Semiconductor Devices (EE336)

Lec. 5: Fermi level and Drift motion of carriers

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Lecture Outline

- Fermi level and its change relative to its intrinsic level with doping
- Temperature dependence of carrier concentration
- Charge neutrality and compensation
- Carrier transport
- Thermal velocity of carriers
- Drift of carriers due to external applied electric field
- Mobility of charge carriers

Carrier concentration n_0 and p_0 at thermal equillibrium

$$\boxed{n_0 = N_c e^{-(E_c - E_f)/kT}}$$

$$N_c \equiv 2\left[\frac{2\pi m_n^* kT}{h^2}\right]^{3/2}$$

N_c is called the ***effective density of states of the conduction band***

$$\boxed{p_0 = N_v e^{-(E_f - E_v)/kT}}$$

$$N_v \equiv 2\left[\frac{2\pi m_p^* kT}{h^2}\right]^{3/2}$$

N_v is called the ***effective density of states of the valence band***

- Looking at formulas, as n_0 increases (due to n-doping for example) E_f moves closer to E_c and similarly p_0 increases as E_f moves closer to E_v
- For Si at T=300 K, $m_n^* = 1.1m_0 \rightarrow N_c = 2.8\times10^{19}$ cm^{-3}
- For Si at T=300 K, $m_p^* = 0.57m_0 \rightarrow N_v = 1.07\times10^{19}$ cm^{-3}

Product of n_0 and p_0 (Either intrinsic or extrinsic)

$$\left.\begin{aligned} n_0 &= N_c e^{-(E_c - E_f)/kT} \\ p_0 &= N_v e^{-(E_f - E_v)/kT} \end{aligned}\right\} \quad \begin{aligned} n_0 p_0 &= N_c N_v e^{-(E_c - E_v)/kT} \\ &= N_c N_v e^{-E_g/kT} \end{aligned}$$

- Product of n_0 and p_0 formula holds even if the SC is doped since it only depends on N_c, N_v and E_g where none of them changes with doping !! (**Very important**)

- Remember that we used Boltzmann approximation in derivation of above formulas

$$f(E) \approx e^{-\left(E - E_f\right)/kT} \qquad \begin{aligned} E - E_f &>> kT \\ E - E_f &> 3kT \end{aligned}$$

→ What does that mean physically? (SC is lightly doped or non-degenerate such that E_f is at least 3kT below E_c (n-type) or 3kT above E_v (p-type))

→ In lightly doped SC, donor or acceptor energy levels are discrete (not bands)

Intrinsic carrier concentration n_i at thermal equillibrium

$$\left.\begin{aligned} n_0 &= N_c e^{-(E_c - E_f)/kT} \\ p_0 &= N_v e^{-(E_f - E_v)/kT} \end{aligned}\right\} \quad \begin{aligned} n_0 p_0 &= N_c N_v e^{-(E_c - E_v)/kT} \\ &= N_c N_v e^{-E_g/kT} \end{aligned}$$

$$\Downarrow$$

$$\boxed{\begin{aligned} & n_0 = p_0 = n_i \qquad \text{(Intrinsic)} \\ & n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT} \end{aligned}}$$

- For Si at T=300 K, $N_c = 2.8 \times 10^{19}$ cm^{-3} , $N_v = 1.07 \times 10^{19}$ cm^{-3}, Eg = 1.1 eV substitute above to get

$$\boxed{n_i = 10^{10} \quad \text{cm}^{-3}}$$

Intrinsic Fermi level E_i

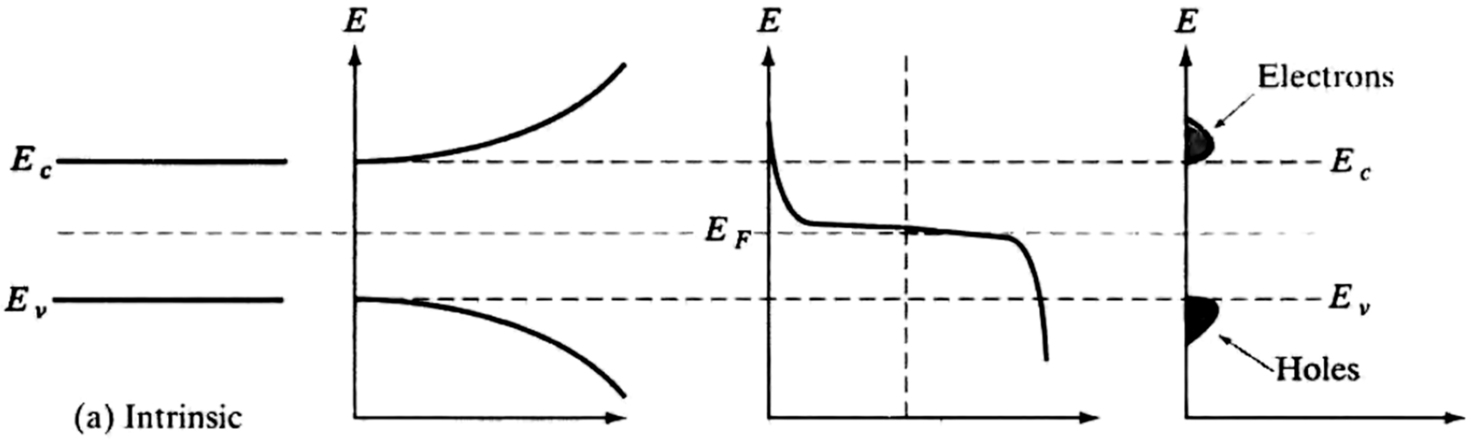
$$n_0 = p_0$$

$$N_c e^{-(E_c - E_i)/kT} = N_v e^{-(E_i - E_v)/kT}$$

$$E_i = \frac{E_c + E_v}{2} + \frac{kT}{2}\ln\left(\frac{N_v}{N_c}\right) = \frac{E_c + E_v}{2} + \frac{3kT}{4}\ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$E_i \approx \frac{E_c + E_v}{2}$$

E_i lies (almost) in the middle between E_c and E_v (provided hole and electron effective masses are approximately the same)



What happens to E_f if we dope the SC (change n_0 from n_i)?

