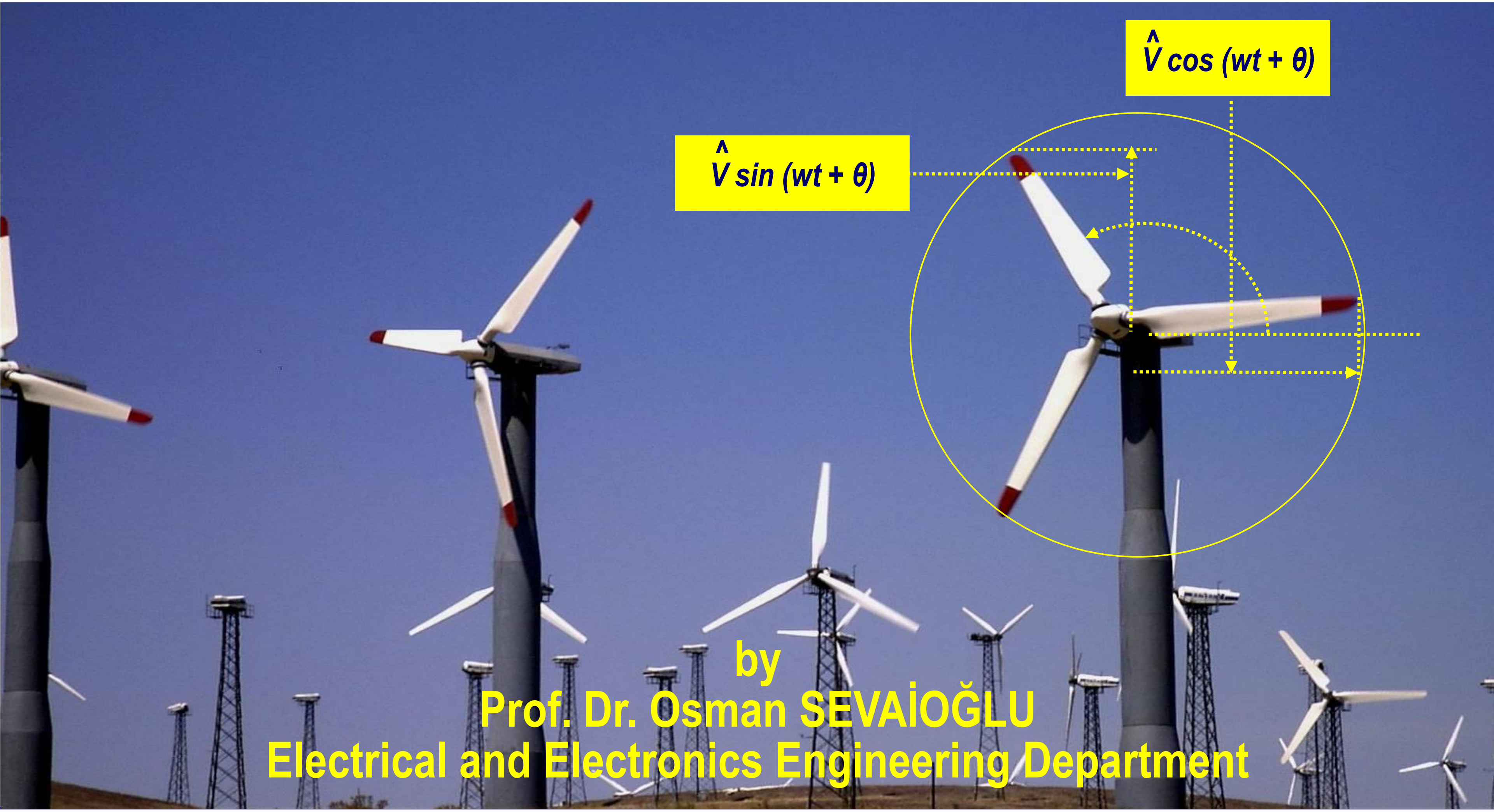
Phasors



by
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Electrical and Electronics Engineering Department

Phasors

Vector

Definition

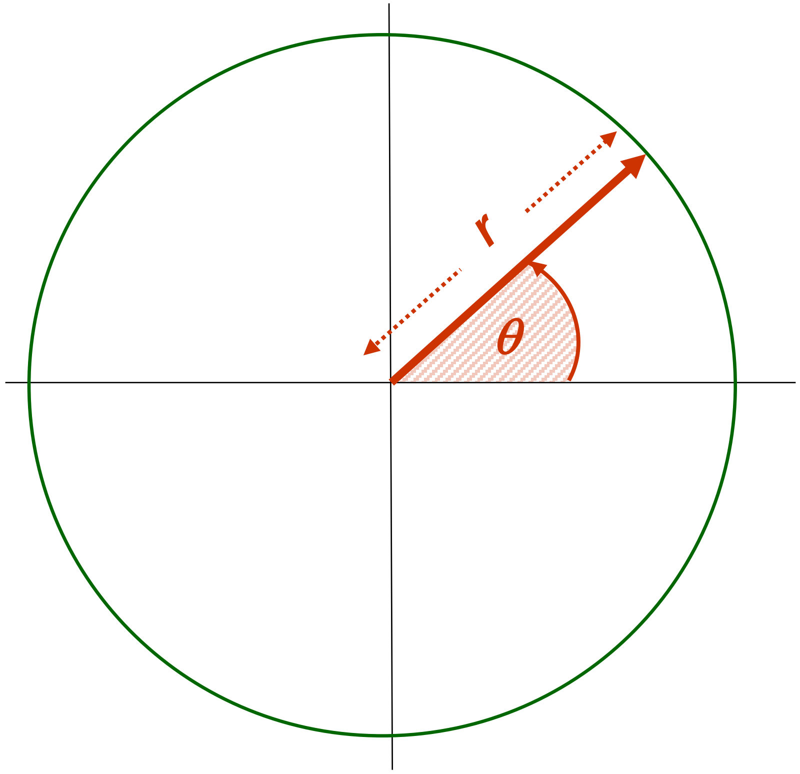
A vector is a magnitude directed in a certain direction (angle)

A vector is shown as

$$r \angle \theta$$

where,
r is the magnitude, known as radius,
θ is the angle

The above representation is known as ***"the polar representation"*** of a vector



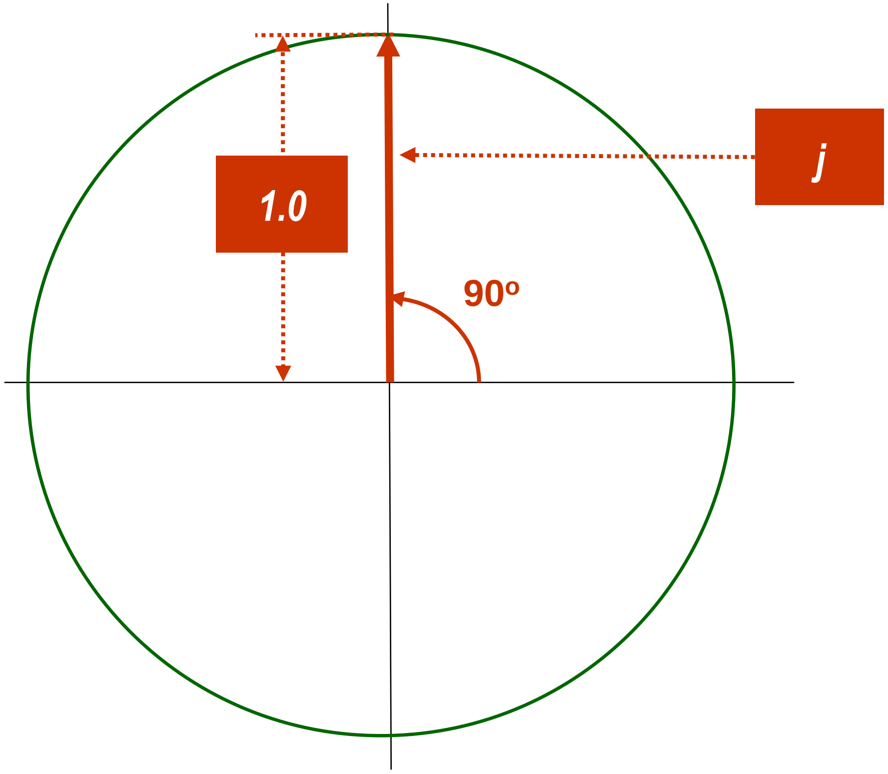
Phasors

j Operator

Definition

"j operator" is a vector with unity magnitude, directed in vertical direction, i.e. in 90° angle

$$j = 1 \angle 90^o$$



Phasors

Complex Number

Definition

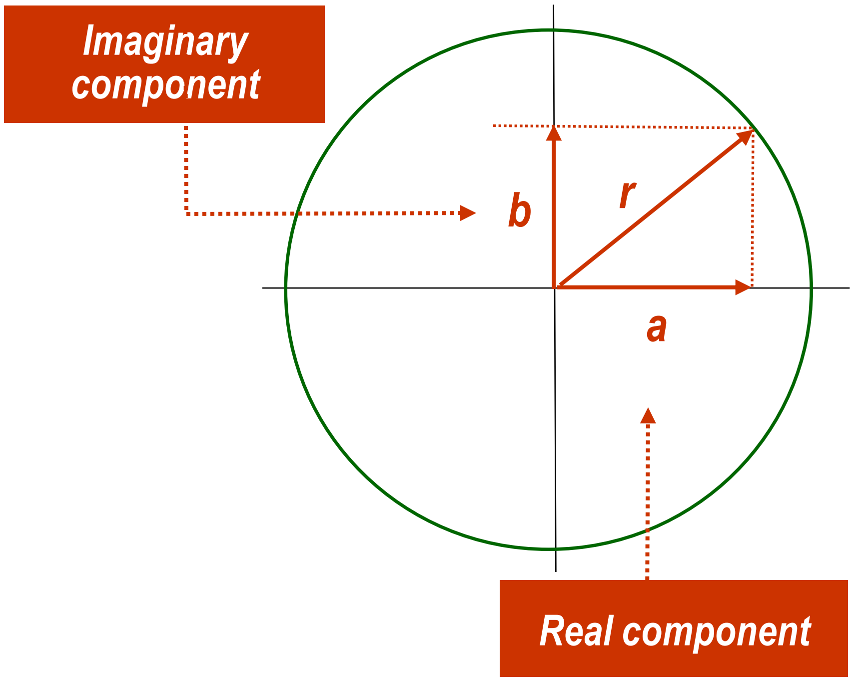
A complex number is a number with two components;

- Real component (an ordinary number),
- Imaginary component (a number multiplied by the j operator)

a + j b

The above representation is known as *"the rectangular representation"*

Graphical Representation



Phasors

Polar Representation of Complex Number

Definition	Graphical Representation

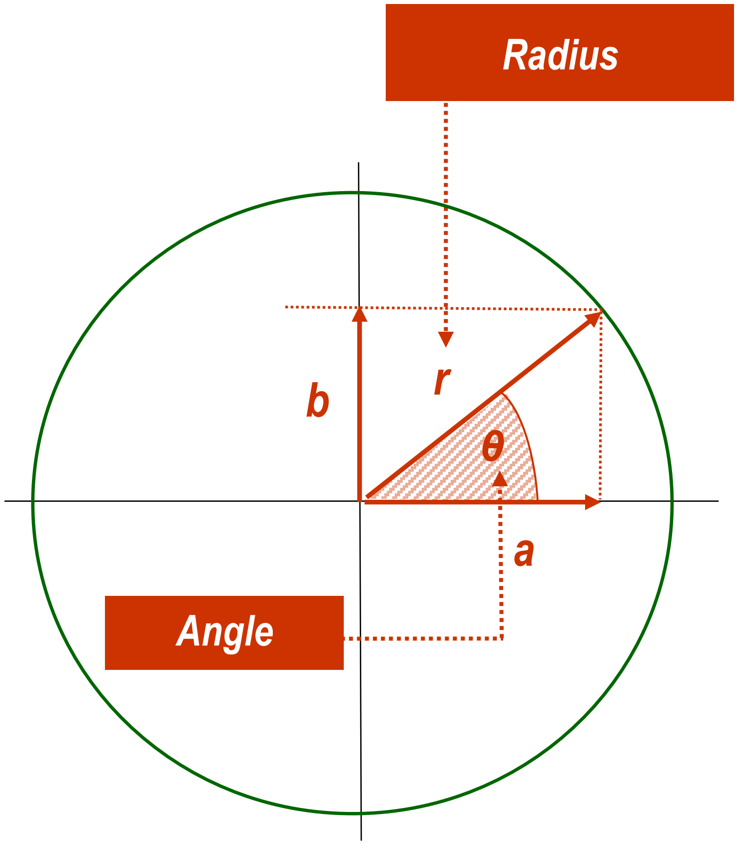
A complex number may be expressed in ***"polar representation"*** by employing the following conversion

$$r = \sqrt{a^2 + b^2}$$

$$\theta = Tan^{-1}(b / a)$$

$$r \angle \theta$$

"Polar representation" of a vector



Phasors

Conversion from Rectangular Representation to Polar Representation

Rule	Graphical Representation
A complex number expressed in rectangular coordinates can be converted into a number expressed in polar coordinates as follows;	

Let the complex number expressed in rectangular coordinates be

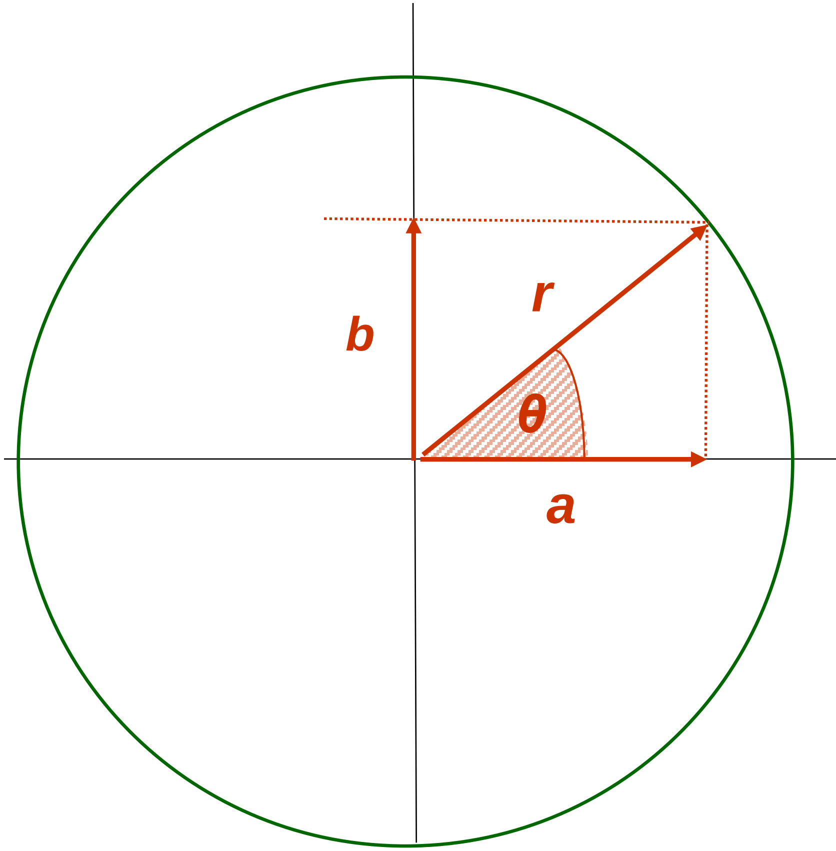
$$a + j\,b$$

Then

$$a + j\,b = r \angle \theta$$

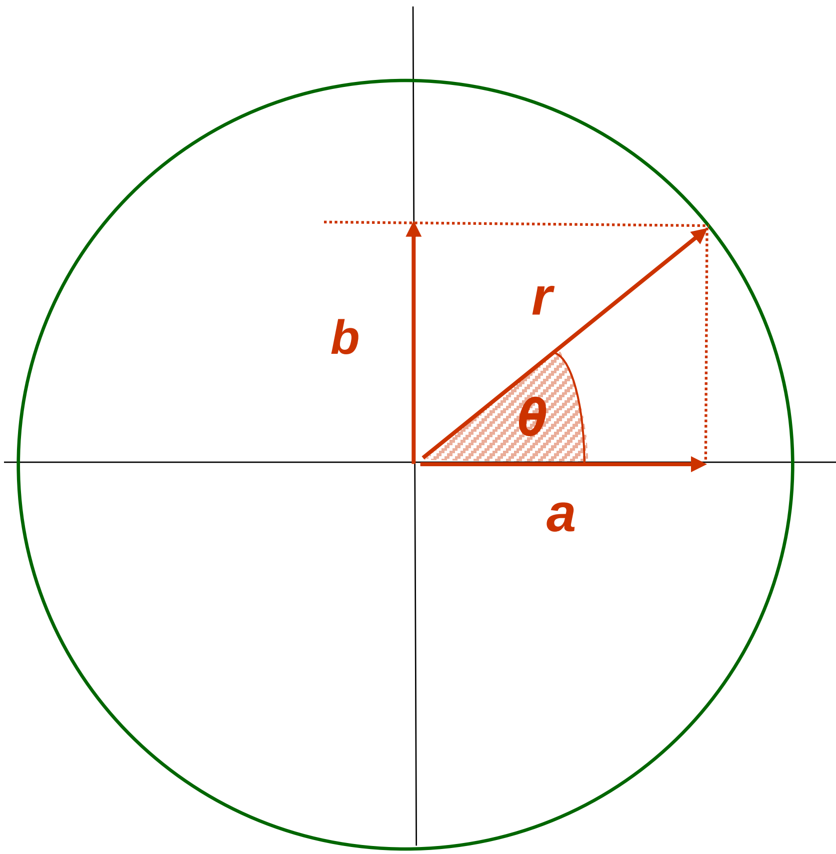
where,

$$r = \sqrt{a^2 + b^2} \qquad \theta = Tan^{-1} (b / a)$$



Phasors

Conversion from Polar Representation to Rectangular Representation

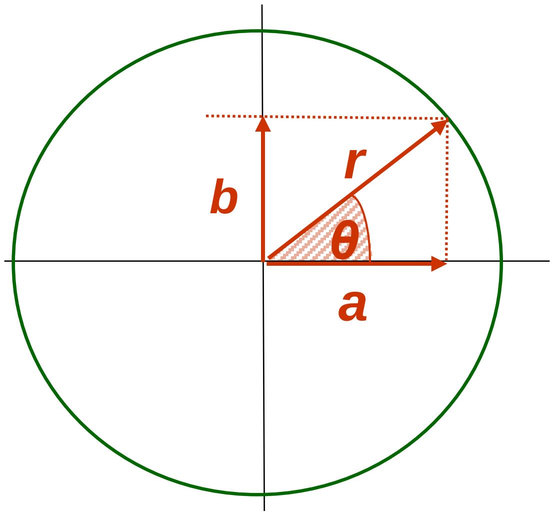
Rule	Graphical Representation
A complex number expressed in polar coordinates can be converted into a number expressed in rectangular coordinates as follows;	
Let the complex number expressed in polar (phasor) coordinates be	
$r\angle\theta$ Then $r\angle\theta = a + j\,b$	
where, $a = r\cos\theta \qquad b = r\sin\theta$	

