Operational Amplifiers



 Ideal op-amps
 Negative feedback
 Applications
 Useful designs
 Integrators, differentiators and filters

Chapter 14: Operational Amplifiers

Introduction: Ideal Operational Amplifier

Operational amplifier (Op-amp) is made of many transistors, diodes, resistors and capacitors in integrated circuit technology. Ideal op-amp is characterized by:

- Infinite input impedance
- Infinite gain for differential input
- Zero output impedance
- Infinite frequency bandwidth



Figure 14.1 Circuit symbol for the op amp.

Ideal Operational Amplifier

Equivalent circuit of the ideal op-amp can be modeled by:

- Voltage controlled source with very large gain A_{OL} known as open loop gain
- •Feedback reduces the gain of op-amp
- Ideal op-amp has no nonlinear distortions



Figure 14.2 Equivalent circuit for the ideal op amp. The open-loop gain A_{OL} is very large (approaching infinity).

Ideal Operational Amplifier

A real op-amp must have a DC supply voltage which is often not shown on the schematics



Figure 14.3 Op-amp symbol showing the dc power supplies, V_{CC} and V_{EE} .

Inverting Amplifier

Op-amp are almost always used with a negative feedback:

Part of the output signal is returned to the input with negative signFeedback reduces the gain of op-amp

•Since op-amp has large gain even small input produces large output, thus for the limited output voltage (lest than V_{CC}) the input voltage v_x must be very small.

•Practically we set v_x to zero when analyzing the op-amp circuits.



Inverting Amplifier

Since
$$v_o = -i_2 R_2 = -v_{in} R_2 / R_1$$

Then we see that the output voltage does not depend on the load resistance and behaves as voltage source.

Thus the output impedance of the inverting amplifier is zero. The input impedance is R_1 as $Z_{in} = v_{in}/i_1 = R_1$



Figure 14.5 We make use of the summing-point constraint in the analysis of the inverting amplifier.

Inverting amplifier gain $v_o = -i_2 R_2 = -v_{in} R_2/R_1$ Is limited due to fact that it is hard to obtain large resistance ratio.

Higher gains can be obtained in the circuit below where we have: $i_1 = v_{in} / R_1 = i_2$ from KCL at N₂ we have: $i_2 + i_3 = i_4$ R_1 + $v_{
m ip}$ v_i v_o

Figure 14.6 An inverting amplifier that achieves high gain magnitude with a smaller range of resistance values than required for the basic inverter. See Example 14.1.

Higher gains can be obtained in the circuit below where we have: $i_1 = v_{in}/R_1 = i_2$ from KCL at N2 we have: $i_2 + i_3 = i_4$

Also from KVL1: -vand from KVL2: $i_2 F$



Figure 14.6 An inverting amplifier that achieves high gain magnitude with a smaller range of resistance values than required for the basic inverter. See Example 14.1.

Finally using: $i_2 + i_3 = i_4$ and $i_4 = (-v_0 - i_2 R_2)/R_4$ $i_3 = i_2 R_2 / R_3$ we have

 $i_2 + i_2 R_2 / R_3 = (-v_0 - i_2 R_2) / R_4 = i_2 (1 + R_2 / R_3 + R_2 / R_4) = -v_0 / R_4$



Figure 14.6 An inverting amplifier that achieves high gain magnitude with a smaller range of resistance values than required for the basic inverter. See Example 14.1.

$$i_2(1+R_2/R_3+R_2/R_4) = -v_0/R_4$$

Substitute

 $i_2 = v_{in}/R_1 => v_{in}/R_1 *(1+R_2/R_3+R_2/R_4) = -v_o/R_4$ to get the voltage gain

 $v_o/v_{in} = -R_4/R_1 * (1 + R_2/R_3 + R_2/R_4)$



Figure 14.6 An inverting amplifier that achieves high gain magnitude with a smaller range of resistance values than required for the basic inverter. See Example 14.1.

So if we chose $R_1 = R_3 = 1k\Omega$ and $R_2 = R_4 = 10 k\Omega$ then the voltage gain is

$$A_{v} = v_{o} / v_{in} =$$

$$= -R_{4} / R_{1} * (1 + R_{2} / R_{3} + R_{2} / R_{4}) =$$

$$= -10 * (1 + 10 + 1) =$$

$$= -120$$



Figure 14.6 An inverting amplifier that achieves high gain magnitude with a smaller range of resistance values than required for the basic inverter. See Example 14.1.

The output voltage in summing amplifier is

$$v_o = -i_f * R_f$$
 since $v_i = 0$



Figure 14.7 Summing amplifier. See Exercise 14.1.

