



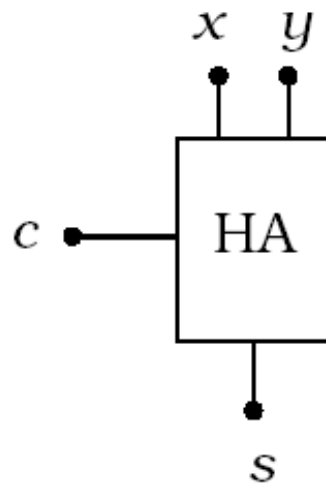
ARITHMETIC CIRCUITS IN CMOS VLSI



**Faculty of Engineering
Alexandria University**



Adders

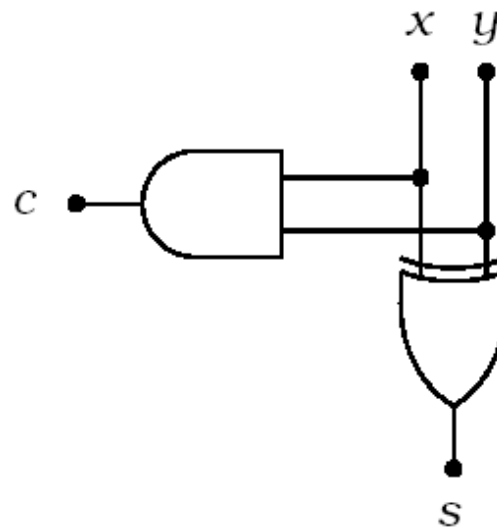


x	y	s	c
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Half-adder symbol and operation.



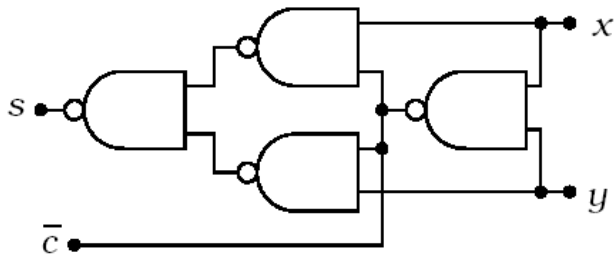
Adders (2)



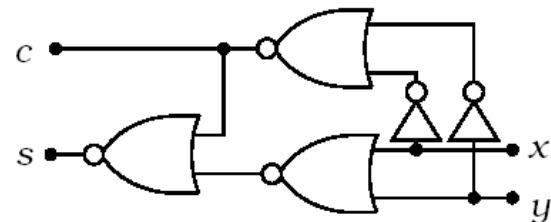
Half-adder logic diagram.



Adders (3)



(a) NAND2 logic

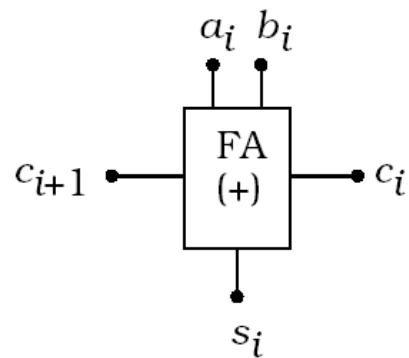


(b) NOR-based network

Alternate half-adder logic networks.



Adders (4)

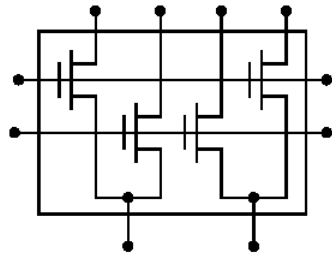


a_i	b_i	c_i	s_i	c_{i+1}
0	0	0	0	0
0	1	0	1	0
1	0	0	1	0
1	1	0	0	1
0	0	1	1	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	1

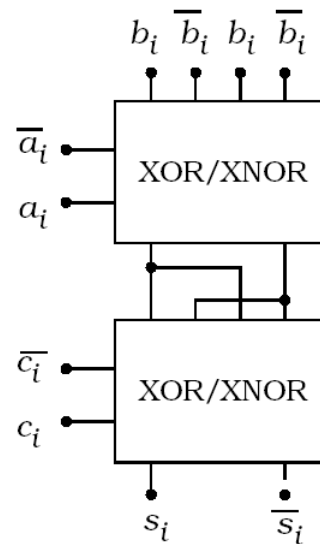
Full-adder symbol and function table.



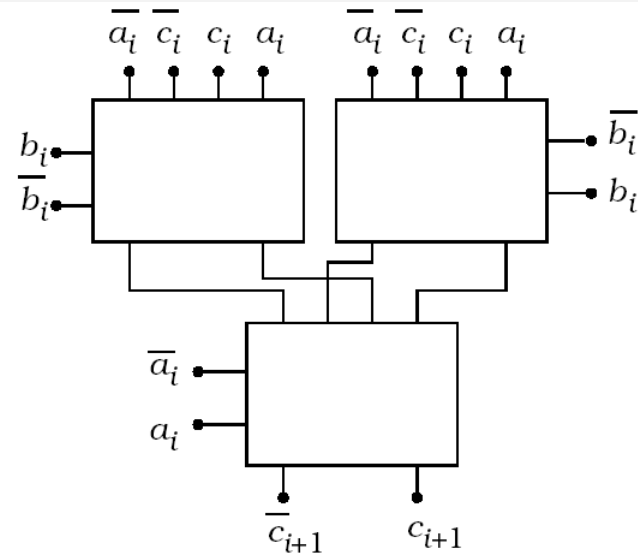
Adders (5)



(a) 2-input array



(b) Sum circuit

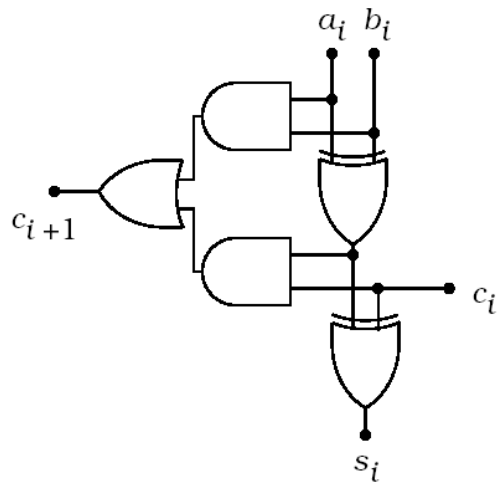


(c) Carry circuit

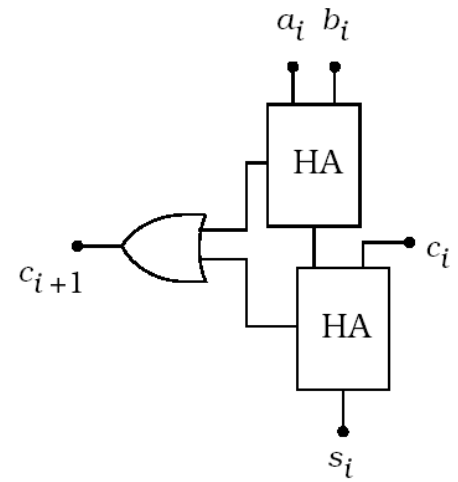
CPL full-adder design.



Adders (6)



(a) Gate-level logic

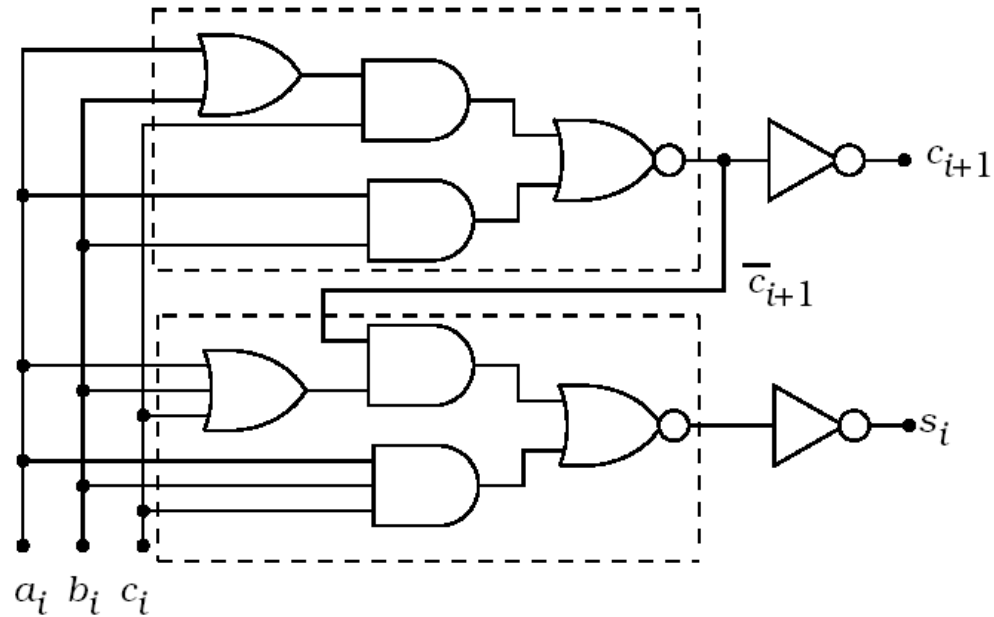


(b) HA-based design

Full-adder logic networks.