Phasors

Polar and Rectangular Representations - Summary

Conversion Rules

Polar Representation		Rectangular Representation
$r\angle\theta$	$a = r\cos\theta, \quad b = r\sin\theta$	$a + jb$
	$r = \sqrt{a^2 + b^2}, \quad \theta = Tan^{-1}(b/a)$	



Phasors

Addition of two Complex Numbers

Method

Suppose that two phasors are to be added

$$r_1 \angle \theta_1 + r_2 \angle \theta_2 = r_{tot} \angle \theta_{tot}$$

- First express the phasors in rectangular coordinates,
- and then perform the addition

Polar Representation	$a = r\cos\theta,\ b = r\sin\theta$	Rectangular Representation
$r_1 \angle \theta_1$	→	$a_1 + j\,b_1$
$r_2 \angle \theta_2$	→	$a_2 + j\,b_2$

+

$a_{tot} + j\,b_{tot}$ → $r = \sqrt{a^2 + b^2}$, $\theta = Tan^{-1}(b/a)$ → $r_{tot} \angle \theta_{tot}$

Isn't there an easier way of doing that ?

Phasors

Subtraction of two Complex Numbers

Method

Suppose that two phasors are to be subtracted

$$r_1 \angle \theta_1 - r_2 \angle \theta_2 = r_{tot} \angle \theta_{tot}$$

- First express the phasors in polar coordinates,
- and then perform the subtraction

Polar Representation	$a = r \cos\theta,\ b = r \sin\theta$	Rectangular Representation
$r_1 \angle \theta_1$	→	$a_1 + j\, b_1$
$r_2 \angle \theta_2$	→	$a_2 + j\, b_2$
	–	

$(a_1 - a_2) + j\,(b_1 - b_2)$ → $r = \sqrt{a^2 + b^2}$, $\theta = Tan^{-1}(b/a)$ → $r_{tot} \angle \theta_{tot}$

I'm afraid NOT !

Phasors

Multiplication of two Complex Numbers

Method

Suppose that two phasors are to be multiplied

$$(a_1 + j\,b_1) \times (a_2 + jb_2) = a_{result} + j\,b_{result}$$

- First express the phasors in polar coordinates,
- and then perform the multiplication

Rectangular Representation	$r = \sqrt{a^2 + b^2}$, $\theta = Tan^{-1}(b/a)$	Polar Representation
$a_1 + j\,b_1$	→	$r_1 \angle \theta_1$
$a_2 + j\,b_2$	→ x	$r_2 \angle \theta_2$

$r_1 r_2 \angle \theta_1 + \theta_2$ → $a = r\cos\theta$, $b = r\sin\theta$ → $a_{result} + j\,b_{result}$

Phasors

Division of two Complex Numbers

Method

Suppose that two phasors are to be divided

$$(a_1 + j\,b_1)\,/\,(a_2 + jb_2) = a_{result} + j\,b_{result}$$

- First express the phasors in polar coordinates,
- and then perform the division

Rectangular Representation	$r = \sqrt{a^2 + b^2}$, $\theta = Tan^{-1}(b/a)$	Polar Representation
$a_1 + j\,b_1$	→	$r_1 \angle \theta_1$
$a_2 + j\,b_2$	→ ÷	$r_2 \angle \theta_2$
		$r_1 / r_2 \angle \theta_1 - \theta_2$

$r_1 / r_2 \angle \theta_1 - \theta_2$ → $a = r\cos\theta,\ b = r\sin\theta$ → $a_{result} + j\,b_{result}$

Properties of j Operator

Definition

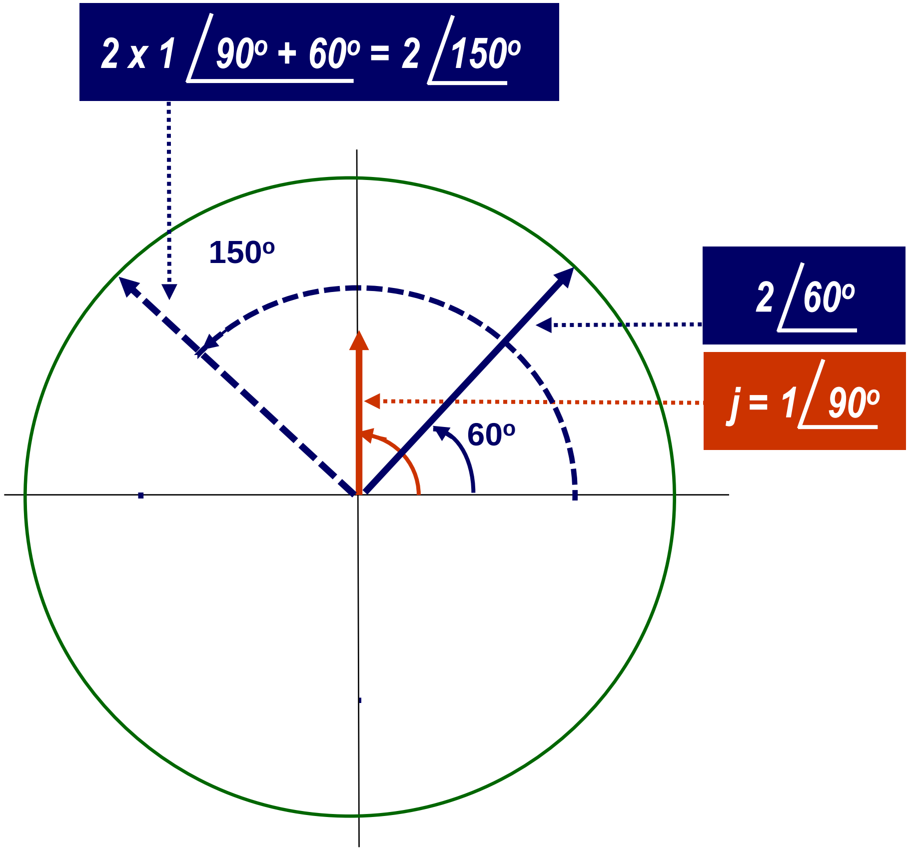
Multiplication of a complex number by j operator shifts (rotates) the angle of vector by 90°, while the magnitude is unchanged

$j = 1 \angle 90^o$

Example

Multiply complex number $2 \angle 60^o$ by j

$$2 \angle 60^o \times j = 2 \angle 60^o \times 1 \angle 90^o$$
$$= 2 \times 1 \angle 60^o + 90^o$$
$$= 2 \angle 150^o$$



Properties of j Operator

Powers of j

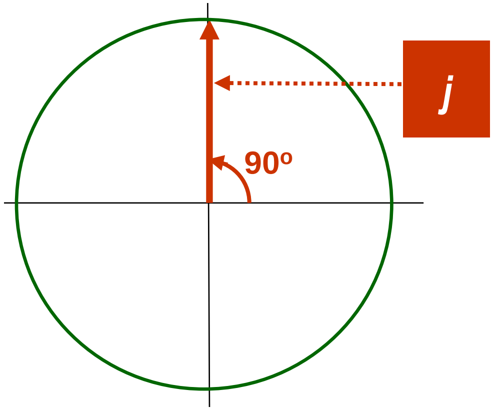
$$j = 1\angle 90^o$$

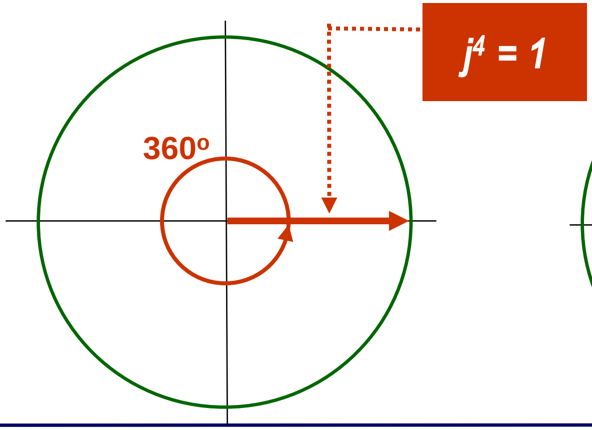
$$j^2 = 1\angle 90^o \times 1\angle 90^o = 1\angle 180^o = -1$$

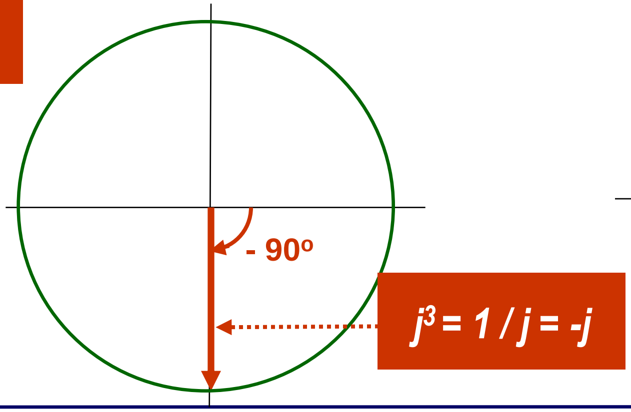
$$j^3 = 1\angle 270^o = 1\angle -90^o = -j$$

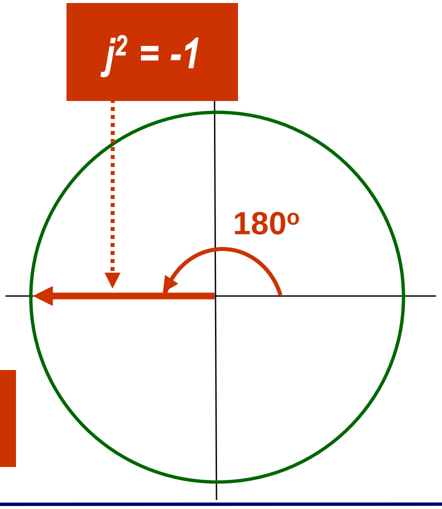
$$j^4 = 1\angle 4 \times 90^o = 1\angle 360^o = 1$$

$$1/j = 1 / 1\angle 90^o = 1\angle -90^o = -j$$









Phasors

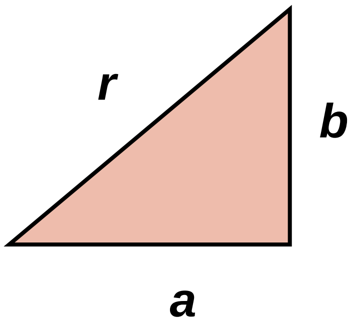
Euler's Identity

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$|e^{j\theta}| = |\cos\theta + j\sin\theta|$$

$$= \sqrt{|\cos\theta|^2 + |\sin\theta|^2}$$

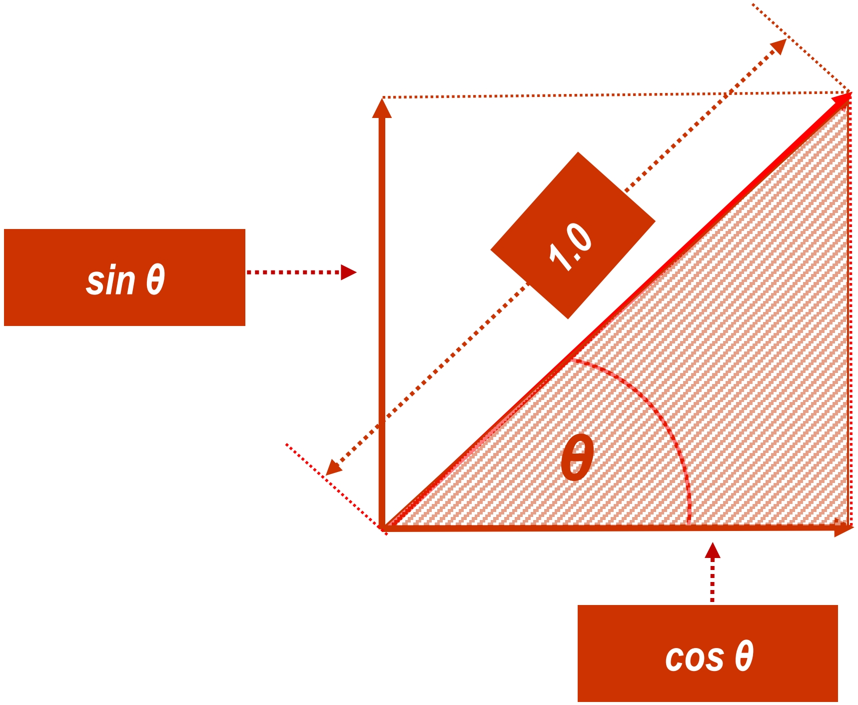
$$= 1$$



$$a^2 + b^2 = r^2$$

$$r = \sqrt{a^2 + b^2}$$

Graphical Representation



Phasors

Leonhard EULER
(1707-1783)
Swiss Mathematician

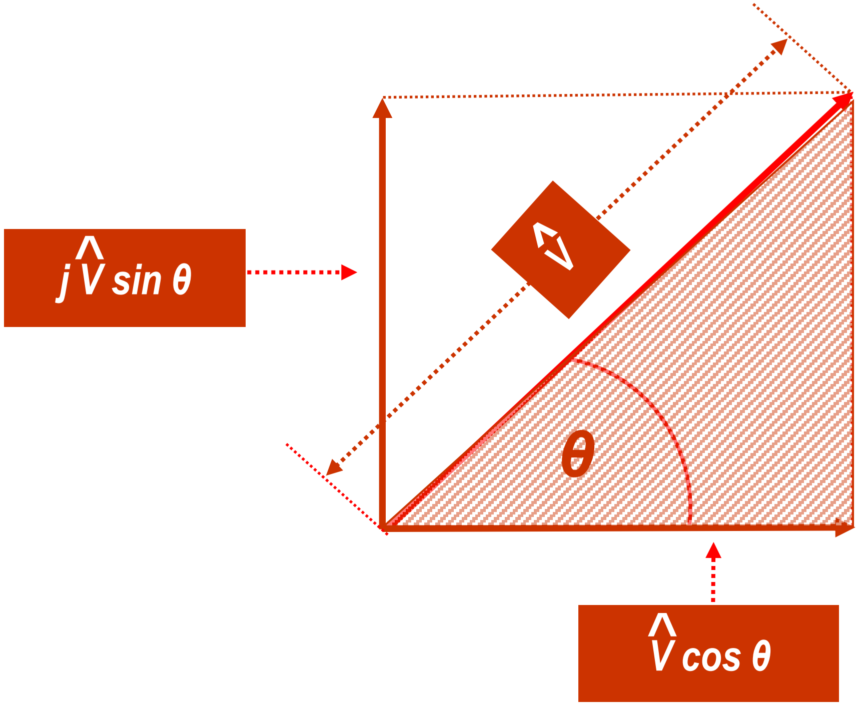
Leonhard Euler was born in Basel, Switzerland, but the family moved to Riehen when he was one year old and it was in Riehen, not far from Basel, that Leonard was brought up. Paul Euler had, as we have mentioned, some mathematical training and he was able to teach his son elementary mathematics along with other subjects.