$$n_0 = N_c e^{-(E_c - E_f)/kT}$$

$$n_i = N_c e^{-(E_c - E_i)/kT}$$

$$N_c = n_i e^{(E_c - E_i)/kT}$$

$$n_0 = n_i e^{(E_c - E_i)/kT} . e^{-(E_c - E_f)/kT}$$

$$\boxed{n_0 = n_i e^{(E_f - E_i)/kT}}$$

$$\boxed{\therefore \quad E_f = E_i + kT \ln\left(\frac{n_0}{n_i}\right)}$$

$$p_0 = N_v e^{-(E_f - E_v)/kT}$$

$$n_i = N_v e^{-(E_i - E_v)/kT}$$

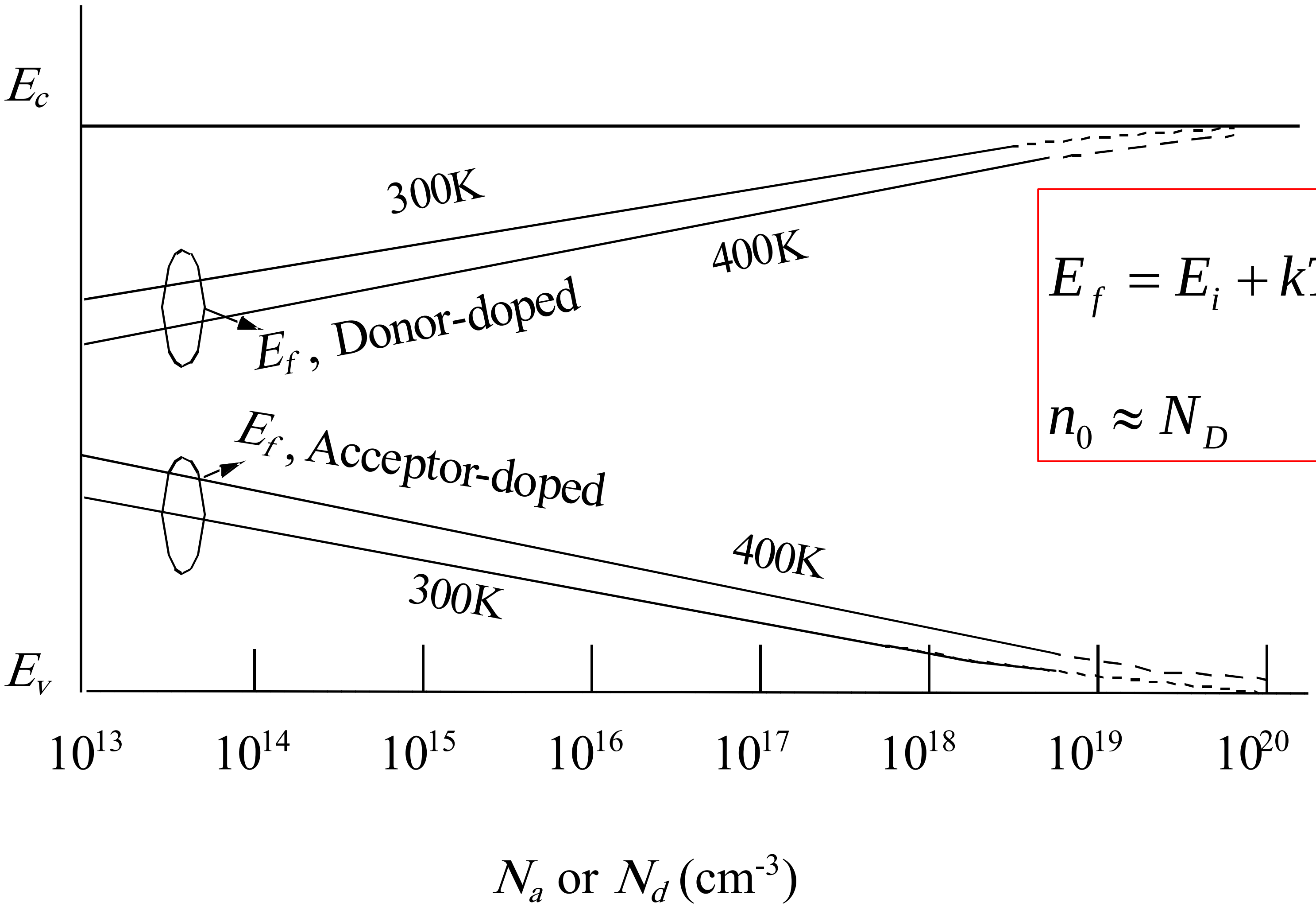
$$N_v = n_i e^{(E_i - E_v)/kT}$$

$$p_0 = n_i e^{(E_i - E_v)/kT} . e^{-(E_f - E_v)/kT}$$

$$\boxed{p_0 = n_i e^{(E_i - E_f)/kT}}$$

$$\boxed{\therefore \quad E_f = E_i - kT \ln\left(\frac{p_0}{n_i}\right)}$$

What happens to E_f if we dope the SC (change n_0 from n_i)?

