Optical Code-Division Multiplexing (OCDM) Networks Adopting Code-Shift Keying/Overlapping PPM Signaling: Proposal and Performance Analysis

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Abstract—Optical code-division multiplexing (OCDM) systems use short pulses compared with bit-duration to achieve high transmission rate. The use of short pulses poses several problems as a result of group velocity dispersion (GVD), intersymbol interference (ISI) (due to avalanche photodiode (APD) buildup time), and receivers limited bandwidth. In this paper, an OCDM system employing code-shift keying (CSK) and overlapping pulse-position modulation (OPPM) signaling is proposed and theoretically investigated. By using CSK while maintaining same data rate, the chip duration can be increased to counteract the GVD effect in 2D OCDM systems. Moreover, by increasing the chip duration, the chip rate is decreased and the stringent requirement on receiver bandwidth is relaxed. In addition, using overlapping property in OPPM allows for further chip duration increase. We consider using correlation receivers with hard-limiters and APDs at the receiver side. The bit error probability (BEP) of the proposed system is derived taking into account the impacts of APD noise, thermal noise, GVD, ISI, and multiple-access interference (MAI). A performance comparison between OOK-, PPM-, OPPM-OCDM and the proposed system is carried out. Our results reveal that the use of CSK/OPPM-OCDM with data rate constraint allows the reduction of MAI, GVD and ISI effects with improved spectral efficiency.

Index Terms—Optical code division multiplexing (OCDM), APD noise, intersymbol interference (ISI), code-shift keying (CSK), overlapping pulse-position modulation (OPPM).

I. INTRODUCTION

Optical code division multiplexing (OCDM) is a very promising candidate for all-optical local and metropolitan area networks, as it enables fully asynchronous transmission, low access delay, potentially secure data transmission, support for bursty traffic, and soft-capacity on-demand.

Two-dimensional (2D) wavelength-hopping time-spreading (WHTS) OCDM codes, where encoding is performed in both time and wavelength domains, can support larger code cardinality using much shorter code lengths than one-dimensional (1D) time spreading codes under same bit-rate constraint [1]–[5]. This is particularly useful in high bit-rate OCDM applications, where the number of time slots is restricted.

Avalanche photodiodes (APDs) are commonly used in many high-speed optical receivers due to their internal optoelectronic gain [6]. The three main competing factors that govern APD-based optical receivers performance at high speed are: (i) the avalanche noise of the APD; (ii) the buildup time which limits APD speed by inducing intersymbol interference (ISI); and (iii) the APD dark current [7].

Group velocity dispersion (GVD) is one of the physical impairments that limits the performance of 2D OCDM systems [8], [9]. Due to the differing group velocities of a codeword’s wavelength components, GVD causes temporal spreading of chips, which results in chip peak-power reduction. Moreover, in TS OCDM systems the chip rate is very large (>Gchip/s) and ISI resulting from the APD’s buildup time cannot be neglected.

Code-shift keying (CSK)-OCDM, where different sequences of log2Mc bits of a message are encoded into different Mc codewords, has been demonstrated to increase the data rate, increase the spectral efficiency, enhance the OCDM network confidentiality, and relax the effects of GVD [10]–[13]. However, CSK requires codes with very large cardinality. In addition, compared with on-off keying (OOK)-OCDM, CSK would produce significantly more multiple-access interference (MAI) for a given number of simultaneous active users as a result of data bit “0” being transmitted also [10].

One possible solution to reduce MAI is the use of pulse-position modulation (PPM)-OCDM as transmission occurs only in a small fraction of the frame, thus reducing the probability of chip interference [14]. However, in PPM techniques, the laser pulsewidth must be shortened to be able to achieve the requirements on the throughput while using very long code sequences. Thus, PPM is more susceptible to GVD and ISI. In [15] the authors combine multi-code modulation (MCM) and PPM in a new modulation scheme, namely multi-code PPM (MCPPM) to simultaneously mitigate the impact of both GVD and MAI. In overlapping PPM (OPPM) techniques, where
overlapping between neighboring slots is allowable, it is possible to offer higher throughput without the need to decrease the laser pulsewidth [16], [17].

