Semiconductor Devices (EE336)

Lec. 4: Carrier concentration and Fermi level

Wed. Oct. 19th, 2016

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Lecture Outline

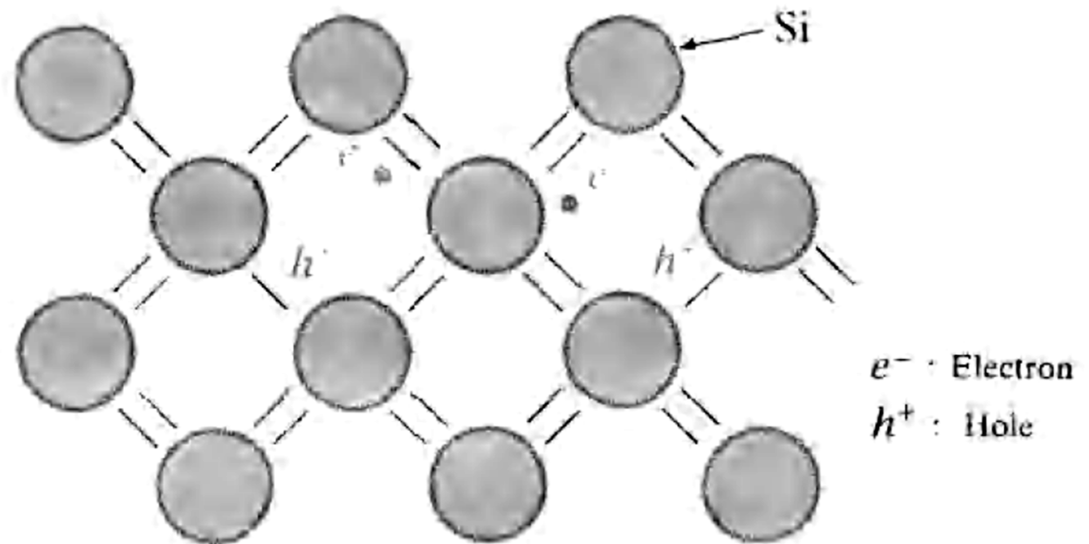
- Intrinsic versus extrinsic SCs
- Donor and acceptor ionization energy
- Derivation of carrier concentrations based on Fermi-Dirac statistics and density of states
- Fermi level and its change relative to its intrinsic level with doping

Intrinsic semiconductors

- Perfect Semiconductor

(no impurities, No lattice defects).

- Thermal excitation breaks one covalent bond producing an electron and a hole free to move across the lattice



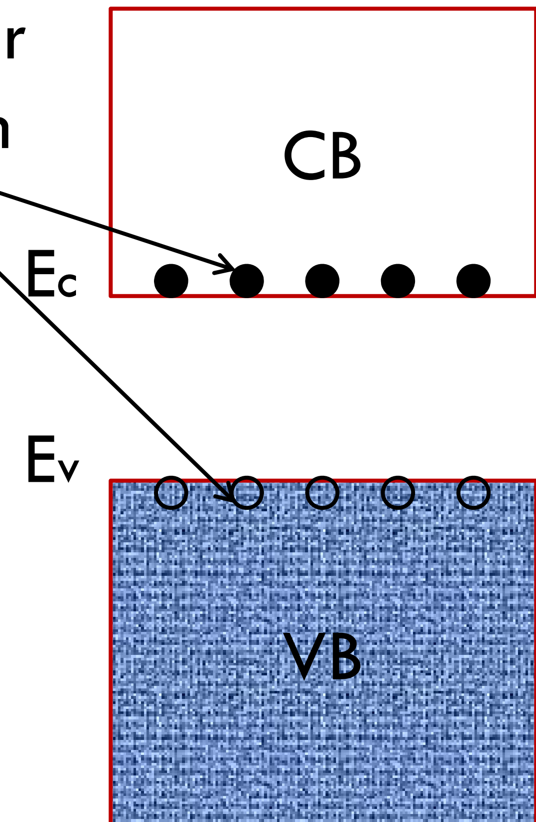
Chemical bond model of intrinsic Si

Intrinsic semiconductors

- Perfect Semiconductor (no impurities, No lattice defects).

- rate of generation, g_i due to thermal excitation at $T > 0$

Electron-Hole Pair (EHP) generation



n electrons/ unit volume in CB
p holes/unit volume in VB

$$n = p = n_i$$

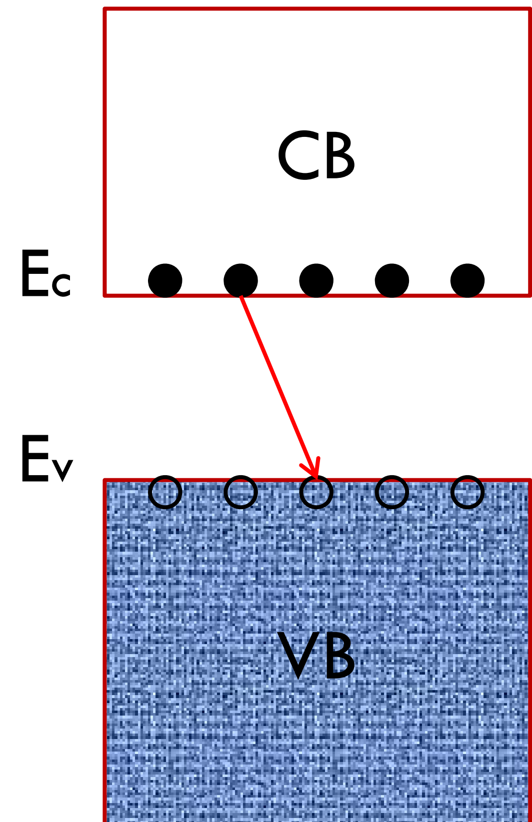
At temperature ($T > 0$ K)

Intrinsic semiconductors

- Perfect Semiconductor

(no impurities, No lattice defects).