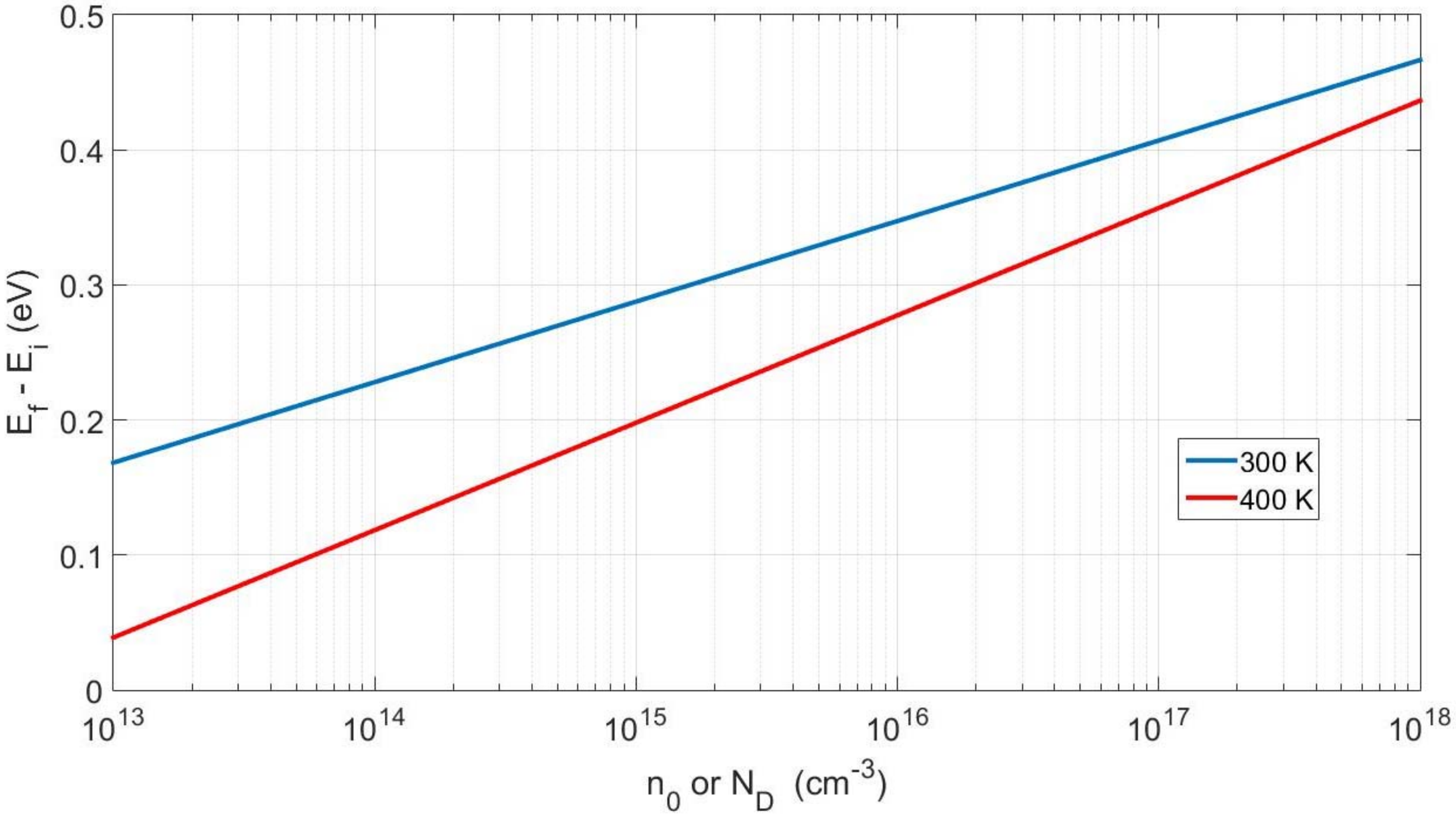


$$E_f = E_i + kT \ln\left(\frac{n_0}{n_i}\right)$$

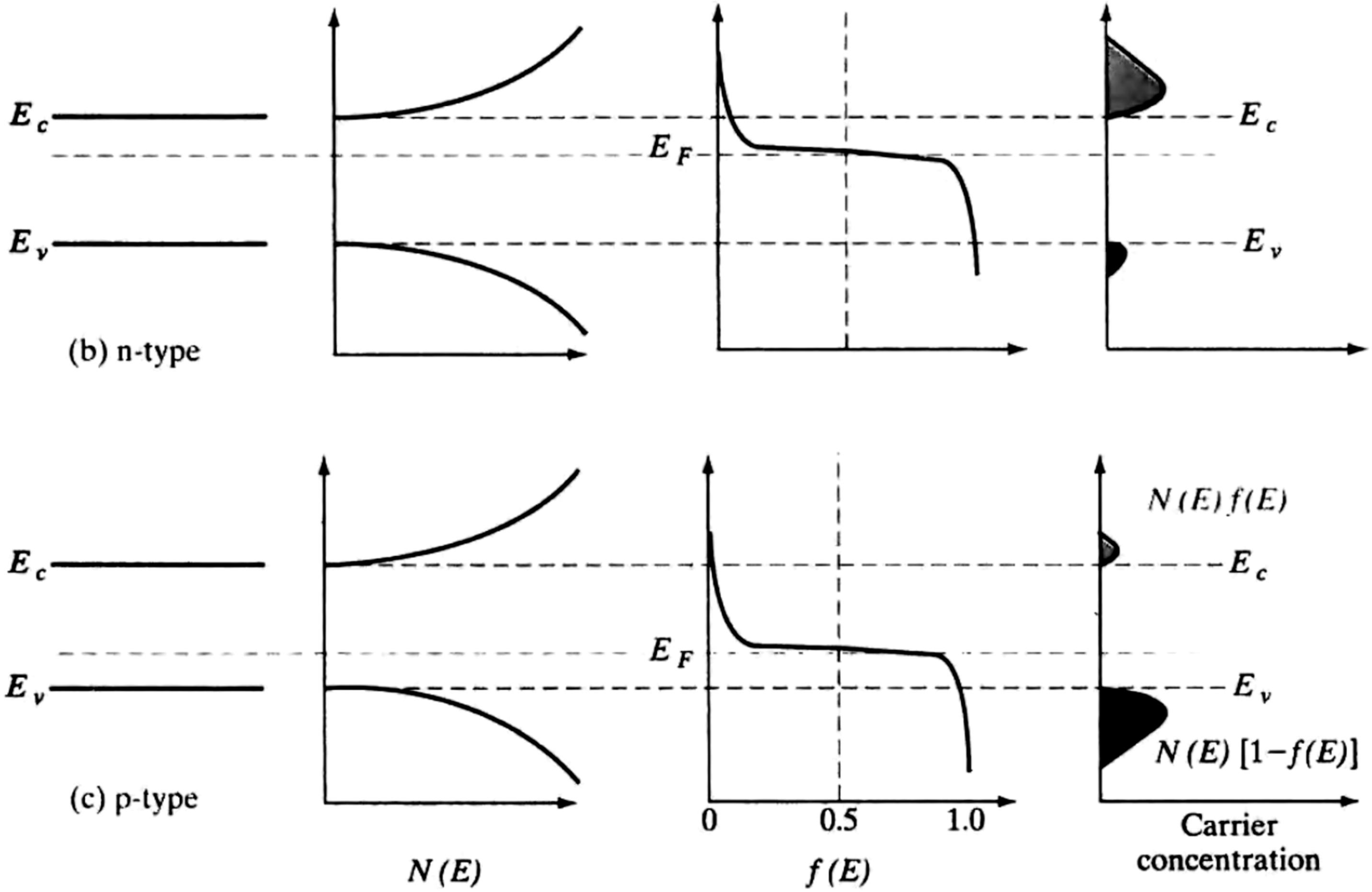
$$n_0 \approx N_D$$

Take home exercise

- Plot on MATLAB E_f-E_i for Si at both T = 300K and 400K (<u>Hint</u>: you have to find n_i at 300K and 400K using $m_n^* = 1.1m_0$, $m_p^* = 0.57m_0$, Eg = 1.1 eV)



What happens to E_f if we dope the SC?



Numerical example

- A Si sample is doped with 10^{17} As atoms/cm^3. What is the equilibrium hole concentration at 300 K? Where is E_f relative to E_i? Where is E_f relative to E_c? (use $n_i = 1.5\times10^{10}$ cm^{-3} at this temperature)

$$n_0 \approx N_D = 10^{17} \quad \text{cm}^{-3}$$

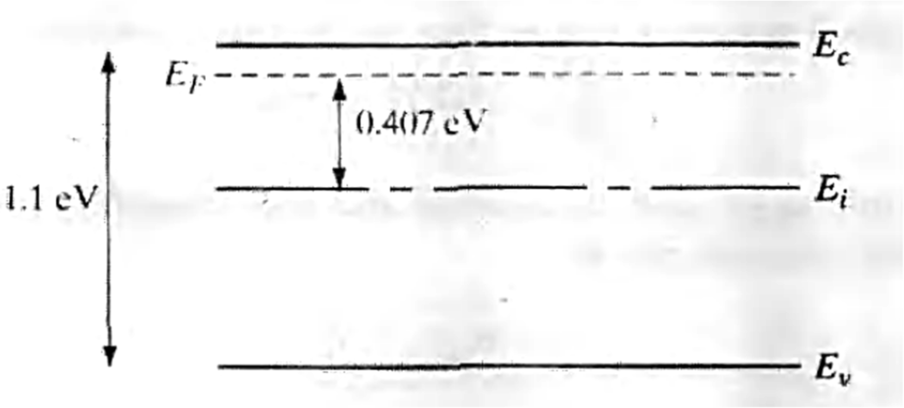
$$p_0 = \frac{n_i^2}{n_0} = 2.25\times10^3 \quad \text{cm}^{-3}$$

$$\because \quad n_0 = n_i e^{(E_f - E_i)/kT}$$

$$\therefore \quad E_f = E_i + kT\ln\left(\frac{n_0}{n_i}\right)$$

$$E_f - E_i = 0.0259\ln\left(\frac{10^{17}}{1.5\times10^{10}}\right) = 0.407 \quad \text{eV}$$

$$\therefore \quad E_c - E_f = 0.5E_g - (E_f - E_i) = 0.55 - 0.407 = 0.143 \ \text{eV}$$



General theory of n_0 and p_0 (charge neutrality and compensation)

What if SC is doped with both donors and acceptors?

Charge neutrality: $\boxed{n_0 + N_a^- = p_0 + N_d^+}$

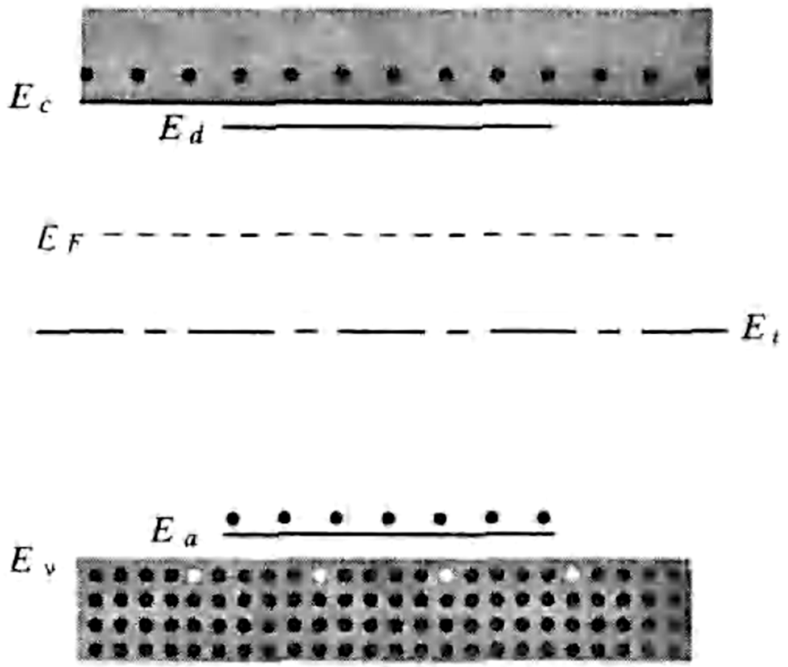
$$n_0 p_0 = n_i^2$$

$$p_0 = \frac{N_a - N_d}{2} + \left[\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2\right]^{1/2}$$

$$n_0 = \frac{N_d - N_a}{2} + \left[\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2\right]^{1/2}$$

General theory of n_0 and p_0 (charge neutrality and compensation)

How does compensation physically happen?



