# Control Systems And Their Components (EE391)

# Lec. 12: Combined state feedback and state estimator

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# Lecture Outline



- Combining regulator with state estimator
- Combining state feedback + state estimator + pre-scaling reference input

# Combining regulator with state estimator



- Now we will study the performance when combining a regulator **designed as** $u=-\mathbf{K}\mathbf{x}$ **but implemented as** $u=-\mathbf{K}\hat{\mathbf{x}}$

$$\begin{aligned}
\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t)+\mathbf{B}u(t) && \Leftarrow \quad \text{plant state equation}\\
&= \mathbf{A}\mathbf{x}(t)-\mathbf{B}\mathbf{K}\hat{\mathbf{x}}(t) && \Leftarrow \quad \text{apply state feedback } u=-\mathbf{K}\hat{\mathbf{x}}\\
&= \mathbf{A}\mathbf{x}(t)-\mathbf{B}\mathbf{K}\left(\mathbf{x}(t)-\mathbf{e}(t)\right)\\
&= \left(\mathbf{A}-\mathbf{B}\mathbf{K}\right)\mathbf{x}(t)+\mathbf{B}\mathbf{K}\mathbf{e}(t)
\end{aligned}$$

$$\dot{\hat{\mathbf{x}}}(t)=\mathbf{A}\hat{\mathbf{x}}(t)+\mathbf{B}u(t)+\mathbf{L}\mathbf{C}\left(\mathbf{x}(t)-\hat{\mathbf{x}}(t)\right)$$

$$\dot{\mathbf{e}}(t)=\left(\mathbf{A}-\mathbf{L}\mathbf{C}\right)\mathbf{e}(t)$$

$$\begin{bmatrix}\dot{\mathbf{x}}(t)\\ \dot{\mathbf{e}}(t)\end{bmatrix}=\begin{bmatrix}\mathbf{A}-\mathbf{B}\mathbf{K} & \mathbf{B}\mathbf{K}\\ 0 & \mathbf{A}-\mathbf{L}\mathbf{C}\end{bmatrix}\begin{bmatrix}\mathbf{x}(t)\\ \mathbf{e}(t)\end{bmatrix} \quad \Leftarrow \quad \text{augmented equation}$$

$$y(t)=\begin{bmatrix}\mathbf{C} & 0\end{bmatrix}\begin{bmatrix}\mathbf{x}(t)\\ \mathbf{e}(t)\end{bmatrix}$$

# Combining regulator with state estimator



$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \dot{\mathbf{e}}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{A}-\mathbf{BK} & \mathbf{BK} \\ 0 & \mathbf{A}-\mathbf{LC} \end{bmatrix}}_{\mathbf{A}_{cl}} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{e}(t) \end{bmatrix} \quad \Leftarrow \quad \text{augmented equation}$$

- This says that the dynamics of both the state vector of the closed loop systems after introducing feedback as well as the estimation error are determined by the eigenvalues of $\mathbf{A}_{cl}$
- Since this is a block upper diagonal matrix, its eigenvalues are given by

$$\boxed{\left|s\mathbf{I}-\mathbf{A}_{cl}\right| = \left|s\mathbf{I}-\left(\mathbf{A}-\mathbf{BK}\right)\right| \cdot \left|s\mathbf{I}-\left(\mathbf{A}-\mathbf{LC}\right)\right|}$$

- This means that **the poles of the closed loop system are the union of the regulator and estimator poles**

# Combining regulator with state estimator



$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \dot{\mathbf{e}}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{A}-\mathbf{BK} & \mathbf{BK} \\ 0 & \mathbf{A}-\mathbf{LC} \end{bmatrix}}_{\mathbf{A}_{cl}} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{e}(t) \end{bmatrix} \quad \Leftarrow \quad \text{augmented equation}$$

