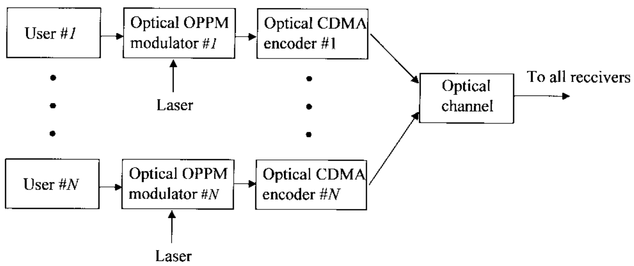

Fig. 1. Optical overlapping PPM-CDMA transmitter model.

receivers and OPPM-CDMA systems with correlator receivers only. Moreover, they perform very close to the OPPM-CDMA systems with optimum receivers.

The rest of the paper is organized as follows. The optical asynchronous OPPM-CDMA system models are described in Section II. Section III is devoted for the derivation of the bit error probabilities of the OPPM-CDMA correlator receiver with and without double optical hardlimiters and for the derivation of the bit error probability of the OPPM-CDMA optimum receiver. Numerical results and comparisons between the performance of the above systems and the performance of the optimum OOK-CDMA system are presented in Section IV for different values of design parameters. Finally, the conclusion is given in Section V.

II. Optical OPPM-CDMA System Models

A. Optical OPPM-CDMA Transmitter Model

The model for an optical OPPM-CDMA transmitter is shown in Fig. 1. It consists of N simultaneous users. Each user transmits M-ary data symbols continuously and asynchronously. Each data symbol is then passed to an optical OPPM modulator, where a narrow optical pulse (of width T_c) is generated and time delayed in accordance to the input symbol. Each optical pulse is then passed to a CDMA encoder where it is spread into w shorter laser pulses with same width T_c according to the signature sequence (of length L and weight w) which characterizes the encoder. Since overlapping is allowed in this type of signal format, the spreading interval is usually greater than the unit time delay of the OPPM modulator leading to an overlap between any two adjacent spreading intervals. The CDMA encoder can be realized simply with the aid of an optical splitter, optical delayers, and an optical combiner in exactly the same way as was done in optical PPM-CDMA systems [6]. Finally, all spreading signals from all users are combined together to form one optical signal which is transmitted across the optical network to all receivers.

Given a code length L, an index of overlap γ, and a pulse-position multiplicity M, the signal formats can be generated as follows. A spreading interval (called slot) of duration $\tau = LT_c$ is subdivided into γ smaller subintervals of width τ/γ each. The unit time delay of the OPPM modulator is set equal to τ/γ. This allows an overlap with depth $(1 - 1/\gamma)\tau$ to occur between any two adjacent spreading intervals. Since there are M possible OPPM pulse positions within the time frame, the full width of any time frame T is given by

$$T = (M-1)\frac{\tau}{\gamma} + \tau = (M-1+\gamma)\frac{L}{\gamma}T_c.$$

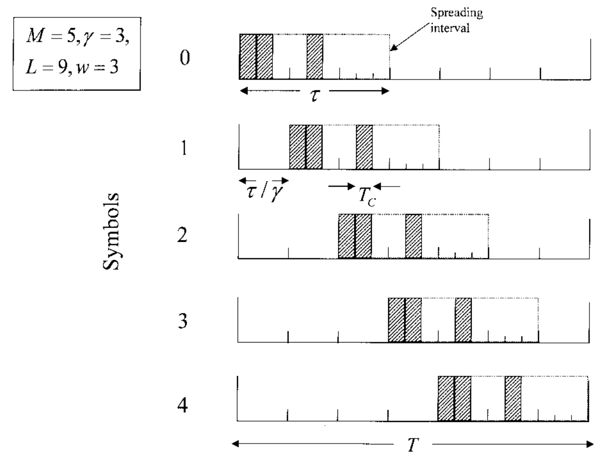

Fig. 2. An example of the transmitted signal formats of single user in an OPPM-CDMA system with $M = 5$, $\gamma = 3$, $L = 9$, and $w = 3$. A signature code of 110010000 is assumed.

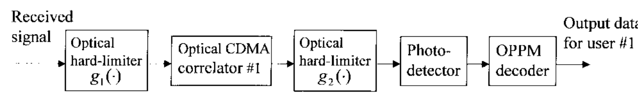

Fig. 3. Optical direct detection OPPM-CDMA correlator receiver with double optical hardlimiters.

An example of the transmitted signal formats of a single user is shown in Fig. 2. For the spreading sequence to fit properly within the spreading interval, the following condition must be satisfied

$$\frac{\tau}{\gamma} = \text{integer} \times T_c \qquad \text{or} \qquad \frac{L}{\gamma} = \text{integer}.$$

It should be noted that PPM-CDMA is a special case of OPPM-CDMA when $\gamma = 1$.

B. Optical OPPM-CDMA Receiver Models

1) OPPM-CDMA Correlator Receiver: At the receiver side, the received optical signal from all N users is correlated with the same signature sequence which characterizes the desired user and then converted (using a photodetector) into an electrical signal which is passed to the OPPM demodulator to decide on the data. The correlator can also be an optical tapped delay line matched to the corresponding code sequence [6]. The OPPM demodulator is a comparator device which selects the slot that contains the greatest photon count among all the M time slots.