- rate of generation, g_i due to thermal excitation at $T > 0$



n electrons/ unit volume in CB

p holes/unit volume in VB

$$n = p = n_i$$

At temperature ($T > 0$ K)

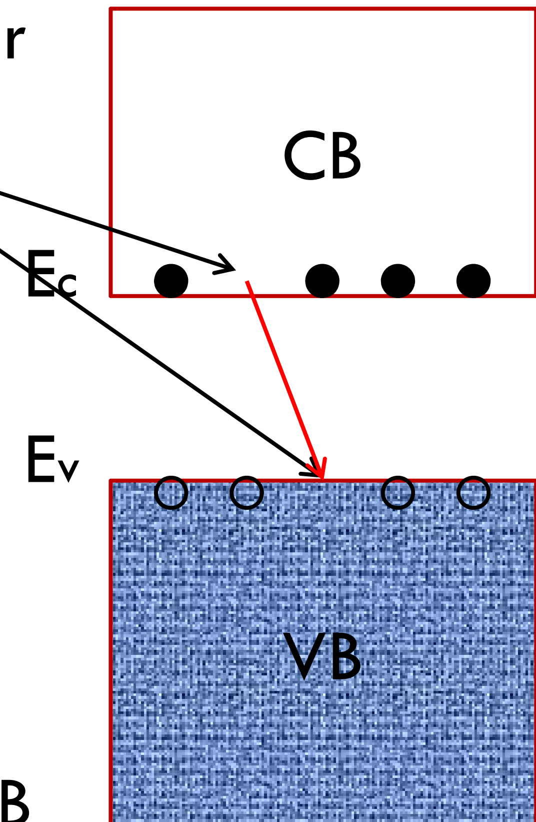
Intrinsic semiconductors

- Perfect Semiconductor

(no impurities, No lattice defects).

- rate of generation, g_i due to thermal excitation at $T > 0$
- rate of recombination, r_i must equal g_i to have a constant steady state carrier concentration **(thermal equilibrium)**

Electron-Hole Pair (EHP) recombination



n electrons/ unit volume in CB

p holes/unit volume in VB

$$n = p = n_i$$

At temperature ($T > 0$ K)

Intrinsic semiconductors

$r_i = g_i$ $n_o = p_o = n_i$

$$r_i \propto n_o p_o$$

$$r_i = \alpha_r n_o p_o = g_i = \alpha_r n_i^2$$

$$\alpha_r \text{ : constant}$$

- Of course n_i is temperature dependent and will later be derived
- Also g_i and r_i are temperature dependent
- At room temperature, Si has n_i of about 10^{10} EHP/cm^3 which is too small compared to its atomic density of 5×10^{22} atoms/cm^3
- Via doping, we can change this carrier density and hence manipulate the conductivity of Si

Extrinsic semiconductors

- Impurities introduced via doping process to vary conductivity of SC

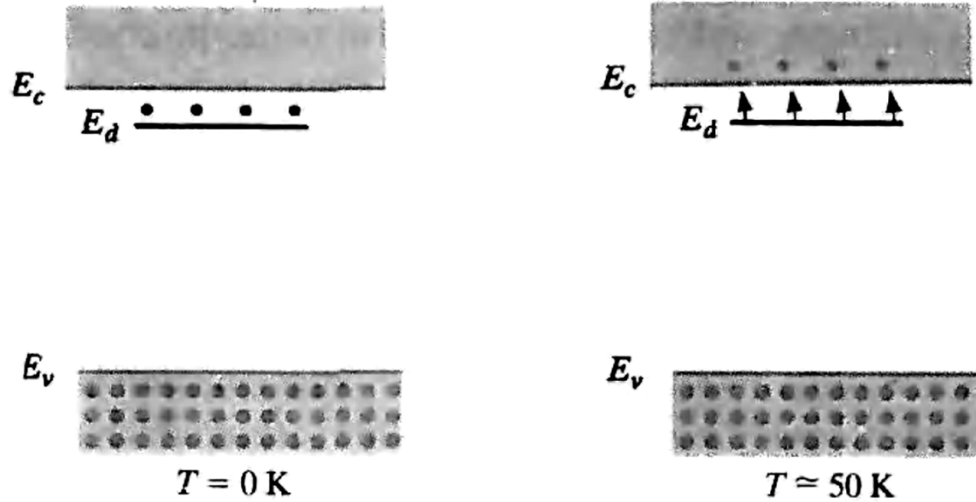
- Doping increases the concentration of one type of carrier (either electrons or holes) compared to the intrinsic concentration n_i due to thermal excitation and hence increases the conductivity of SC

- N-type doping by adding pentavalent (valency = 5) atoms such as As and P

- P-type doping by adding trivalent (valency = 3) atoms such as B, Al, Ga

- Introducing impurities creates additional energy levels typically lying within the bandgap of SC either very near its CB (n-type doping) or its VB (p-type doping)

Extrinsic semiconductors (n-type)

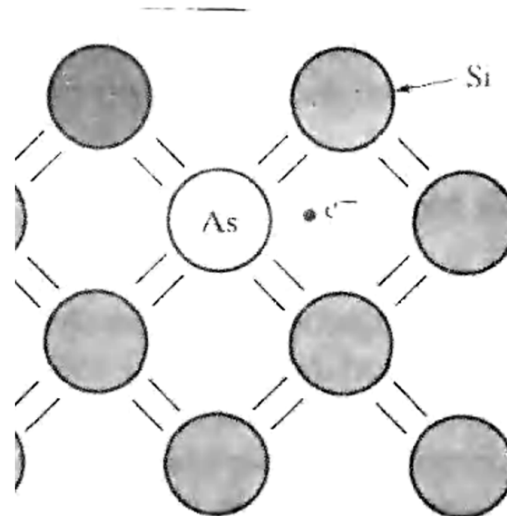


$$n_0 >> n_i$$

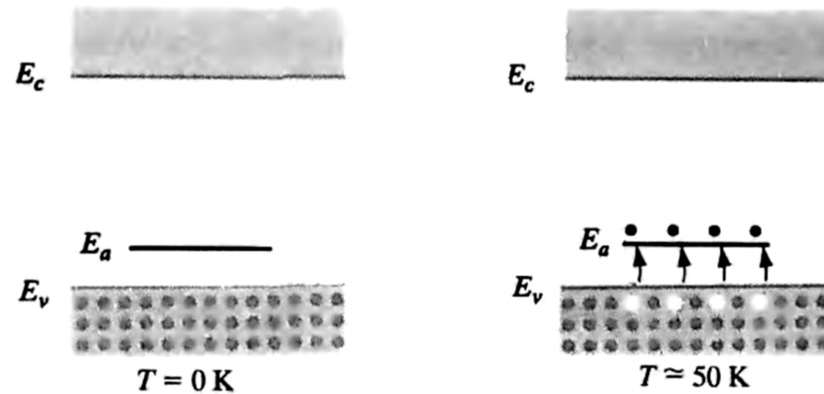
$$n_0 >> p_0$$

$$n_0 \approx N_D$$

Donor concentration (cm^{-3})



Extrinsic semiconductors (p-type)