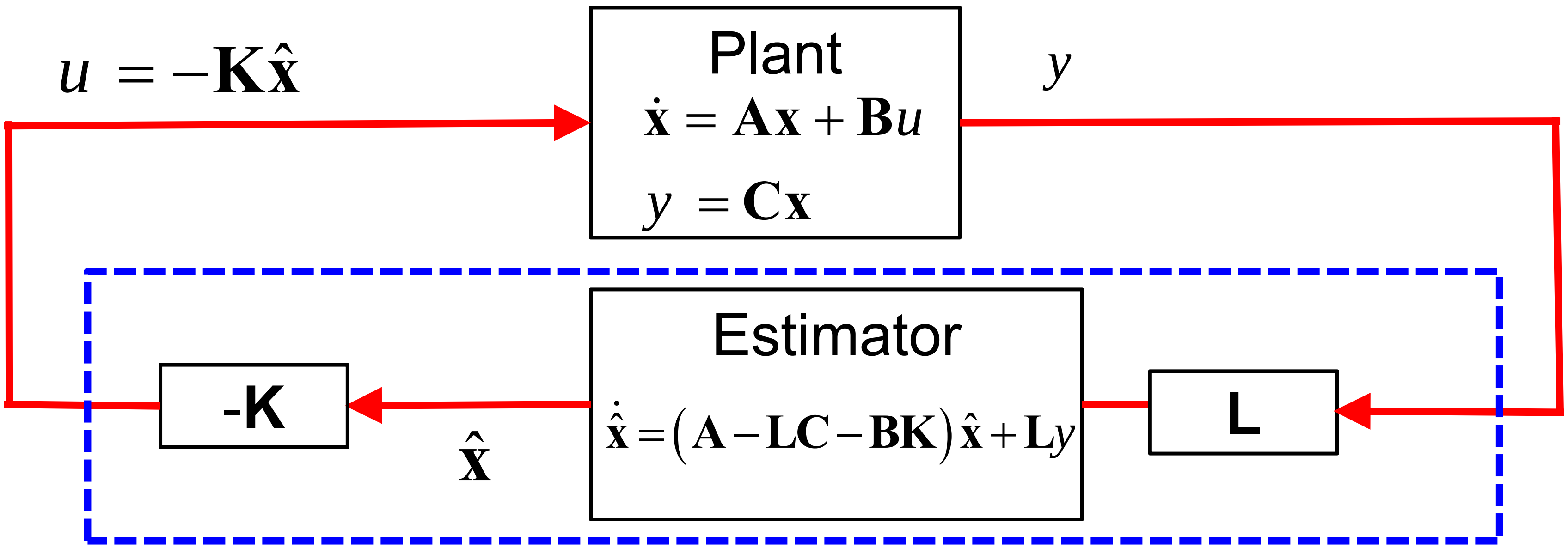
$$\boxed{\left| s\mathbf{I}-\mathbf{A}_{cl} \right| = \left| s\mathbf{I}-\left(\mathbf{A}-\mathbf{BK}\right)\right| \cdot \left| s\mathbf{I}-\left(\mathbf{A}-\mathbf{LC}\right)\right|}$$

- This also means that you can design the compensator and estimator separately just as we did before and then combine them **(separation principle)**
- As a design rule, you should place the estimator poles at >2 the real part of the regulator poles (found from transient specs required) to ensure the estimator converges fast and hence the estimated values used for feedback are good