2) OPPM-CDMA Correlator Receiver with Double Optical Hardlimiters: This receiver model is similar to the correlator receiver except that two optical hardlimiters are added, one is placed before the correlator and the second is placed after the correlator, Fig. 3.

3) OPPM-CDMA Optimum Receiver: Rather than using an optical correlator at the receiving end, an optimum decision rule is designed (based on the MAP criterion) so as to minimize the probability of error.

III. Bit Error Probabilities

A. The Interference Probability

Given a desired user and a single interference user, the latter can interfere with the former at 1 or 2 mark positions or does not interfere at all (cf., Fig. 8). The probability of occurrence of one hit is of order $o(\gamma w^2/ML)$, where $o(t) \to 0$ as $t \to 0$, whereas the probability of occurrence of two hits is of order $o(\gamma^2 w^4/M^2L^2)$ which is negligible with respect to the one hit probability since the values of M and L are usually large enough. In our analysis, we assume that if a user interferes with the desired user, then it will cause interference at only one mark position. In the Appendix, we derive the average probability that a single user interferes with the desired user at any mark position of its signature code. Denoting this probability by p, we have

$$p = \frac{\gamma w^2}{(M-1+\gamma)L}. \tag{1}$$

Further we conclude that the average probability that a single user interferes with the desired user at one particular mark position of its signature code is given by

$$p_1 = \frac{p}{w} = \frac{\gamma w}{(M-1+\gamma)L}.$$

If we denote by $\kappa_j,\ j \in \mathcal{M} \stackrel{\text{def}}{=} \{0, 1, \ldots, M-1\}$, the number of other users that cause interference to the desired user within slot j, then the random variable κ_j admits a binomial distribution with parameters p and $N-1$:

$$\Pr\{\kappa_j = l_j\} = \binom{N-1}{l_j} p^{l_j}(1-p)^{N-1-l_j}$$

where $l_j \in \{0, 1, \ldots, N-1\}$ is a realization for κ_j. Moreover, let $\kappa_{ij},\ i \in \mathcal{X} \stackrel{\text{def}}{=} \{1, 2, \ldots, w\},\ j \in \mathcal{M}$, be the number of pulses (from other users) that cause interference to chip i of the mark positions (of the desired user's code) in slot j and let the vector $(\kappa_{1j}, \kappa_{2j}, \ldots, \kappa_{nj})^T,\ n \in \mathcal{X}$, be denoted by κ_j^n. Then κ_j^n is a multinomial random vector with parameters $p_1 = p/w$ and $N-1$

$$\Pr\{\kappa_j^n = l_j^n\} = \frac{(N-1)!}{l_{1j}! l_{2j}! \ldots l_{nj}! s_{nj}!} \times \left(\frac{p}{w}\right)^{N-1-s_{ij}} \left(1 - \frac{np}{w}\right)^{s_{nj}}$$

where

$$l_j^n = (l_{1j}, l_{2j}, \ldots, l_{nj})^T$$

and

$$s_{nj} = N - 1 - \sum_{i=1}^{n} l_{ij}, \qquad n \in \mathcal{X},\ j \in \mathcal{M}.$$

B. The Decision Rule and the Probability of Error

We describe here the decision rule for the OPPM-CDMA correlator receiver with and without double hardlimiters. Each receiver counts the photons collected from all the mark positions of its signature code within each slot in the time frame. The decision will be in favor of the slot which contains the largest count. We denote the photon count collected from slot $i \in \mathcal{M}$ by Y_i. Symbol i is thus declared to be the true one if $Y_i > Y_j$ for every $j \neq i$. Hence, if D denotes the transmitted data symbol, then the probability of error can be written as

$$P[E] = \sum_{i=0}^{M-1} \Pr\{Y_j \geq Y_i,\ \text{some } j \neq i | D = i\} \Pr\{D = i\}$$

where $\Pr\{D = i\} = 1/M$ for equiprobable data symbols.

C. Optical OPPM-CDMA Correlator Receiver

We derive here an upper bound on the error probability of the correlator receiver. Denoting the vector $(\kappa_i, \kappa_{i+1}, \ldots, \kappa_j)^T$ by $\mathbf{K}_i^j$ and its realization $(l_i, l_{i+1}, \ldots, l_j)^T$ by $\mathbf{L}_i^j$, we have

$$\begin{aligned}
&\Pr\{Y_j \geq Y_i,\ \text{some } j \neq i | D = i\} \\
&\quad = \sum_{l_0^{M-1}} \Pr\{Y_j \geq Y_i,\ \text{some } j \neq i | D = i,\ \mathbf{K}_0^{M-1} = \mathbf{L}_0^{M-1}\} \\
&\qquad \times \Pr\{\mathbf{K}_0^{M-1} = \mathbf{L}_0^{M-1}\} \\
&\quad \leq \sum_{l_0^{i-1},\, l_{i+1}^{M-1}} \Pr\{Y_j \geq Y_i,\ \text{some } j \neq i | D = i,\ \kappa_i = 0, \\
&\qquad\qquad \mathbf{K}_0^{i-1} = \mathbf{L}_0^{i-1},\ \mathbf{K}_{i+1}^{M-1} = \mathbf{L}_{i+1}^{M-1}\} \\
&\qquad\qquad \times \Pr\{\mathbf{K}_0^{i-1} = \mathbf{L}_0^{i-1},\ \mathbf{K}_{i+1}^{M-1} = \mathbf{L}_{i+1}^{M-1}\}.
\end{aligned}$$