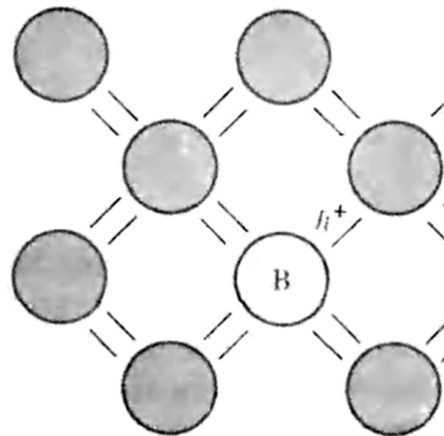


$$p_0 >> n_i$$

$$p_0 >> n_0$$

$$p_0 \approx N_A$$

Acceptor concentration (cm^{-3})



Donor and acceptor ionization energy calculation

- What we mean by ionization energy E_{ion} is the energy (thermal) required to excite an electron from E_d to E_c, i.e. E_c-E_d, or from E_v to E_a, i.e. E_a-E_v

- This is typically a very small quantity relative to the bandgap of SC
- The calculation is made by treating the donor atom, which has 5 valence electrons, as a Hydrogen atom and use the Bohr model
- The four tightly bound electrons that contributed to four covalent bonds will be shielded with the nucleus and all core electrons leaving only a single electron in a Hydrogen-like orbit
- We can then use Bohr's model relationship to calculate the energy to move this electron from n =1 to n=∞ and become a free electron thus ionizing the donor atom as

$$E_{ion} = \frac{mq^4}{2(4\pi\varepsilon_0\varepsilon_r)^2\hbar^2} = \frac{mq^4}{8(\varepsilon_0\varepsilon_r)^2 h^2}$$

Donor and acceptor ionization energy calculation

Ex: Calculate E_{ion} for n-doped Si crystal

$$E_{ion} = \frac{mq^4}{8(\varepsilon_0 \varepsilon_r)^2 h^2}$$

$$m_n^* = \frac{1}{3} m_o, \varepsilon_r = 11.9$$

$$E_{ion} = \frac{9.11 * 10^{-31} * (1.6 * 10^{-19})^4}{3 * 8 * 11.9^2 * (8.85 * 10^{-12})^2 * (6.63 * 10^{-34})^2}$$

$$= 0.000005102 * 10^{-15} \text{ J}$$

$$= \frac{0.000005102 * 10^{-15}}{1.6 * 10^{-19}} \text{ eV}$$

$$= 0.032 \text{ eV}$$

Donor and acceptor ionization energy actual measured values

Ionization energy of selected donors and acceptors in silicon

	Donors			Acceptors		
Dopant	Sb	P	As	B	Al	In
Ionization energy, $E_c - E_d$ or $E_a - E_v$ (meV)	39	44	54	45	57	160

- Because the ionization energy is too small, all electrons due to donors will be present in CB even at very low temperatures whereas EHP generation due to excitation of electrons from VB to CB will typically start at higher temperature because E_g is larger than E_{ion}
- Remember that each donor atom contributes one electron and vice versa each acceptor atom contributes one hole

Carrier concentration (Electrons and holes)

$$n_0 = \int_{E_c}^{\text{top of conduction band}} f(E) N_c(E) dE$$

Electron concentration in CB per unit volume (cm^{-3}) at thermal equilibrium

Probability of occupancy of states as a function of their energy

Density of states (number of states per unit volume that may or may not be filled with electrons in CB)

- The integration will be made from E_c to infinity because as we will see *f*(*E*) will become negligibly small as energy increases beyond top of CB
- Both density of states and probability of occupancy should be derived separately before making integration above
- We will use final expressions of f(E) and N(E) → for complete derivation of both <u>see appendices IV and V in textbook</u>

Carrier concentration (Electrons and holes)

$$n_0 = \int_{E_c}^{\text{top of conduction band}} f(E) N_c(E) dE$$

Electron concentration in CB per unit volume (cm^{-3}) at thermal equilibrium

Probability of occupancy of states as a function of their energy

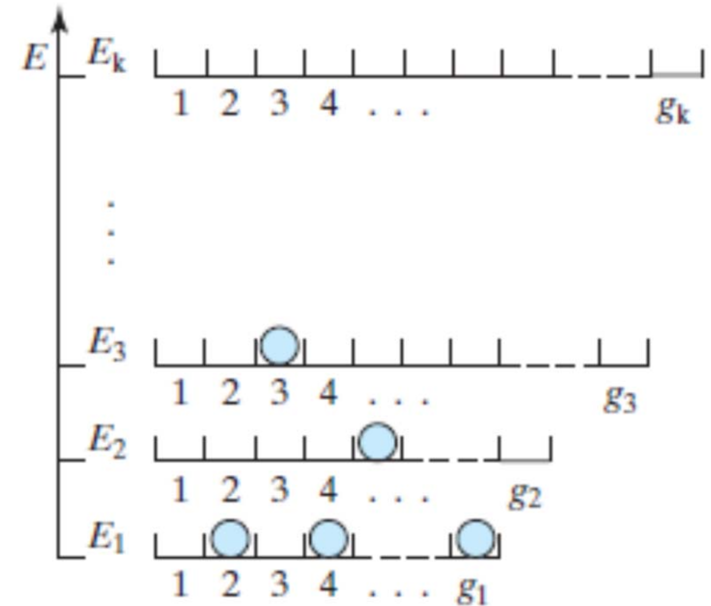
Density of states (number of states per unit volume per unit energy that may or may not be filled with electrons in CB) → $eV^{-1}.cm^{-3}$

$$p_0 = \int_{\text{bottom of valence band}}^{E_v} [1 - f(E)] N_v(E) dE$$

Probability that states are empty, i.e. probability that holes exist at these states

Fermi-Dirac distribution f(E)

- Band comprises discrete energy levels
- There are g_1 states at E_1, g_2 states at E_2… There are N electrons, which constantly shift among all the states but the average electron energy is fixed at 3kT/2.



- There are many ways to distribute N among n_1, n_2, n_3….and satisfy the 3kT/2 condition.