In the present paper, we propose and analyze an alternative technique that combines both CSK and OPPM-OCMD techniques; namely CSK/OPPM-OCMD. In this work, we only consider unwrapped OPPM signaling scheme [17]. The idea is that, in OCDM, at specified bit error probability (BEP) level, there is always a set of unused codes that can be exploited by assigning a set of $M_t$ codewords to each user. 2D one-coincidence frequency-hopping code/optical orthogonal code (OCFHC/OOC) codewords are used as the signature sequences in the proposed OCDM system [5]. In order to increase the discrimination between the desired codeword and interfering codewords in addition to minimizing MAI between active users, the auto- and cross-correlation are bounded by one. Under the condition of the same bit rate, employing CSK- or CSK/OPPM-OCMD permits increase of the chip duration. Thus, the effects of GVD are greatly reduced. In addition, as the chip duration increases, the chip rate decreases, relaxing the receiver bandwidth constraints and reducing the ISI effect.

The remainder of the paper is organized as follows. The system model of CSK/OPPM-OCMD is described in Section II. In Section III, an upper bound on BEP is derived taking into account the effects of MAI and GVD. In addition, the analysis includes the effects of APD noise, dark current, thermal noise, and ISI. Next, we numerically compare the BEP of the proposed system with that of OOK-, PPM-, and OPPM-OCMD systems in Section IV under the peak transmitted power constraint. Finally, conclusions are drawn in Section V.

II. PROPOSED CSK/OPPM-OCMD SCHEME

In this section we describe the transmitter and receiver architectures of our network, which is composed of $N$ users.

A. Transmitter Side

The transmitter of each user $k \in \{1, 2, \ldots, N\}$ is composed of one OPPM modulator and $M_c$ 2D OCDM encoders (or one tunable). OPPM modulator only involves time delaying of the optical pulses into one of the $M_p$ overlapping spreading intervals (called slots) that constitute an OPPM frame [17], [18]. In OPPM technique, a slot of duration $\tau = LT_c$, where $L$ is the code length and $T_c$ is the chip duration, is divided into $\gamma \in \{1, 2, \ldots, M_p − 1\}$ subslots of width $\tau/\gamma$ each, where $\gamma$ is the overlapping index. Each slot is forced to overlap with its adjacent slots so that there is an overlap of $(\gamma − 1)\tau/\gamma$ between any two adjacent slots. An optical pulse coming from a multiwavelength light source of width $T_c$, repetition rate $1/TOPPM$, and peak power $w_P$ is time delayed depending on the incoming data, where $TOPPM = (M_p − 1 + \gamma)LT_c/\gamma$ is the full width of OPPM time frame, $P_c$ is the transmitted peak power per chip, and $w$ is the code weight. The CSK/OPPM electronic logic unit is merely a serial-to-parallel (S/P) converter where a serial data bit stream $\{d_0, d_1, d_2, \ldots\}$, $d_i \in \{0, 1\}$, is segmented every $\log_2 M$ bits, where $M = M_c \times M_p$. The segmented bits are then parallized to generate $a_i$, $i \in \{0, 1, \ldots, M − 1\}$ trigger signals that will employed for configuring the electro-optic (EO) switches as shown in Fig. 1. The S/P converters can be implemented using low cost field-programmable gate array (FPGA) logic chip taking advantage of its high-speed serializers/deserializers (SerDes) [19]. Moreover, one FPGA chip can control many transmitters in parallel which results in a simpler design and overall system cost reduction. The control bits $a_i$, $i \in \{\log_2 M_c, \log_2 M_c + 1, \ldots, \log_2 M − 1\}$ are used to control the relative optical delays between the EO switches, which are either 0 or $\frac{\tau}{2^{i−\log_2 M_c}}$, depending on the value of $a_i$, while the control bits $a_i$, $i \in \{0, 1, \ldots, \log_2 M_c − 1\}$ control which one of the $M_c$ OCDM encoders will be used. The maximum bit rate per user supported by the system is limited by the EO switches configuring latency (equal FPGA processing time and EO switch switching time). It is worthwhile noting that this latency should be $\leq TOPPM$. One advantage of using CSK under same bit constraint is the increase of $T_c$ and consequently $TOPPM$, hence eliminating the need for high-switching speeds. High-speed compact Mach-Zehnder-type EO switches, with switching times $< 2.5$ ns, have been successfully fabricated [20]. A silicon EO switch, consisting of double-ring assisted Mach-Zehnder interferometer with GHz speed, has been demonstrated in [21]. In OCDM encoder, the frequency components of the optical pulse are selectively demultiplexed into $w$ wavelength components according to the wavelength hopping pattern in the corresponding signature code. Then each component is appropriately time-delayed using optical tapped delay lines (TDLs) in accordance with the time pattern of the

![Fig. 1. CSK/OPPM-OCMD encoder for $M_c = 2, M_p = 4, \gamma = 3, L = 15$, and $w = 3$.](image-url)
signature code and then recombined using a multiplexer to generate the CSK/OPPM-OCDM signal. The CSK/OPPM-OCDM signal is then combined with signals from other transmitters and transmitted across the passive $N \times N$ star coupler to all receivers. Fig. 1 schematically shows the proposed CSK/OPPM-OCDM transmitter for $M_c = 2$, $M_p = 4$ and $\gamma = 3$, where bits 110 are mapped into slot $M_p - 1$ and code $C_0$.