Euler made substantial contributions to differential geometry, investigating the theory of surfaces and curvature of surfaces. Many unpublished results by Euler in this area were rediscovered by Gauss. Other geometric investigations led him to fundamental ideas in topology such as the Euler characteristic of a polyhedron.

In 1736 Euler published Mechanica which provided a major advance in mechanics.

Phasors

Euler's Identity	Graphical Representation
$\hat{V} e^{j\theta} = \hat{V} (\cos\theta + j \sin\theta)$	
$\hat{V} \, \vert e^{j\theta} \vert = \hat{V} \, \vert \cos\theta + j\sin\theta \vert$ $= \hat{V} \sqrt{\cos^2\theta + \sin^2\theta}$ $= \hat{V}$	



Phasors

Definition of Basic Terms

Now, Let θ be a linear function of time t, i.e. rotate it clockwise

$\theta = wt$

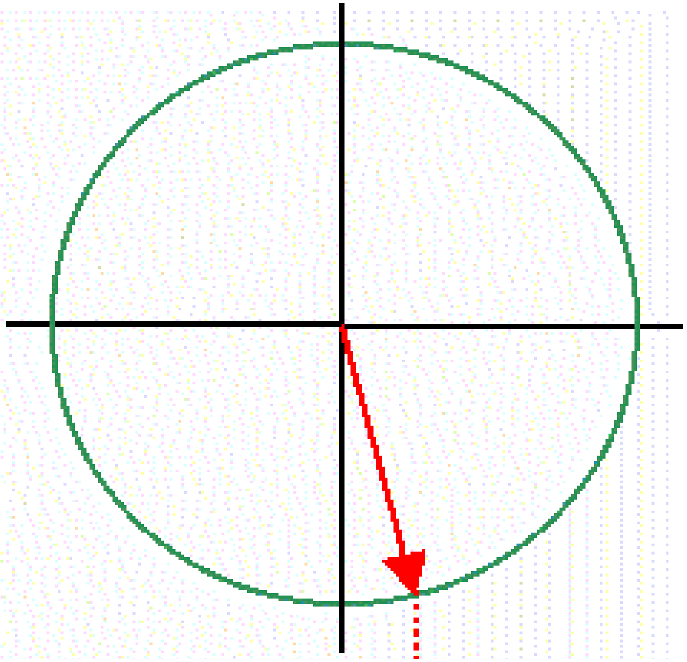
$w = 2\pi f$

$= 2 \times \pi \times 50 = 314$ Radians/sec

($f = 50$ Hz)

1 Radian = $360^{o} / (2\pi) = 57.29^{o}$

Graphical Representation



Phasors

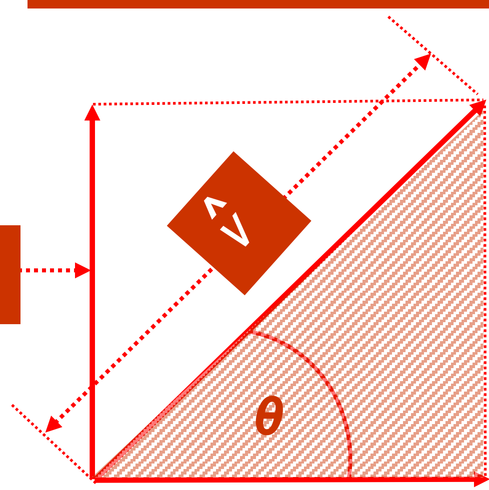
Euler's Identity

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\hat{V} e^{jwt} = \hat{V}(\cos wt + j\sin wt)$$

$V(t) = \hat{V}\sin\theta = \hat{V}\sin wt$

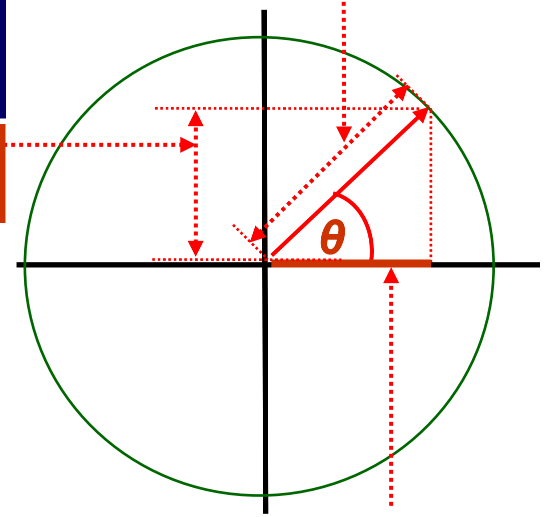
$V(t) = \hat{V}\sin\theta$



$V(t) = \hat{V}\cos\theta$

Graphical Representation

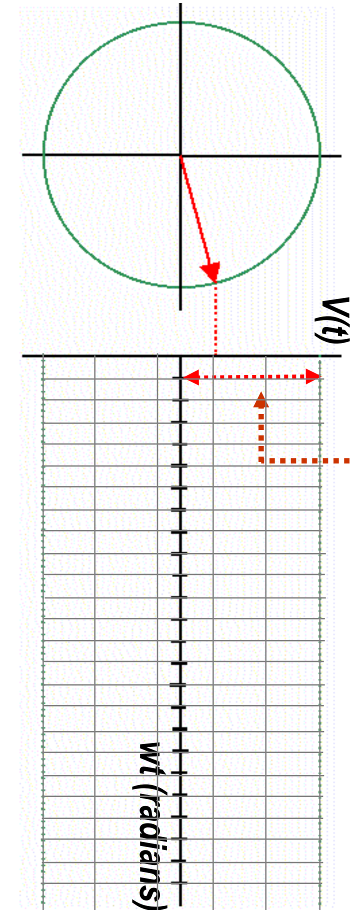
$\hat{V}$ = Magnitude



$V(t) = \hat{V}\cos\theta$

$\theta = wt$

$V(t) = \hat{V}\cos wt$



$\hat{V}$

Phasors

Symbolic Representation

Mathematical Notation

Now let a phasor be located at an angular position ϕ initially, i.e.

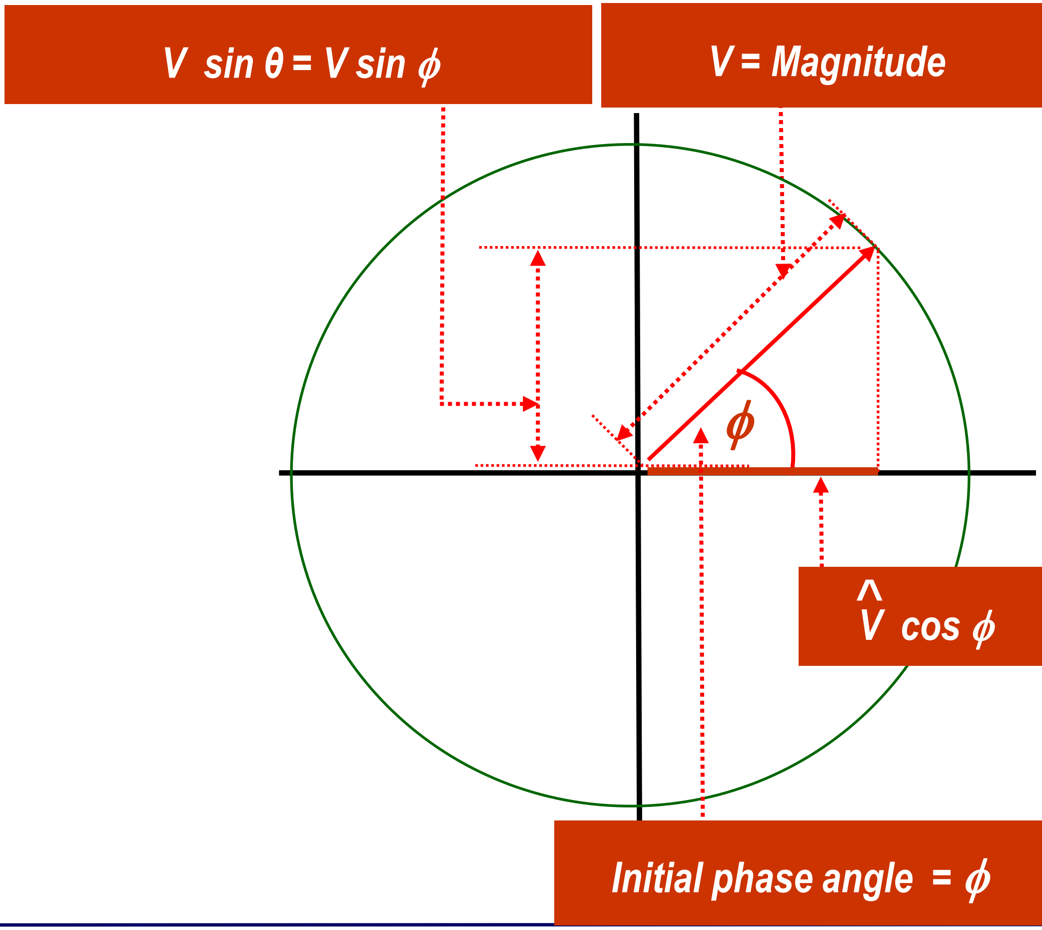
$$V(t) = \hat{V} \cos(wt + \phi) \Big|_{t=0}$$

In other words the phasor is at an angular position ϕ at $t = 0$

The phasor on the RHS is then represented mathematically by the following notations;

$$\theta = wt \,\Big|_{t=0} + \phi = \phi$$

Graphical Representation



Phasors

Angular Displacement

Total Angular Displacement	Graphical Representation

Total angular displacement at time $t = t_1$ may then be expressed as;

$$\theta(t_1) = w\,t_1 + \phi$$

Initial phase angle $= \phi$

Then the horizontal component becomes;

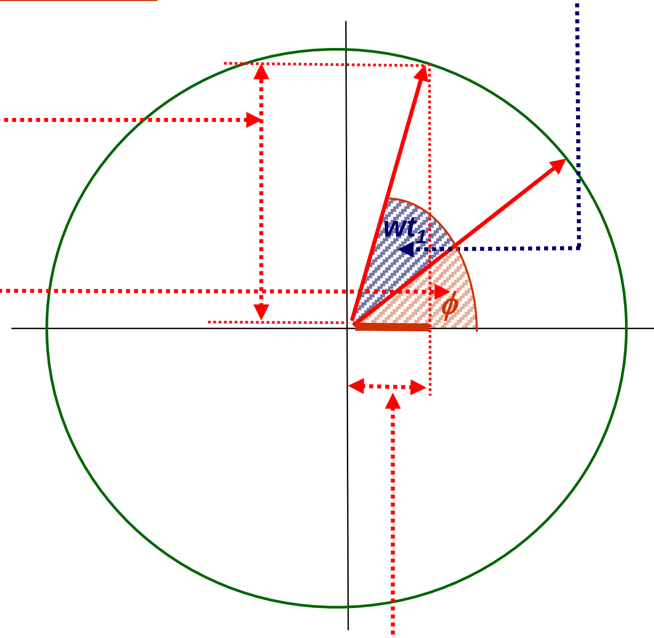
$$V(t) = \hat{V}\cos(wt + \phi)\Big|_{t = t_1}$$

$$= \hat{V}\cos(wt_1 + \phi)$$

In other words, the phasor will be at an angular position $wt_1 + \phi$ at $t = t_1$

$\hat{V}\sin\theta = \hat{V}\sin(wt_1 + \phi)$

Angle swept during $t = t_1$



$\hat{V}\cos\theta = \hat{V}\cos(wt_1 + \phi)$

Phasors

Mathematical Notation

Notation	Graphical Representation

The phasor on the RHS may be represented mathematically as;