N-type material

$N_d > N_a$

General theory of n_0 and p_0 (charge neutrality and compensation)

I. $N_d - N_a \gg n_i$ (i.e., N-type)

$$\boxed{\begin{aligned} n_0 &= N_d - N_a \\ p_0 &= n_i^2 / n_0 \end{aligned}}$$

Majority conc.

Minority conc.

If $N_d \gg N_a$, $n_0 = N_d$ and $p_0 = n_i^2 / N_d$

II. $N_a - N_d \gg n_i$ (i.e., P-type)

$$\boxed{\begin{aligned} p_0 &= N_a - N_d \\ n_0 &= n_i^2 / p_0 \end{aligned}}$$

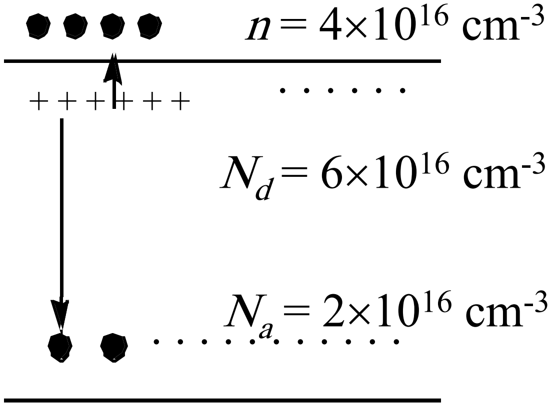
If $N_a \gg N_d$, $p_0 = N_a$ and $n_0 = n_i^2 / N_a$

General theory of n_0 and p_0 (charge neutrality and compensation)

What are n and p in Si with (a) $N_d = 6\times10^{16}$ cm^{-3} and $N_a = 2\times10^{16}$ cm^{-3} and (b) additional 6×10^{16} cm^{-3} of N_a?

(a) $n = N_d - N_a = 4\times10^{16}\,\text{cm}^{-3}$

$$p = n_i^{\,2} / n = 10^{20} / 4\times10^{16} = 2.5\times10^{3}\,\text{cm}^{-3}$$



$n = 4\times10^{16}$ cm^{-3}

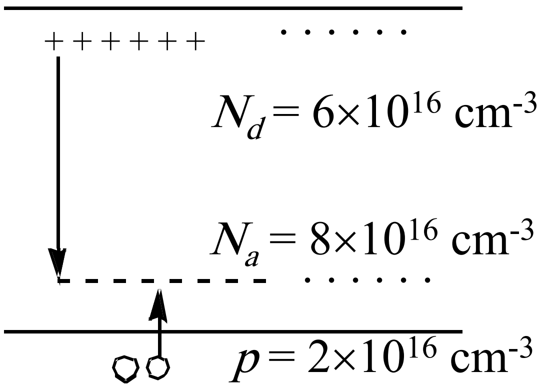
$N_d = 6\times10^{16}$ cm^{-3}

$N_a = 2\times10^{16}$ cm^{-3}

(b) $N_a = 2\times10^{16} + 6\times10^{16} = 8\times10^{16}$ cm^{-3} $> N_d$

$$p = N_a - N_d = 8\times10^{16} - 6\times10^{16} = 2\times10^{16}\,\text{cm}^{-3}$$