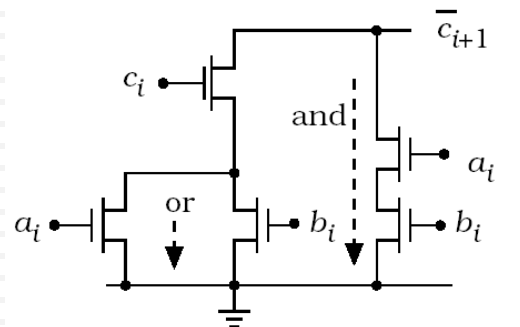
Adders (7)



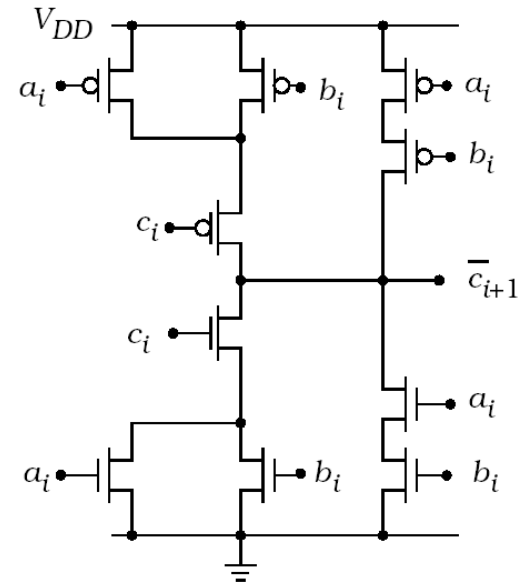
AOI full-adder logic.



Adders (8)



(a) Standard nFET logic

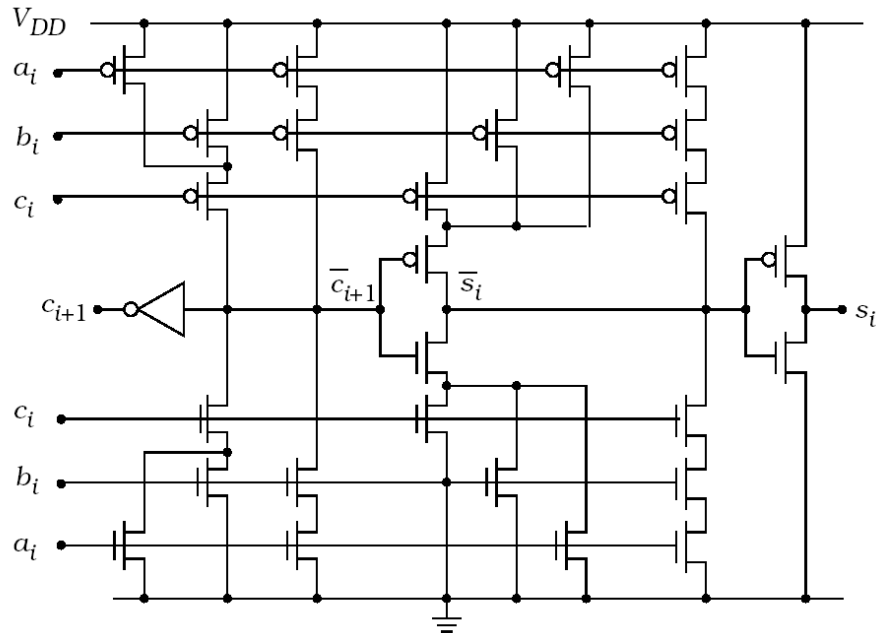


(b) Mirror circuit

Evolution of carry-out circuit.



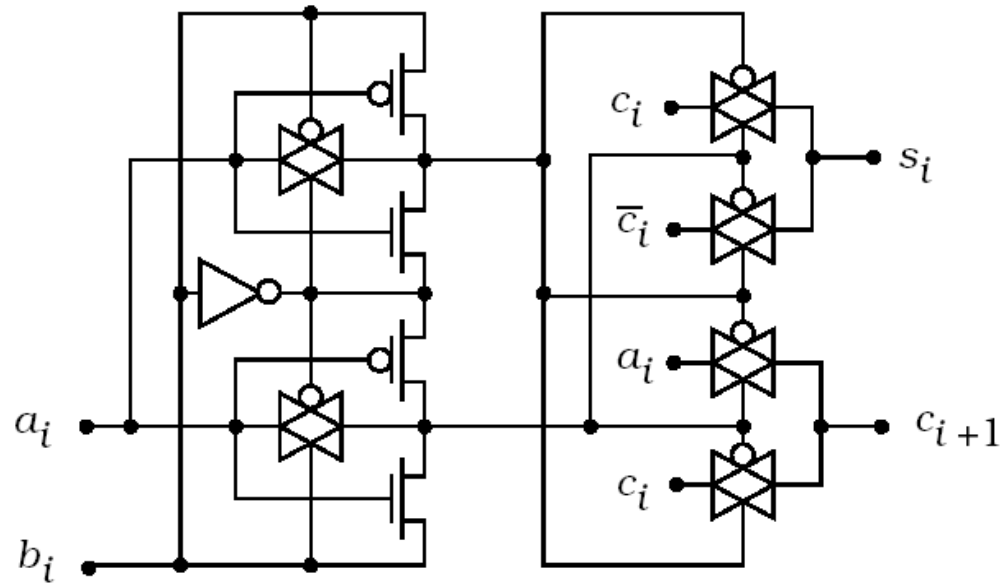
Adders (9)



Mirror AOI CMOS full-adder.



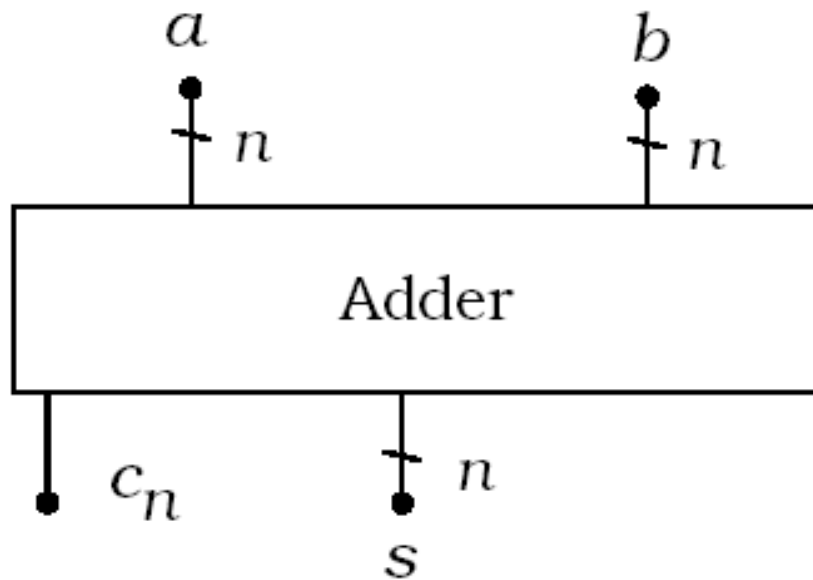
Adders (10)



Transmission-gate full-adder circuit.



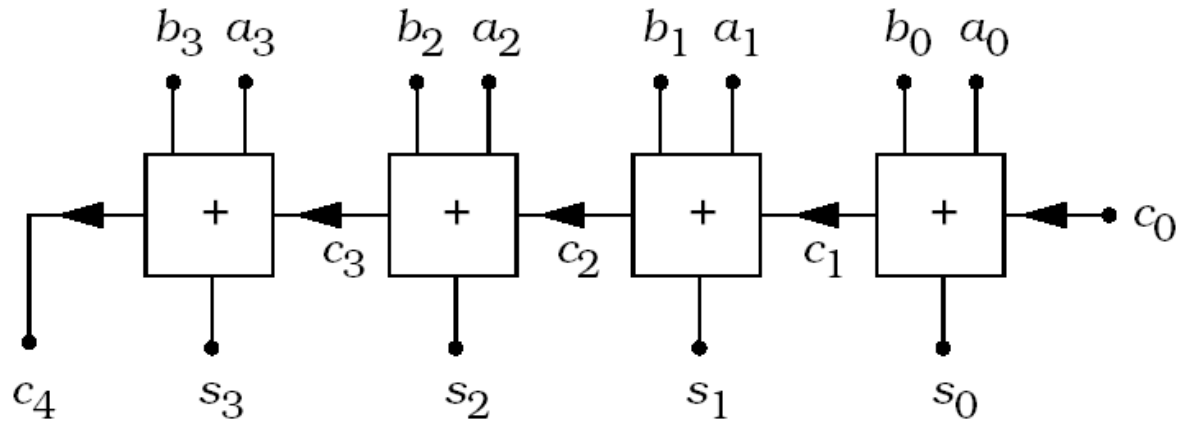
Adders (II)



An n -bit adder.



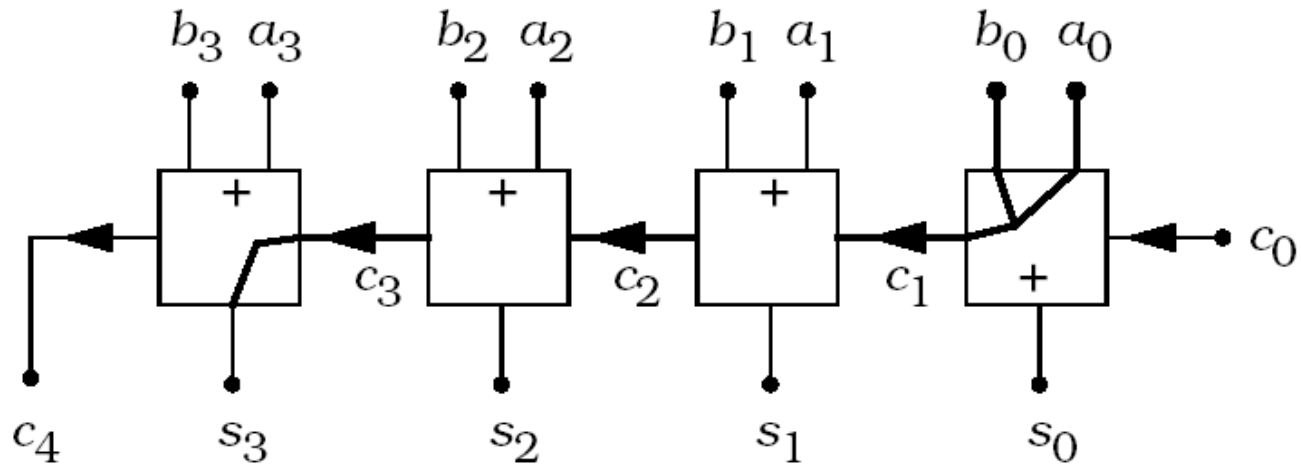
Adders (12)



A 4-bit ripple-carry adder.