$$\hat{V} e^{j\theta(t_1)} = \hat{V} e^{j(w t_1 + \phi)}$$

$\hat{V}\sin\theta = \hat{V}\sin(wt_1 + \phi)$

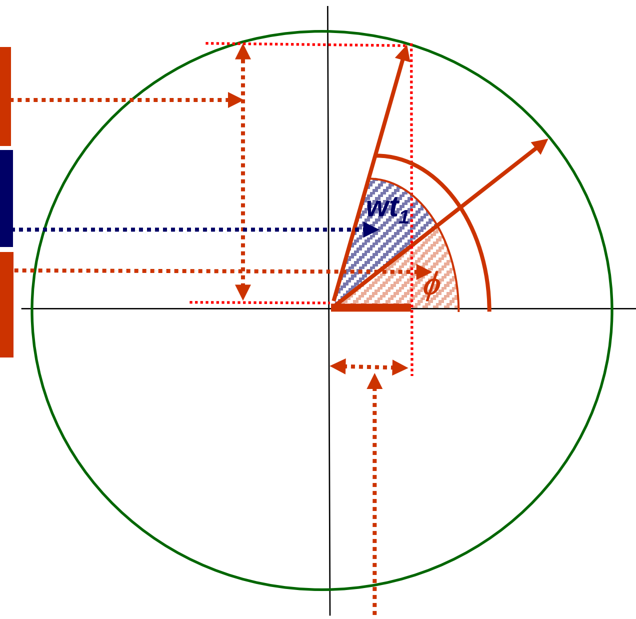
Angle swept during $t = t_1$

Initial phase angle $= \phi$

$$\hat{V} e^{j(w t_1 + \phi)} \longleftrightarrow \hat{V}\angle\phi$$

Amplitude

Phase angle

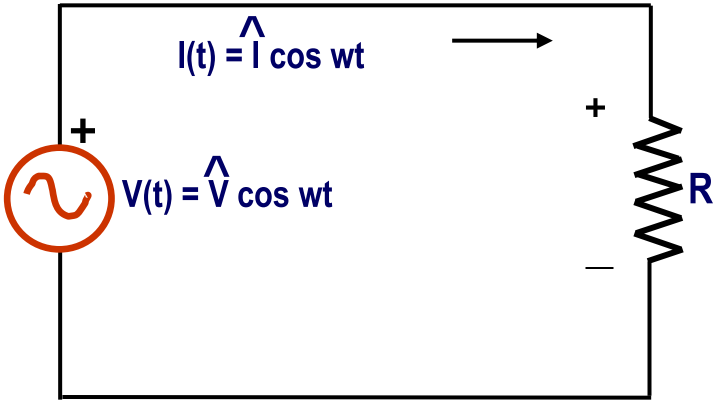


$\hat{V}\cos(wt_1 + \phi)$

Waveform Representation of Resistive Circuits

Waveform Representation of Resistive Circuits

Consider the resistive circuit shown on the RHS



$V(t) = \hat{V} \cos wt$

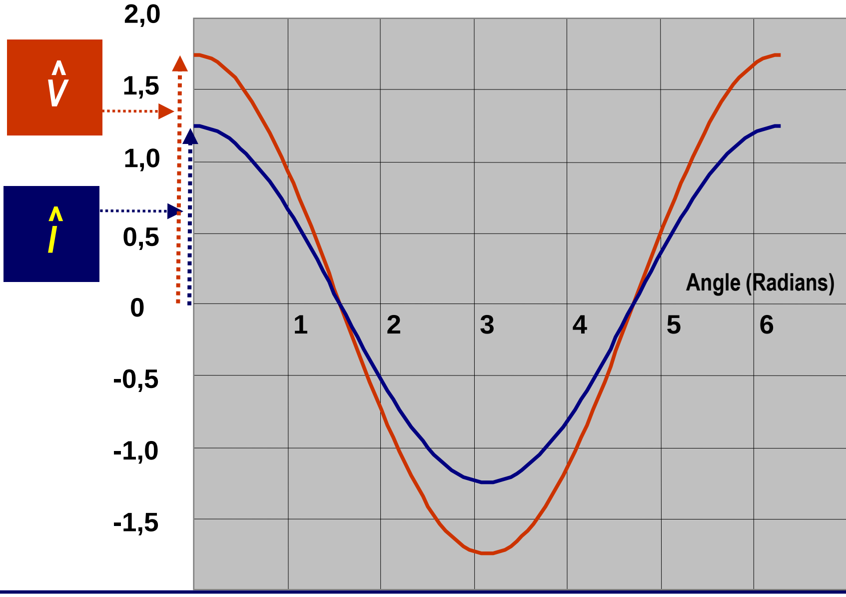
$I(t) = \hat{V}_s(t) / R$ ← Ohm's Law

$= \hat{V} \cos wt / R$

$= \hat{I} \cos wt$

where,

$\hat{I} = \hat{V} / R$



Phasor Representation of Resistive Circuits

Phasor Representation of Resistive Circuits

Consider the resistive circuit shown on the RHS

Ohm's Law

Waveform Representation

$V(t) = \hat{V} \cos (wt + 0^o)$

$I(t) = \hat{I} \cos (wt + 0^o)$

Phasor Representation

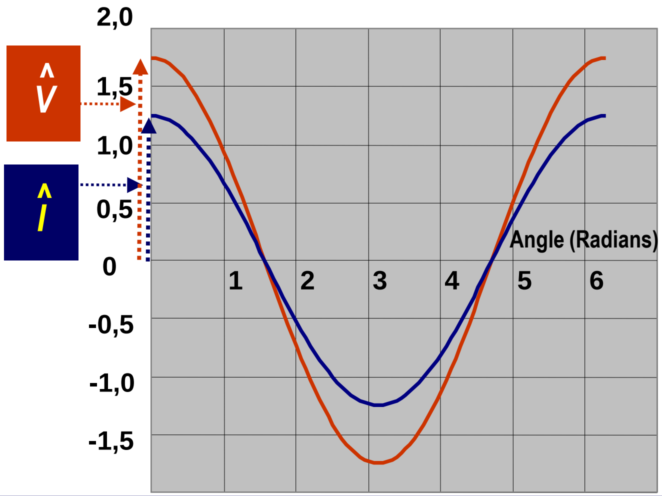
$V = \hat{V} \angle 0^o$

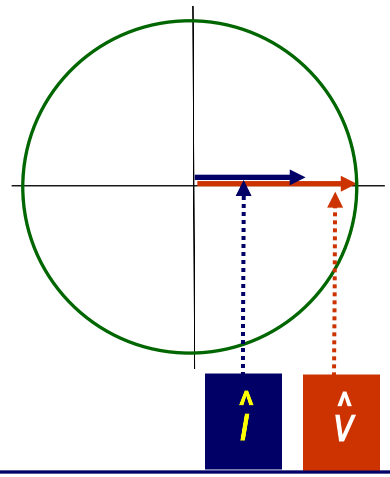
$I = \hat{I} \angle 0^o$

$$Z = \frac{\hat{V} \angle 0^o}{\hat{I} \angle 0^o} = Z \angle 0^o = R$$

$$| Z | = \sqrt{R^2} = R$$

$$\angle Z = Tan^{-1} (0 / R) = 0$$

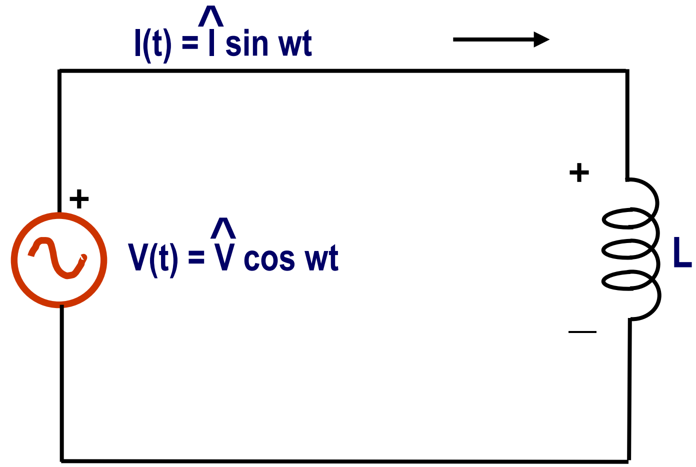




Waveform Representation of Inductive Circuits

Waveform Representation of Inductive Circuits

Consider the inductive circuit shown on the RHS



Let now,

$$I(t) = \hat{I} \sin wt$$

$$V(t) = L\, dI(t) / dt = L w \hat{I} \cos wt = \hat{V} \cos wt$$

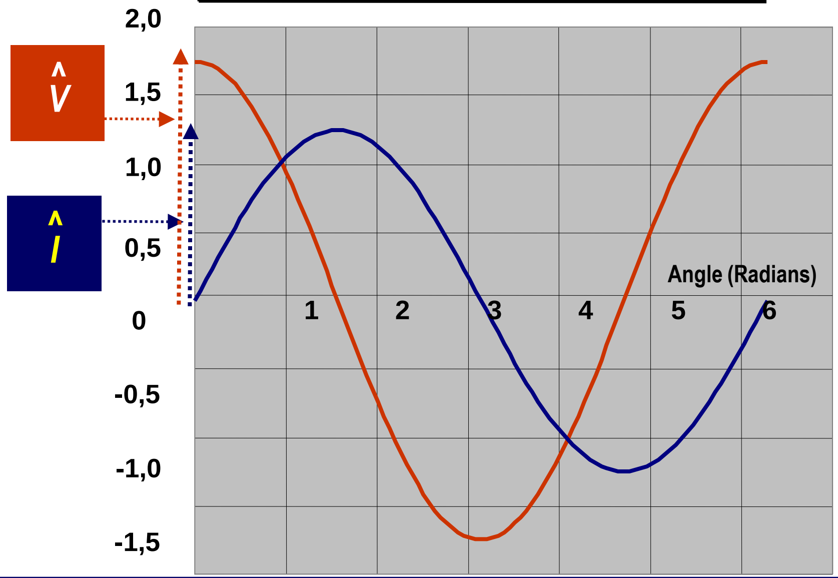
where,

$$\hat{V} = L w \hat{I}$$

$\hat{V}$ 0   1

$$\cos(wt - 90^o) = \cos wt \cos 90^o + \sin wt \sin 90^o$$

$$I(t) = \hat{I} \sin wt = \hat{I} \cos(wt - 90^o)$$



Phasors

Phasor Representation of Inductive Circuits

Phasor Representation of Inductive Circuits

Waveform Representation

$V(t) = \hat{V} \cos wt$

$I(t) = \hat{I} \cos (wt - 90^o)$

Phasor Representation

$V = \hat{V} \angle 0^o$

$I = \hat{I} \angle -90^o$

Ohm's Law

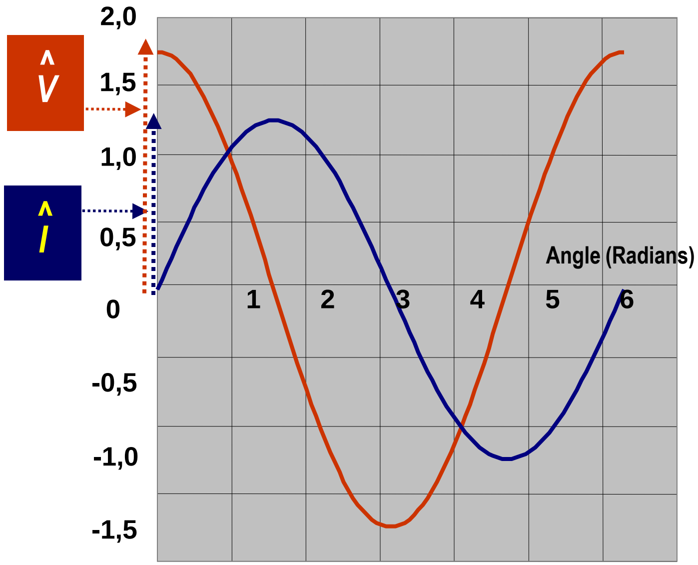
$$Z = \frac{\hat{V} \angle 0^o}{\hat{I} \angle -90^o} = Z \angle 90^o = Lw \angle 90^o$$

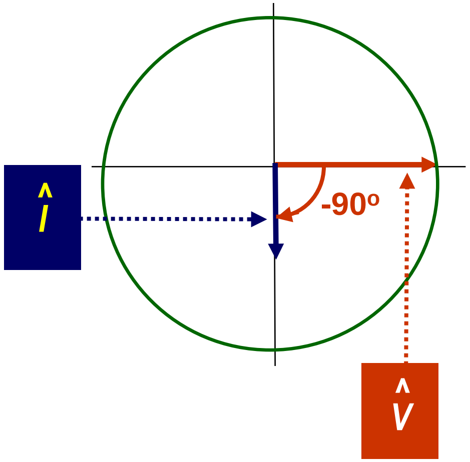
$$Z = Lw \angle 90^o = j\,wL$$

$$|Z| = \sqrt{R^2 + (wL)^2} = wL$$

$$\angle Z = Tan^{-1} (wL / 0) = 90^o$$

$$\hat{V} = L\,w\,\hat{I}$$





Waveform Representation of Capacitive Circuits

Waveform Representation of Capacitive Circuits

Consider the capacitive circuit shown on the RHS

$V(t) = \hat{V} \cos wt$

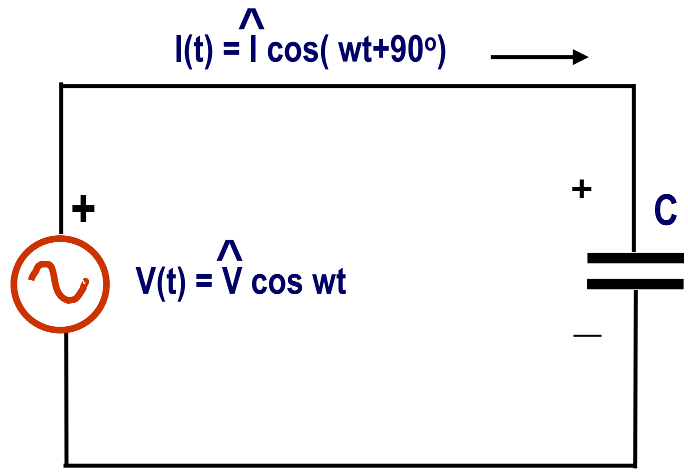
$I(t) = C\, dV_s(t) / dt = - C w \hat{V} \sin wt$

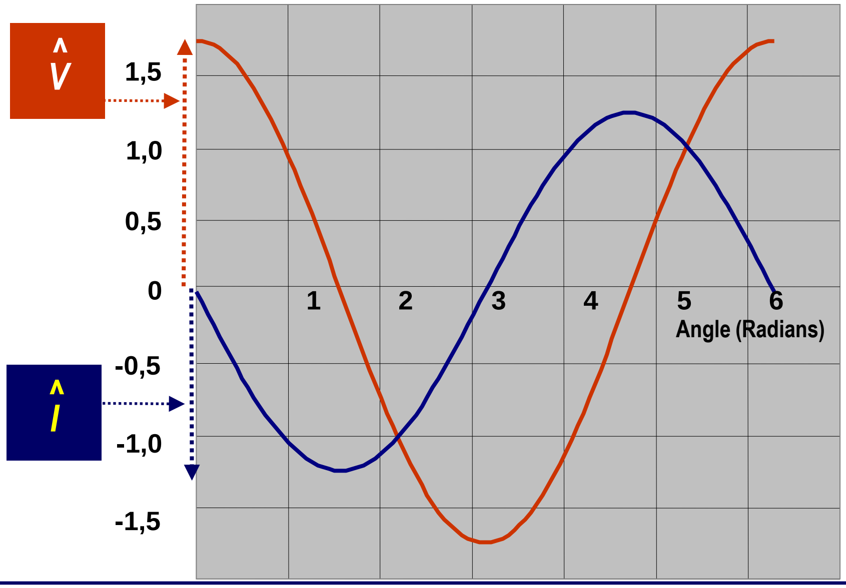
$\cos (wt + 90^o) = \cos wt \cos 90^o - \sin wt \sin 90^o$ (0, 1)

$= \hat{I} \cos (wt + 90^o)$

where,

$\hat{I} = C w \hat{V}$





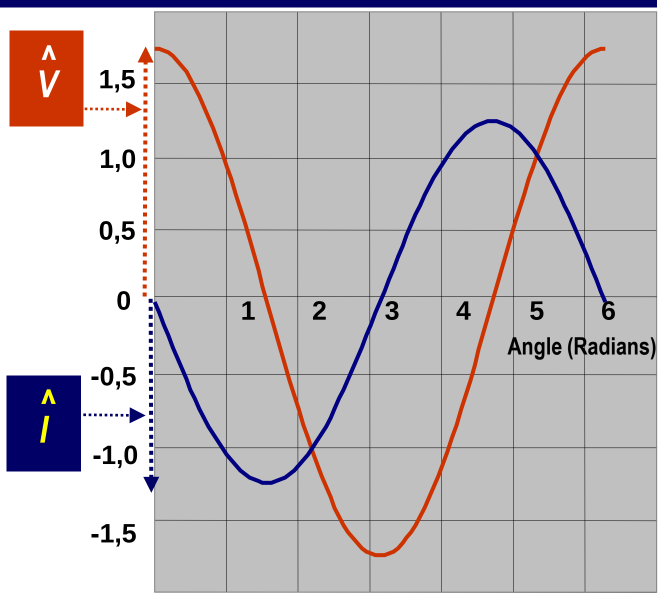
Phasor Representation of Capacitive Circuits

Phasor Representation of Capacitive Circuits

Waveform Representation

$V(t) = \hat{V} \cos wt$

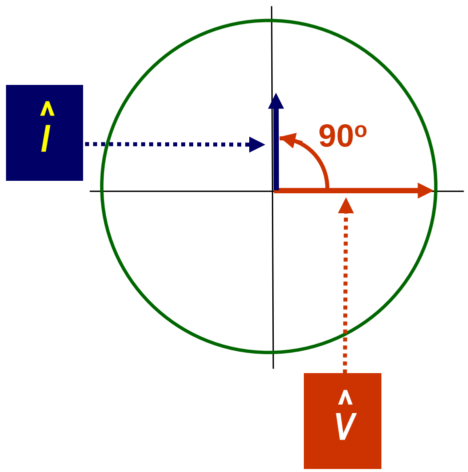
$I(t) = \hat{I} \cos (wt + 90^o)$



Phasor Representation

$V = \hat{V} \angle 0^o$

$I = \hat{I} \angle 90^o$



Ohm's Law

$$Z = \frac{\hat{V} \angle 0^o}{\hat{I} \angle 90^o} = Z \angle -90^o = 1 / Cw \angle -90^o$$