$$n = n_i^{\,2} / p = 10^{20} / 2\times10^{16} = 5\times10^{3}\,\text{cm}^{-3}$$



$N_d = 6\times10^{16}$ cm^{-3}

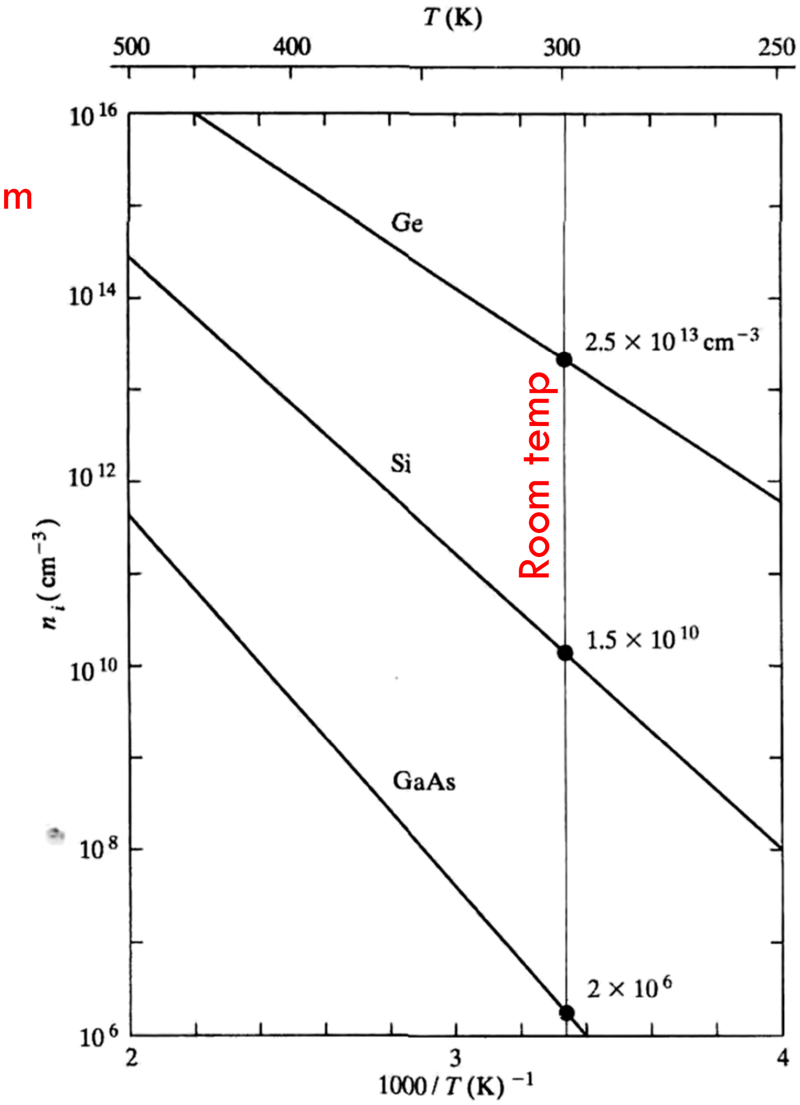
$N_a = 8\times10^{16}$ cm^{-3}

$p = 2\times10^{16}$ cm^{-3}

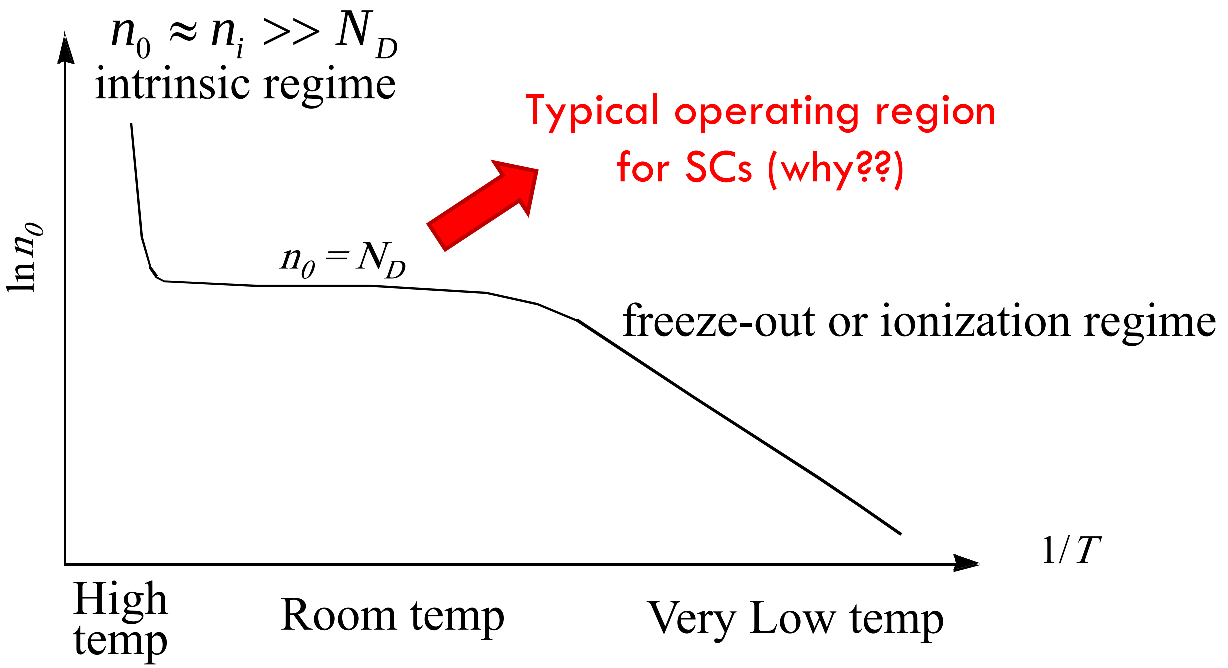
Effect of temperature on n_i

$$n_i(T) = 2\left(\frac{2\pi kT}{h^2}\right)^{3/2}(m_n^* m_p^*)^{3/4}\underbrace{e^{-E_g/2kT}}_{\text{Dominant term}}$$

Semilog plot of n_i versus $1/T$ is linear



Effect of temperature on n_0 for a n-type SC



Typical operating region for SCs (why??)

$$\text{Very high T: } n_0 = p_0 = n_i = \sqrt{N_c N_v} e^{-E_g/2kT}$$

➡ Thermally gen. Intrinsic EHPs dominate donor electrons

$$\text{Very low T: } n_0 = \left[\frac{N_c N_D}{2}\right]^{1/2} e^{-(E_c - E_d)/2kT}$$

➡ Donor electrons are the only free electrons in CB (no intrinsic EHP)

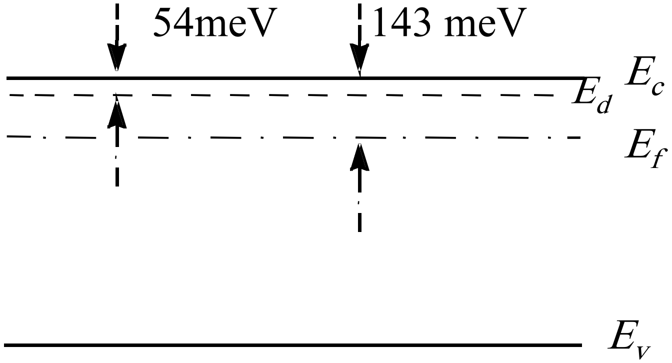
Final note about 100% ionization of impurity atoms

Illustrative Example (continue example in slide 11)

N_d = 10^{17} cm^{-3} (As donor atoms) What fraction of the donors are not ionized?

***Solution:** First assume that all the donors **are** ionized.*

$$n_0 = N_d = 10^{17}\,\mathrm{cm}^{-3} \Rightarrow E_c - E_f = 143\,\mathrm{meV}$$



Probability of not being ionized

$$\approx \frac{1}{1+\frac{1}{2}e^{(E_d-E_f)/kT}} = \frac{1}{1+\frac{1}{2}e^{((143-54)\mathrm{meV})/26\mathrm{meV}}} = 0.061$$

Therefore, it is reasonable to assume complete ionization, i.e., $n_0 = N_d$.

Carrier transport

This part is concerned with motion of charge carriers (electrons and holes)

External applied
electric field
(Drift)

Changes in carrier
concentration with
position or its
gradient
(Diffusion)

For this part, I am closely following chapter 2 in this book

"Modern semiconductor devices for integrated circuits," by Chenming Hu Prentice Hall, 2010 [https://people.eecs.berkeley.edu/~hu/Book-Chapters-and-Lecture-Slides-download.html]

Thermal motion

- Even without applied Electric field, carriers are not at rest and possess finite kinetic energy due to thermal excitation

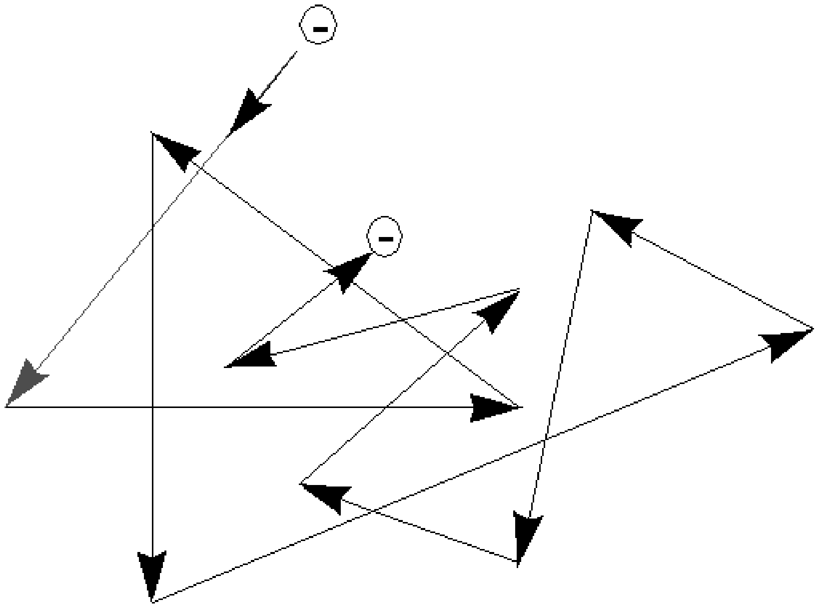
$$\text{Average electron K.E in CB} = \frac{\text{Total K.E.}}{\text{Elect. conc. in CB}} = \frac{\int_{E_c}^{\infty} f(E) N_c(E)(E - E_c)\,dE}{n_0}$$

$$\text{Average electron or hole kinetic energy} = \frac{3}{2}kT = \frac{1}{2}mv_{th}^2$$

$$v_{th} = \sqrt{\frac{3kT}{m_{eff}}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23}\ \text{JK}^{-1} \times 300\ \text{K}}{0.26 \times 9.1 \times 10^{-31}\ \text{kg}}}$$