B. Receiver Side

At the receiver side, there are $M_c$ parallel OCDM decoders with double optical hard-limiters, each matched to one of the $M_c$ assigned codes to user $k$. Fig. 2 illustrates the schematic of the CSK/OPPM-OCDM receiver for user $k$ for $M_c = 2$, $M_p = 4$, and $\gamma = 3$. In the OPPM demodulation after each OCDM decoder, a time-gate opens a time window of $T_c$ at the start of each $M_p$ slots constituting the OPPM frame. The gated pulses are then photodetected for decision making. The OPPM demodulator is merely a comparator that determines which slot in the $M_p$ time slots for which one of the $M_c$ codes has the largest photon count to be declared as the transmitted symbol.

Fig. 3 illustrates an example of the transmitted CSK/OPPM-OCDM signal formats of a single user. In this figure two 2D OCFHC/OOC codes (i.e., $M_c = 2$) and four overlapping positions (i.e., $M_p = 4$) with overlapping index $\gamma = 3$ are used to create 2–4 CSK/OPPM-OCDM. In the figure, $S_{ij}$, $i \in \{0, 1, \ldots, M_c - 1\}$, $j \in \{0, 1, \ldots, M_p - 1\}$, represents the symbol $ij$ which use code $i$ within slot $j$.

III. PERFORMANCE ANALYSIS

In this section, we analyze the performance of the proposed system using APD. In our derivation, we assume a slot-asynchronous chip-synchronous system for ease of computation. In the more realistic case of chip-asynchronous, the performance will be superior to the case of chip-synchronous since the latter provides BEP upper bound [22]. We start by the BEP of traditional OOK-OCDM system, followed by the proposed CSK/OPPM-OCDM system.

A. BEP of OOK-OCDM System

The correlation receiver with hard-limiters decides a data bit “1” was transmitted if the collected photon count in any bit interval is greater than a decision threshold $\theta$. A data bit “0” is decided otherwise. For large number of incident photons, the APD output can be approximated by a Gaussian random variable [23]. Let $Y$ denote the photon count collected in one bit interval, then the BEP can be evaluated as described in [24] as follows:

$$
P_e = \frac{1}{2} \left[ \Pr[Y \leq \theta|1] + \Pr[Y > \theta|0] \right]$$

$$= \frac{1}{2} \left[ Q \left( \frac{\mu_1 - \theta}{\sigma_1} \right) + Q \left( \frac{\theta - \mu_1}{\sigma_1} \right) \Pr[Z = 0] \right] \quad \text{(1)}$$

where $Q(x) \equiv (1/\sqrt{2\pi}) \int_{x}^{\infty} e^{-y^2/2} dy$. Moreover, $\mu_0/1$

![Fig. 2. CSK/OPPM-OCDM decoder for $M_c = 2$, $M_p = 4$, $\gamma = 3$, $L = 15$, and $w = 3$.](image)

![Fig. 3. CSK/OPPM-OCDM signaling scheme with $M_c = 2$, $M_p = 4$, $\gamma = 3$, $L = 15$, and $w = 3$. A codeset of $C_1 \in \{(\lambda_1, \lambda_2, 00000000000000), (\lambda_1, \lambda_2, 00000000000000)\}$ is assumed.](image)
and $\sigma^2_{0/1}$ denote the mean and variance of the receiver output conditional on the transmitted bit being 0/1, respectively. The random variable $Z \in \{0, 1, \ldots, w\}$ denotes the number of interfered mark positions in the bit interval of the desired user. Since \( \alpha_t \), \( t \in \chi \) denotes the number of interference hits (or coincidences) occurred in the th marked chip of the desired user, since $\sum_{z=0}^w \Pr[Z = z] = 1$, then

$$\Pr[Z \neq w] = 1 - \Pr[Z = w].$$

To minimize the BEP, the decision threshold $\theta$ is taken as $\theta = (\mu_0 \sigma_1 + \mu_1 \sigma_0) / (\sigma_0 + \sigma_1)$ [26], then

$$P_e = \frac{1}{2} \Pr[Z = w] + Q\left(\frac{\mu_1 - \mu_0}{\sigma_1 + \sigma_0}\right) \Pr[Z \neq w].$$