$$Z = 1 / jwC = 1 \angle -90^o / Cw = -j / Cw$$

$$| Z | = \sqrt{R^2 + (1/Cw)^2} = 1 / Cw$$

$$\angle Z = Tan^{-1} (-Cw / 0) = -90^o$$

$$\hat{I} = C w \hat{V}$$

Phasor Representation of AC Circuits

Phasor Representation of R-L Circuits

Consider the following R-L circuit

$$V = \hat{V} \angle 0^o$$

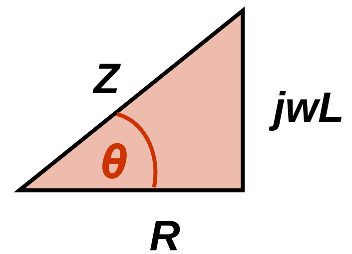
$Z = R + j\,wL$

$|Z| = \sqrt{R^2 + (wL)^2}$

$\angle Z = Tan^{-1}(wL / R) = \theta$

$R^2 + (wL)^2 = Z^2$

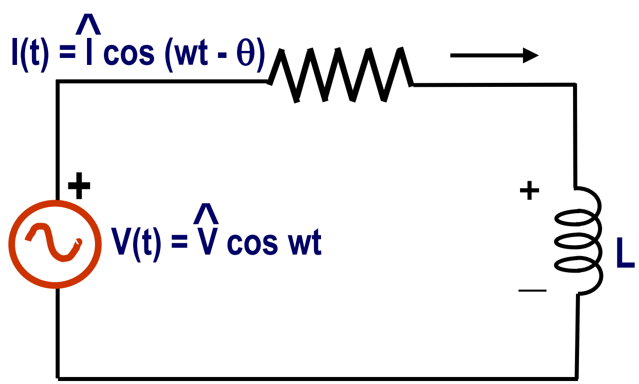
$Z = \sqrt{R^2 + (wL)^2} \angle \theta$

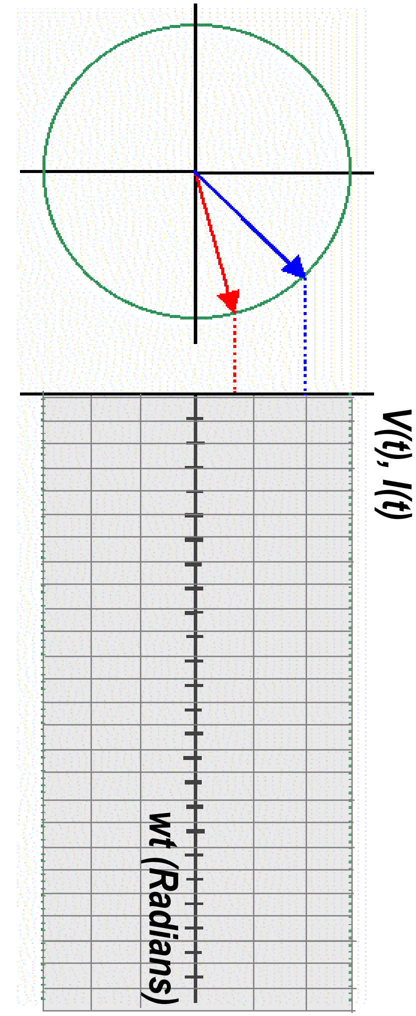


$$I = \hat{V} \angle 0^o \,/\, (\sqrt{R^2 + (wL)^2} \angle \theta)$$

$$= \hat{I} \angle -\theta$$

$$\hat{I} = \hat{V} / \sqrt{R^2 + (wL)^2}$$





Phasors

Phasor Representation of AC Circuits

Phasor Representation of R-C Circuits

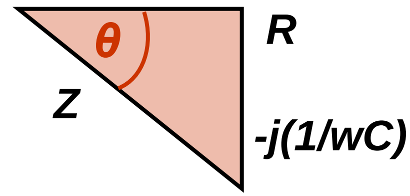
Consider the R-C circuit shown below

$$V = \hat{V} \angle 0^o$$

$$Z = R + (1 / jwC) = R - j\,(1 / wC)$$

$$|Z| = \sqrt{R^2 + (1 / wC)^2}$$

$$\angle Z = Tan^{-1}\,(1 / (wCR)) = \theta < 0$$



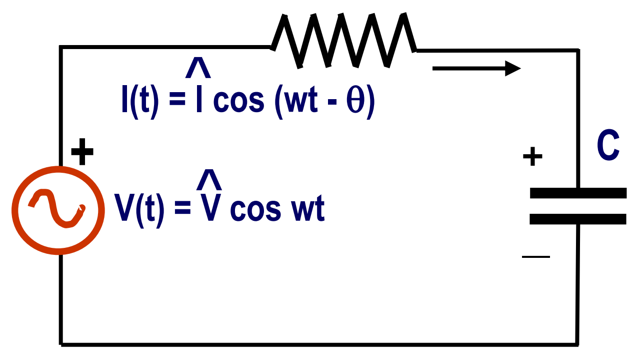
$$R^2 + (1/wC)^2 = Z^2$$

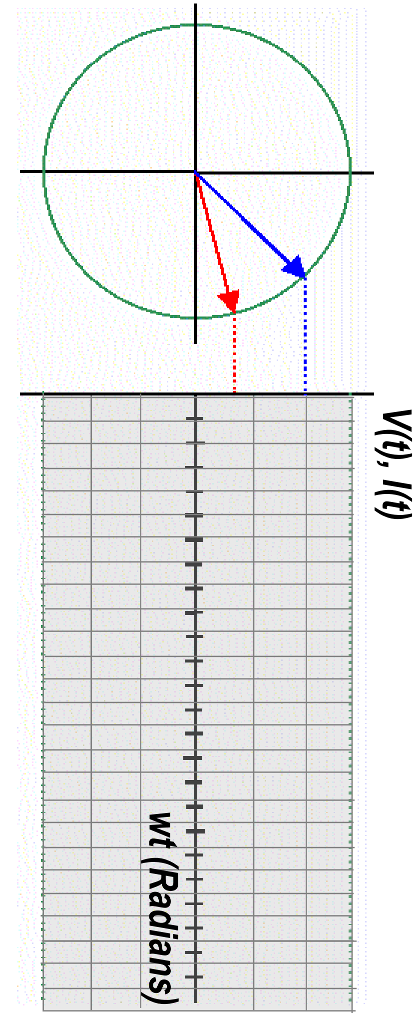
$$Z = \sqrt{R^2 + (1/wC)^2} \angle \theta$$

$$I = \hat{V} \angle 0^o \,/\, (\sqrt{R^2 + (1 / wC)^2} \angle -\theta\,)$$

$$= \hat{I} \angle \theta$$

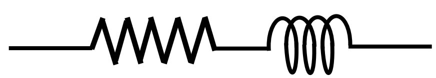
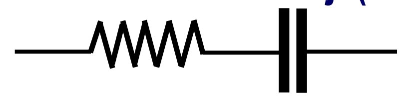
$$\hat{I} = \hat{V} / \sqrt{R^2 + (1 / wC)^2}$$

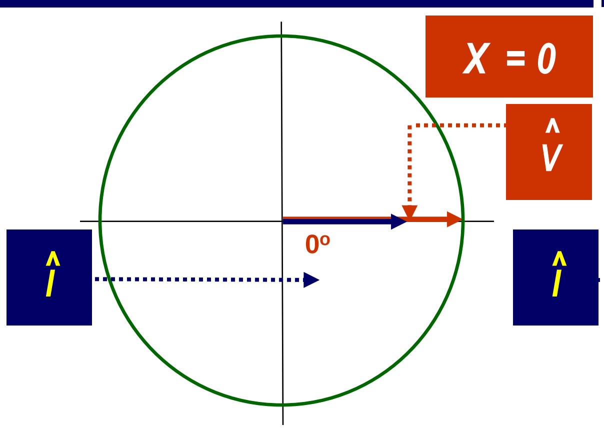


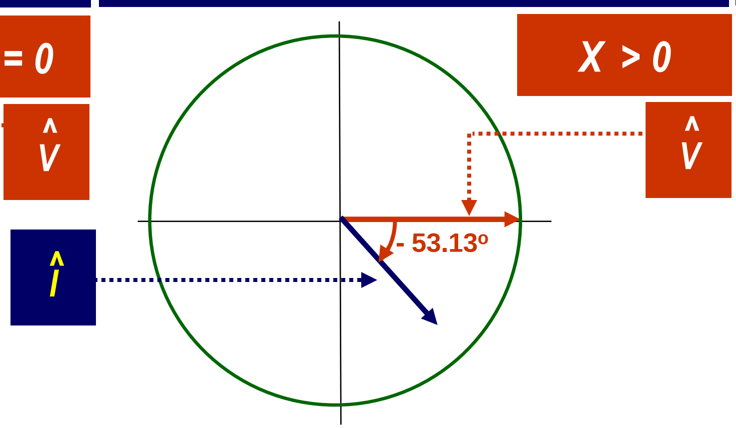


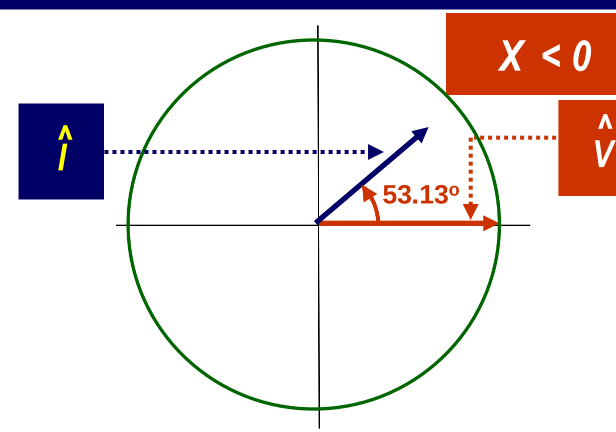
Phasors

Phasor Representation of AC Circuit Elements

Resistance	R-L Element	R-C Element
		
$\lvert Z \rvert = \sqrt{R^2} = R$	$\lvert Z \rvert = \sqrt{R^2 + (wL)^2}$	$\lvert Z \rvert = \sqrt{R^2 + (1/wC)^2}$
$\angle Z = Tan^{-1}(0 / R) = 0$	$\angle Z = Tan^{-1}(wL / R)$	$\angle Z = Tan^{-1}((1/wC) / R)$
$Z = R \angle 0^o$	$Z = \sqrt{R^2 + (wL)^2} \angle Tan^{-1}(wL / R)$	$Z = \sqrt{R^2 + (1/wC)^2} \angle -Tan^{-1}((1/wC)/R)$







Phasors

Phasor Representation of R-L-C Circuits

Phasor Representation of R-L-C Circuits

Consider the R-L-C circuit shown on the RHS

$V = \hat{V} \angle 0^o$

$Z = R + jwL - j / wC = R + j\,(wL - 1/wC)$

$|Z| = \sqrt{R^2 + (wL - 1/wC)^2}$

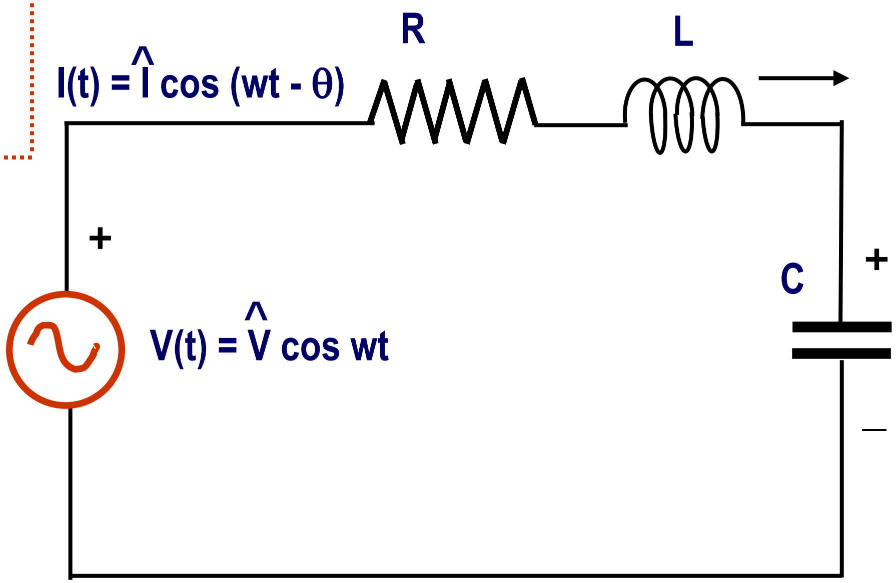
$\angle Z = Tan^{-1}\,((wL - 1/(wC))/R) = \theta$

$I = \hat{V} \angle 0^o \,/\, (\sqrt{R^2 + (wL - 1/wC)^2} \angle \theta\,)$

$= \hat{I} \angle -\theta$

$\hat{I} = \hat{V} / \sqrt{R^2 + (wL - 1/wC)^2}$

$1/j = -j$



Ohm's Law: $\hat{I} = \hat{V} / Z$

Phasor Representation of R-L-C Circuits

R-L-C Circuits

Solve the R-L-C circuit shown on the RHS for current phasor

$V = \hat{V}\ \angle 0^o = 220\sqrt{2}\ \angle 0^o$

$Z = R + jwL - j / wC = 2 + j(3.14 - 1.59)\ \Omega$

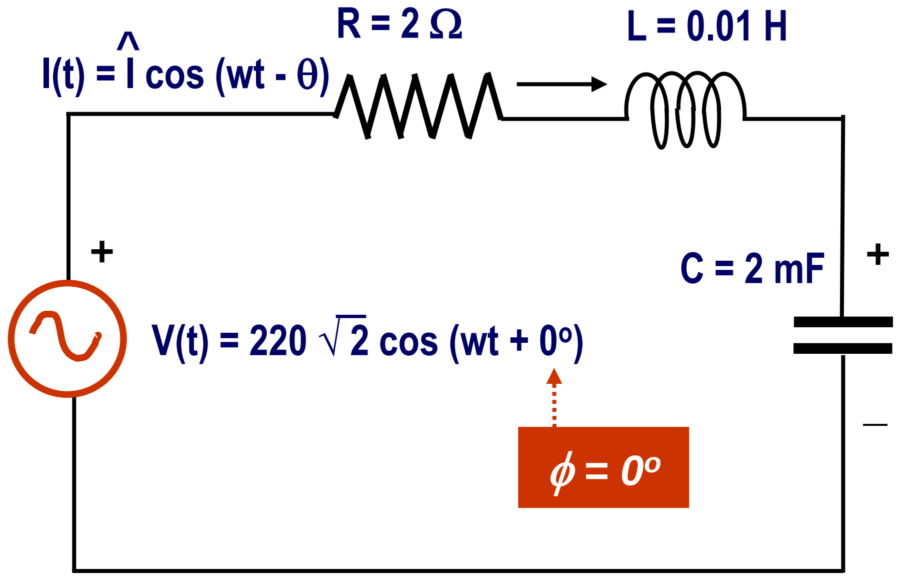
$= 2 + j\ 1.55\ \Omega$

$|Z| = \sqrt{R^2 + (wL - 1/wC)^2} = \sqrt{2^2 + 1.55^2}$

$= 2.098\ \Omega$

$\angle Z = Tan^{-1}((wL - 1/(wC))/R) = Tan^{-1}(1.55 / 2) = \theta$
$= 17.58^o$

$Z = 2.098 \angle 17.58^o\ \Omega$



$w = 2\pi f = 2 \times 3.14 \times 50 = 314$ radians /sec

$X_L = j\ wL = j\ 0.01 \times 314 = j\ 3.14\ \Omega$

$X_C = 1 / (jwC) = -j / wC = -j / (314 \times 2 \times 10^{-3})$

$= -j\ 1.59\ \Omega$

$\theta > 0$ (Inductive)

Phasor Representation of R-L-C Circuits

Example

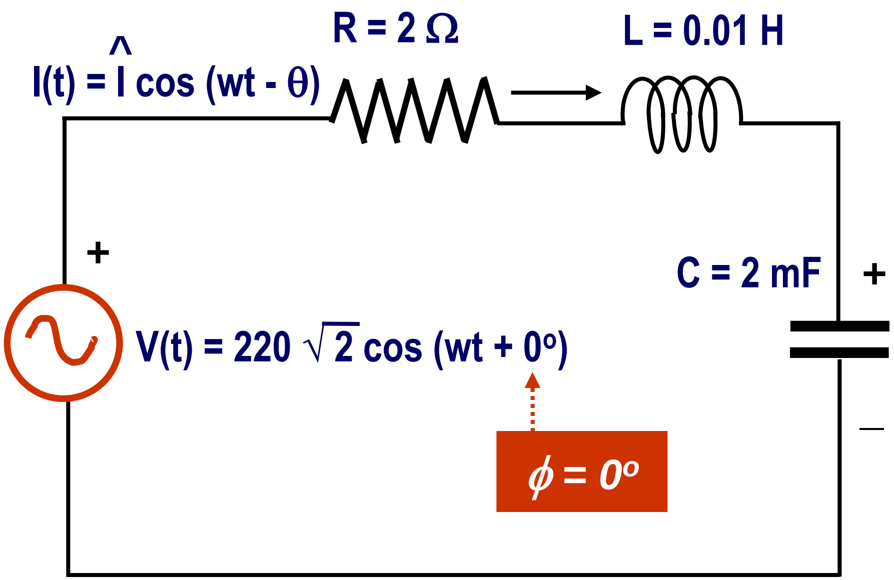
Solve the R-L-C circuit shown on the RHS for current phasor

$$I = \hat{V}\angle 0^o \;/\; (\sqrt{R^2 + (wL - 1/wC)^2}\angle\theta)$$

$$= 220\sqrt{2}\angle 0^o \;/\; 2.098\angle 17.58^o$$

$$= \hat{I}\angle -\theta$$

$$= 148.30\angle -17.58^o \text{ Amp}$$



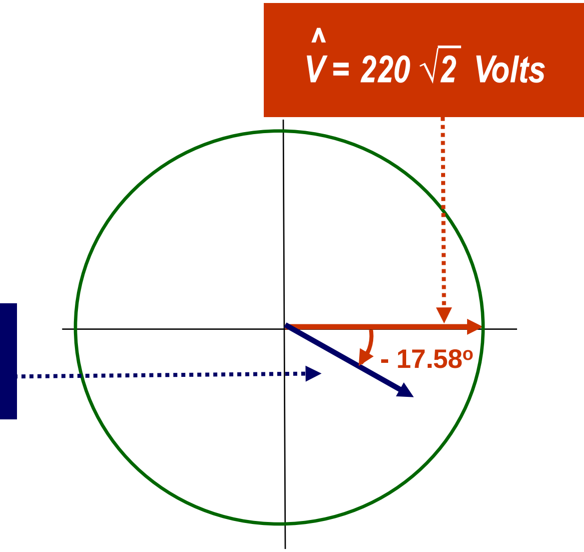
Phasor Representation of R-L-C Circuits

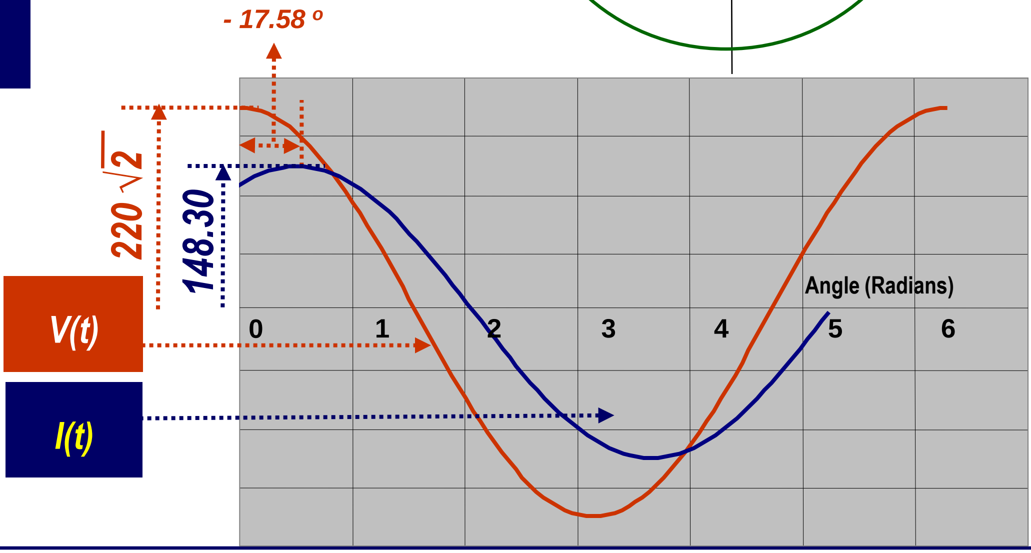
Example (Continued)