- The equilibrium distribution is the distribution that maximizes the number of combinations of placing n_1 in g_1 slots, n_2 in g_2 slots…. :

$$n_i/g_i = \frac{1}{1+e^{(E_i-E_f)/kT}}$$

E_F is a constant determined by the condition $\sum n_i = N$

Fermi-Dirac distribution f(E)

$$f(E) = \frac{1}{1 + e^{(E - E_f)/kT}}$$

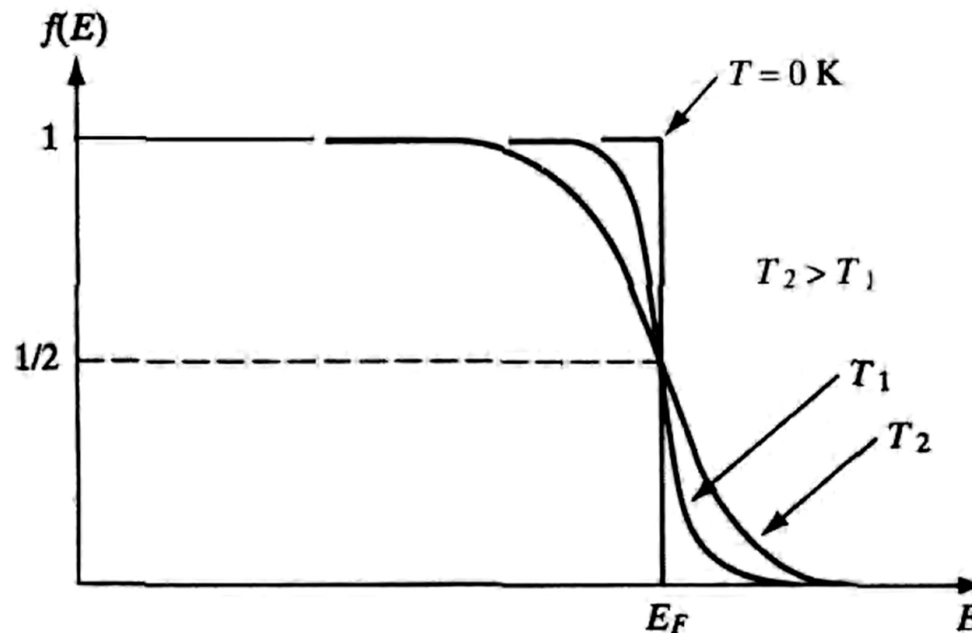
E_f is called the ***Fermi energy*** or the ***Fermi level***

Boltzmann approximation:

$$f(E) \approx e^{-(E - E_f)/kT}$$

$$E - E_f >> kT$$

$$E - E_f > 3kT$$



Fermi-Dirac distribution f(E)

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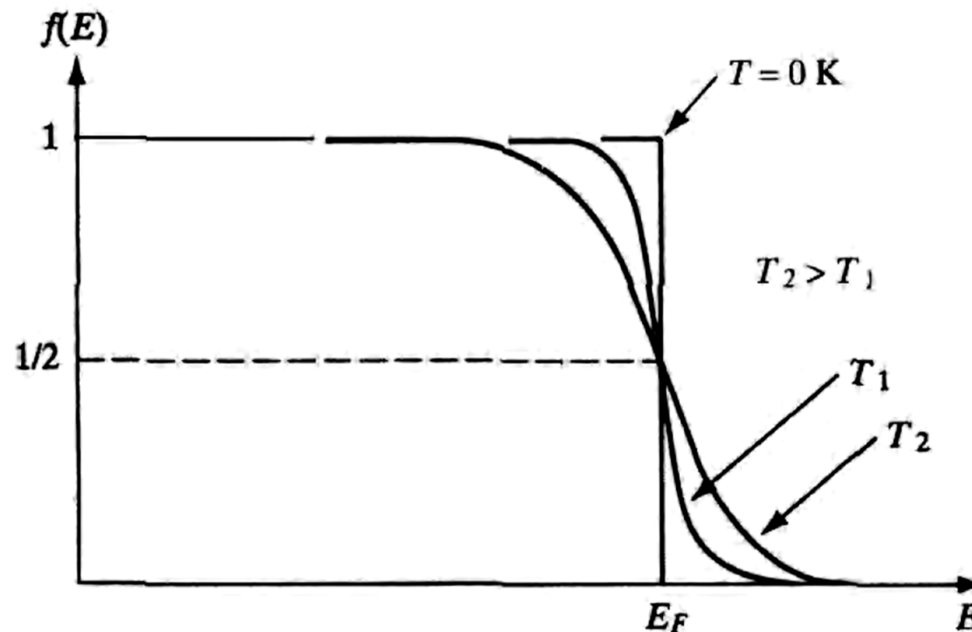
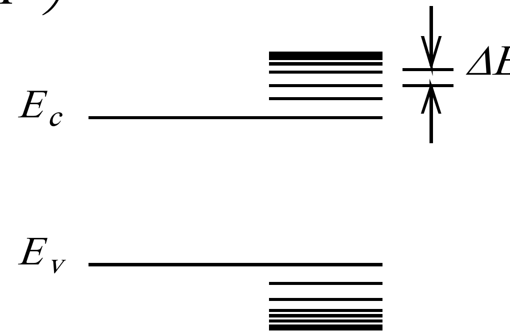
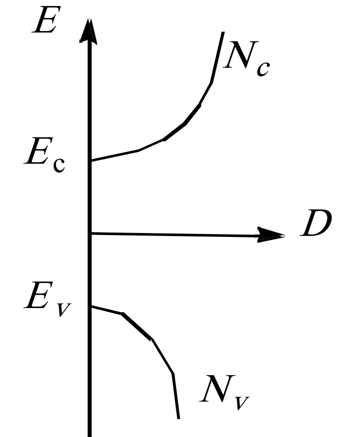


- Fermi level is the energy at which probability of occupance equals ½
- Fermi level is the topmost filled energy level at 0 K
- As T increases, the tail of pdf extends as it becomes more probable that electrons fill states with higher energy due to thermal agitation
- F-D distribution is symmetric about E_f at all T, i.e. $f(E_f + E_1) = 1 - f(E_f - E_1)$

Density of states $N_c(E)$, $N_v(E)$

$$N_c(E) \equiv \frac{\text{number of states in } \Delta E}{\Delta E \cdot \text{volume}} \quad \left(\frac{1}{\text{eV} \cdot \text{cm}^3}\right)$$





$$N_c(E)dE = \frac{8\pi\sqrt{2}}{h^3}\left(m_n^*\right)^{3/2}\sqrt{E - E_c}\,dE$$

$$N_v(E)dE = \frac{8\pi\sqrt{2}}{h^3}\left(m_p^*\right)^{3/2}\sqrt{E_v - E}\,dE$$

- The density of available states increases as their energies increase
- Analogy with for example Hydrogen atom, the larger the energy (higher *n*), the more the available states at these higher energies (more subshells)

Derivation of carrier concentration n_0 and p_0

Boltzmann approximation:

$$f(E) \approx e^{-(E-E_f)/kT}$$

$$n_0 = \int_{E_c}^{\text{top of conduction band}} f(E) N_c(E)\, dE$$

$$n_0 \approx \frac{8\pi\sqrt{2}}{h^3} \left(m_n^*\right)^{3/2} \int_{E_c}^{\infty} \sqrt{E - E_c}\, e^{-(E-E_f)/kT} dE$$

$$= \frac{8\pi\sqrt{2}}{h^3} \left(m_n^*\right)^{3/2} e^{-(E_c-E_f)/kT} \int_{0}^{\infty} \sqrt{E - E_c}\, e^{-(E-E_c)/kT} d(E - Ec)$$

$$= N_c e^{-(E_c-E_f)/kT}$$

$$\int_{0}^{\infty} \sqrt{E - E_c}\, e^{-(E-E_c)/kT} d(E - Ec) = \frac{\sqrt{\pi}}{2} (kT)^{3/2}$$