We further use a union bound, taking into account that the out-of-phase autocorrelation is bounded by one and that at most $\gamma - 1$ hits (self-interference) may occur in other slots due to code pulses in one slot of the desired user

$$\begin{aligned}
&\Pr\{Y_j \geq Y_i,\ \text{some } j \neq i | D = i,\ \kappa_i = 0, \\
&\qquad \mathbf{K}_0^{i-1} = \mathbf{L}_0^{i-1},\ \mathbf{K}_{i+1}^{M-1} = \mathbf{L}_{i+1}^{M-1}\} \\
&\quad \leq (M - \xi) \Pr\{Y_1 \geq Y_0 | D = 0,\ \kappa_0 = 0,\ \kappa_1 = l_1,\ \nu_1 = 0\} \\
&\qquad + \sum_{j=1}^{\xi-1} \Pr\{Y_j \geq Y_0 | D = 0,\ \kappa_0 = 0,\ \kappa_j = l_j\}
\end{aligned}$$

where $\xi = \min\{M, \gamma\}$ and for any $j \in \{1, 2, \ldots, M-1\}$, $\nu_j \in \{0, 1\}$ denotes the number of pulses that cause a hit (self-interference) in slot j due to the signature code pulses sent in slot 0 by the desired user. The first term in the right hand side of the last inequality is due to the $M - 1 - (\xi - 1)$ slots that do not have self-interference with slot 0, i.e., $\nu_j = 0$ with probability 1 for these slots. The second term, however, is due to the remaining $\xi - 1$ slots. These slots interfere with slot 0 at a positive probability. Assuming uniformly distributed marks in the code sequences it is easy to check that $\Pr\{\nu_j = 1\} \leq w(w-1)/(L-1)$. Hence

$$\begin{aligned}
&\Pr\left\{Y_j \geq Y_0,\ \text{some } j \neq i | D = i,\ \kappa_0 = 0, \right. \\
&\qquad \left. \mathbf{K}_0^{i-1} = \mathbf{L}_0^{i-1},\ \mathbf{K}_{i+1}^{M-1} = \mathbf{L}_{i+1}^{M-1}\right\} \\
&\quad \leq (M - \xi) \Pr\{Y_1 \geq Y_0 | D = 0,\ \kappa_0 = 0,\ \kappa_1 = l_1,\ \nu_1 = 0\} \\
&\qquad + r(\xi - 1) \Pr\{Y_1 \geq Y_0 | D = 0,\ \kappa_0 = 0,\ \kappa_1 = l_1,\ \nu_1 = 1\} \\
&\qquad + (1 - r)(\xi - 1) \Pr\{Y_1 \geq Y_0 | D = 0,\ \kappa_0 = 0, \\
&\qquad\qquad \kappa_1 = l_1,\ \nu_1 = 0\}
\end{aligned}$$

where $r \stackrel{\text{def}}{=} w(w-1)/(L-1)$. Let Q denote the average number of photons per pulse. We thus obtain the following upper bound on the word error probability

$$P[E] \le \sum_l [\{M-1-r(\xi-1)\}P_0 + r(\xi-1)P_1] \times \Pr\{\kappa_1 = l\}$$

where

$$\Pr\{\kappa_1 = l\} = \binom{N-1}{l} p^l (1-p)^{N-1-l},$$

$$P_0 = \Pr\{Y_1 \ge Y_0 | D=0, \kappa_0 = 0, \kappa_1 = l, \nu_1 = 0\} = \sum_{y_1=0}^{\infty} e^{-Ql}\frac{(Ql)^{y_1}}{y_1!} \cdot \sum_{y_0=0}^{y_1} e^{-Qw}\frac{(Qw)^{y_0}}{y_0!}$$

and

$$P_1 = \Pr\{Y_1 \ge Y_0 | D=0, \kappa_0 = 0, \kappa_1 = l, \nu_1 = 1\} = \sum_{y_1=0}^{\infty} e^{-Q(l+1)}\frac{(Q(l+1))^{y_1}}{y_1!} \cdot \sum_{y_0=0}^{y_1} e^{-Qw}\frac{(Qw)^{y_0}}{y_0!}.$$

The bit error rate, P_b, can be obtained from the relation $P_b = (M/2)/(M-1)P[E]$, [15, ch. 6].

D. Optical OPPM-CDMA Correlator Receiver with Double Optical Hardlimiters

In this section, we derive an upper bound on the word error probability when double optical hardlimiters are placed before and after the optical correlator. Since Q denotes the average number of photons per pulse, we define the first hardlimiter (before the correlator) as follows:

$$g_1(x) = \begin{cases} Q; & \text{if } x \ge Q \\ 0; & \text{otherwise} \end{cases}$$

where x denotes the input photons per chip. The second hardlimiter (after the correlator) can be defined as follows:

$$g_2(x) = \begin{cases} Qw; & \text{if } x \ge Qw \\ 0; & \text{otherwise.} \end{cases}$$