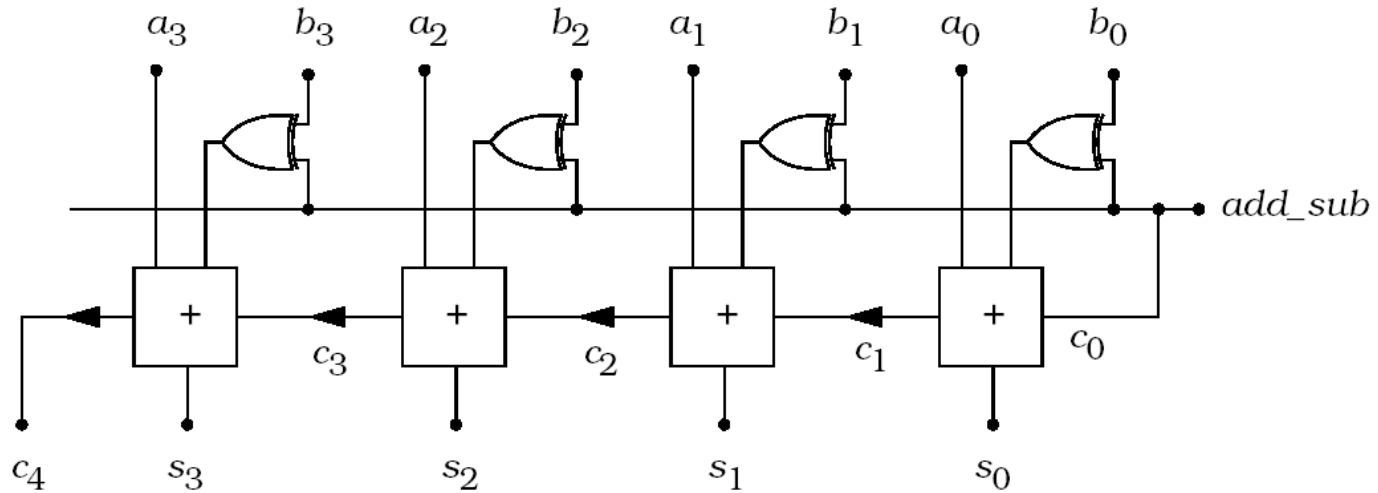
Adders (13)



Worst-case delay through the 4-bit ripple adder.



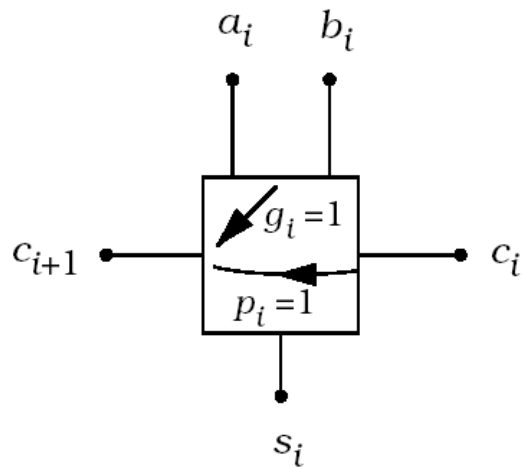
Adders (14)



4-bit adder-subtractor circuit.



Adders (15)



	g_i	P_i
	$a_i \cdot b_i$	$a_i \oplus b_i$
$a_i = b_i = 0$	0	0
$a_i = b_i = 1$	1	0
$a_i \neq b_i$	0	1

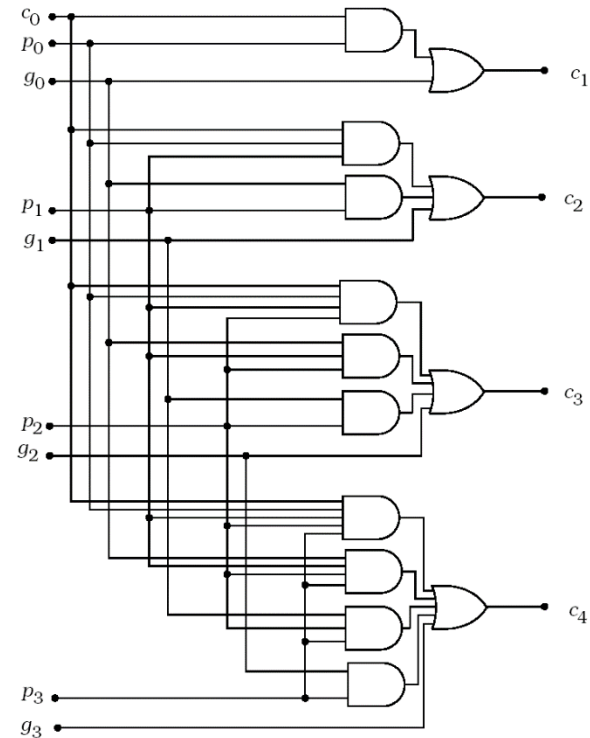
$$c_{i+1} = a_i \cdot b_i + c_i \cdot (a_i \oplus b_i)$$

A basis of the carry look-ahead algorithm.



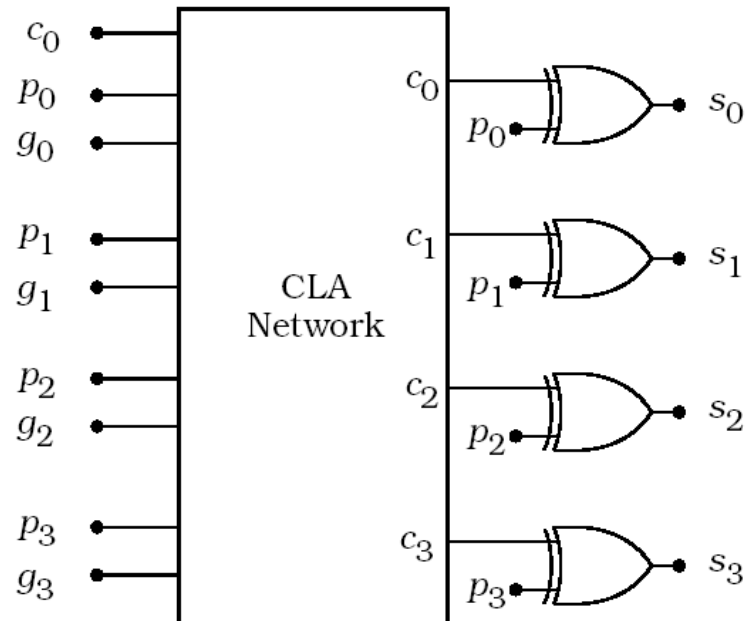
Adders (16)

Logic network for 4-bit CLA carry I





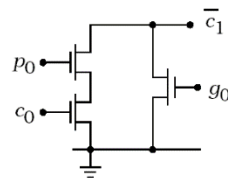
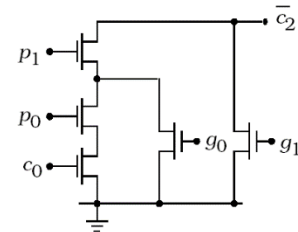
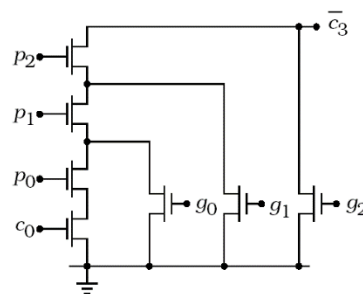
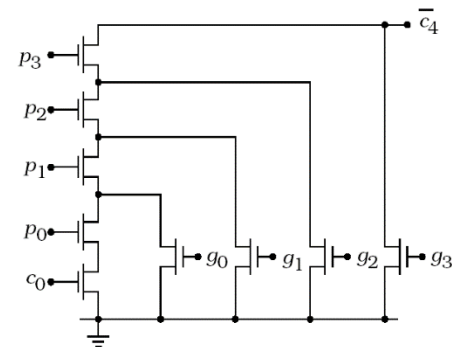
Adders (17)



Sum calculation using the CLA network.



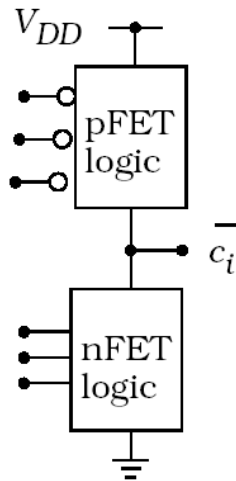
Adders (18)

(a) c_1 logic(b) c_2 logic(c) c_3 logic(d) c_4 logic

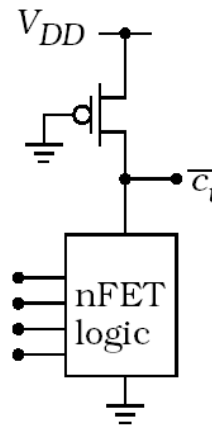
nFET logic arrays for the CLA terms.