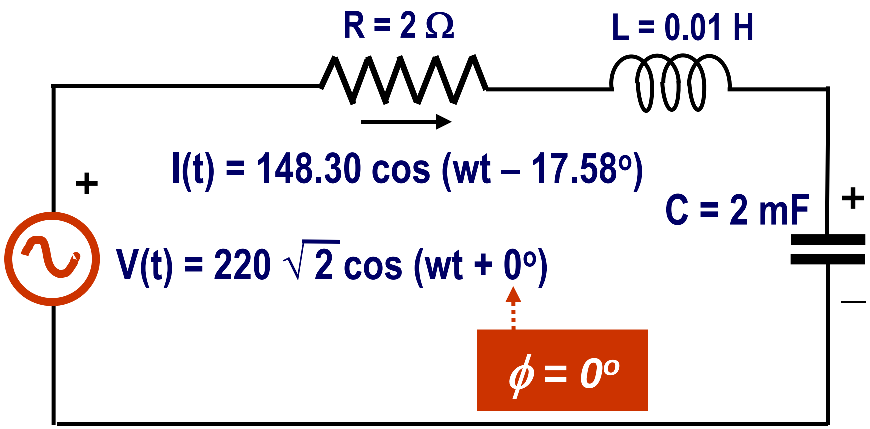
Draw the voltage and current phasors and waveforms

$V = 220\sqrt{2}\angle 0^o$ *Volts (peak)*

$I = 148.30\angle -17.58^o$ *Amp*







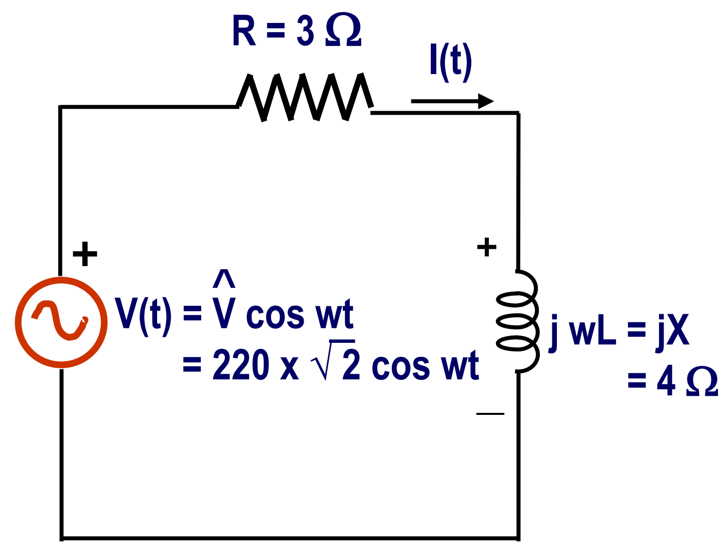
METU

Phasors

Example

Problem

Solve the circuit on the RHS for current waveform by using the phasor method



Solution

Waveform Representation

$V(t) = \hat{V} \cos wt$

$= 220 \times \sqrt{2} \cos wt$

Phasor Representation

$V = \hat{V} \angle 0^o = 220 \times \sqrt{2} \angle 0^o$ Volts

$|Z| = \sqrt{R^2 + X^2}$

$= \sqrt{3^2 + 4^2} = 5\ \Omega$

$\angle Z = Tan^{-1} (4 / 3)$

$= 53.13^o$

$Z = 5 \angle 53.13^o\ \Omega$

$I = 220 \times \sqrt{2} \angle 0^o / 5 \angle 53.13^o$ Amp

$= 62.22 \angle -53.13^o$ Amp

Phasors

Example

Problem

Draw the waveforms and phasors for the previous problem

$I = 62.22 \angle -53.13^o$ Amp

$V_s = \hat{V} \angle 0^o$
$= 220 \times \sqrt{2} \angle 0^o$

Waveform Representation

$V(t) = \hat{V} \cos wt$
$= 220 \times \sqrt{2} \cos wt$

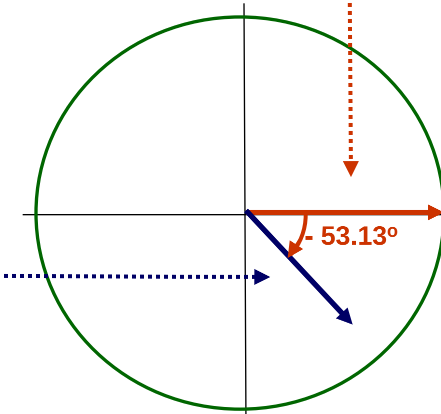
$I(t) = 62.22 \cos (wt - 53.13^o)$

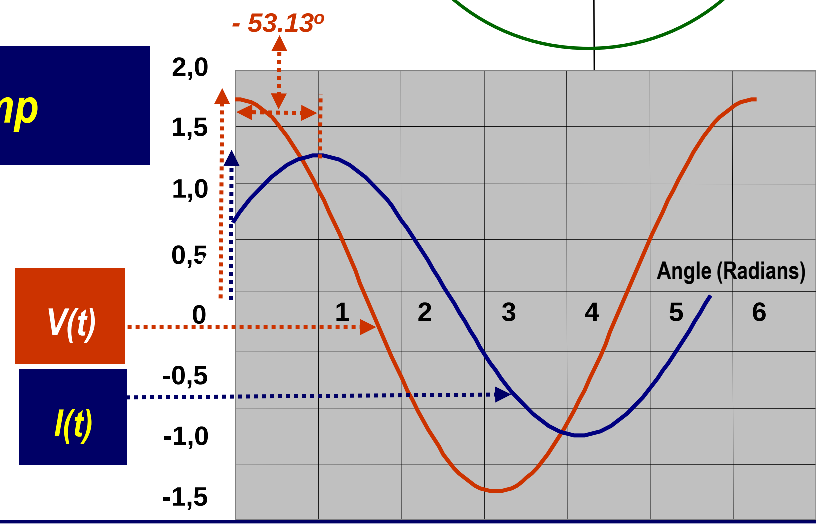
Phasor Representation

$V = \hat{V} \angle 0^o = 220 \times \sqrt{2} \angle 0^o$ Volts

$I = 62.22 \angle -53.13^o$ Amp

$\hat{I}$





Phasors

Example

Calculate the equivalent impedance seen between the terminals A and B of the AC circuit given on the RHS (w = 314 rad / sec)

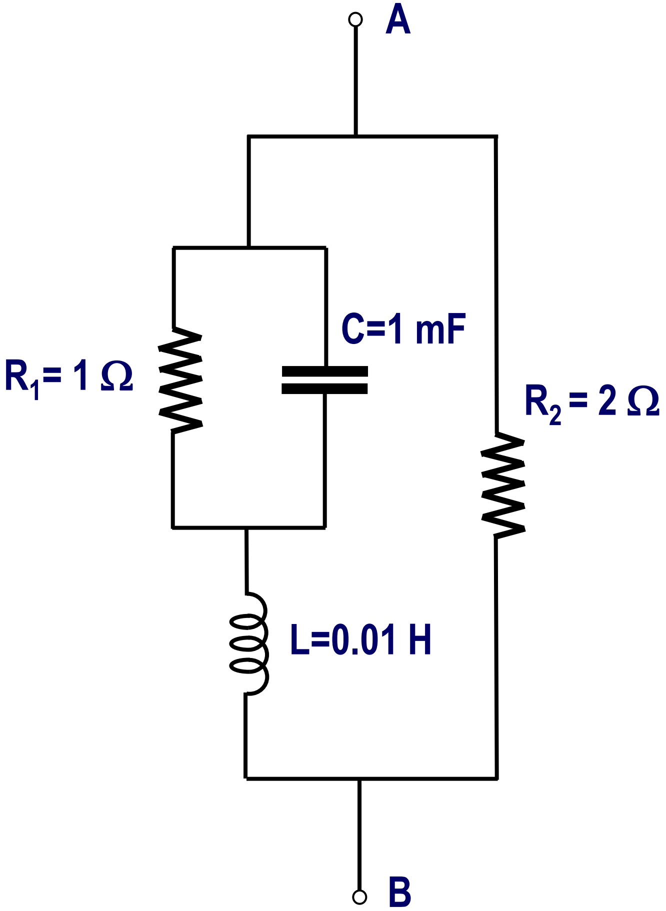
First, let us calculate impedances

$Z_C = 1 / (jwC) = 1 / (j314 \times 1 \times 10^{-3})$
$= - j\, 3.1847\ \Omega$

$Z_L = jwL = j\, 314 \times 0.01 = j\, 3.14\ \Omega$

$Z_C // R_1 = 1 / (1/Z_c + 1/R_1) = 1 / (1/-j3.1847 + 1/1)$
$= 1 /(1 + j0.314) = 1 / (1.04814 \angle 17.43^o)$
$= 0.9540 \angle -17.43^o = 0.91019 - j\, 0.285761\ \Omega$

$(Z_C // R_1) + Z_L = 0.91019 - j\, 0.285761 + j\, 3.14$
$= 0.91019 + j\, 2.854239\ \Omega$



Example (Continued)

(Continued)

Now, let us calculate: $(R_{eq} + j\,Z_{Leq}) \,//\, R_2$

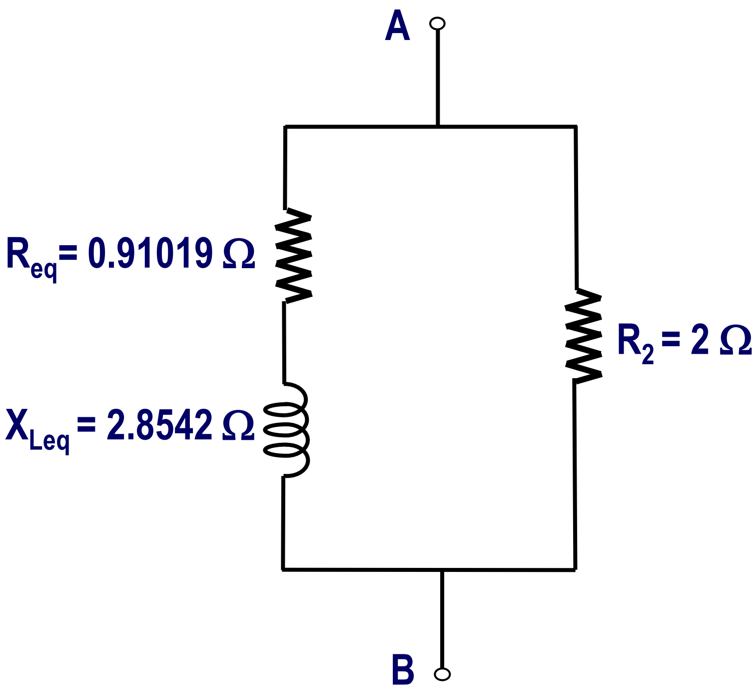
$$(R_{eq} + j\,Z_{Leq}) \,//\, R_2 = (0.91019 + j\,2.854239) \,//\, 2$$

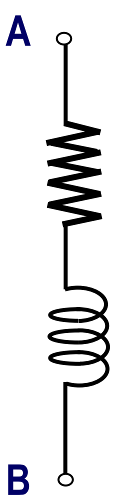
$$= 2.9958\angle 72.31^o \,//\, 2\angle 0^o$$

$$= \frac{2.9958 \times 2\angle 72.31^o}{(0.91019 + 2) + j\,2.854239}$$

$$= \frac{5.99160\angle 72.31^o}{4.07685\angle 44.44^o}$$

$$= 1.46966\angle 27.87 \;\Omega$$



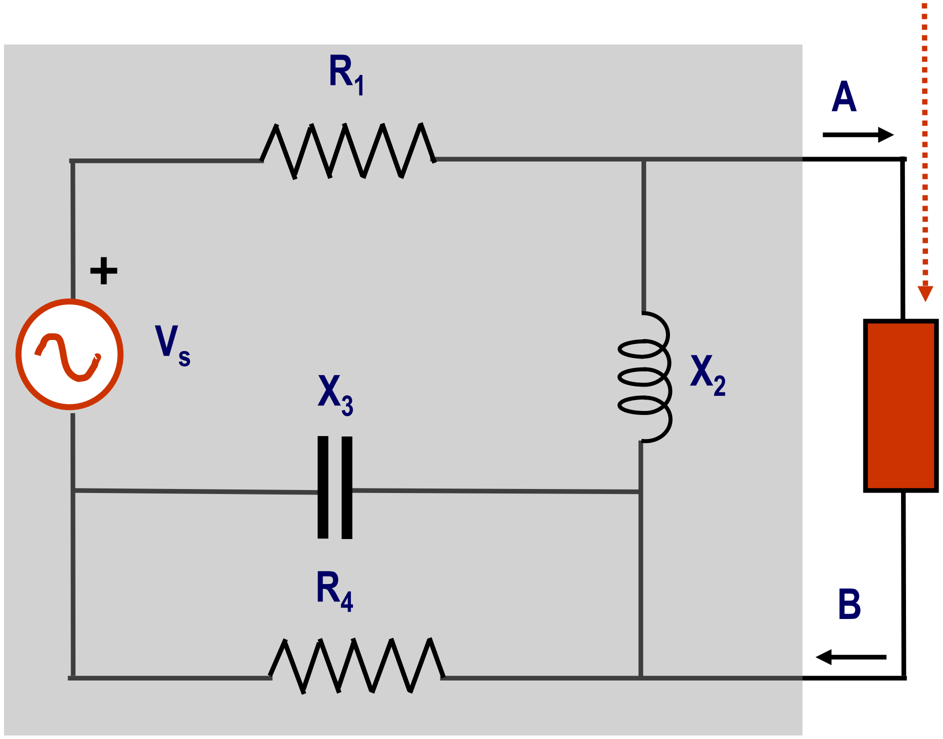


$a = r\cos\theta,\;\; b = r\sin\theta$

$1.299 + j\,0.6870\;\Omega$

Maximum Power Transfer Condition in AC Circuits

Question	Given Circuit
Calculate the value of the impedance Z_L of the load in the AC circuit shown on the RHS, in order to transfer maximum power from source to load	Load Impedance: $Z_L = R_L + j\,X_L$

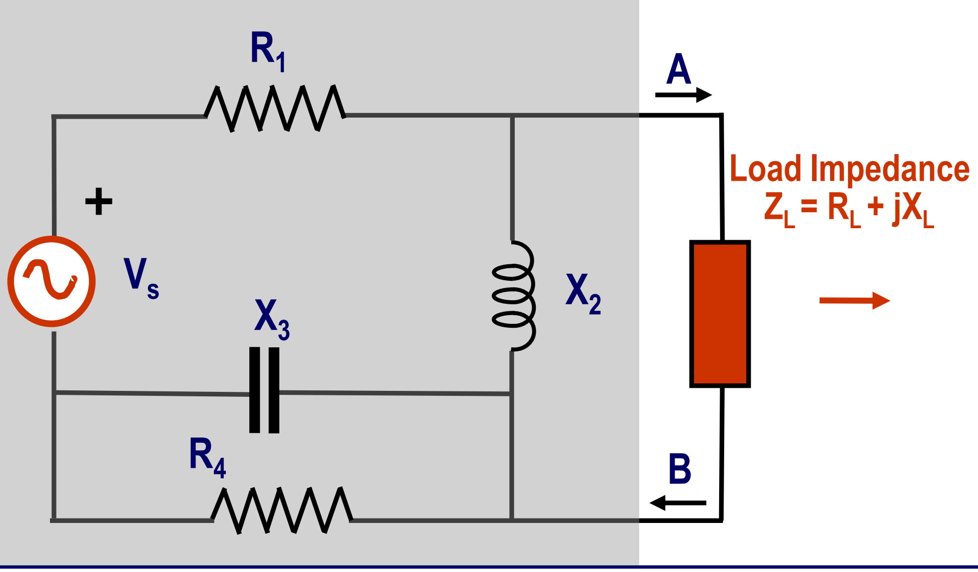


Maximum Power Transfer Condition in AC Circuits

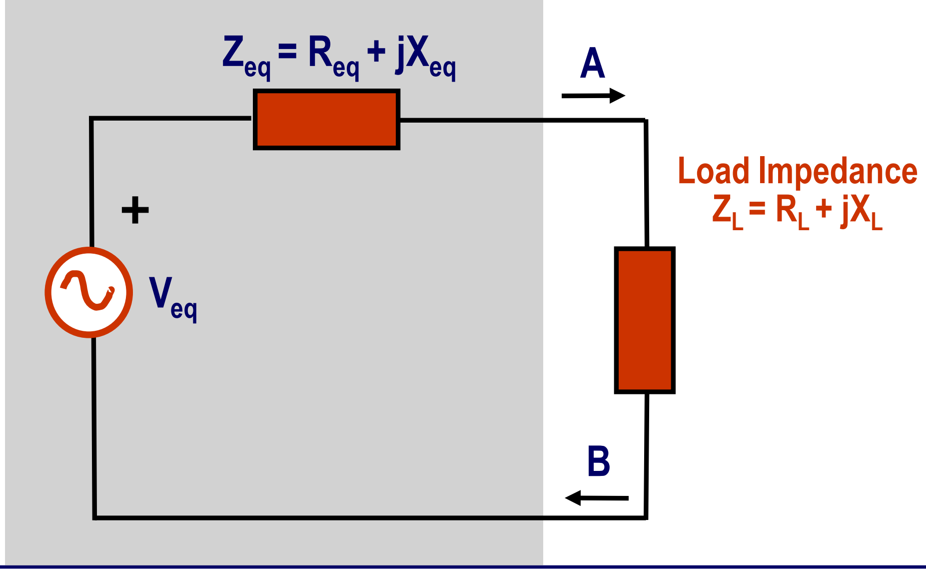
Solution

First simplify the AC circuit to its Thevenin Equivalent Form as shown on the RHS