Consider the probability

$$\begin{aligned}\Pr\{Y_j \ge Y_i,\ \text{some } j \ne i | D=i\} &= \Pr\{Y_j \ge Y_i,\ \text{some } j \ne i, Y_i = 0 | D=i\} \\ &\quad + \Pr\{Y_j \ge Y_i,\ \text{some } j \ne i, Y_i \ge 1 | D=i\} \\ &= \Pr\{Y_i = 0|D=i\} + \Pr\{Y_j \ge Y_i \ge 1, \\ &\quad \text{some } j \ne i | D=i\}.\end{aligned}$$

Following a similar argument to what was done in Section III-C above, we obtain the following upper bound on the word error probability

$$\begin{aligned}P[E] \le \Pr\{Y_0 = 0|D=0\} &+ \{M-1-r(\xi-1)\} \\ &\times \Pr\{Y_1 \ge Y_0 \ge 1 | D=0, \nu_1 = 0\} \\ &+ r(\xi-1)\Pr\{Y_1 \ge Y_0 \ge 1|D=0, \nu_1 = 1\}.\end{aligned}$$

The first probability is given by $\Pr\{Y_0 = 0|D=0\} = \exp[-Qw]$. The last two probabilities can be evaluated as follows. Let the random variable T denote the number of interfered mark positions in slot 1 of the desired user immediately after the first optical hardlimiter. Of course T can take values only in the discrete set $\{0, 1, \ldots, w\}$. The average collected photon count in slot 1 at the input of the second optical hardlimiter in this case is equal to QT and at the input of the photodetector is either Qw if $T = w$ or 0 if $T < w$. The second probability is thus given by

$$\Pr\{Y_1 \ge Y_0 \ge 1 | D=0, \nu_1 = 0\} = \Pr\{T = w\}\sum_{y_1=1}^{\infty} e^{-Qw}\frac{(Qw)^{y_1}}{y_1!} \cdot \sum_{y_0=1}^{y_1} e^{-Qw}\frac{(Qw)^{y_0}}{y_0!}$$

whereas the third probability is given by

$$\Pr\{Y_1 \ge Y_0 \ge 1 | D=0, \nu_1 = 1\} = \Pr\{T' = w-1\}\sum_{y_1=1}^{\infty} e^{-Qw}\frac{(Qw)^{y_1}}{y_1!} \cdot \sum_{y_0=1}^{y_1} e^{-Qw}\frac{(Qw)^{y_0}}{y_0!}$$

where T' is a random variable that denotes the number of interfered mark positions in slot 1 of the desired user which are not self-interfered. T' can take values only in the discrete set $\{0, 1, \ldots, w-1\}$. To evaluate $\Pr\{T = w\}$ we proceed as follows:

$$\begin{aligned}\Pr\{T = w\} &= \Pr\{\kappa_{i1} \ge 1 \forall i \in \mathcal{X}\} \\ &= 1 - \Pr\{\kappa_{i1} = 0,\ \text{some } i \in \mathcal{X}\} \\ &= 1 + \sum_{i=1}^{w} (-1)^i \binom{w}{i} \Pr\{\kappa_{11} = \kappa_{21} = \cdots = \kappa_{i1} = 0\} \\ &= \sum_{i=0}^{w} (-1)^i \binom{w}{i} \left[1 - i\frac{\gamma w}{(M-1+\gamma)L}\right]^{N-1}.\end{aligned}$$

We can evaluate $\Pr\{T' = w-1\}$ in a similar way

$$\Pr\{T' = w-1\} = \sum_{i=0}^{w-1} (-1)^i \binom{w-1}{i} \left[1 - i\frac{\gamma w}{(M-1+\gamma)L}\right]^{N-1}.$$

E. Optical OPPM-CDMA Optimum Receiver

We derive here a lower bound on the error probability of the optimum receiver. Since the error probability can be decreased if we assume an ideal photodetector, we will ignore the effect of the photodetector's shot noise in our analysis throughout this section. Since self-interference will cause some of the optical energy of the data-carrying slot to be collected from other slots, we assume that self-interference events will never occur, and hence, the error probability will further be decreased.

1) The Optimum Decision Rule: Let Z_{ij}, $i \in \mathcal{X}$, $j \in \mathcal{M}$, be the number of optical pulses collected from chip number i of the mark positions (of the desired user's code) of slot number j. We denote the vector $(Z_{1j}, Z_{2j}, \ldots, Z_{wj})^T$ by Z_j^w and denote the supervector $(Z_0^w, Z_1^w, \ldots, Z_{M-1}^w)^T$ by Z^{wM}.

We decide that data symbol "m" was sent if for any $j \in \mathcal{M}$ and $j \neq m$, $\Pr\{m|Z^{wM} = z^{wM}\} > \Pr\{j|Z^{wM} = z^{wM}\}$. For equiprobable data symbols, this is equivalent to

$$\Pr\{Z^{wM} = z^{wM}|m\} > \Pr\{Z^{wM} = z^{wM}|j\}.$$

Notice that for any $j \in \mathcal{M}$

$$\begin{aligned}
&\Pr\{Z^{wM} = z^{wM}|j\}\\
&\quad = \Pr\{\kappa_0^w = z_0^w, \ldots, \kappa_{j-1}^w = z_{j-1}^w\\
&\qquad\quad (\kappa_{1j}+1, \kappa_{2j}+1, \ldots, \kappa_{wj}+1)^T = z_j^w\\
&\qquad\quad \kappa_{j+1}^w = z_{j+1}^w, \ldots, \kappa_{M-1}^w = z_{M-1}^w\}\\
&\quad = \Pr\{\kappa_0^w = z_0^w, \ldots, \kappa_{j-1}^w = z_{j-1}^w, \kappa_j^w\\
&\qquad\quad = (z_{1j}-1, z_{2j}-1, \ldots, z_{wj}-1)^T\\
&\qquad\quad \kappa_{j+1}^w = z_{j+1}^w, \ldots, \kappa_{M-1}^w = z_{M-1}^w\}.
\end{aligned}$$