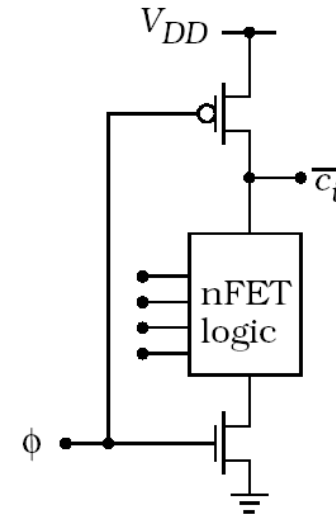
Adders (19)



(a) Complementary



(b) Pseudo nMOS

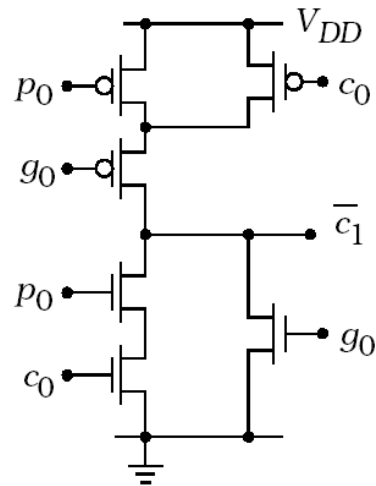


(c) Dynamic

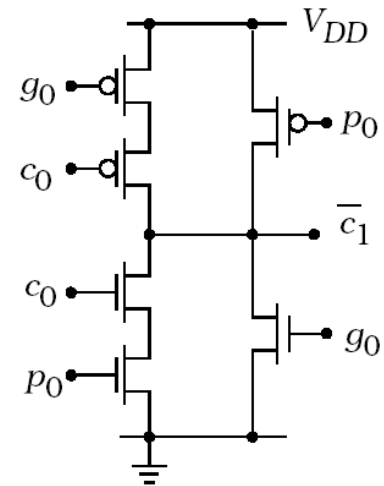
Possible uses of the nFET logic arrays in Figure 12.18.



Adders (20)



(a) Series-parallel circuit



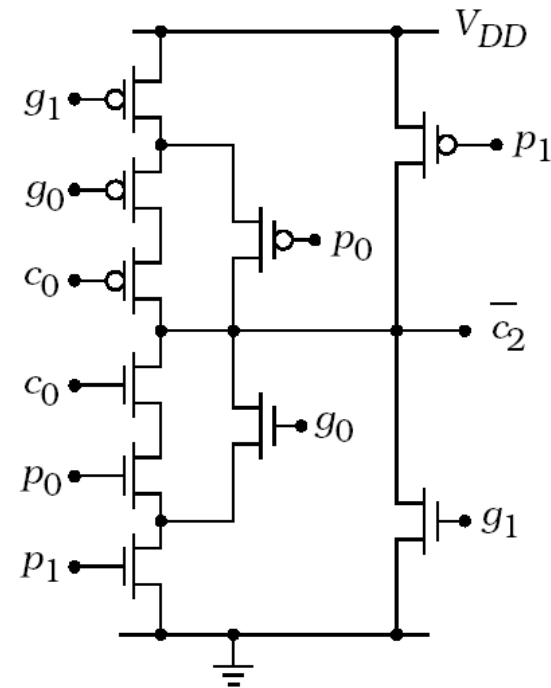
(b) Mirror equivalent

Static CLA mirror circuit.



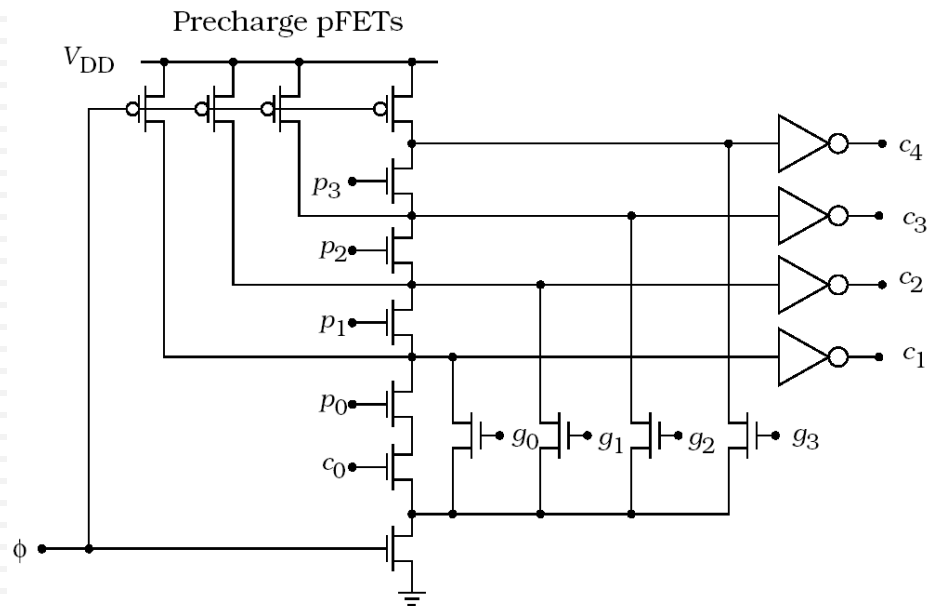
Adders (21)

Static mirror circuit for c_2 .





Adders (22)



MODL carry circuit.



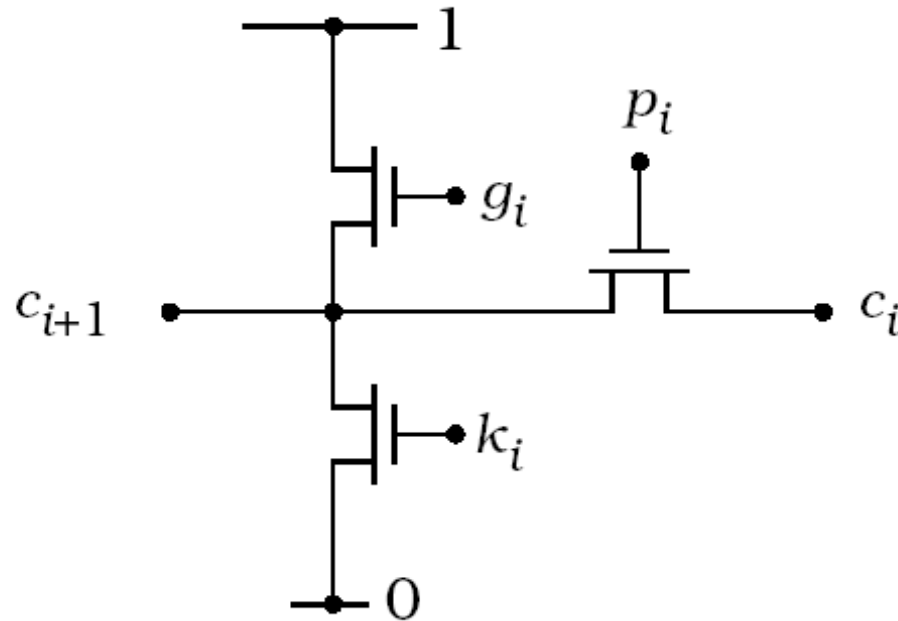
Adders (23)

a_i	b_i	p_i	g_i	k_i
0	0	0	0	1
0	1	1	0	0
1	0	1	0	0
1	1	0	1	0

Propagate, generate, and carry-kill values



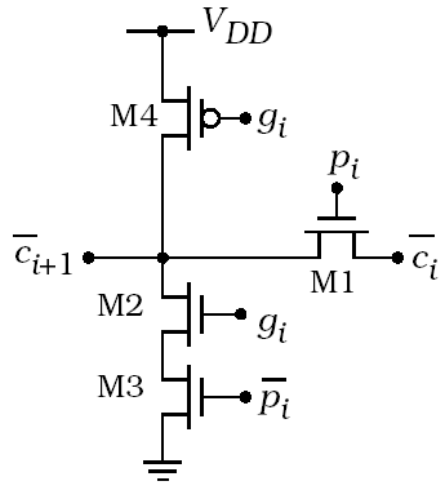
Adders (24)



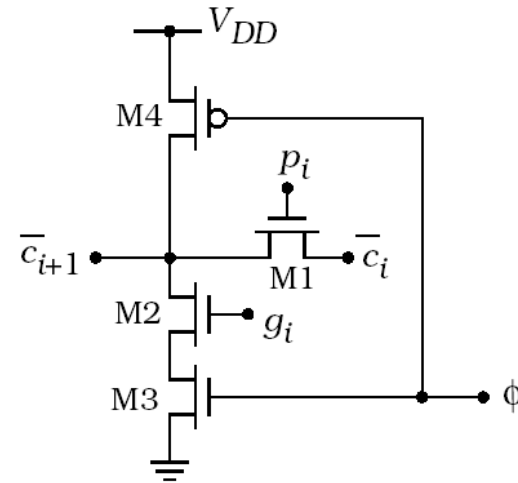
Switching network for the carry-out equation.



Adders (25)



(a) Static circuit

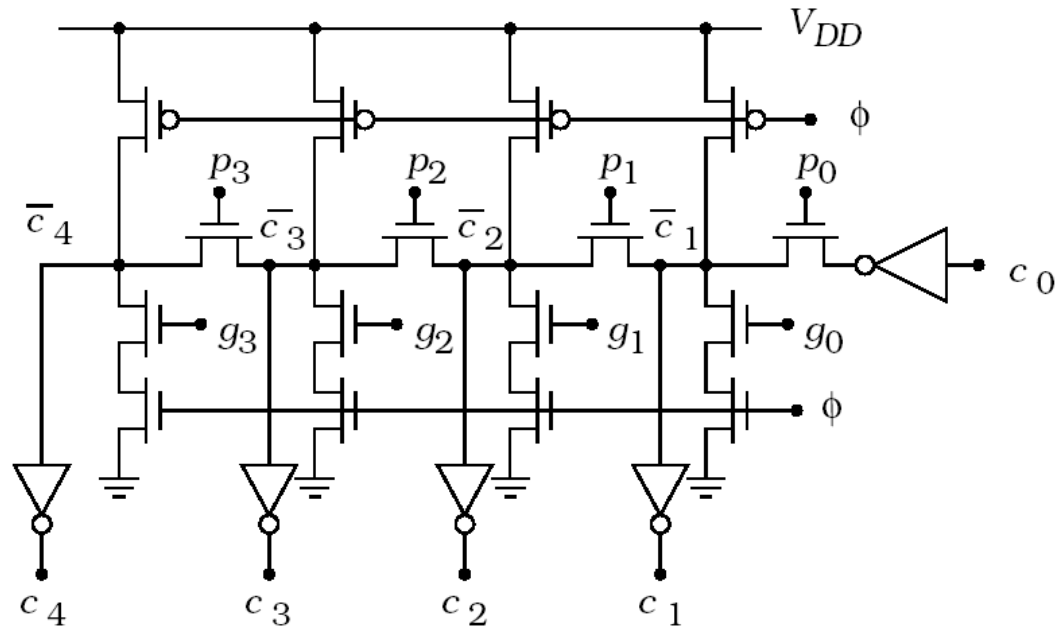


(b) Dynamic circuit

Manchester circuit styles.