Derivation of carrier concentration n_0 and p_0

$$n_0 = N_c e^{-(E_c - E_f)/kT}$$

$$N_c \equiv 2\left[\frac{2\pi m_n^* kT}{h^2}\right]^{3/2}$$

N_c is called the ***effective density of states (of the conduction band) .***

$$p_0 = N_v e^{-(E_f - E_v)/kT}$$

$$N_v \equiv 2\left[\frac{2\pi m_p^* kT}{h^2}\right]^{3/2}$$

N_v is called the ***effective density of states of the valence band.***

- Looking at formulas, as n_0 increases (due to n-doping for example) E_f moves closer to E_c and similarily p_0 increases as E_f moves closer to E_v
- For Si at T=300 K, $m_n^* = 1.1m_0$ → $N_c = 2.8\times10^{19}$ cm^{-3}
- For Si at T=300 K, $m_p^* = 0.57m_0$ → $N_v = 1.04\times10^{19}$ cm^{-3}

Product of n_0 and p_0 (Either intrinsic or extrinsic)

$$n_0 = N_c e^{-(E_c - E_f)/kT}$$

$$p_0 = N_v e^{-(E_f - E_v)/kT}$$

$$n_0 p_0 = N_c N_v e^{-(E_c - E_v)/kT} = N_c N_v e^{-E_g/kT}$$

- Product of n_0 and p_0 formula holds even if the SC is doped since it only depends on N_c, N_v and E_g where none of them changes with doping !! (**Very important**)

Intrinsic carrier concentration n_i

$$n_0 = N_c e^{-(E_c - E_f)/kT}$$

$$p_0 = N_v e^{-(E_f - E_v)/kT}$$

$$n_0 p_0 = N_c N_v e^{-(E_c - E_v)/kT}$$

$$= N_c N_v e^{-E_g/kT}$$

$$n_0 = p_0 = n_i \quad \text{(Intrinsic)}$$

$$n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT}$$

- For Si at T=300 K, N_c = 2.8×10^{19} cm^{-3} , N_v = 1.04×10^{19} cm^{-3}, Eg = 1.1 eV substitute above to get

$$n_i = 10^{10} \quad \text{cm}^{-3}$$

Intrinsic Fermi level E_i

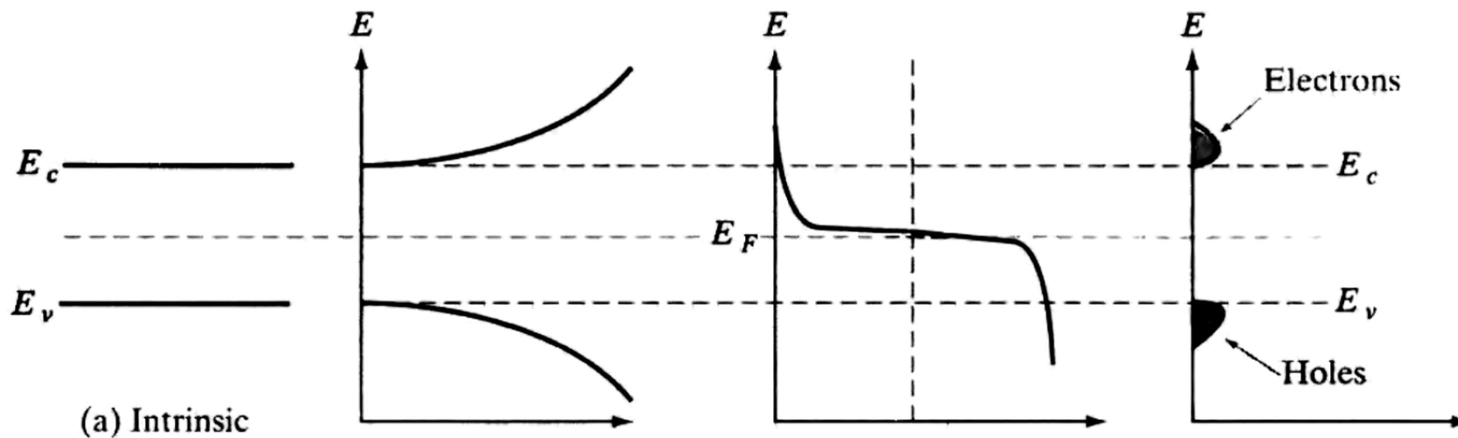
$$n_0 = p_0$$

$$N_c e^{-(E_c - E_i)/kT} = N_v e^{-(E_i - E_v)/kT}$$

$$E_i = \frac{E_c + E_v}{2} + \frac{kT}{2}\ln\left(\frac{N_v}{N_c}\right)$$

$$E_i \approx \frac{E_c + E_v}{2}$$

E_i lies (almost) in the middle between E_c and E_v



(a) Intrinsic

What happens to E_f if we dope the SC (change n_0 from n_i)?