Noticing that, in this case, the vector $(\kappa_0^w, \kappa_1^w, \ldots, \kappa_{M-1}^w)^T$ admits a multinomial distribution and substituting in the last inequality, we can get the following optimum decision rule.

2) The Decision Rule: Data symbol "m" is declared to be sent if for any $j \in \mathcal{M}$ and $j \neq m$

$$\prod_{i\in\mathcal{X}} Z_{im} > \prod_{i\in\mathcal{X}} Z_{ij}.$$

An incorrect decision is otherwise declared. The probability of a word error is thus given by

$$\begin{aligned}
&P[E]\\
&= \frac{1}{M}\sum_{m=0}^{M-1} \Pr\left\{\prod_{i\in\mathcal{X}} Z_{im} \le \prod_{i\in\mathcal{X}} Z_{ij},\ \text{some } j \neq m | D = m\right\}\\
&\ge \Pr\left\{\prod_{i\in\mathcal{X}} Z_{i0} \le \prod_{i\in\mathcal{X}} Z_{ij},\right.\\
&\qquad\qquad \left.\text{some } j \neq 0 | D = 0, \nu_j = 0 \forall j \neq 0\right\}\\
&\ge \Pr\left\{\prod_{i\in\mathcal{X}}(\kappa_{i0}+1) \le \prod_{i\in\mathcal{X}} \kappa_{ij},\ \text{some } j \neq 0\right\}\\
&\ge (M-1)\Pr\left\{\prod_{i\in\mathcal{X}}(\kappa_{i0}+1) \le \prod_{i\in\mathcal{X}} \kappa_{i1}\right\}\\
&\quad - \binom{M-1}{2}\Pr\left\{\prod_{i\in\mathcal{X}}(\kappa_{i0}+1) \le \prod_{i\in\mathcal{X}} \kappa_{i1}\right.\\
&\qquad\qquad \left.\text{and } \prod_{i\in\mathcal{X}}(\kappa_{i0}+1) \le \prod_{i\in\mathcal{X}} \kappa_{i2}\right\}.
\end{aligned}$$

The first probability can be lower bounded as

$$\begin{aligned}
&\Pr\left\{\prod_{i\in\mathcal{X}}(\kappa_{i0}+1) \le \prod_{i\in\mathcal{X}} \kappa_{i1}\right\}\\
&\quad \ge \Pr\left\{\kappa_{i0} = 0 \quad \forall i \in \mathcal{X}, \prod_{i\in\mathcal{X}} \kappa_{i1} \ge 1\right\}\\
&\quad = \Pr\{\kappa_{i0} = 0, \kappa_{i1} \ge 1 \quad \forall i \in \mathcal{X}\}\\
&\quad = \Pr\{\kappa_{i0} = 0 \quad \forall i \in \mathcal{X}\}\\
&\qquad - \Pr\{\kappa_{i0} = 0 \quad \forall i \in \mathcal{X}, \kappa_{n1} = 0, \text{ some } n \in \mathcal{X}\}\\
&\quad = \sum_{n=0}^{w}(-1)^n\binom{w}{n}\\
&\qquad \times \Pr\{\kappa_{i0} = 0 \forall i \in \mathcal{X}, \kappa_{11} = \kappa_{21} = \cdots = \kappa_{n1} = 0\}\\
&\quad = \sum_{n=0}^{w}(-1)^n\binom{w}{n}\left[1-(n+w)\frac{\gamma w}{(M-1+\gamma)L}\right]^{N-1}.
\end{aligned}$$

The second probability can be upper bounded as

$$\begin{aligned}
&\Pr\left\{\prod_{i\in\mathcal{X}}(\kappa_{i0}+1) \le \prod_{i\in\mathcal{X}} \kappa_{i1} \text{ and } \prod_{i\in\mathcal{X}}(\kappa_{i0}+1) \le \prod_{i\in\mathcal{X}} \kappa_{i2}\right\}\\
&\quad \le \Pr\left\{\prod_{i\in\mathcal{X}} \kappa_{i1} \ge 1 \text{ and } \prod_{i\in\mathcal{X}} \kappa_{i2} \ge 1\right\}\\
&\quad = \Pr\{\kappa_{i1} \ge 1 \text{ and } \kappa_{i2} \ge 1 \forall i \in \mathcal{X}\}\\
&\quad \le \Pr\left\{\sum_{i\in\mathcal{X}} \kappa_{i1} + \sum_{i\in\mathcal{X}} \kappa_{i2} \ge 2w\right\}\\
&\quad \le \left[\frac{(N-1)\gamma w e}{(M-1+\gamma)L}\right]^{2w}.
\end{aligned}$$

The last inequality is justified by using Chernoff bound [6].