The output voltage in summing amplifier is

$$v_o = -i_f * R_f$$
 since $v_i = 0$



Figure 14.7 Summing amplifier. See Exercise 14.1.

The output voltage in summing amplifier is

 $v_o = -i_f * R_f$ since $v_i = 0$

$$i_f = i_A + i_B = v_A / R_A + v_B / R_B = v_o = -(v_A / R_A + v_B / R_B) * R_f$$



Figure 14.7 Summing amplifier. See Exercise 14.1.

The output voltage in summing amplifier is

$$v_o = -i_f *R_f \quad \text{since } v_i = 0$$

$$i_f = i_A + i_B = v_A / R_A + v_B / R_B \quad = > \quad v_o = -(v_A / R_A + v_B / R_B) * R_f$$

For n inputs we will have $v_o = -R_f * \Sigma_i (v_i/R_i)$



Figure 14.7 Summing amplifier. See Exercise 14.1.

Find the currents and voltages in these two circuits:



Figure 14.8 Circuits for Exercise 14.2.

Find the currents and voltages in these two circuits:



Figure 14.8 Circuits for Exercise 14.2.

Find expression for the output voltage in the amplifier circuit: $i_1 = v_1 / R_1 = v_1 / 10 k \Omega$

 $i_2 = i_1 = v_1 / 10 mA$

$$v_3 = -i_2 * R_2 = -v_1 / 10 k \Omega * 20 k \Omega = -2v_1$$



Figure 14.9 Circuit for Exercise 14.3.

Find expression for the output voltage in the amplifier circuit:

$$v_3 = -i_2 * R_2 = -v_1 / 10 k \Omega * 20 k \Omega = -2v_1$$

 $i_5 = i_3 + i_4 = v_3 / 10 k \Omega + v_2 / 10 k \Omega$

$$v_o = -i_5 *R_5 = -(v_3/10k\Omega + v_2/10k\Omega) *20k\Omega = -2v_3 - 2v_2 = 4v_1 - 2v_2$$



Figure 14.9 Circuit for Exercise 14.3.

Positive Feedback

When we flip the polarization of the op-amp as shown on the figure we will get a positive feedback that saturates the amplifier output.

This is not a good idea.



Figure 14.10 Circuit with positive feedback.

Noninverting amplifier



Figure 14.11 Noninverting amplifier.

$$v_{1} = v_{in}$$

$$i_{1} = v_{1}/R_{1}$$

$$i_{2} = -i_{1}$$

$$v_{o} = v_{1} - i_{2}*R_{2} = v_{1} + i_{1}*R_{2} =$$

$$= v_{1} + R_{2}*v_{1}/R_{1} = v_{1}(1 + R_{2}/R_{1})$$

Thus the voltage gain of noninverting amplifier is:

$$A_{v} = v_{o} / v_{in} = 1 + R_{2} / R_{1}$$

Voltage Follower

Special case of noninverting amplifier is a voltage follower Since in the noninverting amplifier

 $v_o = v_1(1 + R_2/R_1)$ so when R2=0

 $v_o = v_1$



Figure 14.11 Noninverting amplifier.



Figure 14.12 The voltage follower which has $A_v = 1$.

Find voltage gain $A_v = v_o / v_{in}$ and input impedance

- a. With the switch open
- b. With the switch closed

a.

From KVL: $v_{in} = i_1 R + i_1 R + v_o$



Figure 14.13 Inverting or noninverting amplifier. See Exercise 14.4.

Find voltage gain $A_v = v_o / v_{in}$ and input impedance

- a. With the switch open
- b. With the switch closed

a.

Input impedance: $Z_{in} = v_{in}/i_{in} = v_{in}/0 = inf$



Figure 14.13 Inverting or noninverting amplifier. See Exercise 14.4.

Find voltage gain $A_v = v_o / v_{in}$ and input impedance

- a. With the switch open
- b. With the switch closed



Figure 14.13 Inverting or noninverting amplifier. See Exercise 14.4.

Find voltage gain $A_v = v_o / v_{in}$ and input impedance

- a. With the switch open
- b. With the switch closed



Figure 14.13 Inverting or noninverting amplifier. See Exercise 14.4.

Voltage to Current Converter

Find the output current i_o as a function of v_{in}



Figure 14.14 Voltage-to-current converter (also known as a transconductance amplifier). See Exercise 14.5.

Voltage to Current Converter

Find the output current i_o as a function of v_{in}

$$v_{in} = i_o * R_f$$

so
 $i_o = v_{in} / R_f$



Figure 14.14 Voltage-to-current converter (also known as a transconductance amplifier). See Exercise 14.5.