Given Circuit



Thevenin Equivalent Circuit



Maximum Power Transfer Condition in AC Circuits

Solution (Continued)

Then the problem reduces to the determination of the load impedance in the simplified circuit shown on the RHS

$$P = R_L\, I^2$$

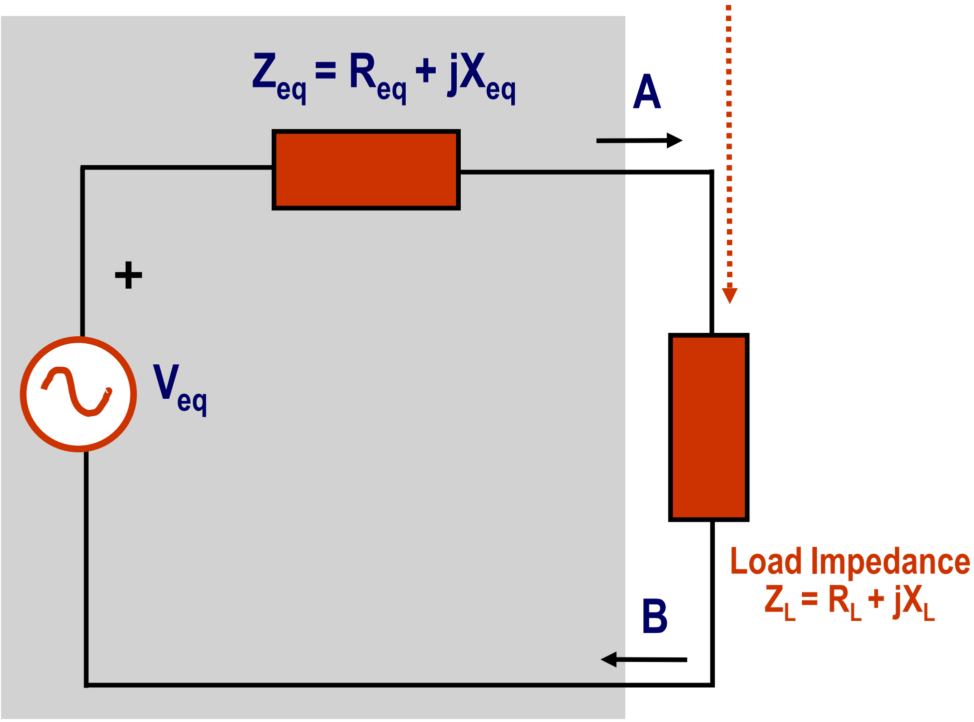
$$I^2 = (\, V_{eq.} / Z_{total})^2 = (\, V_{eq.} / (\, Z_{eq.} + Z_L))^2$$

Hence,

$$P = R_L\ \ V_{eq.}^{\ 2} / (\, Z_{eq.} + Z_L\,)^2$$

$$= V_{eq.}^{\ 2}\, R_L / ((\, R_{eq.} + R_L\,)^2 + (\, X_{eq.} + X_L\,)^2\,)$$

Load Impedance: $Z_L = R_L + jX_L$



Maximum Power Transfer Condition in AC Circuits

Solution (Continued)

Let us now first maximize *P* wrt X_L by differentiating *P* with respect to X_L

$dP / dX_L = 0$

$d/dX_L \; V_{eq.}^2 R_L / ((R_{eq.} + R_L)^2 + (X_{eq.} + X_L)^2) = 0$

or

$V_{eq.}^2 R_L (-2 (X_{eq.} + X_L) / [(R_{eq.} + R_L)^2 + (X_{eq.} + X_L)^2]^2 = 0$

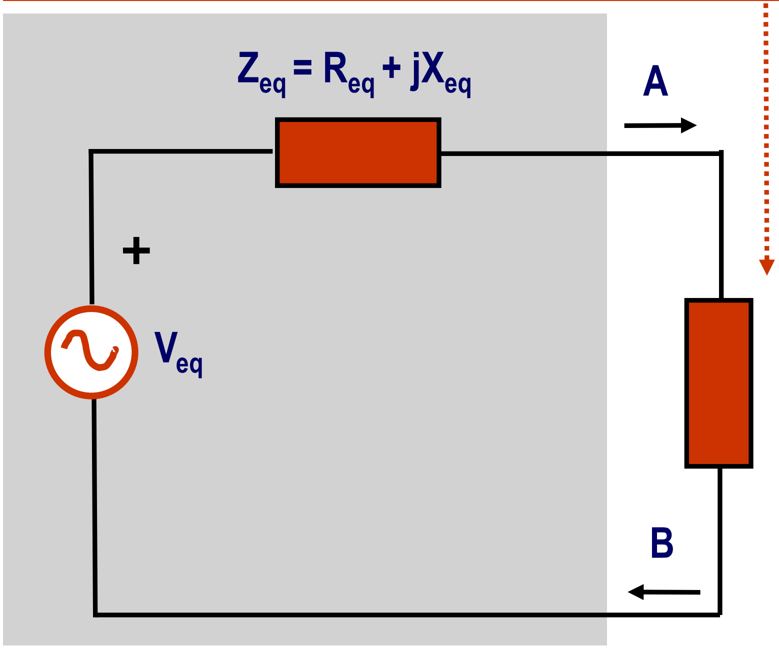
or

$V_{eq.}^2 R_L (-2 (X_{eq.} + X_L) = 0$

The above expression becomes zero when;

$X_{eq.} = - X_L$

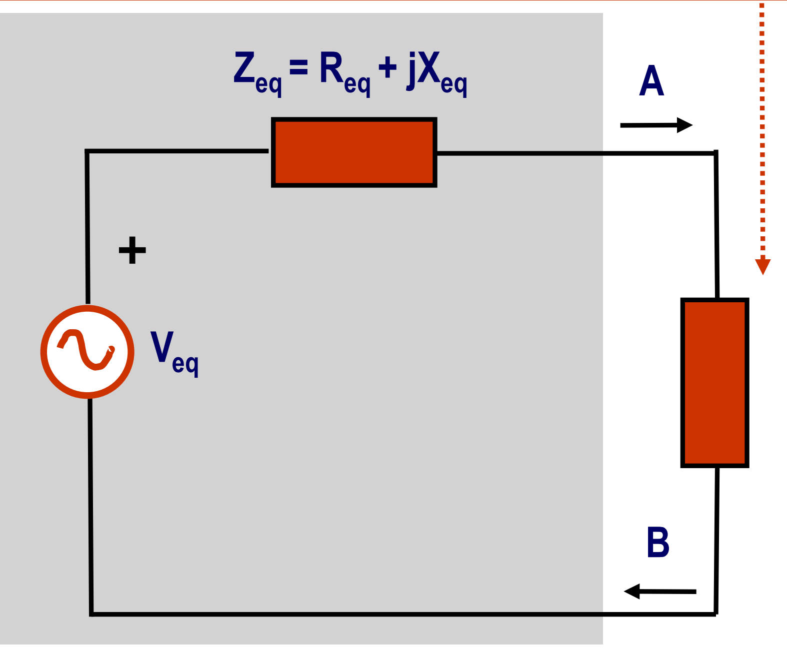
Load Impedance: $Z_L = R_L + jX_L$



Maximum Power Transfer Condition in AC Circuits

Solution (Continued)

Load Impedance: $Z_L = R_L + jX_L$



Hence, power becomes the same as that for the DC case;

$$P = V_{eq.}^{\,2}\, R_L \,/\, ((R_{eq.} + R_L)^2 + (X_{eq.} + X_L)^2)$$

$(X_{eq.} + X_L)^2 = 0$

$$P = R_L \; V_{eq.}^{\,2} \,/\, (R_{eq.} + R_L)^2$$

Maximum Power Transfer Condition in AC Circuits

Solution (Continued)

Now, we must maximize P wrt R_L by differentiating P with respect to R_L

$dP / dR_L = 0$

$d/dR_L \; V_{eq.}^2 R_L / (R_{eq.} + R_L)^2 = 0$

$V_{eq.}^2 [(R_{eq.} + R_L)^2 - 2 (R_{eq.} + R_L) V_{eq.}^2 R_L] / d^2 = 0$

where, $d = (R_{eq.} + R_L)^2$

or

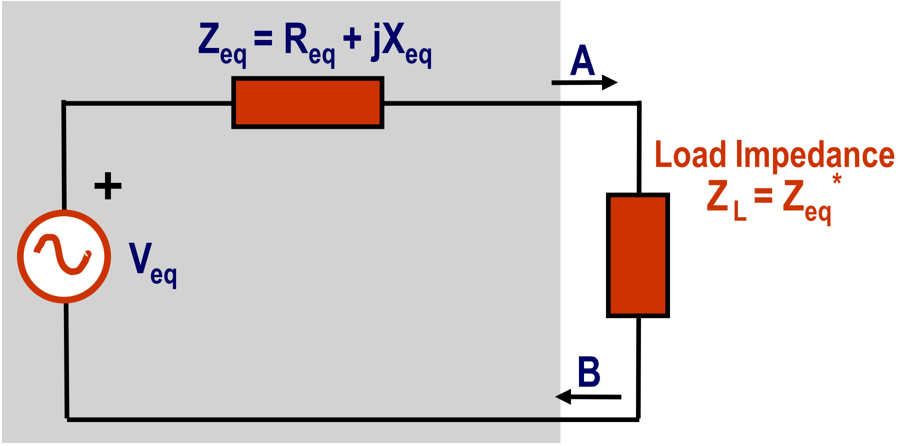
$V_{eq.}^2 [(R_{eq.} + R_L)^2 - 2 (R_{eq.} + R_L) R_L] = 0$

$(R_{eq.} + R_L)^2 - 2 (R_{eq.} + R_L) R_L = 0$

$(R_{eq.} + R_L) - 2 R_L = 0$

$\longrightarrow \; R_{eq.} = R_L$

Thevenin Equivalent Circuit



Conclusions:

For maximum power transfer;

(a) $X_L = - X_{eq}$

(b) Load resistance must be equal to the Thevenin Equivalent Resistance of the simplified circuit; $R_{eq.} = R_L$

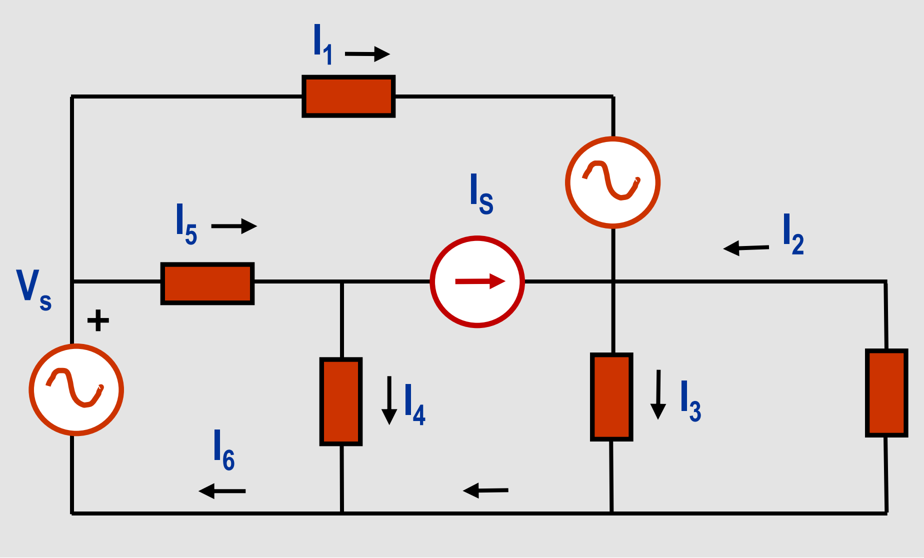
or

(c) $Z_L = Z_{eq}^*$

The Principle of Superposition in AC Circuits

Question

Solve the AC circuit shown on the RHS by using The Principle of Superposition

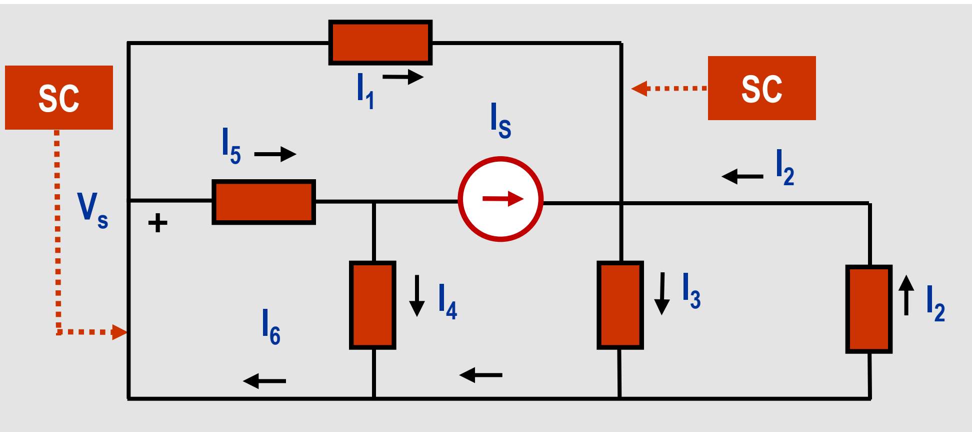


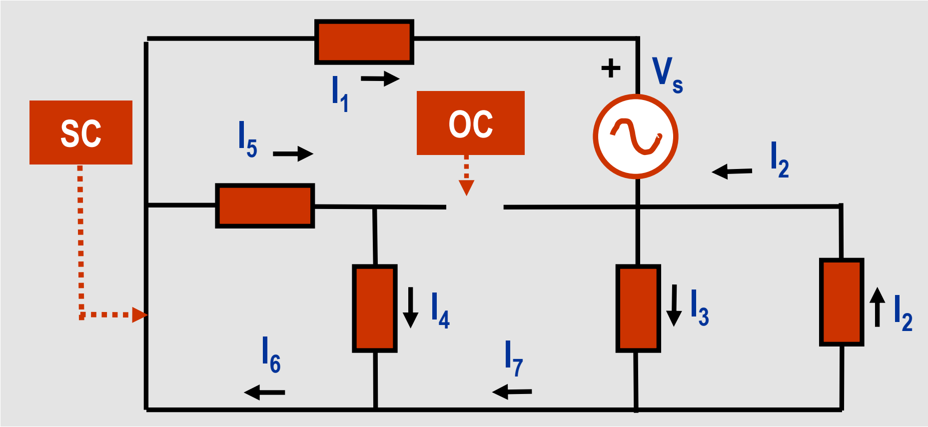
Solution

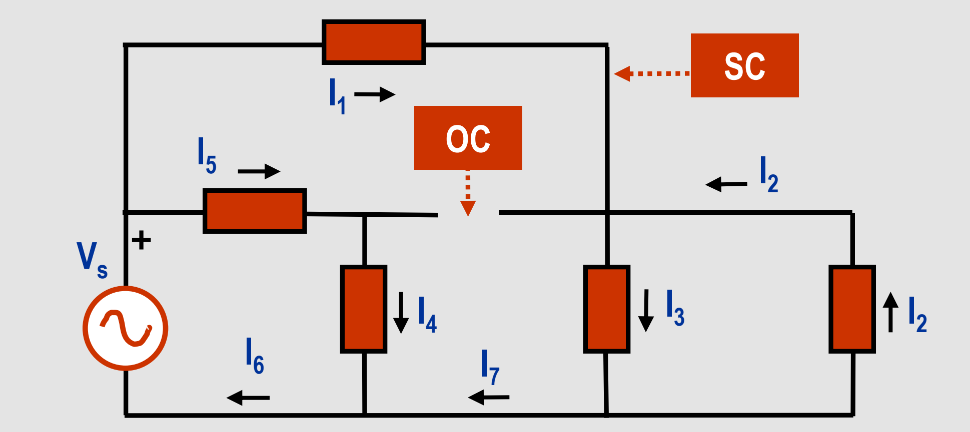
a) Kill all sources, except one,
b) Solve the resulting circuit,
c) Restore back the killed source and kill all sources, except another one,
d) Repeat the solution procedure (a) - (c) for all sources,
e) Then, sum up algebraically all the solutions found