IV. Numerical Results

In our numerical calculations we assume that the throughput per second R_T and the pulsewidth T_c are both held fixed. Thus, the throughput per chip time R_0 is fixed as well

$$\begin{aligned}
R_0 &\overset{\text{def}}{=} R_T \cdot T_c\\
&= \frac{\log M}{T} \cdot T_c\\
&= \frac{\gamma \log M}{(M-1+\gamma)L} \quad \text{nats/chip time.}
\end{aligned} \tag{2}$$

Given a throughput constraint, a pulse-position multiplicity M, and an index of overlap γ, we can obtain a code length L from the last relation along with the constraint $L/\gamma =$ integer. Given a number of simultaneous users N, a maximum code weight w is chosen so as to satisfy the constraint [1]

$$N \le \frac{L-1}{w(w-1)}.$$

The bit error probabilities for the OPPM-CDMA system with and without optical hardlimiters are shown in Fig. 4 (for

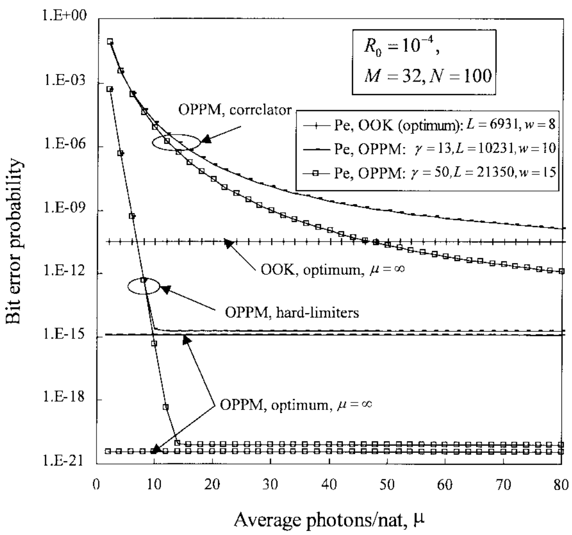

Fig. 4. A comparison between the bit error probabilities of the OPPM-CDMA correlator receivers (with and without optical hardlimiters) and both the OOK- and OPPM-CDMA optimum receivers versus the average photons per nat.

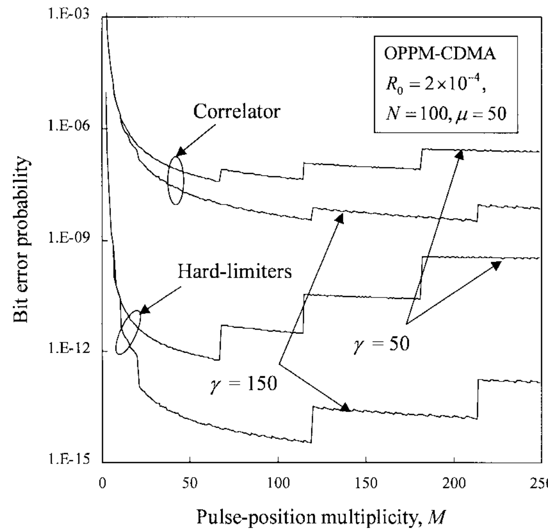

Fig. 6. A comparison between the bit error probabilities of the OPPM-CDMA correlator receivers (with and without optical hardlimiters) for two different values of the overlapping index versus the pulse-position multiplicity.

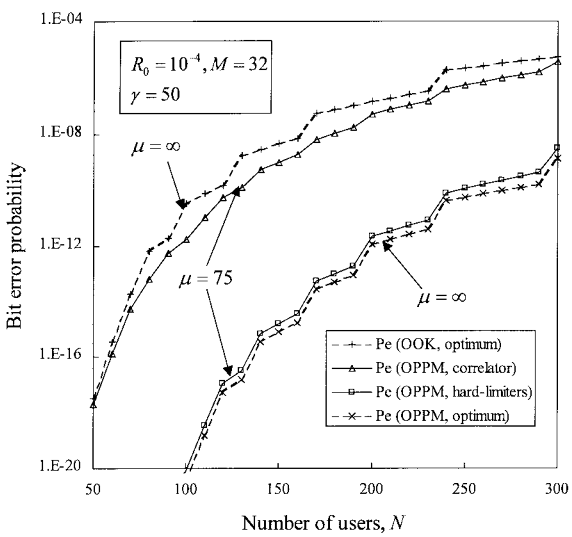

Fig. 5. A comparison between the bit error probabilities of the OPPM-CDMA correlator receivers (with and without optical hardlimiters) and both the OOK- and OPPM-CDMA optimum receivers versus the number of simultaneous user.

