MIMO Equalization for Multi-Core Fiber-Based Systems Using the Affine Projection Algorithm

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Abstract: An adaptive algorithm for MIMO equalization through multi-core fibers based on the affine projection is proposed. Our algorithm improves the convergence speed compared to LMS algorithm while having low computational complexity compared to RLS algorithm.
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1. Introduction

Space-division multiplexing (SDM) based on multi-core fibers (MCFs) is regarded as a promising technology to overcome the capacity limit of single mode fibers (SMFs). To increase the capacity, it is necessary to use high-count MCFs, which definitely require multiple-input multiple-output digital signal processing (MIMO-DSP) to counteract inter-core crosstalk (XT). However, the number of equalizers required for optimal MIMO detection grows quadratically with the number of cores. In addition , the equalizer length should ensure compensation of the differential group delay (DGD) between cores. Hence, the required MIMO-DSP computational complexity is potentially high [1].

Two major algorithms, least mean squares (LMS) and recursive least squares (RLS), have been studied in [2] for adaptive equalization. The LMS algorithm is widely used due to its low computational complexity. However, its main drawback is the slow of convergence speed. Faster convergence speed in MIMO equalization can be achieved using RLS-algorithms. However, they suffer from high computational complexity and instability problems.

In the field of adaptive signal processing, it is well known that the affine projection algorithm (APA) can provide a good trade-off between convergence speed and computational complexity [3]. It has a much improved convergence speed compared to LMS algorithm with much reduced computational complexity compared to RLS algorithm. The APA is a generalization of normalized least mean squares (NLMS) algorithm [4]. The APA exploits multiple regressors for updating the coefficients of an adaptive filter, while the NLMS algorithm uses only the current single regressor. The APA improves the convergence speed of the adaptive filter over the NLMS algorithm by increasing the number of regressors used, but at the expense of an increased computational complexity.

In this paper, the convergence speed of APA is investigated and compared with both the NLMS and RLS algorithms in the context of MCF-based systems. We show that the convergence speed and the tolerable end-to-end XT can be greatly improved with increasing the APA-order, with much less computational complexity compared to the RLS algorithm. We also present a comparison between the complexity of APA, NLMS, and RLS algorithms.

2. Proposed MCF-based systems with MIMO equalization

Fig. 1 shows a generic MCF-based system with MIMO equalization. At the transmitter side, the modulated optical signals are transmitted over the *M* cores of a MCF. For simplicity, we limit the number of cores to two in this investigation. At the receiver, coherent detected signals from different cores are passed to a DSP block, where chromatic dispersion compensation and MIMO equalization are performed.

We model the MCF as in [5] using cascaded SMF segments, where each segment is characterized by a random variation in the propagation constant that captures the effect of bending and twisting. Then the effect of accumulated XT through each segment is added by solving the conventional coupled mode equations (CMEs) analytically within each segment. In addition, the DGD is modeled by introducing a delay between signals at the end of each segment.

To study the proposed APA and derive the updating equation of this algorithm, we assume that the transmitted electrical signal through *M*-cores is $\boldsymbol{b} = [b_{1X}, b_{1Y}, b_{2X}, b_{2Y}, ..., b_{MX}, b_{MY}]^T$ and consider dual polarization multiplexed signals. After chromatic dispersion compensation, the received signal is $\boldsymbol{y} = [y_{1X}, y_{1Y}, y_{2X}, y_{2Y} ...., y_{MX}, y_{MY}]^T$. The APA minimizes the squared Euclidean norm of the change in the weight vector subject to multiple constraints [3] as follows:

$$\min \| \delta\boldsymbol{w}(n) \|^2 = \min \| \boldsymbol{w}(n) - \boldsymbol{w}(n-1) \|^2 \qquad (1)$$