# Combining regulator with state estimator



- The whole system (regulator + estimator) looks like

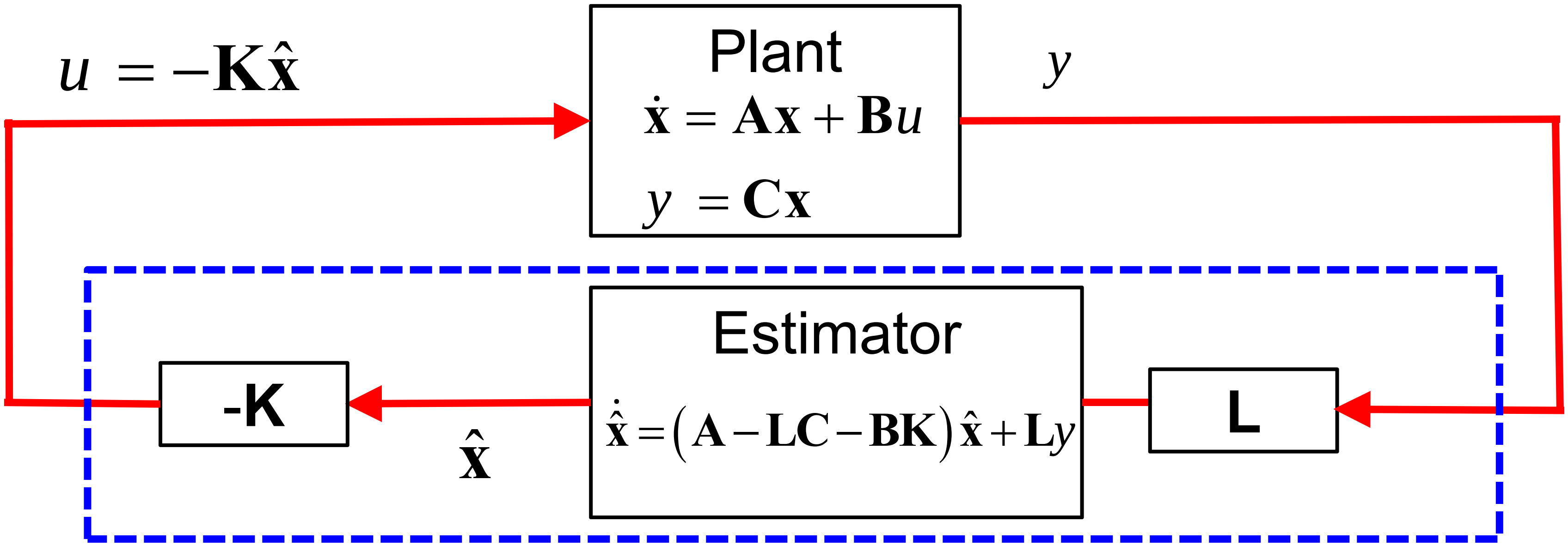



The full compensator
(Regulator + Estimator)

- Compensator accepts the sensor outputs as its inputs and provides at its output the actuator input
- Sometimes called **dynamic output feedback compensator (DOFC)**

# Full compensator equations






The full compensator
(Regulator + Estimator)

$$\dot{\hat{\mathbf{x}}} = (\mathbf{A} - \mathbf{LC} - \mathbf{BK})\hat{\mathbf{x}} + \mathbf{L}y$$

$$u = -\mathbf{K}\hat{\mathbf{x}}$$

SS Matrices of compensator

$$\mathbf{A}_c \equiv \mathbf{A} - \mathbf{LC} - \mathbf{BK}, \quad \mathbf{B}_c \equiv \mathbf{L}, \quad \mathbf{C}_c \equiv -\mathbf{K}$$

$$\text{TF of compensator} = -\mathbf{K}\left(s\mathbf{I} - (\mathbf{A} - \mathbf{LC} - \mathbf{BK})\right)^{-1}\mathbf{L}$$