The average interference hit probability for 2D OCFHC/OOC codes is given by [5]:

$$q = \Phi_{C0}^0 + \Phi_{C1} q_1,$$

where $\Phi_{C0} = p^k \Phi_{OOC}$ and $\Phi_{C1} = p^k (1 - p^k) \Phi_{OOC}$ denote the cardinality of codewords in group $C^0$ and $C^1$, respectively. $\Phi_{OOC} = (L - 1) / w(w - 1)$ represents the cardinality of $(L, w, 1, 1)$ OOC used for TS. The overall cardinality $\Phi_C$ is given by:

$$\Phi_C = \Phi_{C0} + \Phi_{C1} = m^2 \Phi_{OOC},$$

where $m = p^k$ represents the number of available wavelengths, $p$ is a prime number, and $k$ is the degree of the primitive polynomial $f(x)$ over Galois field $GF(p)$. $q_0$ and $q_1$ denote the one hit probabilities between the desired codeword originated from group $C^0$ or group $C^1$ and any interfering codeword in the code set, respectively. They are given by [5]:

$$q_0 = \frac{w^2 m \Phi_{OOC} - 1}{2L(\Phi_C - 1)}, \quad q_1 = \frac{w^2 m \Phi_{OOC} - w}{2L(\Phi_C - 1)}.$$

where the factor 1/2 comes from the assumption of equiprobable on-off data-bit transmission, and $\Phi_C - 1$ is the number of possible interferers. Substituting (7) in (5) we get $q = w^2 R / 2 mL$, where

$$R = \frac{\Phi_C - 1 - (m - 1) / w}{\Phi_C - 1}$$

represents the fraction of the number of interfering codewords contributing one hit (due to cross-correlation function) to the total number of interfering codewords (as there is some codewords in 2D OCFHC/OOC have zero cross-correlation).

The expressions for the parameters $\mu_0 \sigma^2_0$, $\mu_1$, and $\sigma^2_1$ for 2D OCFHC/OOC OCDM system are adapted here from [7, 27]:

$$\mu_0 = \frac{1}{2} \frac{\langle G \rangle \sum_{j=1}^w n_j}{k}\left(1 - e^{-k}\right) + n_d \langle G \rangle,$$

$$\sigma^2_0 = \frac{1}{4} \frac{(G)^2}{k^2 \lambda^2} \left(1 - e^{-2k}\lambda\right) + \frac{\langle G \rangle^2 F \sum_{j=1}^w n_j}{2k\lambda} \left(1 - e^{-k\lambda} - k\lambda e^{-k\lambda}\right) + \frac{(\mu_1 - \mu_0)^2}{\sigma_1 + \sigma_0} \left(k\lambda - 1 + e^{-k}\right) + \sigma^2_2,$$

where brackets represent ensemble averaging, $G$ denotes the stochastic APD gain, $k = 4B_{sneq} / 2\pi B_{3dB}$ represents bandwidth correction factor, $B_{sneq}$ is shot-noise-equivalent bandwidth, $B_{3dB}$ is 3-dB bandwidth of the APD, and $\lambda = 2\pi B_{3dB} / R_e$ is the detector’s relative speed, where $R_e = 1 / T_e$ is the chip transmission rate. Here, $n_j$ is the average number of absorbed photons per received single-user chip at wavelength $\lambda_j$, $j \in \{1, 2, \ldots, m\}$, given by:

$$n_j = \frac{n_j \lambda j P_{p,j} T_e}{h C},$$

where $P_{p,j}$ and $n_j$ denote the peak power of the received chip and the quantum efficiency at $\lambda_j$, respectively, $C = 3 \times 10^8 \text{ m/s}$ is the vacuum speed of light, and $h = 6.626 \times 10^{-34}$ J.s is Planck’s constant. The average single-user received laser power $P_{av}$ for OOK-OCDM will be

$$P_{av} = \frac{\sum_{j=1}^w n_j \lambda j P_{p,j} T_e}{2 \tau},$$

where $\tau = LT_e$ is the OOK bit duration, and the factor 1/2 is multiplied due to data bit “0” is not transmitted. $F$ is the APD excess noise factor, defined as $F \triangleq (G^2 / \langle G \rangle)^2 = k_{eff} (G) + (1 - k_{eff})[2 - (1 / \langle G \rangle)]$, where $k_{eff}$ is the APD effective ionization ratio [28]. Also, $n_j$ is the average number of dark carriers generated per chip time interval $T_e$, given by $n_d = L_d T_e / e$, where $L_d$ is the APD dark current, and $e = 1.6 \times 10^{-19}$ C is the electron charge. Finally, $\sigma^2_2 = 2k T_e / e R_e$ represents the variance of thermal noise within a chip interval where $k_B = 1.38 \times 10^{-23}$ J/K is Boltzmann’s constant, $T_R$ is the receiver noise temperature, and $R_e$ is the receiver load resistance.