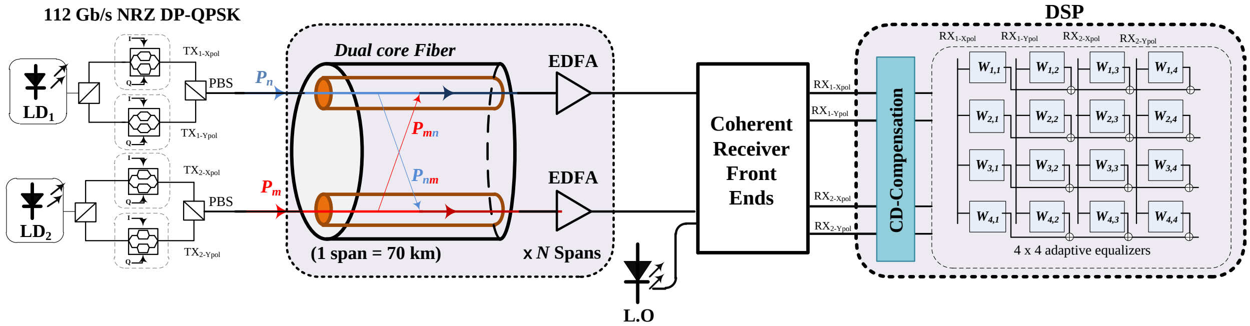

Fig. 1 MCF-based system with MIMO-equalization, EDFA: erbium-doped fiber amplifier, LD: laser diode, L.O: local oscillator, NRZ-DP-QPSK: non-return-to-zero dual polarization quadrature phase shift keying, CD-compensation:chromatic disperion compensation, PBS: polarization beam splitter. Parameters: fiber attenuation = 0.2 dB/km, chromatic dispersion = 17 ps/nm/km, total DGD = 1.96 ns, nonlinear refractive index = $2.6\text{x}10^{-20}$ m^2/w and effective area = 80 μm^2. EDFA noise figure = 4 dB.

subject to constraints

$$\begin{aligned} b(n-k) &= \boldsymbol{w}^{H}(n)\boldsymbol{y}(n-k), \quad k = 0,1,....,L-1 \\ \boldsymbol{b}(n) &= \boldsymbol{X}^{T}(n)\boldsymbol{w}(n) \end{aligned} \qquad (2)$$

where $\boldsymbol{b}(n)=[b_n, b_{n-1},...,b_{n-L-1}]^T$ is the desired response and the $2M\times L$ matrix $\boldsymbol{X}(n)=[\boldsymbol{y}(n), \boldsymbol{y}(n-1),..,\boldsymbol{y}(n-L-1)]$, $\boldsymbol{y}(n)$ is the equalizer input data vector, and L is called order of APA adaptive filter. The optimization problem (1) with the constraints given by (2) can be solved by using the Lagrange multiplier method [4]. Let the Lagrange multiplier vector be $\boldsymbol{\lambda}=[\lambda_1, \lambda_2,....,\lambda_L]^T$, then, the Lagrange cost function is defined as:

$$\boldsymbol{J}(n) = \|\boldsymbol{w}(n)-\boldsymbol{w}(n-1)\|^2 + 2\boldsymbol{\lambda}^T[\boldsymbol{b}(n)-\boldsymbol{X}^T(n)\boldsymbol{w}(n)] \qquad (3)$$

Letting the partial differentiation $\partial \boldsymbol{J}(n)/\partial \boldsymbol{w}(n)=0$, we get $w(n+1)=w(n)+\boldsymbol{X}(n)\boldsymbol{\lambda}$. Next, we multiply both sides by $\boldsymbol{X}^T(n)$ and use the constraints in (2) to get a solution for $\boldsymbol{\lambda}$. Using this solution and introducing the step-size μ, we finally have the update equation of APA. The inversion can then be regularized by small diagonal term such that:

$$\begin{aligned} e(n) &= \boldsymbol{b}(n) - \boldsymbol{X}(n)^T\boldsymbol{w}(n) \\ \boldsymbol{w}(n+1) &= \boldsymbol{w}(n) + \mu\frac{\boldsymbol{X}(n)}{[\boldsymbol{X}^T(n)\boldsymbol{X}(n)+\alpha I]}e(n) \end{aligned} \qquad (4)$$

3. System performance and simulation results

The performance of the proposed APA is evaluated via a MATLAB/OptiSystem co-simulation. Spatially multiplexed 112 Gb/s NRZ-DP-QPSK signals are generated and transmitted through the dual cores of a 980 km fiber link composed of 14 spans. Each span consists of 70 km of MCF followed by a conventional EDFA per core. Each MCF span is modeled by 14 segments of SMF. The value of the DGD is 2 ps/km, therefore, 200 taps per adaptive filter are needed to ensure compensation of the XT effect.

Fig. 2 (a) shows the BER performance of the proposed APA versus the launch power for different projection orders. The accumulated XT level in this simulation is about -6.5 dB and the BER is calculated after running 130,000 iterations for updating the equalizer weights, that was sufficient to have the minimum BER below the FEC threshold. The APA significantly outperforms NLMS-algorithm, with BER decreasing when increasing the projection order especially at lower launch power, as expected due to low convergence speed of the NLMS algorithm. In addition, the APA with higher projection orders has a similar performance as the RLS-algorithm.

The convergence behavior of the proposed algorithm is evaluated in Fig. 2 (b). The APA clearly converges faster than the NLMS algorithm. The RLS algorithm offers the fastest convergence speed but at the expense of the computational complexity, which is large compared to other algorithms. Significant improvement in convergence speed can be obtained; about 72,000 iterations can be saved when increasing the projection order L from 1 up to 10 to obtain the BER at FEC-threshold. Moreover, slightly faster convergence speed is achieved by further increasing the APA-order. Hence, a moderate APA-order provides a reasonable convergence speed.