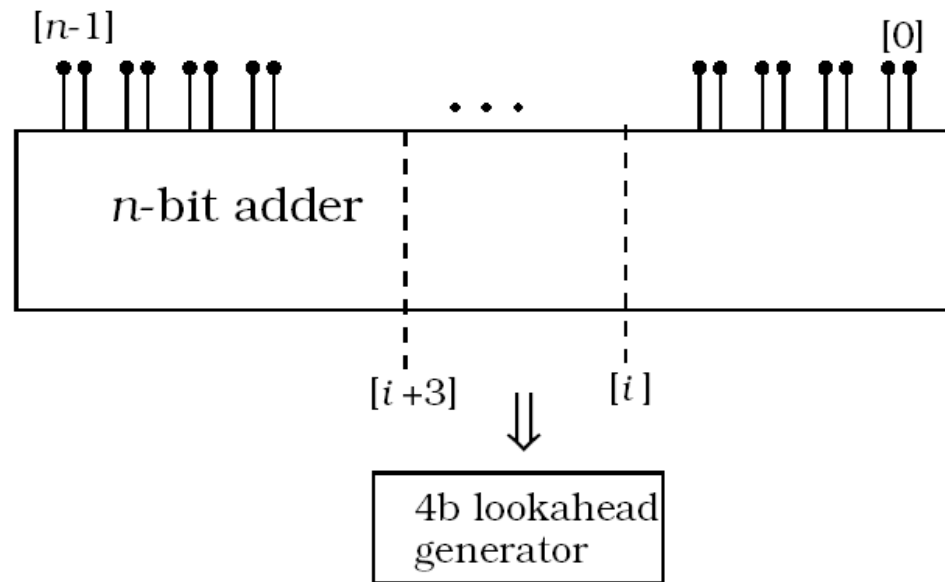
Adders (26)



Dynamic Manchester carry chain.



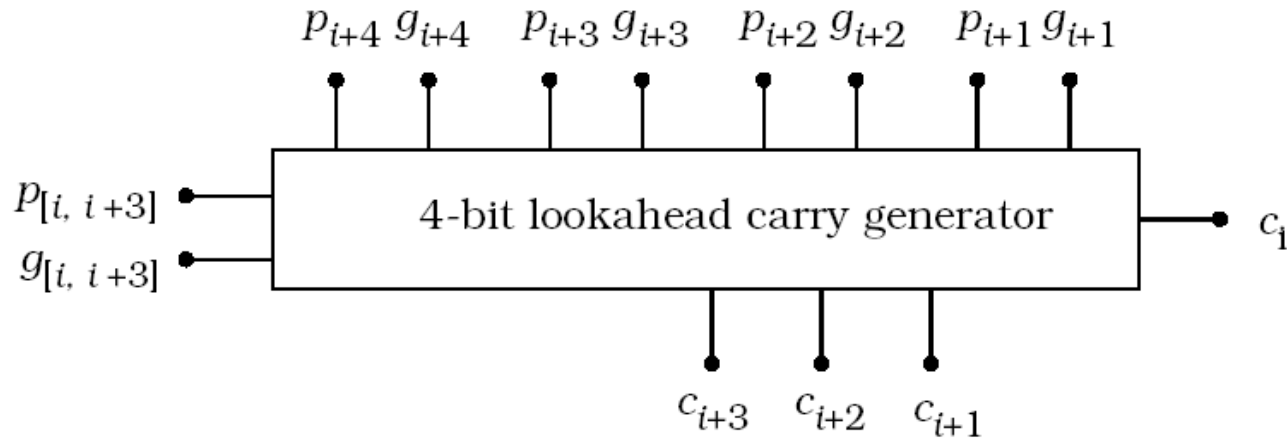
Adders (27)



An n -bit adder network.



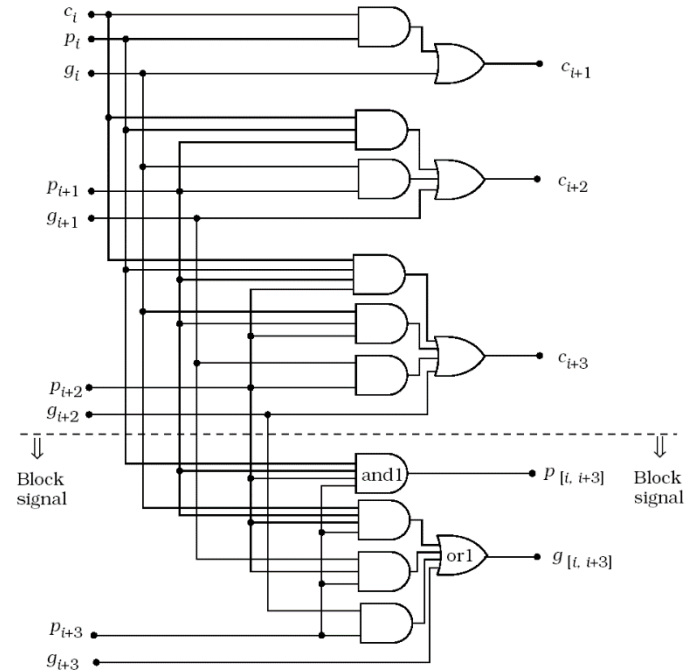
Adders (28)



4-bit lookahead carry generator signals.



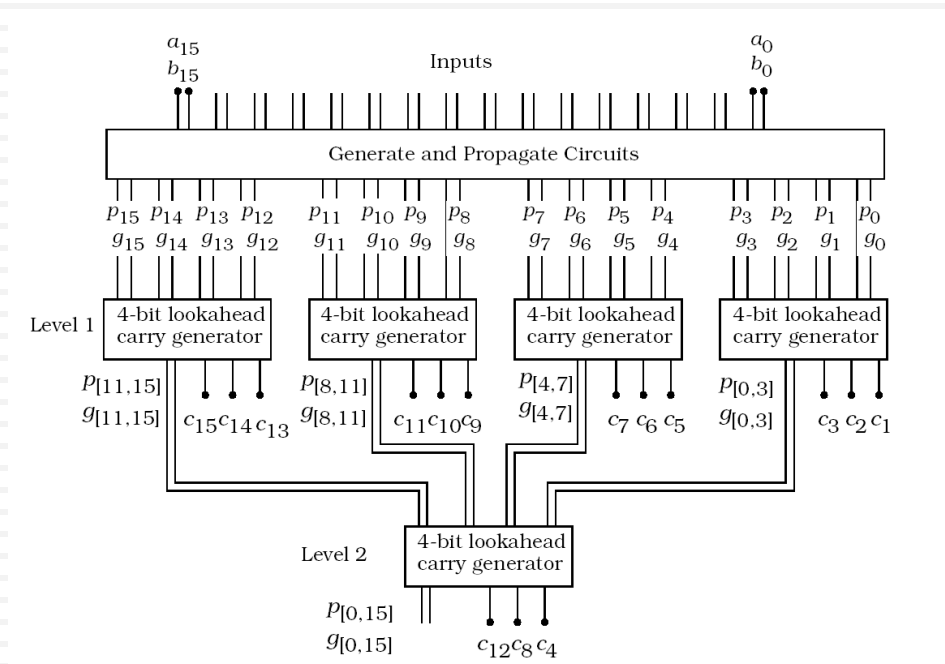
Adders (29)



Block lookahead generator logic.



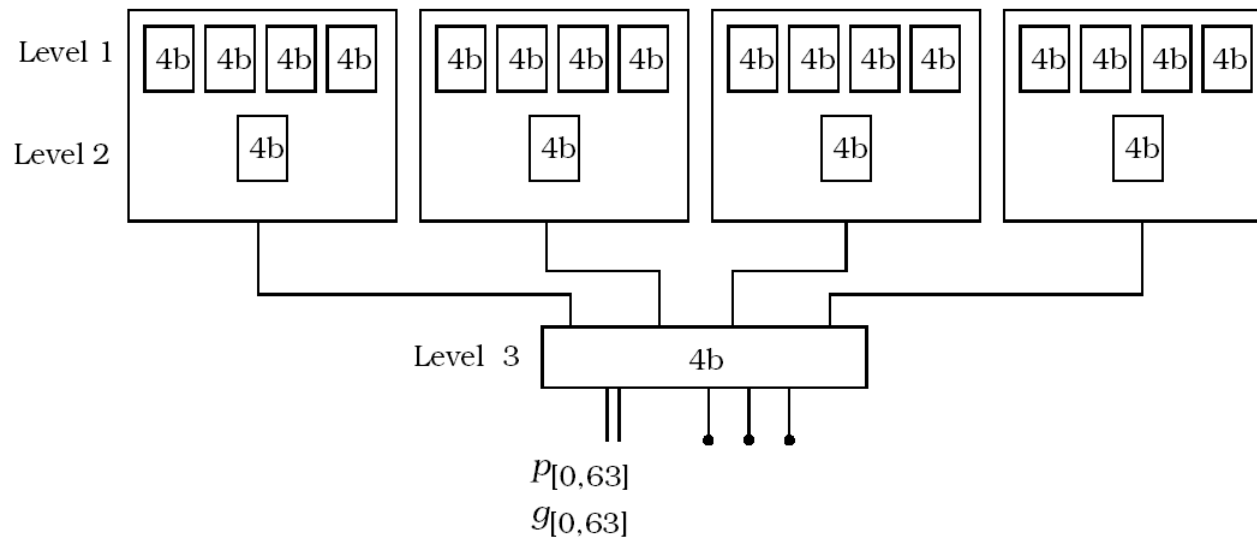
Adders (30)



Multilevel CLA block scheme for a 16-bit adder.