- *a)* Find the voltage gain v_o/v_{in}
- b) Calculate the voltage gain v_o/v_{in} for $R_1=10 k\Omega$, $R_2=100 k\Omega$
- c) Find the input resistance



Figure 14.15 Circuit for Exercise 14.6.

a) Find the voltage gain v_o/v_{in} b) Calculate the voltage gain v_o/v_{in} for $R_1=10 \ k\Omega$, $R_2=100 \ k\Omega$ c) Find the input resistance From KCL1: $v_{in}/R_1 = (v_2 - v_{in})/R_2 = > v_2/R_2 = v_{in}(1/R_2 + 1/R_1)$ From KCL2: $(v_2 - v_{in})/R_2 + v_2/R_1 + (v_2 - v_0)/R_2 = 0 = >$



Figure 14.15 Circuit for Exercise 14.6.

a) Find the voltage gain v_0/v_{in} b) Calculate the voltage gain v_0/v_{in} for $R_1=10 \ k\Omega$, $R_2 = 100 \ k\Omega$ c) Find the input resistance From KCL1: $v_{in}/R_1=(v_2-v_{in})/R_2 => v_2/R_2 = v_{in}(1/R_2+1/R_1)$ (*) From KCL2: $(v_2-v_{in})/R_2+v_2/R_1+(v_2-v_0)/R_2=0 => v_2(2/R_2+1/R_1)=(v_{in}+v_0)/R_2$ (*) $v_2 = v_{in}(1+R_2/R_1) => v_{in}(1+R_2/R_1)(2/R_2+1/R_1)=(v_{in}+v_0)/R_2$ $v_{in}(R_2(1+R_2/R_1)(2/R_2+1/R_1)-1)=v_0$



 $v_0 / v_{in} = 131$

Figure 14.15 Circuit for Exercise 14.6.

a) Find the voltage gain v_0/v_{in} b) Calculate the voltage gain v_0/v_{in} for $R_1=10 \ k\Omega$, $R_2 = 100 \ k\Omega$ c) Find the input resistance From KCL1: $v_{in}/R_1=(v_2-v_{in})/R_2 => v_2/R_2 = v_{in}(1/R_2+1/R_1)$ (*) From KCL2: $(v_2-v_{in})/R_2+v_2/R_1+(v_2-v_0)/R_2=0 => v_2(2/R_2+1/R_1) = (v_{in}+v_0)/R_2$ (*) $v_2 = v_{in}(1+R_2/R_1) => v_{in}(1+R_2/R_1)(2/R_2+1/R_1) = (v_{in}+v_0)/R_2$ $v_{in} (R_2(1+R_2/R_1)(2/R_2+1/R_1)-1) = v_0 => v_0/v_{in} = 100(1+10)0.12-1$



 $v_0 / v_{in} = 131$

Figure 14.15 Circuit for Exercise 14.6.

Matrix equations for op-amp circuits

For Op Amps: write equal voltages at the input terminals (if one is grounded both are equal zero). Do not write KCL equations at the output node of Op Amps.

Example



Example



Nonzero voltages are V_4 , V_5 , V_{out} & E Since we do not write KCL equations at the output nodes of Op Amps, we have for nodes 1, 2 & 3

 $\begin{array}{ll} Node~(1) & -(G_1 + sC_1)V_4 - G_3V_{out} = EG_4 \\ Node~(2) & -G_2V_4 - sG_2V_5 = 0 \\ Node~(3) & -G_5V_5 - G_6V_{out} = 0 \end{array}$

Or in the matrix form

$$\begin{bmatrix} -(G_1 + sC_1) & 0 & -G_3 \\ -G_2 & -sC_2 & 0 \\ 0 & -G_5 & -G_6 \end{bmatrix} \begin{bmatrix} V_4 \\ V_5 \\ V_{out} \end{bmatrix} = \begin{bmatrix} EG_4 \\ 0 \\ 0 \end{bmatrix}$$

Design of Simple Amplifiers

Practical amplifiers can be designed using op-amp with feedback.

We know that for noninverting amplifier $A_v = v_o / v_{in} = 1 + R_2 / R_1$ so to obtain $A_v = 10$ we could use $R_1 = 1\Omega$ and $R_2 = 9\Omega$ But such low output resistance will draw too much current from the power supply



Figure 14.16 If low resistances are used, an excessively large current is required.

Design of Simple Amplifiers

The same gain can be obtained with large resistance values.

$$A_v = v_o / v_{in} = 1 + R_2 / R_1$$

But for high output resistance are sensitive to bias current and we must use a filtering output capacitor to remove the noise.



Figure 14.17 If very high resistances are used, stray capacitance can couple unwanted signals into the circuit.