Assuming an optical chip pulse with a Gaussian shape propagating in a linear dispersive medium. The pulse broadening factor due to the effect of GVD can be expressed as [29]:

$$\sigma_{D,j} = \frac{T_d}{T_0} \left[1 + \frac{C_p \beta_{2,j} d}{2T_0} + \frac{\beta_{2,j} d^2}{2T_0^2}\right]^{1/2},$$

where $\tau = LT_e$ is the OOK bit duration, and the factor 1/2 is multiplied due to data bit “0” is not transmitted.
where $C_p$ is the chirp parameter, $\tau_0 = T_c/(2\sqrt{2\ln 2})$ is the RMS pulse width, $\tau_d$ is pulse power after propagating a certain distance $d$ along fiber, $\beta_2 = -D\lambda_j^2/(2\pi C)$ is the GVD factor at $\lambda_j$, and $D$ is the dispersion parameter. The peak power of the received laser chip is given as [29], [30]:

$$P_{p,j} = \frac{10^2 \left( C_{p,\text{dim}} - \alpha D \lambda_j \right)}{\sigma_{D,j}} \times 10^{-3},$$  

(13)

where $\alpha$ is the attenuation coefficient. It is worth to mention that $P_c$ is assumed to be the same for all wavelengths and OCDM signaling schemes used in this paper.

B. BEP of CSK/OPPM-OCDM System

We define a $M_c \times M_p$ matrix $\mathcal{Y}$ whose element $Y_{ij}$, $i \in \mathcal{M}_c \equiv \{0, 1, \ldots, M_c - 1\}$, $j \in \mathcal{M}_p \equiv \{0, 1, \ldots, M_p - 1\}$, represents the photon count collected from slot $j$ at the receiver matched to codeword $i$ as:

$$\mathcal{Y} \equiv \begin{pmatrix} Y_{00} & Y_{01} & \ldots & Y_{0(M_p-1)} \\ Y_{10} & Y_{11} & \ldots & Y_{1(M_p-1)} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{(M_c-1)0} & Y_{(M_c-1)1} & \ldots & Y_{(M_c-1)(M_p-1)} \end{pmatrix}.$$  

(14)

Data symbol $ij$ is declared to be sent if $Y_{ij} > Y_{Sj}$ for every $x \in \mathcal{M}_c$, $y \in \mathcal{M}_p$, and $xy \neq ij$. An incorrect decision is otherwise declared. The probability of symbol error $P_S$ is given by:

$$P_S = \sum_{i=0}^{M_c-1} \sum_{j=0}^{M_p-1} \Pr[\text{error}|S = ij] \Pr[S = ij],$$  

(15)

where $S$ denotes the transmitted data symbol. Assuming equally probable data symbols, i.e., $\Pr[S = ij] = 1/M$, then $\Pr[\text{error}|S = ij]$ is independent of $ij$. The bit error probability $P_B$ can be obtained from $P_S$ using the formula $P_B = [M/2(M-1)]P_S$ [31]. Without loss of generality, we assume that $S = 00$ (i.e., code 0 in slot 0). Using a union bound, the upper bound on $P_S$ is expressed as:

$$P_S = \Pr[Y_{ij} \geq Y_{00}, \text{ some } ij \neq 00|S = 00]$$

$$\leq \sum_{i=1}^{M_c-1} \Pr[Y_{i0} \geq Y_{00}|S = 00]$$

$$+ \sum_{i=0}^{M_c-1} \sum_{j=1}^{M_p-1} \Pr[Y_{ij} \geq Y_{00}|S = 00]$$

$$\leq (M_c - 1) \Pr[Y_{10} \geq Y_{00}|S = 00]$$

$$+ M_c(M_p - 1) \Pr[Y_{01} \geq Y_{00}|S = 00],$$  

(16)

where the first probability in the last equation is due to the photon count of the symbols that have same position with that of slot 00 (due to CSK effect). It can be evaluated as:

$$\Pr[Y_{10} \geq Y_{00}|S = 00]$$

$$= \Pr[Y_{10} \geq Y_{00}|S = 00, Z_{10} = w] \Pr[Z_{10} = w]$$

$$+ \Pr[Y_{10} \geq Y_{00}|S = 00, Z_{10} \neq w] \Pr[Z_{10} \neq w],$$  

(17)