Fig. 2 (c) illustrates the XT penalty compared to the system with no XT in terms of the optical SNR (OSNR) required to achieve 10^{-3} BER. The NLMS-algorithm can support about -12 dB XT level accumulated over 980 km transmission link. The APA can support -9 dB of accumulated XT with projection order 5, while APA with order 20 support the same tolerable end-to-end XT of RLS-algorithm. The computational complexity per iteration to update

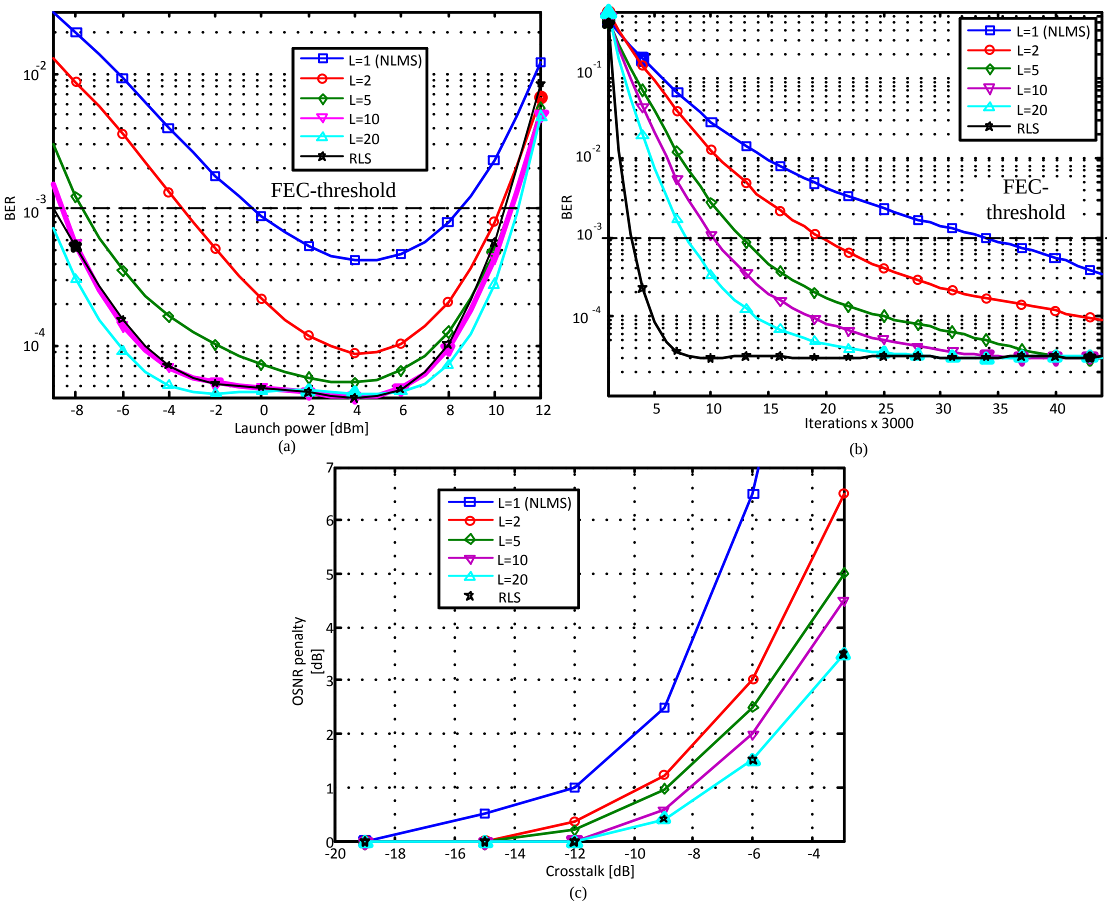

Fig. 2 (a) BER vs. Launch power in dBm for NLMS, APA with μ=0.001 and RLS with forgetting factor λ=0.999 at XT=-6.5 dB, (b) BER vs. number of iterations, and (c) OSNR penalty vs. crosstalk at 0.1 nm resolution bandwidth; launch power is taken at the min BER.

the $2M$ adaptive equalizers of one output for the NLMS, RLS, and APA algorithms are summarized in Table 1, where N is the $2M$-filter length product. Generally for $N >> L$, APA has much less computational complexity than the RLS algorithm, which is of order $O(N^2)$. In this simulation, APA with L=20 needs only 14 % and 19 % of complex multiplications and additions that is needed for RLS-algorithm with almost the same performance.

Table 1. Computational complexity of the different algorithms.

Algorithm	Complex multiplications	Complex additions
NLMS	$4N+2$	$3N+2$
APA	$(L^2+2L+1)N+L^3+L^2$	$(L^2+2L)N+L^3+L$
RLS	$4N^2+4N$	$3N^2+N-1$

4. Conclusion

The performance of MIMO-equalization using APA has been evaluated in MCF-based systems. It has been shown that using a moderate projection order can improve the convergence rate, which minimizes the overhead compared to the LMS-algorithm, while the computational complexity is reduced compared to the RLS-algorithm.

5. References

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