$$n_0 = N_c e^{-(E_c - E_f)/kT}$$

$$n_i = N_c e^{-(E_c - E_i)/kT}$$

$$N_c = n_i e^{(E_c - E_i)/kT}$$

$$n_0 = n_i e^{(E_c - E_i)/kT} . e^{-(E_c - E_f)/kT}$$

$$n_0 = n_i e^{(E_f - E_i)/kT}$$

$$\therefore \quad E_f = E_i + kT \ln\left(\frac{n_0}{n_i}\right)$$

$$p_0 = N_v e^{-(E_f - E_v)/kT}$$

$$n_i = N_v e^{-(E_i - E_v)/kT}$$

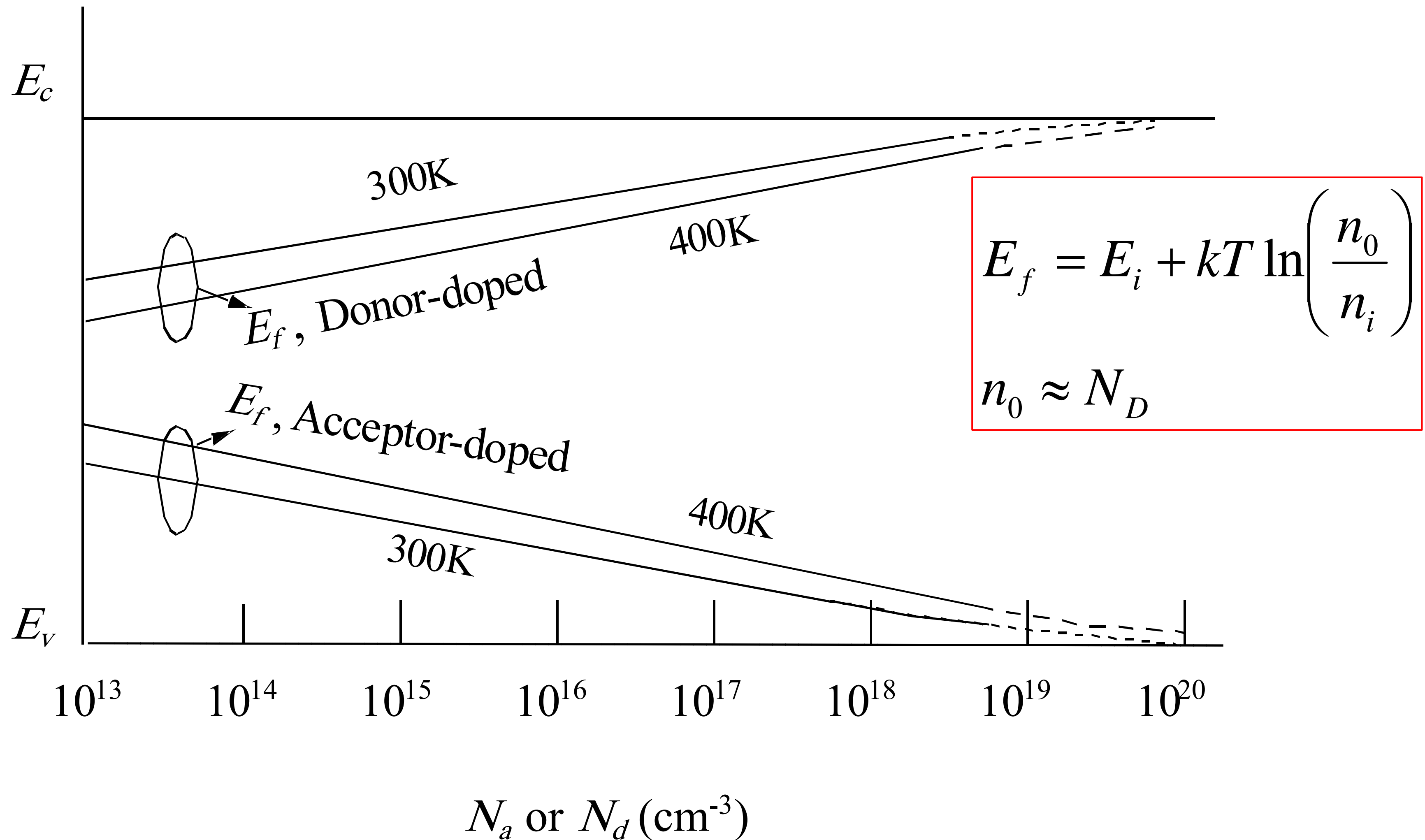
$$N_v = n_i e^{(E_i - E_v)/kT}$$

$$p_0 = n_i e^{(E_i - E_v)/kT} . e^{-(E_f - E_v)/kT}$$

$$p_0 = n_i e^{(E_i - E_f)/kT}$$

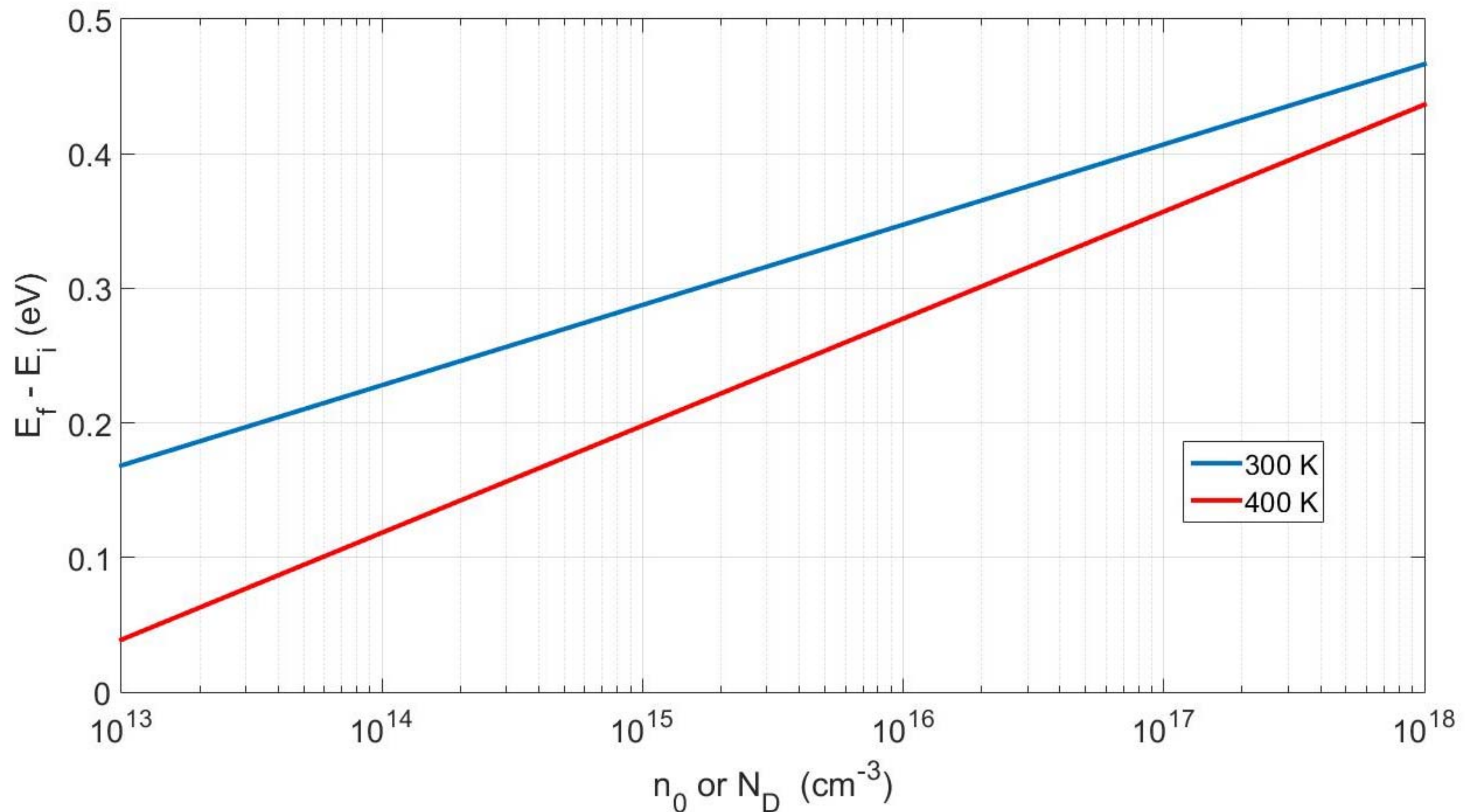
$$\therefore \quad E_f = E_i - kT \ln\left(\frac{p_0}{n_i}\right)$$

What happens to E_f if we dope the SC (change n_0 from n_i)?