The Principle of Superposition in AC Circuits

Procedure



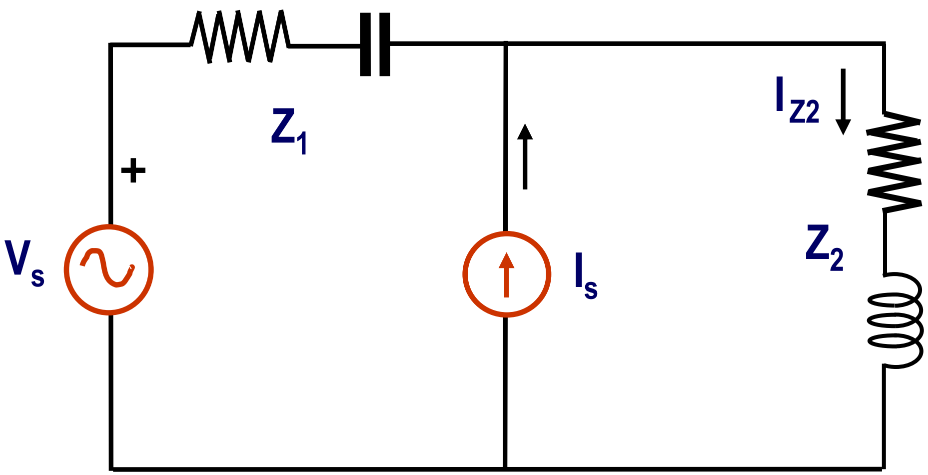




Example 1 - The Principle of Superposition in AC Circuits

Question

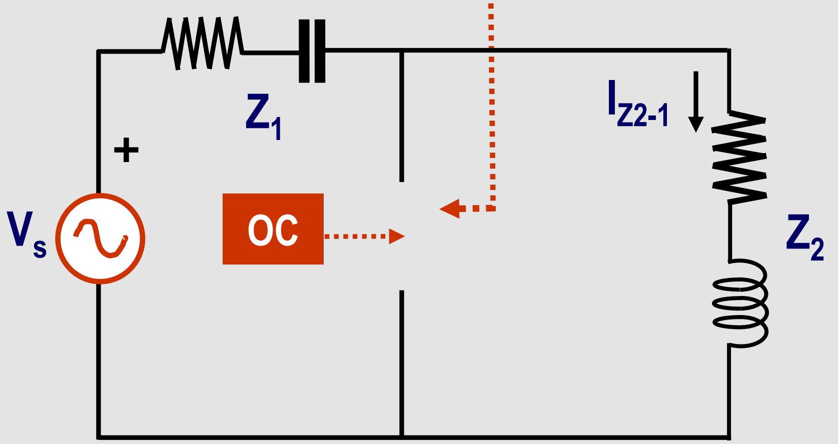
Find the current I_{Z2} flowing in impedance Z_2 in the following circuit by using the Principle of Superposition



This method is particularly useful when there are sources with different frequencies

Solution

Kill all sources except one and solve the resulting circuit



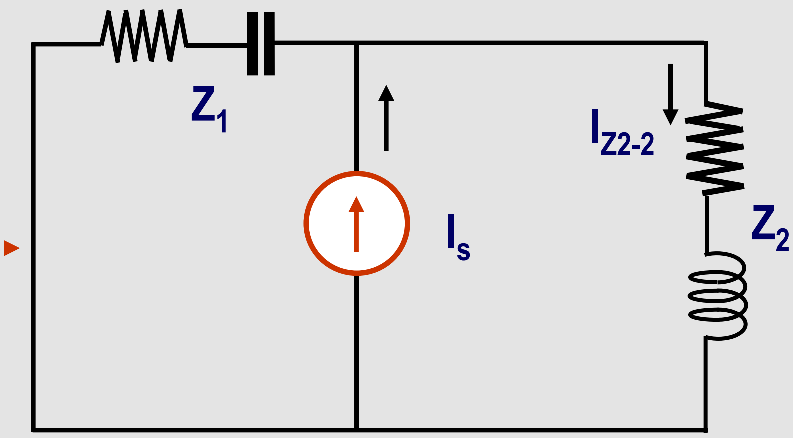
$$I_{Z2-1} = V_s / (Z_1 + Z_2)$$

Example 1 - The Principle of Superposition in AC Circuits

Solution

Kill all sources except one, sequentially and solve the resulting circuits

SC



$$I_{Z2-2} = (I_S / Z_2) / [(1 / Z_1) + (1 / Z_2)]$$

$$= I_S \, g_2 / (g_1 + g_2)$$

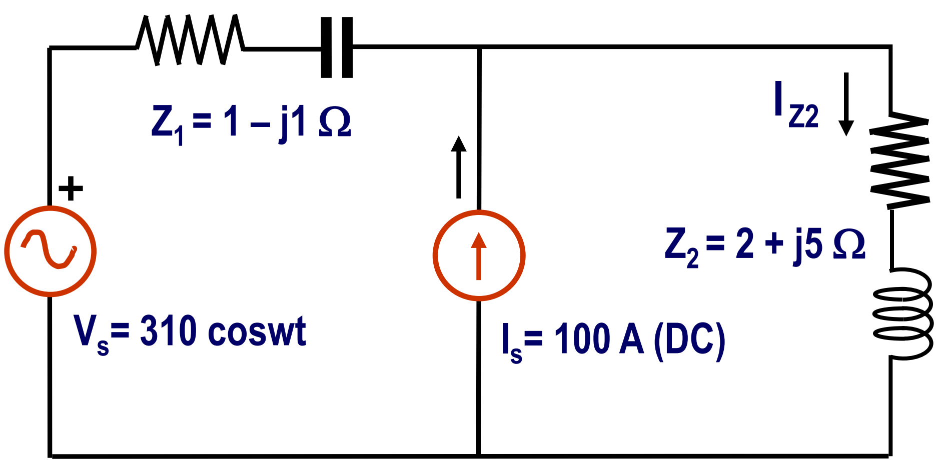
Sum up the resulting currents

$$I_{Z2} = I_{Z2-1} + I_{Z2-2}$$

Example 2 - Sources with Mixed Frequencies

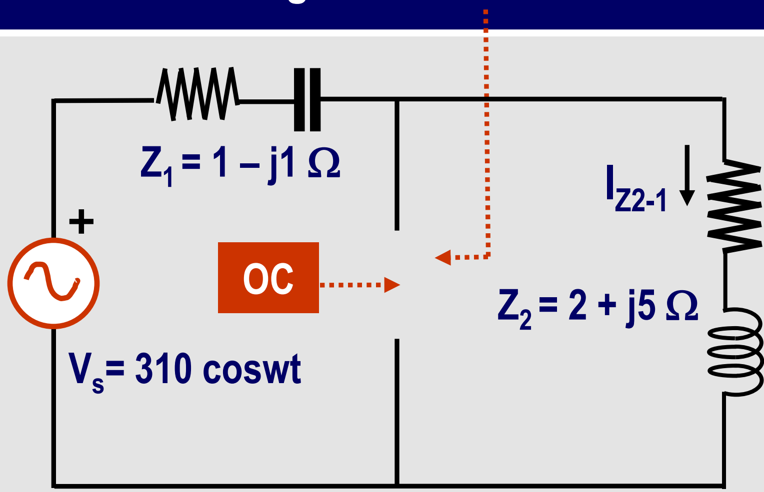
Question

Now, find the steady-state current waveform flowing in impedance Z_2 in the following circuit by using the Principle of Superposition



This method is particularly useful when there are sources with different frequencies

Kill all sources except one, sequentially and solve the resulting circuits



$$I_{Z2-1} = V_s \angle 0^o \ / \ (Z_1 + Z_2)$$

$$= V_s \angle 0^o \ / \ (1 - j1 + 2 + j5)$$

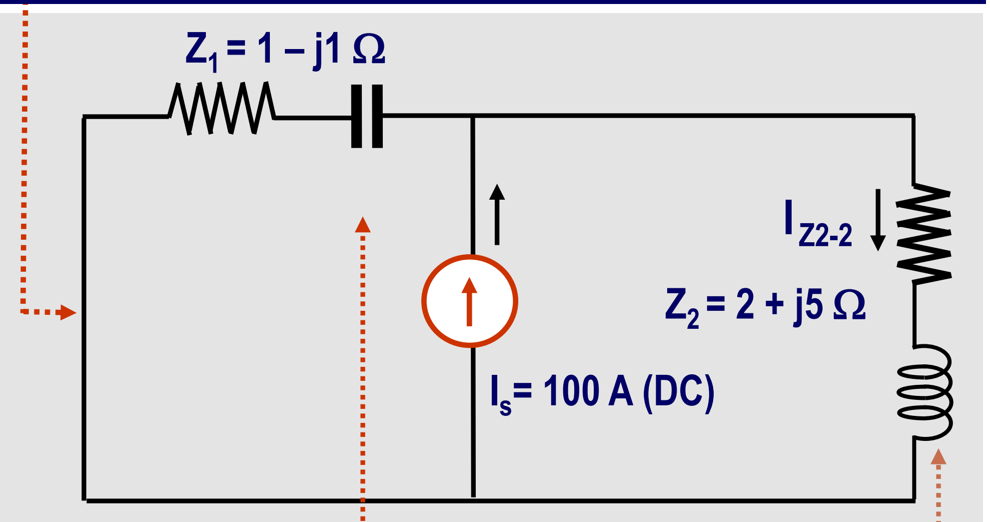
$$= V_s \angle 0^o \ / \ (3 + j4) = V_s \angle 0^o \ / \ 5 \angle 53.13^o$$

$$= 310 / 5 \angle -53.13^o = 62 \angle -53.13^o \ \text{Amp}$$

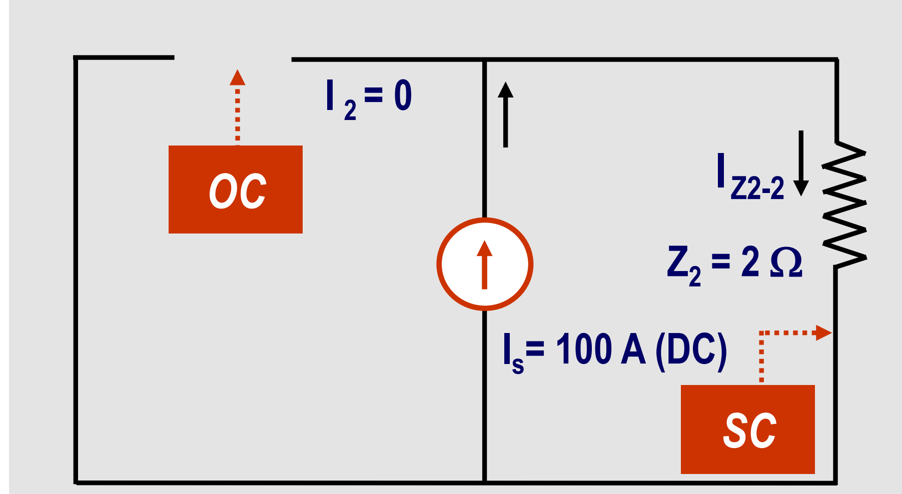
Example 2 - Sources with Mixed Frequencies

Solution

Kill all the sources except one, sequentially and solve the resulting circuits



Please note that the capacitor and inductor in the above circuit respond to DC current source as OC and SC, respectively



$I_2 = 100$ Amp (DC)

Sum up the resulting currents

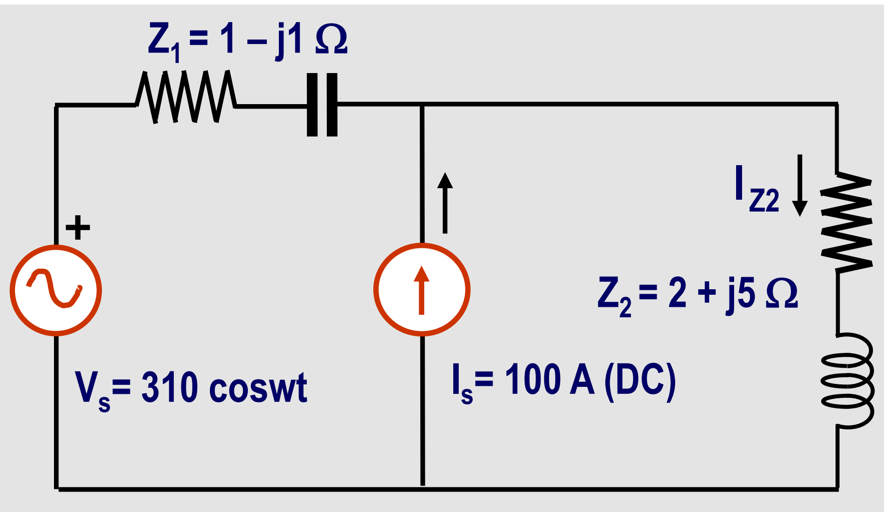
$$I_{Z2} = I_{Z2-1} + I_{Z2-2}$$

$$I_{Z2} = 62\angle{-53.13^o} + 100 \text{ Amp (DC)}$$

Example 2 - Sources with Mixed Frequencies

Waveforms

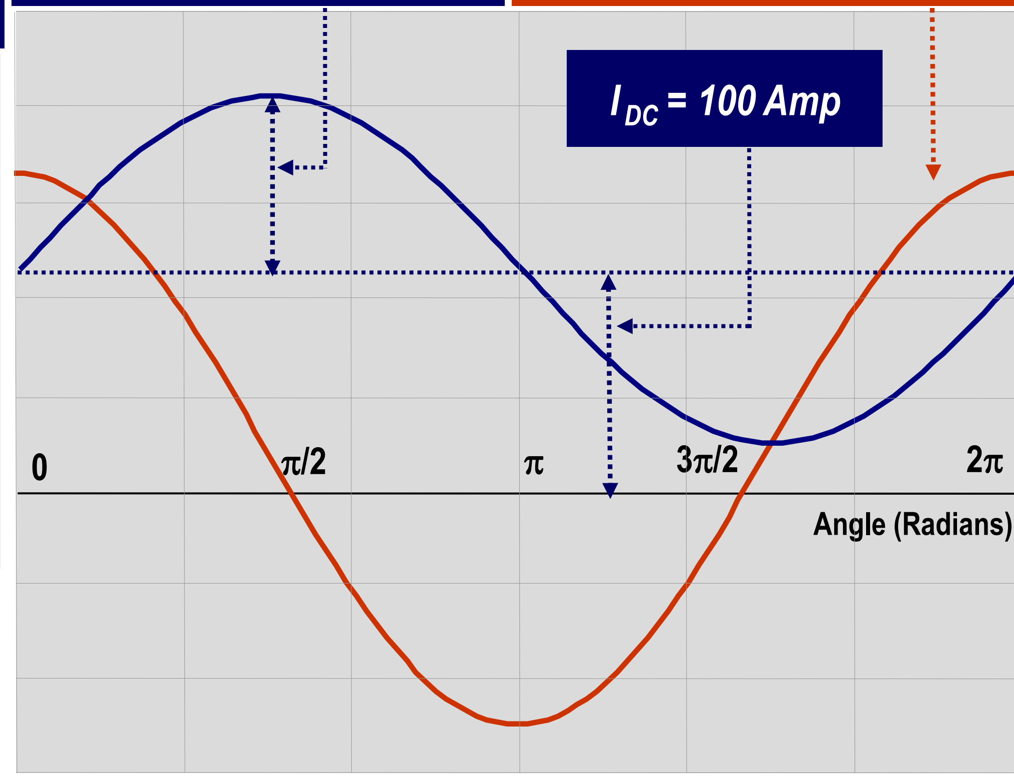
Let us now draw the resulting current and voltage waveforms



$I_{Z2} = 62\angle{-53.13^o} \; + 100 \text{ Amp (DC)}$

$\hat{I} = 62 \text{ Amp}$

$V_{s1}(t) = 310 \cos wt$



Phasors

Any questions please ...