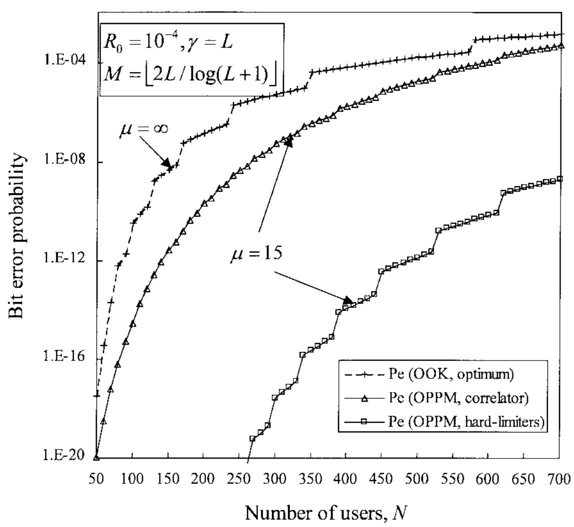

Fig. 7. A comparison between the bit error probabilities of the OPPM-CDMA correlator receivers (with and without optical hardlimiters) and the OOK-CDMA optimum receivers versus the number of simultaneous user when $\gamma = L$ and $M = \lfloor 2L/\log(L+1) \rfloor$.

$R_0 = 10^{-4}$, $M = 32$, $N = 100$, and $\gamma \in \{13, 50\}$) versus the average photons/nat, μ. It should be noted that μ is related to the average photons per chip pulse Q as follows:

$$Q = \frac{\mu \log M}{w}.$$

The asymptotic bit error probabilities for both the optimum OPPM- and OOK-CDMA systems appear in the same figure for the sake of comparison. The latter has been obtained from [9]. The corresponding values of L and w for both OPPM- and OOK-CDMA are listed in the figure as well. It should be emphasized that a longer code length does not lead to a bandwidth expansion. This is because L is chosen so as to satisfy the constraint in (2) which guarantees a fixed pulsewidth for all systems under comparison. From the figure we notice the great improvement in the error probability when using OPPM-CDMA with optical hardlimiters; it becomes very close to the optimum OPPM-CDMA receiver even for small values of μ. This improvement in the error probability can be utilized to increase the capacity of the optical CDMA system. This is explored in Fig. 5, where the probability of error is plotted versus the number of simultaneous users N. Although infinite average energy is used for both the optimum receivers, OPPM-CDMA receiver with hardlimiters is always close to the optimum OPPM-CDMA receiver for all values of N when μ is large enough. It can be extracted from the figure that for a bit error rate not exceeding 10^{-9} the optimum OOK-CDMA system cannot accommodate more that 123 users, whereas the OPPM-CDMA system with hardlimiters can accommodate up to 296 users. Thus more than double

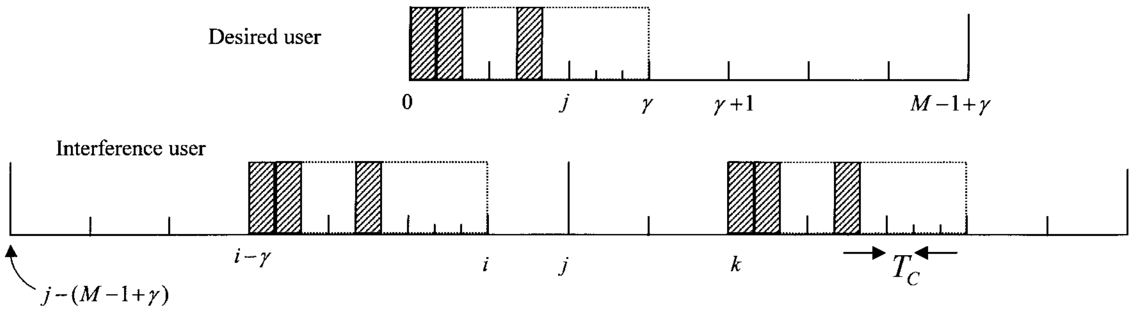

Fig. 8. Interference contribution from a single user to the desired user.

the capacity can be gained. Other choices of the design parameters (L, γ, w, and M) can improve the capacity even more. This can be extracted from Fig. 6, where we plot the error rates for the OPPM-CDMA system with and without optical hardlimiters for two different values of γ versus M. It can been seen that, given an index of overlap γ, there is always a certain value of M that corresponds to a minimum error probability. It is also obvious that the error rate decreases as γ increases. This means that during the design process both values of γ and M should be chosen properly so as to give the best performance. In Fig. 7 we plot the error probabilities for the OPPM-CDMA systems and the optimum OOK-CDMA system when γ is set to a maximum value (which is L) and M is set to $\lfloor 2L/\log(L+1) \rfloor$ (which is very close to the optimum value [14]). The code length L is then chosen so as to satisfy the throughput constraint after substituting the above expressions for both γ and M. From the figure we conclude that for a bit error rate not exceeding 10^{-9} the new choices of the design parameters allow the OPPM-CDMA system with double hardlimiters to accommodate up to 655 users which is about 5.3 times greater than that of the optimum OOK-CDMA system.

V. Concluding Remarks

A direct detection optical OPPM-CDMA communication system with double optical hardlimiters added to the correlator receiver has been studied. The bit error probability of this receiver has been compared with that of the OPPM-CDMA correlator receiver without hardlimiters and with that of both the optimum OOK- and OPPM-CDMA receivers. Upper bounds on the probabilities of error have been derived for the correlator receivers with and without hardlimiters. Nevertheless, lower bounds have been obtained for both optimum receivers. Numerical results have been evaluated for all these receivers under the same environment of prescribed throughput and laser pulsewidth. We can extract the following concluding remarks.