where $Z_{ij} \in \chi$, $i \in \mathcal{M}_c$, $j \in \mathcal{M}_p$, is a random variable that denotes the number of interfered mark positions in slot $ij$ of the desired user immediately after the first optical hardlimiter. Let $\alpha_{10}^w$, $t \in \chi \equiv \{1, 2, \ldots, w\}$, denote the number of hits occurred in the $t$th mark of undesired slot 10. Then $\Pr[Z_{10} = w]$ is given by [25]:

$$\Pr[Z_{10} = w] = \Pr[w \text{ marks interfered in solt 10}].$$

$$= \Pr \left\{ \alpha_{10}^1 \geq 1, \alpha_{20}^1 \geq 1, \ldots, \alpha_{w0}^1 \geq 1 \right\}$$

$$= \sum_{i=0}^{w} (-1)^i \binom{w}{i} \left( 1 - \frac{p_X}{w} \right)^{N-1}$$  

(18)

where $p_X$ is the probability of an external interference from other users in the network adopted for 2-D OCFHC/OOC OCDM system, given by [17]:

$$p_X = \frac{w^2 R}{(M_p - 1 + \gamma)mL v^2}.$$  

(19)

Modeling the discrete outputs of APD, $Y_{00}$ and $Y_{10}$, as continuous Gaussian random variables [23], we get

$$\Pr[Y_{10} \geq Y_{00}|S = 00, Z_{10} = w]$$

$$\leq \Pr[V = Y_{10} - Y_{00} \geq 0|S = 00, Z_{10} = w]$$

$$= Q \left( -\frac{\mu_v}{\sigma_v} \right)$$  

(20)

where $\mu_v$ and $\sigma_v^2$ denote the mean and variance of the random variable $V = Y_{10} - Y_{00}$ conditioned on $Z_{10} = w$. Also $\mu_u$ and $\sigma_u^2$ are the mean and variance of the random variable $U = Y_{10} - Y_{00}$ conditioned on $Z_{10} \neq w$. The expressions for the parameters $\mu_v$, $\sigma_v^2$, $\mu_u$ and $\sigma_u^2$ are [24]:

$$\mu_v = \mu_1 - \mu_1 = 0, \quad \mu_u = \mu_0 - \mu_1,$$

$$\sigma_v^2 = 2\sigma_1^2, \quad \sigma_u^2 = \sigma_0^2 + \sigma_1^2,$$  

(21)

respectively. It is worth noting that in OPPM, $P_{av} = \sum_{j=1}^{M_p} P_{p,j} T_c/T_{OPPM}$. The last probability in (16) is due to the
the number of interfered mark positions in slot 01 of the desired self interference on a specific mark of an undesired slot is further bound:

\[
\Pr(Y_{01} \geq Y_{00}|S = 00) = \Pr(Y_{01} \geq Y_{00}|S = 00, Z_{01} = w) \Pr(Z_{01} = w) + \Pr(Y_{01} \geq Y_{00}|S = 00, Z_{01} \neq w) \Pr(Z_{01} \neq w) \tag{22}
\]

\[
+ \Pr(Y_{01} \geq Y_{00}, \nu_I = 1|S = 00, P_2) \times \Pr(P_2) + \Pr(Y_{01} \geq Y_{00}, \nu_I = 1|S = 00, P_3) \times \Pr(P_3),
\]

where \(\Pr(Z_{01} = w)\) is given by (18) and we have the following further bound:

\[
\Pr(Y_{01} \geq Y_{00}|S = 00, Z_{01} = w) \Pr(Z_{01} = w)
\]

\[
+ \Pr(Y_{01} \geq Y_{00}|S = 00, Z_{01} \neq w) \Pr(Z_{01} \neq w) \leq \frac{1}{2} \Pr(Z_{01} = w) + Q\left( \frac{\mu_u}{\sigma_u} \right) \Pr(Z_{01} \neq w). \tag{23}
\]

In the third term in (22), \(P_2\) represents the event that \(w - 1\) marked chips from the \(w\) marked chips in slot 01 are hit at last by one interfering pulse. Hence, \(\Pr(P_2)\) is given by:

\[
\Pr(P_2) = \Pr[w - 1 \text{ from } w \text{ marks interfered in slot 01}]
\]

\[
= w \times \Pr\left[ \alpha_{01}^{01} \geq 1, \ldots, \alpha_{w-1}^{01} \geq 1, \alpha_w^{01} = 0 \right]
\]

\[
= w \times \left[ \Pr\left[ \alpha_{01}^{01} \geq 1, \ldots, \alpha_{w-1}^{01} \geq 1 \right] - \Pr\left[ \alpha_{01}^{01} \geq 1, \ldots, \alpha_{w-1}^{01} \geq 1, \alpha_w^{01} \geq 0 \right] \right]
\]