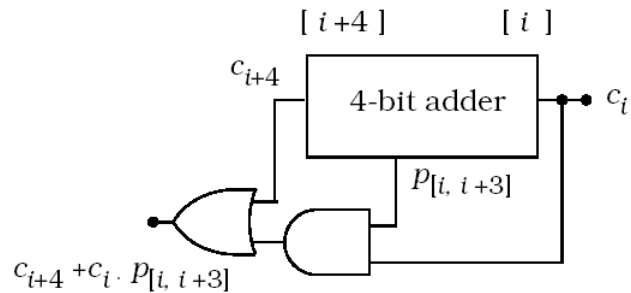
Adders (31)



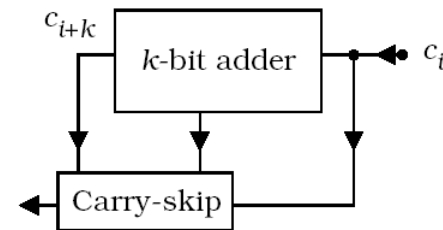
64-bit CLA adder architecture.



Adders (32)



(a) Carry-skip logic

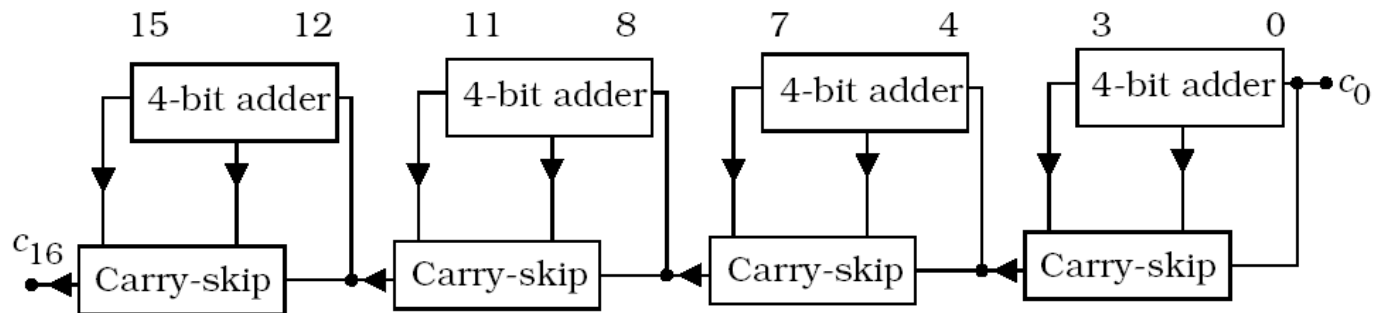


(b) Generalization

Carry-skip circuitry.



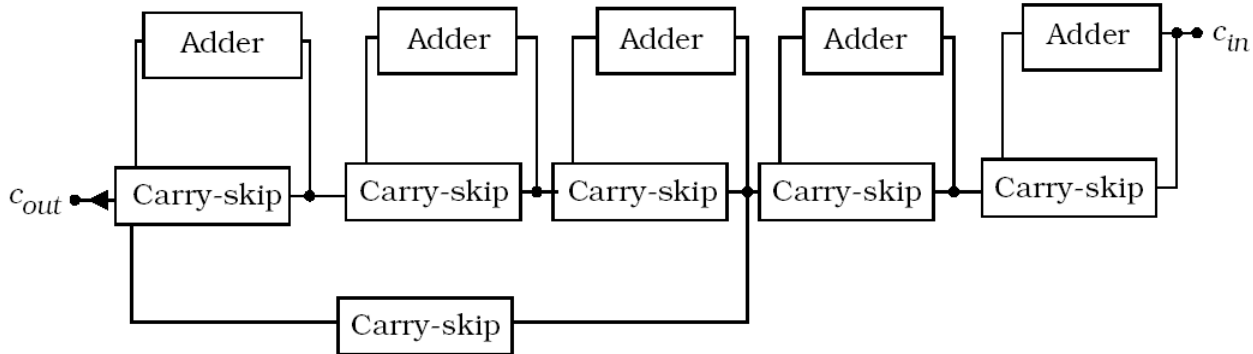
Adders (33)



A 16-bit adder using carry-skip circuits.



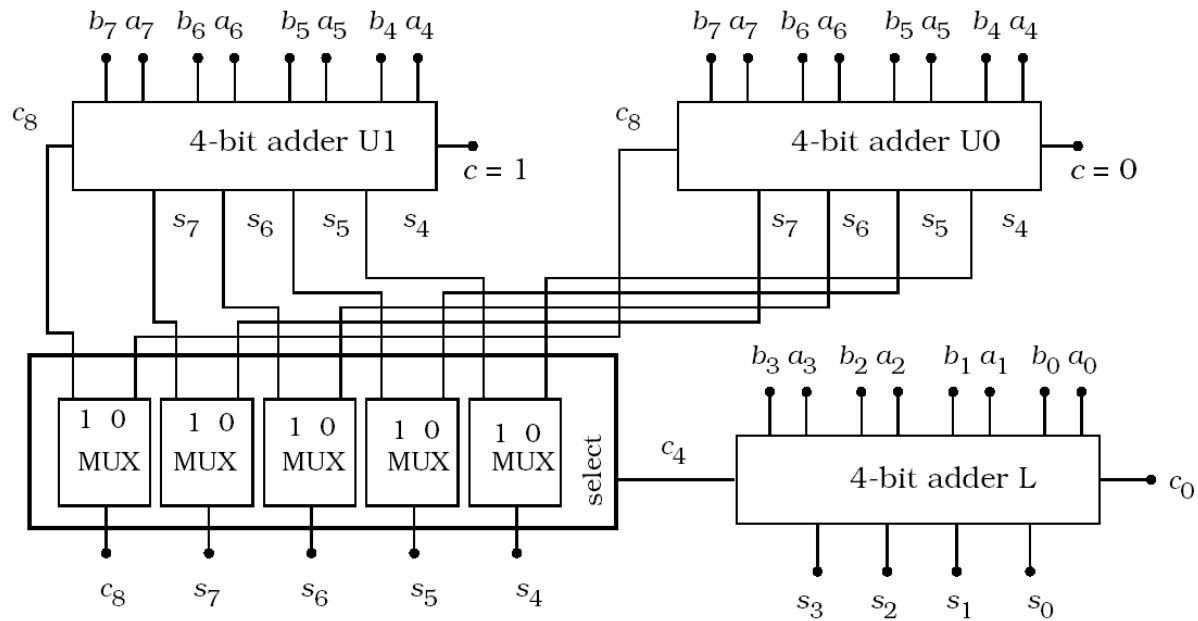
Adders (34)



A 2-level carry-skip adder.



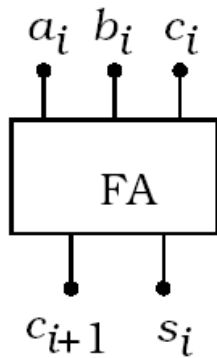
Adders (35)



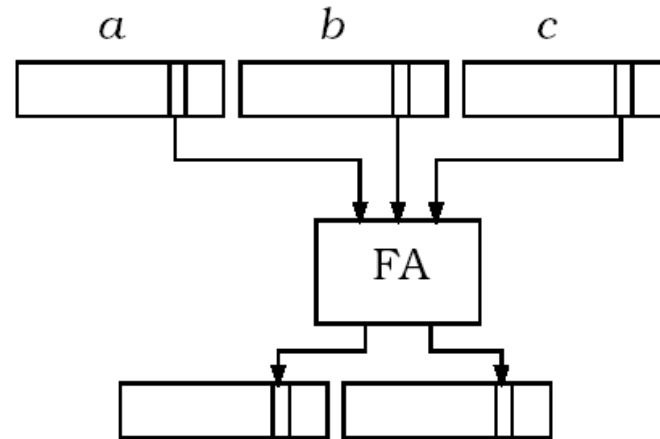
A8-bit carry-select adder.



Adders (35)



(a) Symbol

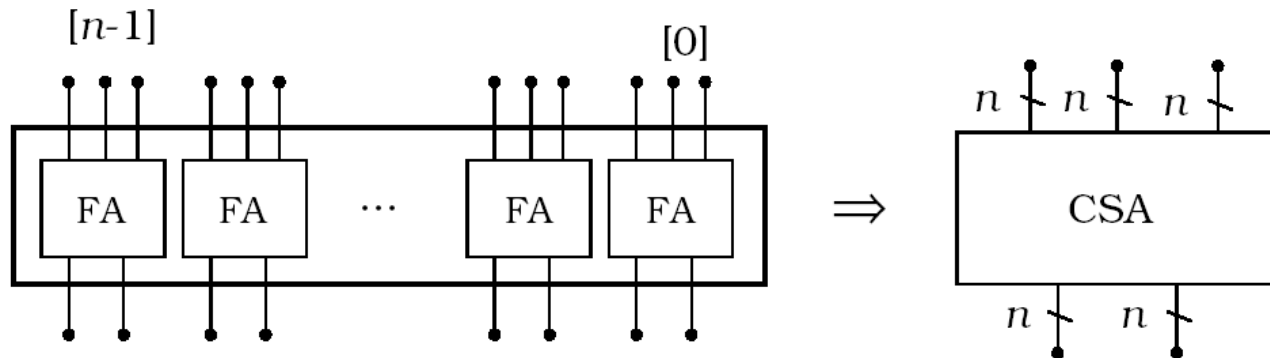


(b) 3-to-2 reduction

Basis of a carry-save adder.