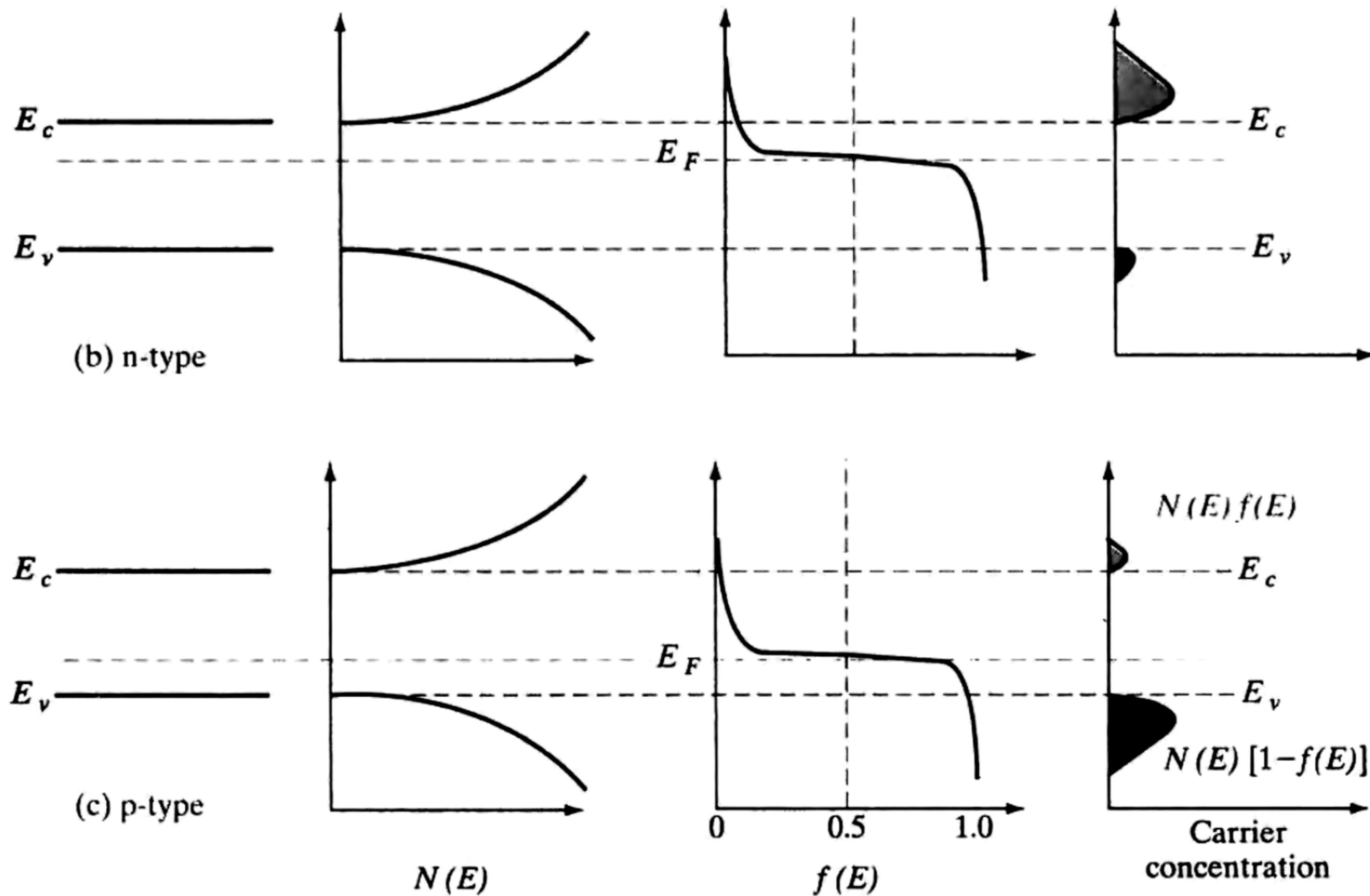


Take home exercise

- Plot on MATLAB E_f-E_i for Si at both T = 300K and 400K (<u>Hint</u>: you have to find n_i at 300K and 400K using $m_n^* = 1.1m_0$, $m_p^* = 0.57m_0$, Eg = 1.1 eV)



What happens to E_f if we dope the SC?



Numerical example

- A Si sample is doped with 10^{17} As atoms/cm^3. What is the equilibrium hole concentration at 300 K? Where is E_f relative to E_i? Where is E_f relative to E_c? (use $n_i = 1.5\times10^{10}$ cm^{-3} at this temperature)

$$n_0 \approx N_D = 10^{17} \quad \text{cm}^{-3}$$

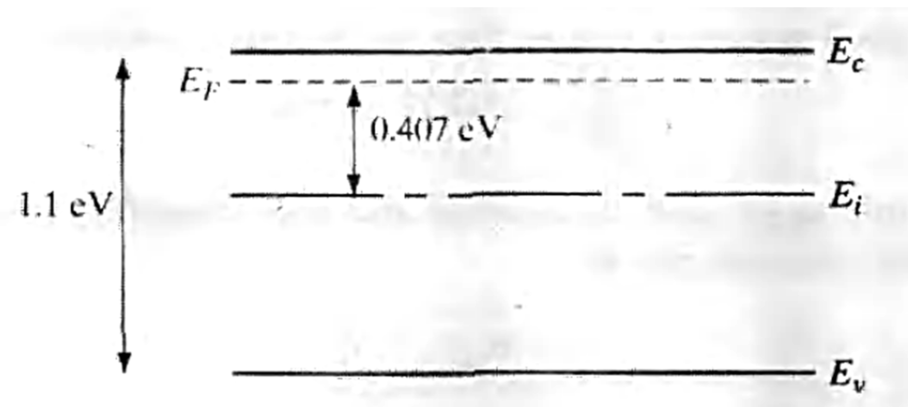
$$p_0 = \frac{n_i^2}{n_0} = 2.25\times10^3 \quad \text{cm}^{-3}$$