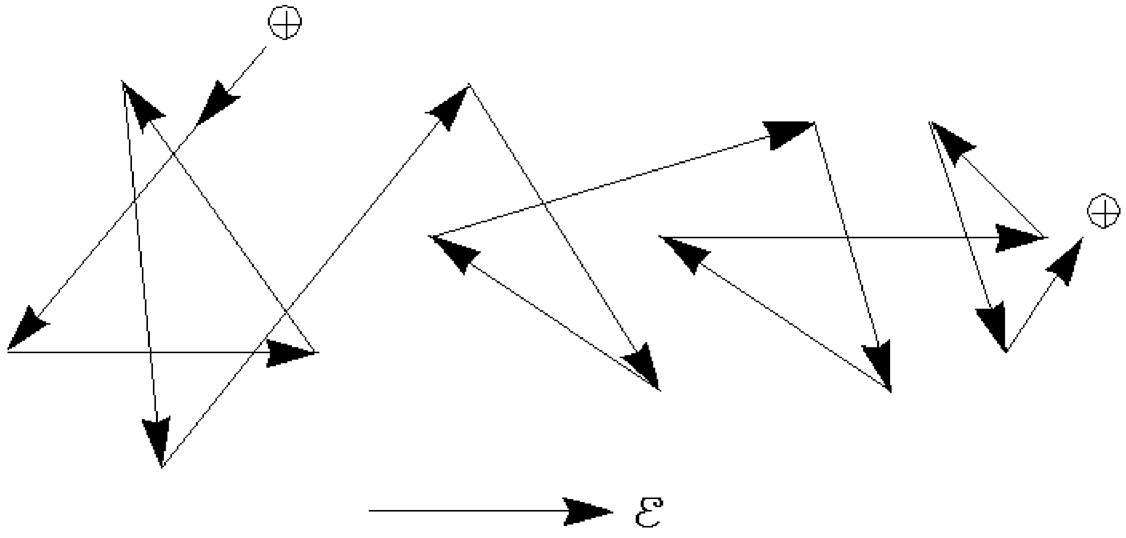
$$= 2.3 \times 10^{5}\ \text{m/s} = 2.3 \times 10^{7}\ \text{cm/s}$$

Thermal motion



- Zig-zag motion is due to collisions or scattering with imperfections in the crystal.
- Net thermal velocity is zero (averaged over many electrons at given time) and hence steady state current due to thermal motion is zero → only causes thermal noise
- Mean time between collisions is $\tau_m \sim 0.1\text{ps}$ (Mean free time)

Carrier drift



- ***Drift*** is the motion caused by an electric field.

Electron and hole mobility

$$m_p^* v_p = q\mathrm{E}\tau_{mp}$$

$$v_p = \frac{q\mathrm{E}\tau_{mp}}{m_p^*}$$

Momentum lost due to collision or scattering equals momentum gain between two scattering events due to external applied force (at steady state)

$$v_p = \mu_p \mathbf{E}$$

$$\mu_p = \frac{q\tau_{mp}}{m_p^*}$$

$$v_n = -\mu_n \mathbf{E}$$

$$\mu_n = \frac{q\tau_{mn}}{m_n^*}$$

- μ_p is the hole mobility and μ_n is the electron mobility
- τ_{mp} is the mean free time for holes and τ_{mn} is the mean free time for electrons which is the average time between two scattering events

Electron and hole mobility

$$v = \mu \mathbf{E}\ ; \quad \mu \text{ has the dimensions of } v/\mathbf{E} \quad \left[\frac{\text{cm/s}}{\text{V/cm}} = \frac{\text{cm}^2}{\text{V}\cdot\text{s}}\right].$$

Electron and hole mobilities of selected semiconductors

	Si	**Ge**	**GaAs**	**InAs**
μ_n (cm^2/V·s)	1400	3900	8500	30000
μ_p (cm^2/V·s)	470	1900	400	500

Based on the above table alone, which semiconductor and which carriers (electrons or holes) are attractive for applications in high-speed devices?

Numerical Example

Given $\mu_p = 470\ cm^2/V{\cdot}s$, what is the hole drift velocity at $\mathbf{E} = 10^3\ V/cm$*? What is τ_{mp} and what is the distance traveled between collisions (called the* ***mean free path****)? Hint: When in doubt, use the MKS system of units.*

Solution: $v = \mu_p \mathbf{E} = 470\ \text{cm}^2/\text{V}\cdot\text{s} \times 10^3\ \text{V/cm} = 4.7\times 10^5\ \text{cm/s}$

$$\tau_{mp} = \mu_p m_p / q = 470\ \text{cm}^2/\text{V}\cdot\text{s} \times 0.39 \times 9.1\times10^{-31}\ \text{kg}/1.6\times10^{-19}\ \text{C}$$

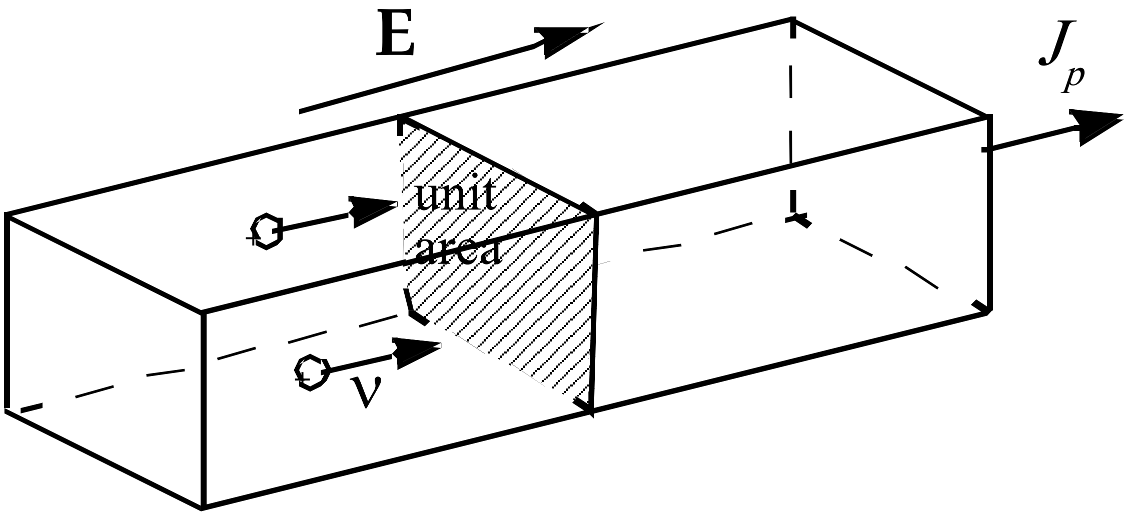
$$= 0.047\ \text{m}^2/\text{V}\cdot\text{s} \times 2.2\times10^{-12}\ \text{kg/C} = 1\times10^{-13}\text{s} = 0.1\ \text{ps}$$

$$\textit{mean free path} = \tau_{mh} v_{th} \sim 1\times 10^{-13}\ \text{s} \times 2.2\times10^{7}\ \text{cm/s}$$

$$= 2.2\times10^{-6}\ \text{cm} = 220\ \text{Å} = 22\ \text{nm}$$

This is smaller than the typical dimensions of devices, but getting close.

Drift current density



Hole current density $\boxed{J_p = qpv}$ A/cm² or C/cm²·sec

EXAMPLE: If $p = 10^{15}\text{cm}^{-3}$ and $v = 10^4$ cm/s, then

$$J_p = 1.6\times10^{-19}\text{C} \times 10^{15}\text{cm}^{-3} \times 10^4\text{cm/s}$$

$$= 1.6\ \text{C/s}\cdot\text{cm}^2 = 1.6\ \text{A/cm}^2$$

Drift current density

$$J_{p,drift} = qpv = qp\mu_p\mathbf{E}$$

$$J_{n,drift} = -qnv = qn\mu_n\mathbf{E}$$

Ohm's law

$$J_{drift} = J_{n,drift} + J_{p,drift} = \sigma\mathbf{E} = (qn\mu_n + qp\mu_p)\mathbf{E}$$

$\therefore$ **conductivity** (1/ohm-cm or S/cm) of a semiconductor is $\sigma = qn\mu_n + qp\mu_p$

$1/\sigma =$ is resistivity (ohm-cm)