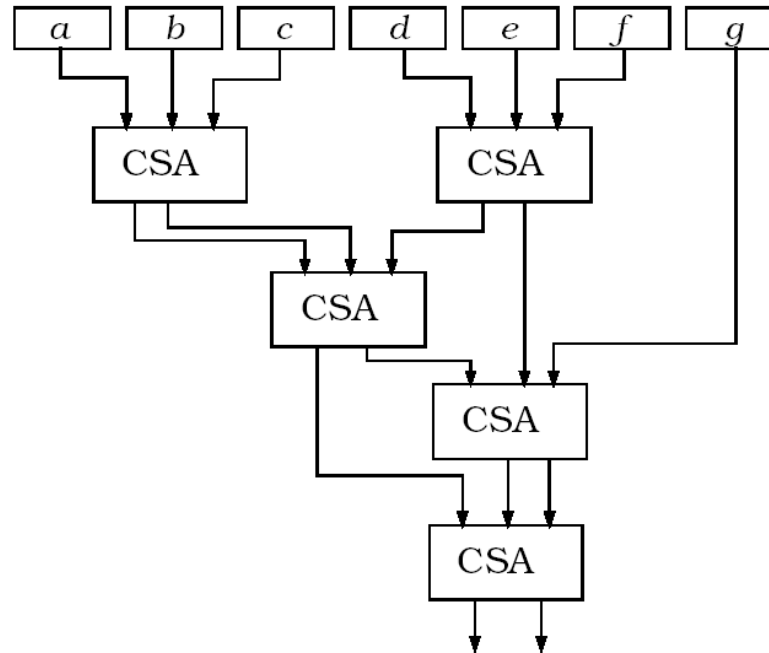
Adders (36)



Creation of an n -bit carry-save adder.



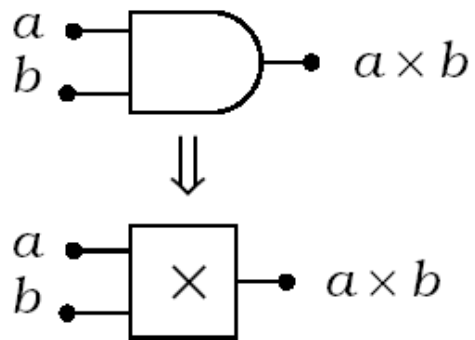
Adders (37)



A 7-to-12 reduction using carry-save adders.



Multipliers



a	b	$a \times b$
0	0	0
0	1	0
1	0	0
1	1	1

Bit-level multiplier.



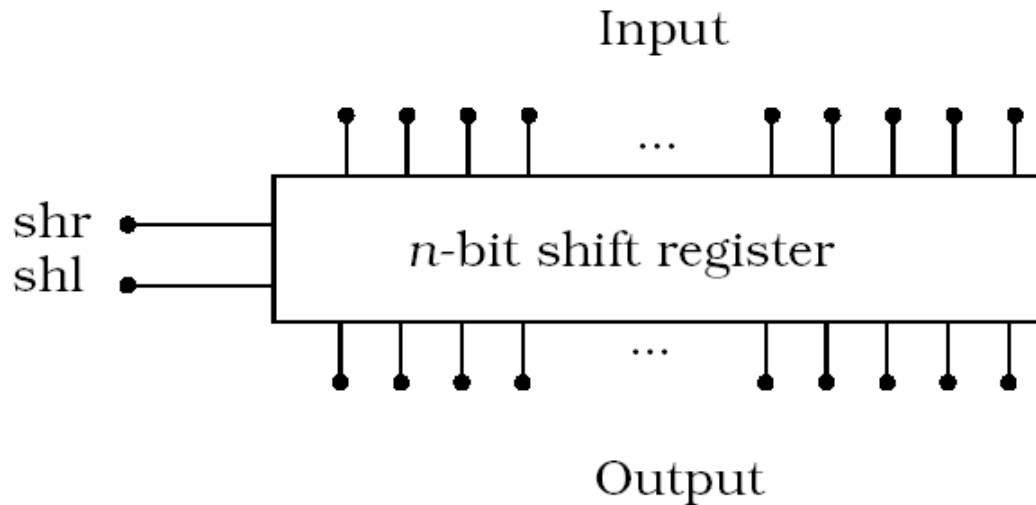
Multipliers (2)

				a_3	a_2	a_1	a_0	multiplicand	
				\times	b_3	b_2	b_1	b_0	multiplier
				$a_3 b_0$	$a_2 b_0$	$a_1 b_0$	$a_0 b_0$		
		+	$a_3 b_1$	$a_2 b_1$	$a_1 b_1$	$a_0 b_1$			
	+	$a_3 b_2$	$a_2 b_2$	$a_1 b_2$	$a_0 b_2$				
+	$a_3 b_3$	$a_2 b_3$	$a_1 b_3$	$a_0 b_3$					
p_7	p_6	p_5	p_4	p_3	p_2	p_1	p_0	product	

Multiplication of two 4-bit words.



Multipliers (3)



Shift register for multiplication or division by a factor of 2.



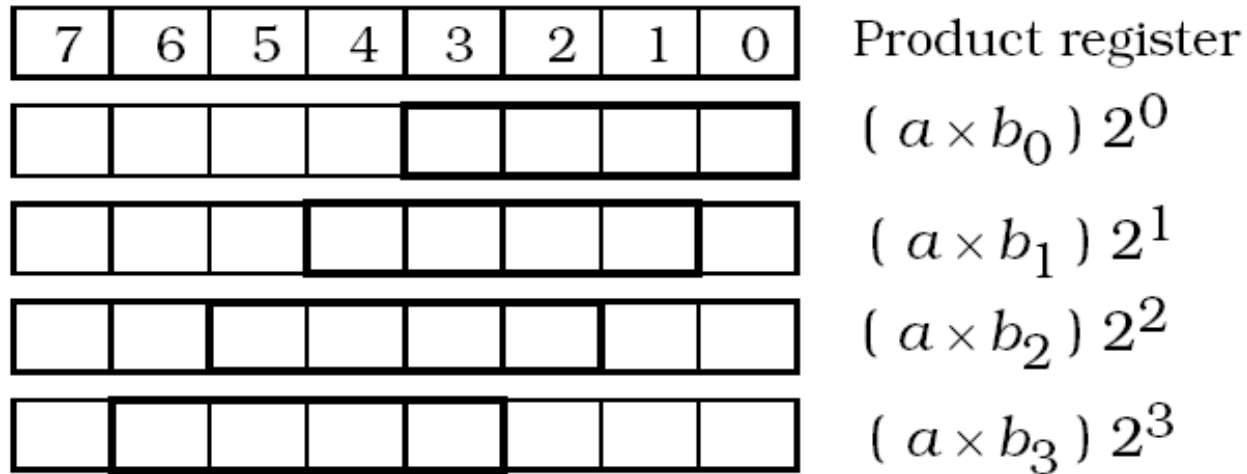
Multipliers (4)

				a_3	a_2	a_1	a_0		multiplicand
			\times	b_3	b_2	b_1	b_0		multiplier
				$(a_3$	a_2	a_1	$a_0)$	$\times b_0$	$(a \times b_0) 2^0$
				$(a_3$	a_2	a_1	$a_0)$	$\times b_1$	$(a \times b_1) 2^1$
				$(a_3$	a_2	a_1	$a_0)$	$\times b_2$	$(a \times b_2) 2^2$
			+	$(a_3$	a_2	a_1	$a_0)$	$\times b_3$	$(a \times b_3) 2^3$
	p_7	p_6	p_5	p_4	p_3	p_2	p_1	p_0	product

Alternate view of multiplication process.



Multipliers (5)

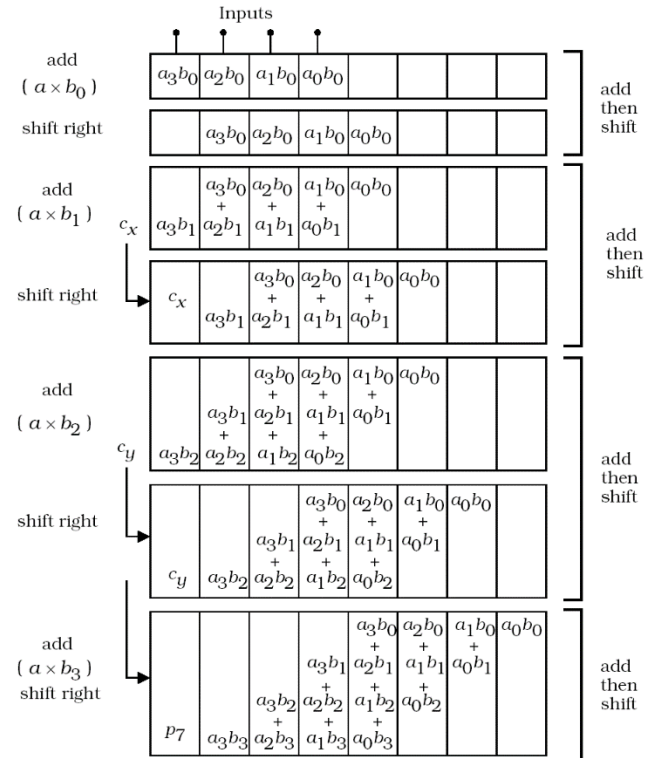


Using a product register for multiplication.