\[
= w \times \left[ \sum_{i=0}^{w-1} (-1)^i \binom{w-1}{i} \left( 1 - \frac{q}{w} \right)^{w-1-i} \right]
\]

\[
= w \times \left[ \Pr[Z_{01} = w - 1] - \Pr[Z_{01} = w] \right], \tag{24}
\]

where \(Z_{01}^i \in \{0, 1, \ldots, w - 1\}\) is a random variable that denotes the number of interfered mark positions in slot 01 of the desired user immediately after the first optical hardlimiter. Also \(P_3\) in (22) represents the event that the number of marked chips interfered at least by one interfering pulse not equal \(w - 1\), and it is given by:

\[
\Pr(P_3) = \Pr[\text{marks interfered in slot 01} \neq w - 1]
\]

\[
= \sum_{i=0}^{w} \binom{w}{i} \Pr\left[ \alpha_{01}^{01} \geq 1, \ldots, \alpha_{w-i}^{01} \geq 1, \alpha_{w-i+1}^{01} = 0, \ldots, \alpha_w^{01} = 0 \right]
\]

\[
= 1 - \binom{w}{1} \Pr\left[ \alpha_{01}^{01} \geq 1, \ldots, \alpha_{w-1}^{01} \geq 1, \alpha_w^{01} = 0 \right]
\]

\[
= 1 - w \times \left[ \Pr[Z_{01} = w - 1] - \Pr[Z_{01} = w] \right]. \tag{25}
\]

In (22), \(\nu_I \in \{0, 1\}\) is the number of pulses that cause self interference from slot 00 onto other undesired slots with different positions from that of symbol 00. The average probability of self interference on a specific mark of an undesired slot is \(p_I/w\).
The system parameters used in the analysis are provided in Table I. In our numerical calculations we have defined the target BEP to be $10^{-6}$. Moreover, we assume that the data rate per user $R_b$ b/s is held fixed, and is given by:

$$R_b = \frac{\log_2 M}{T_{\text{OPPM}}} = \frac{\gamma \log_2(M_c \times M_p)}{(M_p - 1 + \gamma)LT_c}$$

(31)

The BEP for both OOK-, PPM- and OPPM-OCDM systems for correlation receiver with hard-limiters considering only MAI is plotted in Fig. 4(a) against the number of simultaneous users $N$. We set $m = 8$, $L = 200$ and $w = 6$ for OOK-OCDM system (i.e., $\Phi_{\text{OOC}} = \lfloor (L - 1)/w(w - 1) \rfloor = 6$ and $\Phi_{\text{C}} = 384$). In order to maintain the same $R_b$ using PPM system as in the OOK case with constraint on $T_c$, from (31) the code length for PPM system $L_{\text{PPM}}$ will be $(L \log_2 M_p)/M_p$. As a result the overall cardinality will be reduced as shown in Fig. 4(a). The OPPM parameters $M_p$ and $\gamma$ should be properly chosen as overlapping between slots may cause an increase in the error rate. The BEP is plotted for different values of $M_p$ in Fig. 4(b) (Top) with $N = 384$ and $\gamma_p$ calculated using (31) such that $L$, $w$, $T_c$, and $R_b$ are the same with that of OOK-OCDM. The proper values of $M_p$ and $\gamma$ are chosen such that the performance of OPPM system is better than that of OOK-OCDM. Fig. 4(b) (Bottom) shows the fluctuations in achieved $R_b$ because $\gamma_p$ should be integer. From Fig. 4(a), OOK-OCDM system can support 65 users ($\approx 17\%$ of $\Phi_{\text{C}} = 384$) at BEP of $10^{-6}$, whereas OPPM-OCDM can support 90 users ($\approx 23\%$ of $\Phi_{\text{C}} = 384$). This is because OPPM-OCDM achieves higher tolerance to MAI, which results in improvement of BEP. From Fig. 4(a), the performance of OPPM is nearly the same as that of OOK-OCDM for $M_p = 8$ case (it supports $\approx 67$ users at the target BEP of $10^{-6}$). For a much reduction of MAI, $M_p$ should be increased further. As a result, $L_{\text{PPM}}$ will be reduced further limiting the number of available codewords.