# Guidelines of designing full compensator

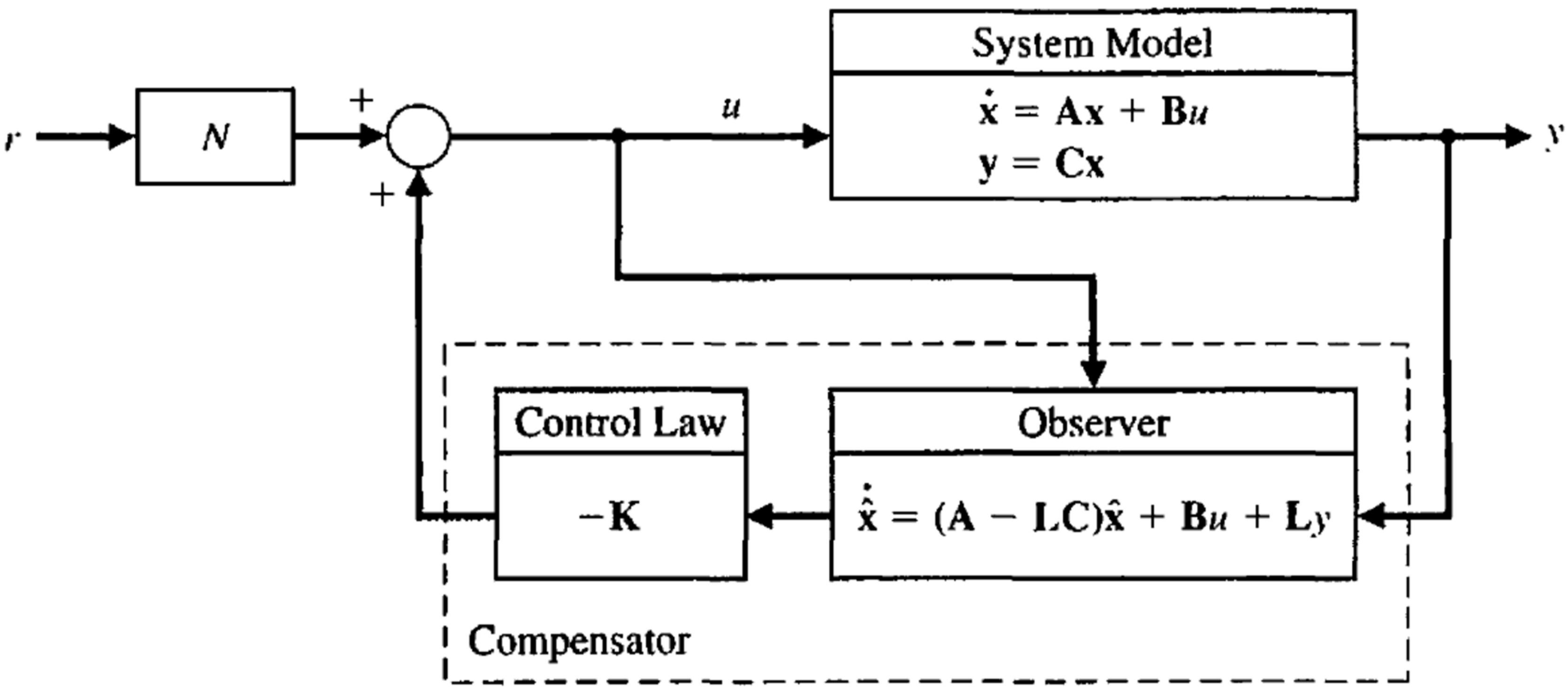


- Applying the separation principle, we simply design the controller matrix **K** and the observer matrix **L** separately each to satisfy a desired set of poles
- **K** is found from the desired poles of closes loop system which can be obtained from the required specifications of the closed loop system (Max overshoot, settling time,…) similar to Lecture 9
- **L** is found from the desired poles estimator which are either given or chosen to be 2-10 times larger than the system poles on the real axis
- Do not expect the feedback of the estimated state vector $u = -\mathbf{K}\hat{\mathbf{x}}$ to operate just as good as $u = -\mathbf{K}\mathbf{x}$ especially at the initial transient period where the estimation error hasn't yet decayed from its initial nonzero value (refer to the MATLAB code of Lec.12 for more insight)

# Pre-Scaling the full compensator (reference input)



- See section 11.6 page 857 of Dorf's book for more details



# Pre-Scaling the full compensator (reference input)



$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) \qquad \Leftarrow \quad \text{plant state equation}$$

$$= \mathbf{A}\mathbf{x}(t) - \mathbf{B}\mathbf{K}\hat{\mathbf{x}}(t) + \mathbf{B}Nr \qquad \Leftarrow \quad \text{apply state feedback with pre-scaled reference i/p } u = Nr - \mathbf{K}\hat{\mathbf{x}}$$

$$= \mathbf{A}\mathbf{x}(t) - \mathbf{B}\mathbf{K}\left(\mathbf{x}(t) - \mathbf{e}(t)\right) + \mathbf{B}Nr$$

$$= \left(\mathbf{A} - \mathbf{B}\mathbf{K}\right)\mathbf{x}(t) + \mathbf{B}\mathbf{K}\mathbf{e}(t) + \mathbf{B}Nr$$

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{A}\hat{\mathbf{x}}(t) + \mathbf{B}u(t) + \mathbf{L}\mathbf{C}\left(\mathbf{x}(t) - \hat{\mathbf{x}}(t)\right)$$

$$= \left(\mathbf{A} - \mathbf{L}\mathbf{C}\right)\hat{\mathbf{x}}(t) + \mathbf{B}u(t) + \mathbf{L}y(t) \qquad \Leftarrow \quad \text{estimator state equation}$$

$$\dot{\mathbf{e}}(t) = \left(\mathbf{A} - \mathbf{L}\mathbf{C}\right)\mathbf{e}(t)$$

$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \dot{\mathbf{e}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{B}\mathbf{K} & \mathbf{B}\mathbf{K} \\ 0 & \mathbf{A} - \mathbf{L}\mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{e}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{B}N \\ 0 \end{bmatrix} r \qquad \Leftarrow \quad \text{augmented equation}$$

$$y(t) = \begin{bmatrix} \mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{e}(t) \end{bmatrix}$$