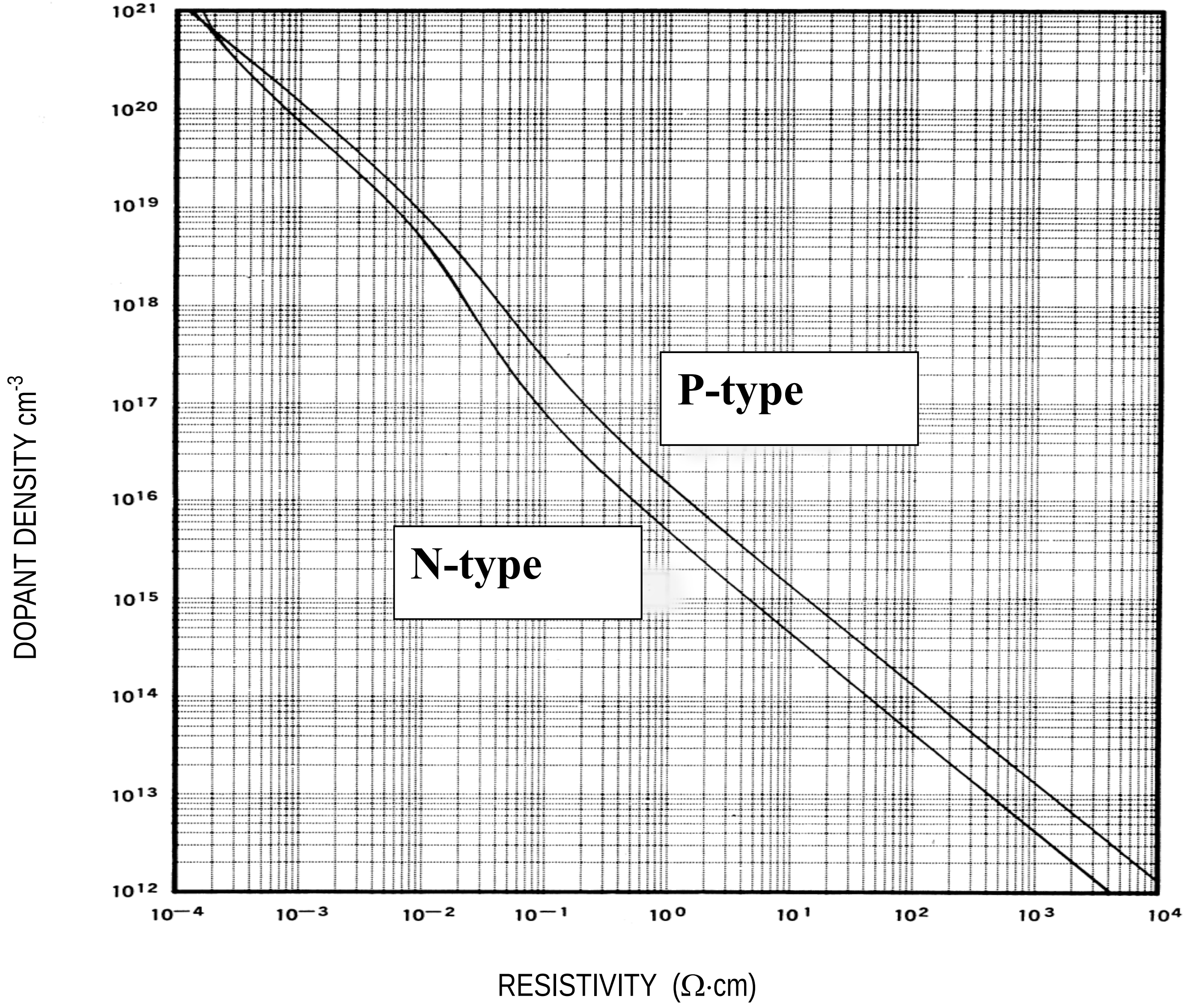
Numerical example

What is the resistivity of intrinsic Si? Use $\mu_n = 1350$ and $\mu_p = 480$ cm^2/V.s and $n_i = 1.5\times10^{10}$ cm^{-3}

$$\rho = \frac{1}{\sigma} = \frac{1}{q\left(\mu_n + \mu_p\right)n_i} = 2.28\times10^5 \quad \Omega.\text{cm}$$

- This number is expected to decrease when doping is made because of the increase in carrier concentration
- Be careful that the mobility will also decrease as the doping concentration increases due to larger impurity scattering as will be seen

Resistivity versus doping concentration for Si at room temp



Mechanisms of carrier scattering

There are two main causes of carrier scattering which impact carrier mobility:

1. Phonon Scattering (Phonon = lattice vibrations)
2. Ionized-Impurity (Coulombic) Scattering

Phonon scattering mobility decreases when temperature rises*:*

$$\mu_{phonon} \propto \tau_{phonon} \propto \frac{1}{phonon \quad density \times carrier \quad thermal \quad velocity} \propto \frac{1}{T \times T^{1/2}} \propto T^{-3/2}$$