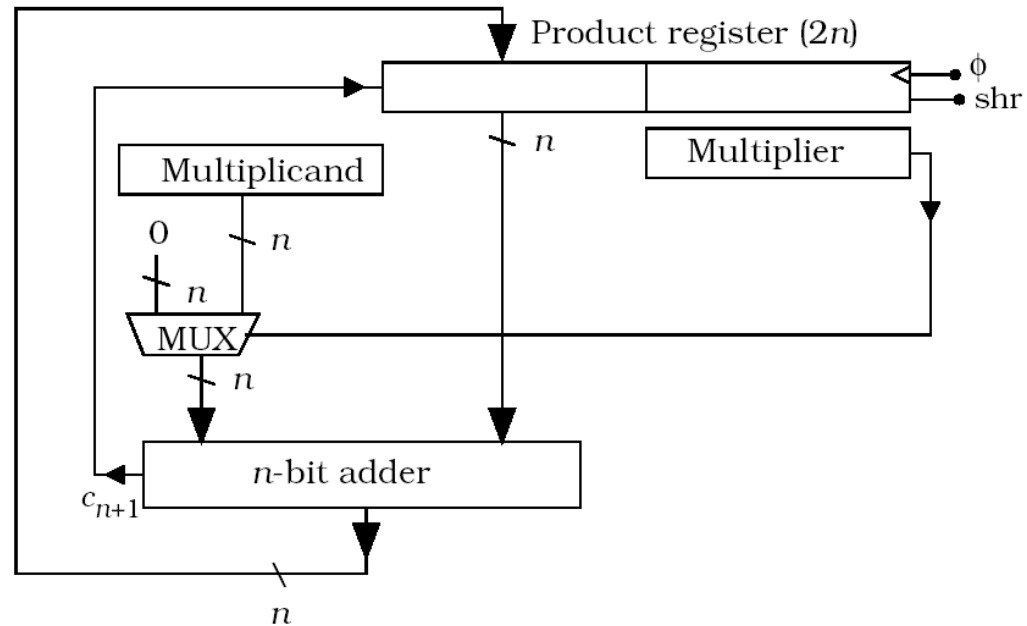
Multipliers (6)

Shift-right multiplication sequence.





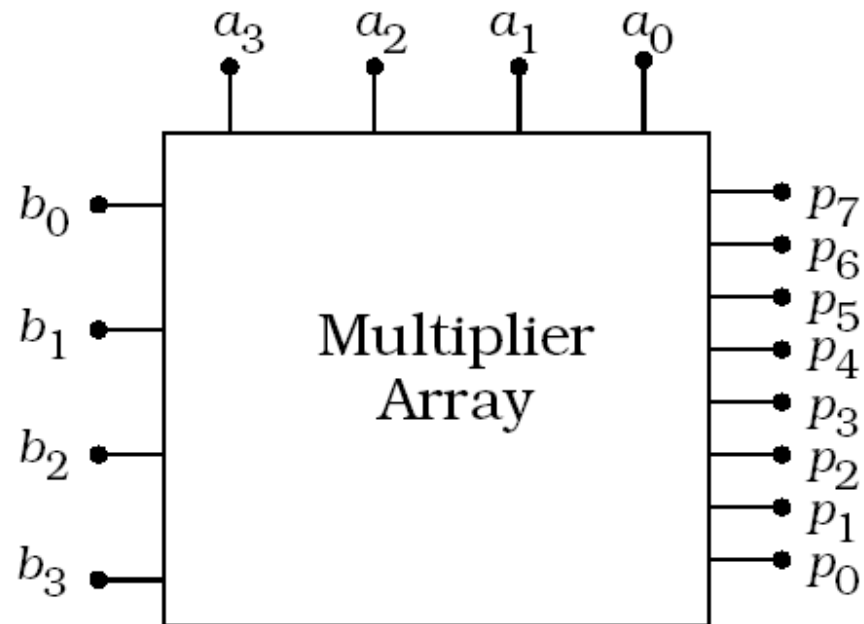
Multipliers (7)



Register-based multiplier network.



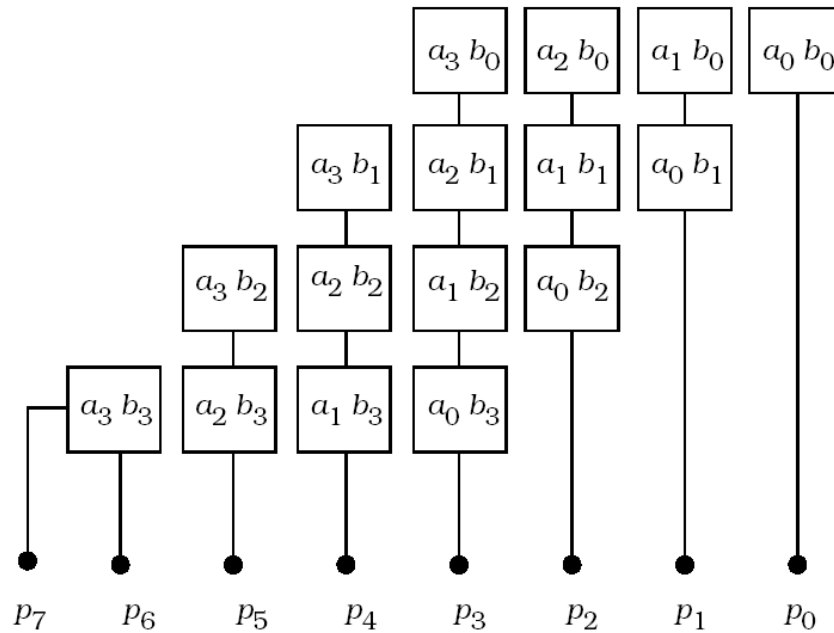
Multipliers (8)



An array multiplier.



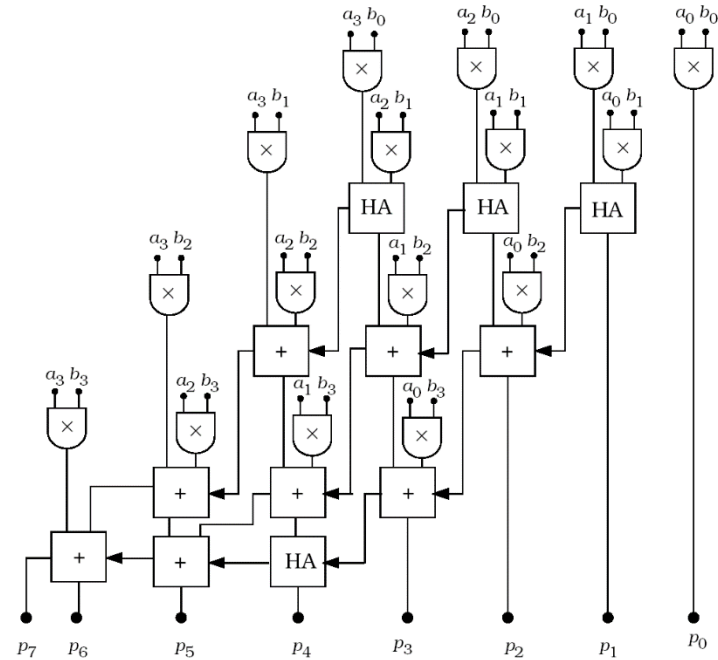
Multipliers (9)



Modularized view of the multiplication sequence.



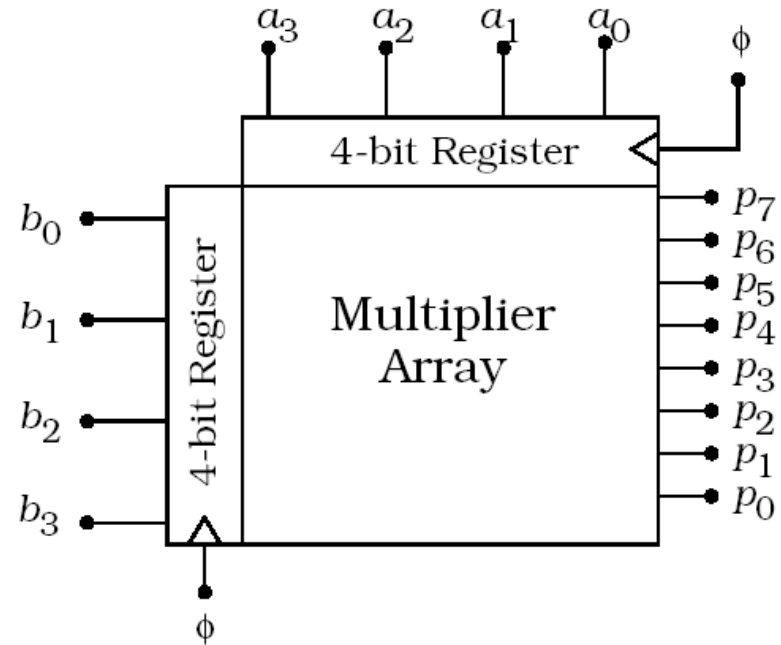
Multipliers (10)



Details for a 4 x 4 array multiplier.



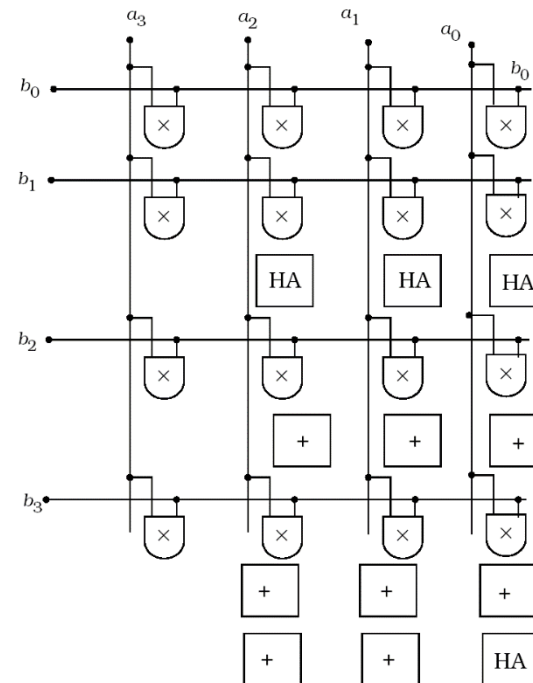
Multipliers (II)



Clocked input registers.



Multipliers (12)



Initial cell placement for the array.



Multipliers (13)

b_{2k+1}	b_{2k}	b_{2k-1}	E_k	Effect on sum
0	0	0	0	add 0
0	0	1	+ 1	add A
0	1	0	+ 1	add A
0	1	1	+ 2	shift A left, add
1	0	0	- 2	take two's (A), shift left, add
1	0	1	- 1	add two's (A)
1	1	0	- 1	add two's (A)
1	1	1	0	add 0

Summary of booth encoded digit operations.