$$\because \quad n_0 = n_i e^{(E_f - E_i)/kT}$$

$$\therefore \quad E_f = E_i + kT\ln\left(\frac{n_0}{n_i}\right)$$

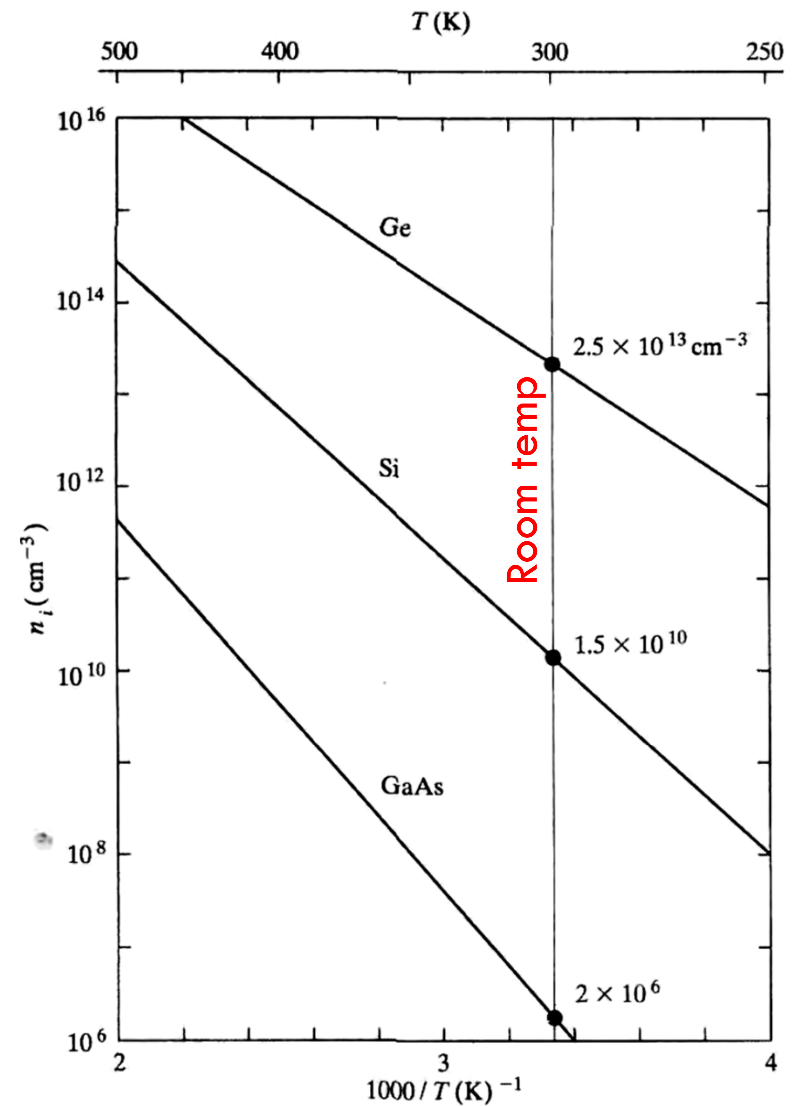
$$E_f - E_i = 0.0259\ln\left(\frac{10^{17}}{1.5\times10^{10}}\right) = 0.407 \quad \text{eV}$$

$$\therefore \quad E_c - E_f = 0.5E_g - \left(E_f - E_i\right) = 0.55 - 0.407 = 0.143 \ \text{eV}$$

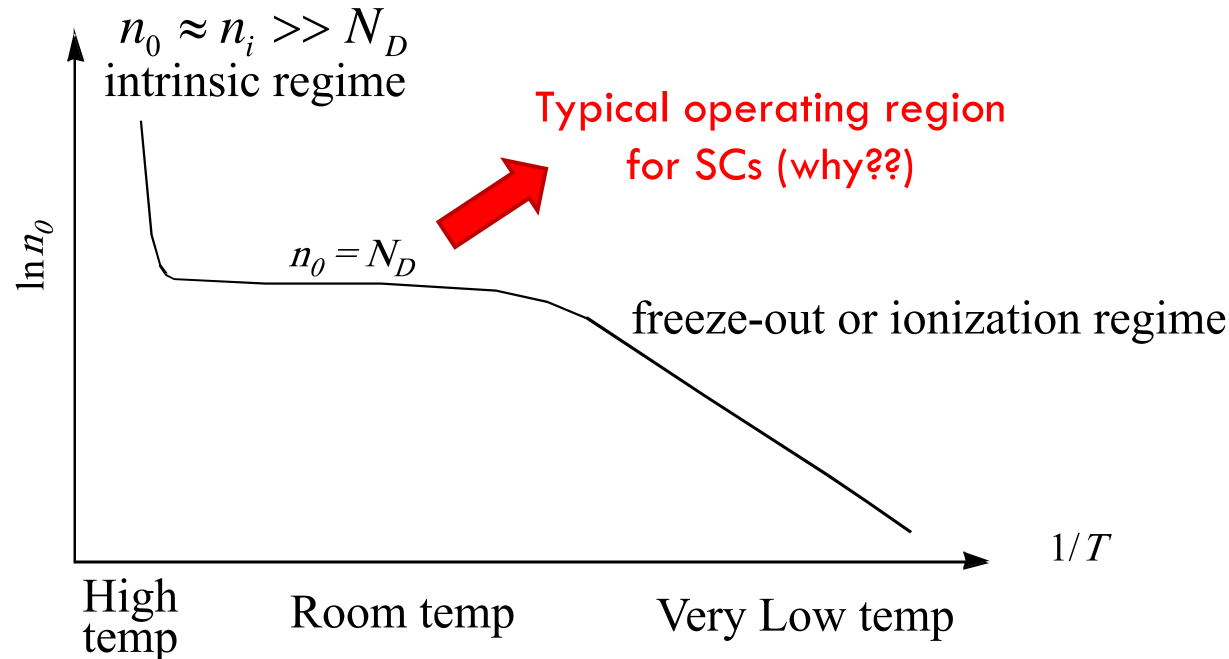


Effect of temperature on n_i

$$n_i(T) = 2\left(\frac{2\pi kT}{h^2}\right)^{3/2}(m_n^* m_p^*)^{3/4} e^{-E_g/2kT}$$



Effect of temperature on n_0 for a n-type SC



high T: $n = p = n_i = \sqrt{N_c N_v} e^{-E_g/2kT}$ ➡ Thermally gen. Intrinsic EHPs dominate donor electrons

low T: $n = \left[\frac{N_c N_D}{2}\right]^{1/2} e^{-(E_c - E_d)/2kT}$ ➡ Donor electrons are the only free electrons in CB (no intrinsic EHP)