Fig. 5 assess the effects of (i) APD noise and thermal noise, (ii) GVD, and (iii) device bandwidth and ISI on the OOK-OCDM system performance by plotting the BEP versus $P_c$ for the three different cases with $N = 65$. Two different values of receiver noise temperatures $T_n \in \{300, 1000\}$ K are considered. The ratio $r = B_{\text{3dB}}/B_{\text{3dB}}$ is assumed to be 1.3, and $B_{\text{3dB}}$ to be 29 GHz. GVD can be excluded from the calculations by setting $\beta_2 = 0$ in (12). APD bandwidth constraint and ISI was removed by assuming an instantaneous APD (i.e., $B_{\text{3dB}} = \infty$). The effects of APD noise and thermal noise is predominant in the range of small transmitted power. However, when GVD effect is included, high transmitted power is required due to GVD power reduction. For the case of complete analysis (i.e., including APD noise, thermal noise, GVD, and ISI) the BEP of OOK-OCDM system greatly degraded because of the additional attenuation in the receiver output resulting from the APD’s bandwidth constraint. Hence, the main factor that limits the system performance is ISI.

The corresponding curves for PPM-OCDM with $N = 67$ and for OPPM-OCDM with $N = 90$ systems are plotted in Fig. 6. For all three cases, PPM-OCDM has slightly better performance than that of OPPM-OCDM. This is because of
overlapping between slots in OPPM which results in increasing the effects of MAI, APD noise, and thermal noise. However, it should be taking into consideration that these plots are at $N = 67$ for PPM and $N = 90$ for OPPM.

Fig. 7 compares the BEP of OOK and different cases of CSK (i.e., increasing the code length, increasing the chip duration, or both) versus the transmitted laser power per chip when $N = 65$. In the figure, the numbers in the parentheses represent $M_c$, $T_c$, and $L$, respectively. Increasing $L$ while keeping $T_c$ constant for the same transmission rate, the BEP of CSK gets worse because the added MAI from CSK effect. Increasing $T_c$ helps to reduce the chip transmission rate, hence relaxing the effects of both GVD and ISI. Simultaneously increasing $L$ and $T_c$ can reduce the effects of MAI, GVD, and ISI.

Fig. 8 shows the BEP for CSK/OPPM-OCDM system versus $P_c$ when $N = 90$ for the different cases (i.e., increasing $L$, $T_c$, or both). Increasing $L$ only degrades the system performance due to the added MAI from CSK effect. With constraint on $R_c$, $L$, and $T_c$ and using $M_c = 2$, the increase in $T_c$ is small (from 0.02 ns to 0.0237 ns), hence the improvement in BEP is little. With constraints on both $R_c$ and $L$, $T_c$ can be increased largely by increasing $\gamma$, thus greatly reducing the effects of GVD and ISI (case $M_c = 4$, $L = 200$, and $T_c = 0.04$ ns which corresponds to $\gamma = 12$). However, increasing $T_c$ by further increasing $\gamma$ (while $M_p$ is fixed) will increase the effect of MAI on BEP which may be worse than GVD effect (case $M_c = 4$, $L = 250$, and $T_c = 0.05$ ns which corresponds to $\gamma = 25$). This happens because the value of $M_p (= 34)$ is not optimized for this large value of overlapping index. In order to overcome the effect of overlapping the parameters should be reoptimized to meet the new requirements. Due to the limited cardinality, only the case of $M_c = 2$, $M_p = 8$ and $N = 67$ for CSK/PPM-OCDM is included in the figure as well.

Fig. 9 shows the BEP versus the number of simultaneous users when $P_c = -15$ dBm and $d = 20$ km.
and OPPM-OCDM systems.

Using the definition of spectral efficiency $\eta = KR_b/m\Delta f$, (b/s/Hz), where $\Delta f = 1/T_c$ is the minimum required bandwidth [33] and $K$ is the number of supported users at the target BEP, the achieved spectral efficiency for the different signaling schemes is presented in Table II at BEP of $10^{-5}$ with the help of Fig. 9. From the figure, the proposed CSK/OPPM-OCDM system achieves higher spectral efficiency than other systems.

V. Conclusion

An OPPM-OCDM system using CSK signaling has been proposed and theoretically investigated. Upper bound on the BEP for the proposed system has been obtained for correlation receiver with double-hard-limiters. The analysis includes the effects of MAI, GVD, APD noise, APD dark current, and thermal noise. Moreover, the effect of ISI has been also taken into account. Our numerical results indicate that ISI is the main performance limiting factor for OOK- and OPPM-OCDM systems. The proposed CSK/OPPM-OCDM system can mitigate the impacts of GVD, ISI, and MAI, consequently improving the overall performance of the network as well as achieving higher spectral efficiency when compared to traditional OOK-, PPM-, and OPPM-OCDM systems.

**REFERENCES**


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