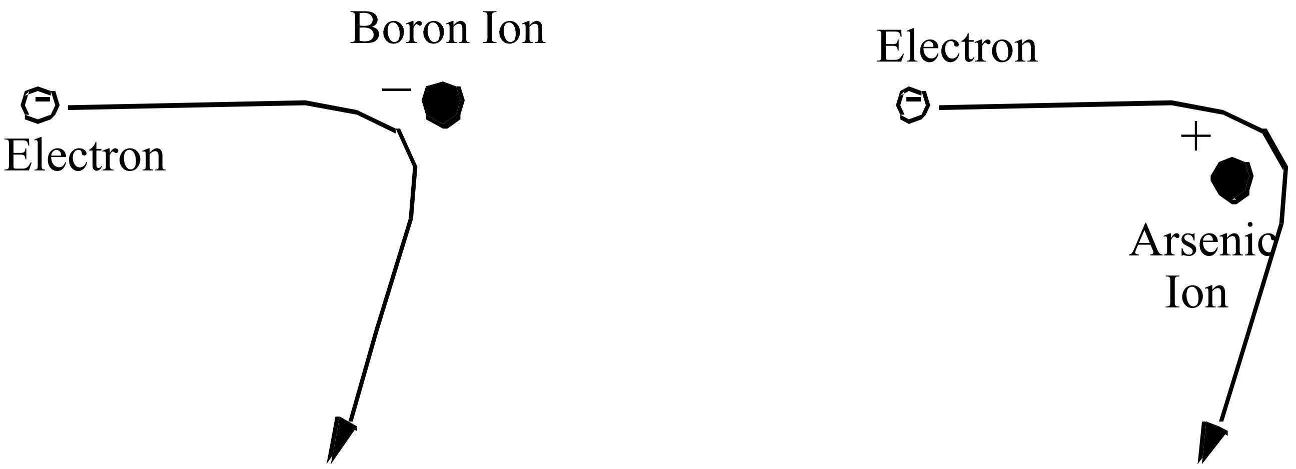
$$\mu = q\tau/m$$

$$\propto T$$

$$v_{th} \propto T^{1/2}$$

Mechanisms of carrier scattering

Impurity (Dopant)-Ion Scattering or Coulombic Scattering



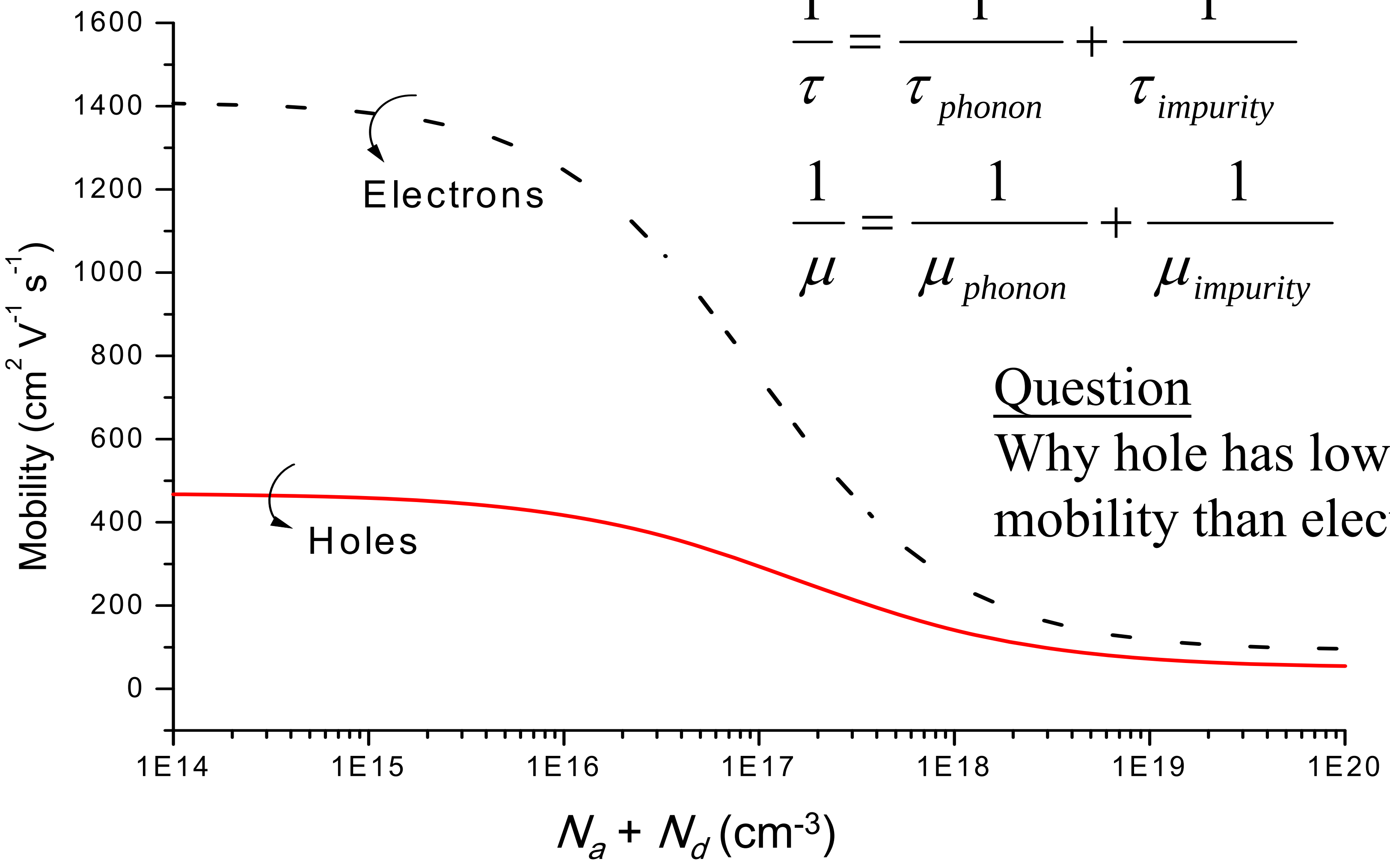
There is less change in the direction of travel if the electron zips by the ion at a higher speed.

$$\mu_{impurity} \propto \frac{v_{th}^{3}}{N_a + N_d} \propto \frac{T^{3/2}}{N_a + N_d}$$

Total Mobility (sum of rates of two mechanisms)

$$\frac{1}{\tau} = \frac{1}{\tau_{phonon}} + \frac{1}{\tau_{impurity}}$$

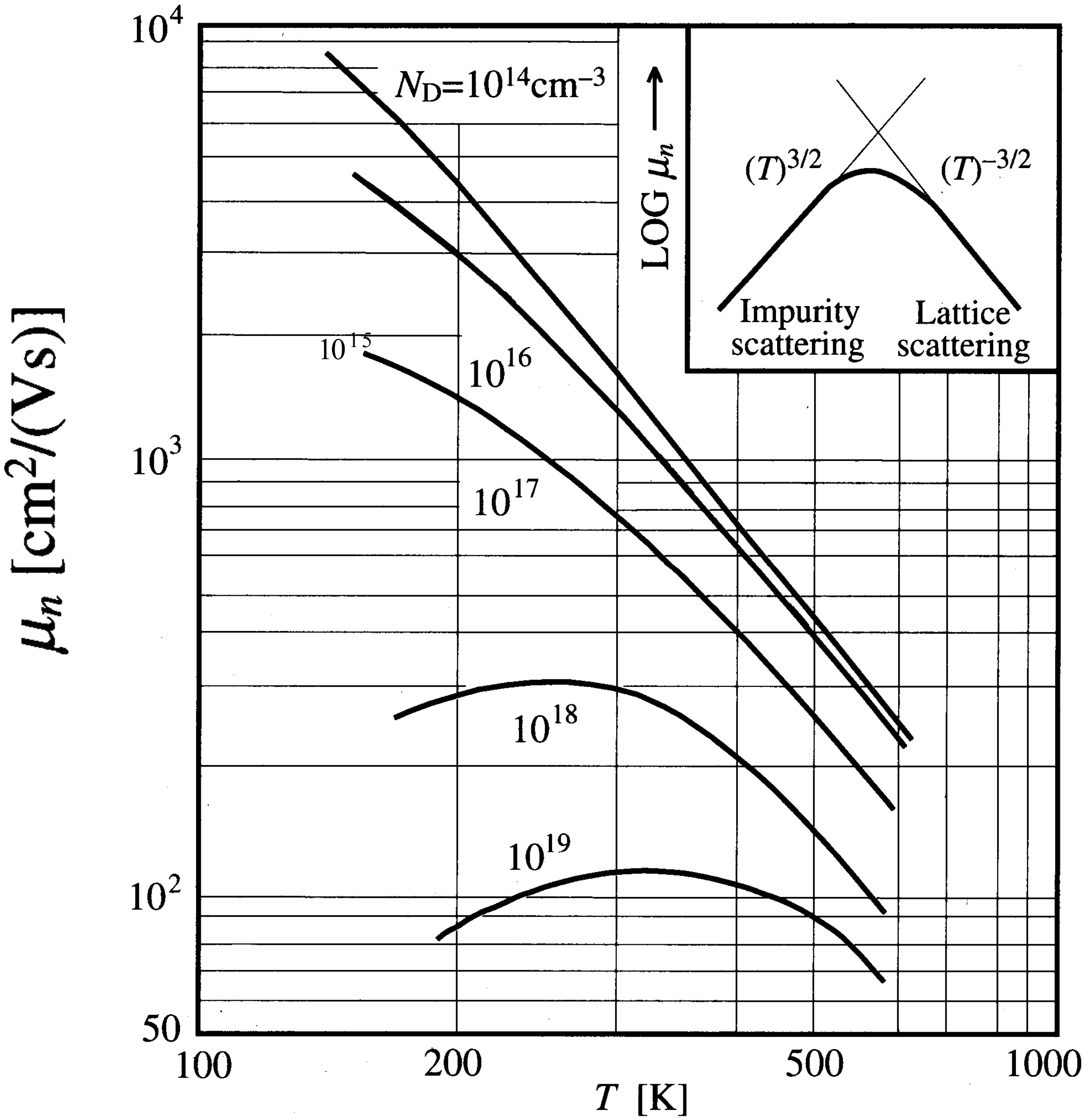
$$\frac{1}{\mu} = \frac{1}{\mu_{phonon}} + \frac{1}{\mu_{impurity}}$$



<u>Question</u>

Why hole has lower mobility than electrons?

Temperature effect on mobility



Velocity saturation (High field effects)

- When the kinetic energy of a carrier exceeds a critical value, it generates an optical phonon and loses the kinetic energy.
- Therefore, the kinetic energy is capped at large **E,** and the velocity does not rise above a saturation velocity, v_{sat} (scattering limited velocity)
- ***Velocity saturation*** affects badly device speed