1) The performance of OPPM-CDMA correlator receiver with optical hardlimiters is superior to that of the optimum OOK-CDMA receiver and to that of the OPPM-CDMA correlator without hardlimiters.
2) The performance of OPPM-CDMA correlator receiver with optical hardlimiters is very close to the OPPM-CDMA optimum receiver even for small values of optical energies.
3) If all other design parameters (i.e., R_0, N, γ, and μ) are held fixed, there is a certain value of M that corresponds to a minimum error probability. These values of M could be selected from curves like that of Fig. 6.
4) The capacity of the OPPM-CDMA system with double optical hardlimiters is about 5.3 times greater than that of the optimum OOK-CDMA system under the constraints of $R_0 \geq 10^{-4}$ and $P_b \leq 10^{-9}$.

Appendix
Derivation of (1)

Assume without loss of generality that the desired user has sent a zero. Consider any other interference user and let $j(L/\gamma)T_c$ be the delay of the interference user with respect to the desired user. Since we are considering chip-synchronous case, we can assume (without loss of generality) that j takes values in the discrete set $\{0, 1, \ldots, M-2+\gamma\}$. The probability that j takes any value in the above set is thus $1/(M-1+\gamma)$. Moreover in our analysis we assume that the marks of any user sequence are uniformly distributed within the active slot period. The probability p that a mark of the interference user hits a mark of the desired user is given by

$$p = \frac{1}{M-1+\gamma} \sum_{j=0}^{M-2+\gamma} \Pr\{H|j\}$$

where $\Pr\{H|j\}$ denotes the probability that a mark of the interference user hits a mark of the desired user given a relative delay of $j(L/\gamma)T_c$ between them. Referring to Fig. 8 the following four cases may arise as long as $M \geq 2\gamma - 2$.

Case 1) $0 \leq j \leq \gamma$*:* In this case two time frames of the interference user may contribute to the hit probability. Consider the time frame to the left and assume that the data-carrying slot extends over the time interval

$$\left[(i-\gamma)\cdot\frac{L}{\gamma}T_c,\, i\cdot\frac{L}{\gamma}T_c\right],\; i \in \{j-M+1,\, j-M+2,\, \ldots,\, j\}.$$

This event happens with probability $1/M$. If $i \geq 0$, then the interference slot will overlap with the desired slot and there will be wi/γ marks (on the average) of the desired user that might be interfered. Each of these marks suffers a probability of interference (which is equal to w/L) from the other user. Hence, the left-hand side frame contributes an interference probability of

$$\sum_{i=1}^{j} \frac{1}{M} \cdot \frac{wi}{\gamma} \cdot \frac{w}{L}.$$

In a similar way, the right-hand side frame contributes an interference probability of

$$\sum_{k=j}^{\gamma-1} \frac{1}{M} \cdot \frac{w(\gamma-k)}{\gamma} \cdot \frac{w}{L}.$$

Thus, $\Pr\{H|j\}$ is given by

$$\begin{aligned}\Pr\{H|j\} &= \sum_{i=1}^{j} \frac{wi}{\gamma} \cdot \frac{w}{ML} + \sum_{k=j}^{\gamma-1} \frac{w(\gamma-k)}{\gamma} \cdot \frac{w}{ML} \\ &= \frac{w^2}{ML}\left[\frac{j(j+1)}{2\gamma} + \frac{(\gamma-j)(\gamma-j+1)}{2\gamma}\right].\end{aligned}$$

Case 2) $\gamma+1 \le j \le 2\gamma-2$: In this case, and all the following cases, only the left-hand side frame contributes to the hit probability

$$\begin{aligned}\Pr\{H|j\} &= \sum_{i=1}^{\gamma} \frac{wi}{\gamma} \cdot \frac{w}{ML} + \sum_{i=\gamma+1}^{j} \frac{w(2\gamma-i)}{\gamma} \cdot \frac{w}{ML} \\ &= \frac{w^2}{ML}\left[\frac{\gamma+1}{2} + \frac{(j-\gamma)(3\gamma-j-1)}{2\gamma}\right].\end{aligned}$$

Case 3) $2\gamma-1 \le j \le M$:

$$\begin{aligned}\Pr\{H|j\} &= \sum_{i=1}^{\gamma} \frac{wi}{\gamma} \cdot \frac{w}{ML} \\ &\quad + \sum_{i=\gamma+1}^{2\gamma-1} \frac{w(2\gamma-i)}{\gamma} \cdot \frac{w}{ML} = \gamma \frac{w^2}{ML}.\end{aligned}$$

Case 4) $M+1 \le j \le M-2+\gamma$:

$$\begin{aligned}\Pr\{H|j\} &= \sum_{i=j-M+1}^{\gamma-1} \frac{wi}{\gamma} \cdot \frac{w}{ML} + \sum_{i=\gamma}^{2\gamma-1} \frac{w(2\gamma-i)}{\gamma} \cdot \frac{w}{ML} \\ &= \frac{w^2}{ML}\left[\frac{\gamma+1}{2} + \frac{(j+\gamma-M)(\gamma-j+M-1)}{2\gamma}\right].\end{aligned}$$

Hence

$$p = \frac{1}{M-1+\gamma} \sum_{j=0}^{M-2+\gamma} \Pr\{H|j\} = \frac{\gamma w^2}{(M-1+\gamma)L}.$$

Furthermore, following the same argument as above (with slight modifications) it can be shown that the above result remains true even if $2 \le M \le 2\gamma - 2$.

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