We consider the following op-amp imperfections:

- 1) Nonideal linear operation,
- 2) Nonlinear characteristics
- 3) Dc offset values.

Input and output impedances: 1) Ideal opamp have $R_{in}=0$,

$$R_{in} = \infty; \quad R_{out} = 0\Omega$$

2) Real op-amp has $R_{in} = 1M\Omega - 10^{12}\Omega;$ $R_{out} = 1\Omega - 100\Omega$

We consider the following op-amp imperfections:

- 1) Nonideal linear operation,
- 2) Nonlinear characteristics
- *3) Dc offset values.*



Voltage gain:

- 1) Ideal op-amp has infinite gain and bandwidth,
- 2) Real op-amp has the gain that changes with frequency.
- 3) Open loop gain:



Terminal frequency f_t – gain drops to 1

Figure 14.20 Bode plot of open-loop gain for a typical op amp.

Negative feedback is used to lower the gain and extend the bandwidth.



of closed-loop bandwidth.



So we will get closed loop dc gain

$$A_{0CL} = \frac{V_o}{V_{in}} = \frac{A_{0OL}}{1 + \beta A_{0OL}}$$

closed loop bandwidth

$$f_{BCL} = f_{BOL} \left(1 + \beta A_{0OL} \right)$$

closed loop voltage gain

$$A_{CL}(f) = \frac{A_{0CL}}{1 + j \frac{f}{f_{BCL}}}$$

Comparing to open loop, the closed loop gain is reduced And closed loop bandwidth is larger

The gain*bandwidth product stays the same Gain (dB) $A_{0CL}f_{BCL} =$ $A_{\rm OL}$ $= \frac{A_{0OL}}{1 + \beta A_{0OL}} \times f_{BOL} (1 + \beta A_{0OL})$ $= A_{0OL} \times f_{BOL}$ Bode plot for Exercise 14.12 $A_{\rm CL}$ f_t f_{BCL} f_{BOL} f 40 kHz 400 kHz 400 Hz 4 kHz 4 MHz 40 Hz

Figure 14.22 Bode plots for Example 14.5 and Exercise 14.12.

100

80

60

40

20

0

Nonlinear Limitations

Nonlinear limitations:

1) Output voltage swing is limited and depend on power supply voltage for $V_{DD} \in (-15V, +15V), \quad v_o(t) \in (-12V, +12V)$

2) Maximum output current is limited

for $\mu A741$ amplifier $i_o(t) \in (-40mA, +40mA)$



 $V_{im} = 1$ V. None of the limitations are exceeded, and $v_o(t) = 4v_s(t)$.

limitations of op amps.

Nonlinear Limitations

When voltage or current limits are exceeded, clipping of the output signal occurs causing large nonlinear distortions



Nonlinear Limitations

Another nonlinear limitation is limited rate of change of the output signal known as the slew-rate limit SR



Using slew rate we can find maximum frequency known as full-power bandwidth. Assuming

$$v_o(t) = V_{om} \sin(\omega t)$$
$$\frac{dv_o}{dt} = \omega V_{om} \cos(\omega t) \le \le 2\pi f V_{om} \le SR$$

So the full-power bandwidth





Dc offset values

There are three dc offset values related to op-amp design:

- 1) Bias currents I_{B+} , I_{B-} related to differential inputs
- 2) Offset current ideally zero value

3) Offset voltage – results in nonzero output for zero input They can be represented as additional dc sources in the op-amp model



Figure 14.29 Three current sources and a voltage source model the dc imperfections of an op amp.

Industrial op-amp

741 Amplifier is the most popular amplifier it has A_{OL} =100000





Industrial op-amp

741 Amplifier BJT transistor level schematic





Figure 14.29 Three current sources and a voltage source model the dc imperfections of an op amp.



Figure 14.30 Circuits of Example 14.7.



Figure 14.31 Adding the resistor R_{bias} to the inverting amplifier circuit causes the effects of bias currents to cancel.



Figure 14.32 Noninverting amplifier, including resistor R_{bias} to balance the effects of the bias currents. See Exercise 14.15.



Figure 14.33 Differential amplifier.



Figure 14.34 Instrumentation-quality differential amplifier.



Figure 14.35 Integrator.



Figure 14.36 Square-wave input signal for Exercise 14.17.



Figure 14.37 Answer for Exercise 14.17.



Figure 14.38 Differentiator.



for lowpass Butterworth filters.



Figure 14.40 Equal-component Sallen–Key lowpass active-filter section.



Figure 14.41 Fourth-order Butterworth lowpass filter designed in Example 14.8.



lowpass filter of Example 14.8.