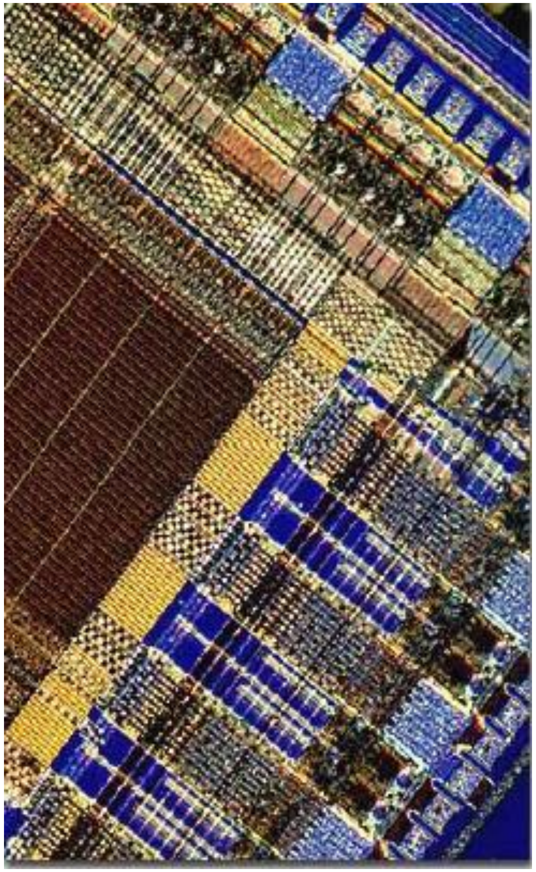
CMOS Digital Integrated Circuits



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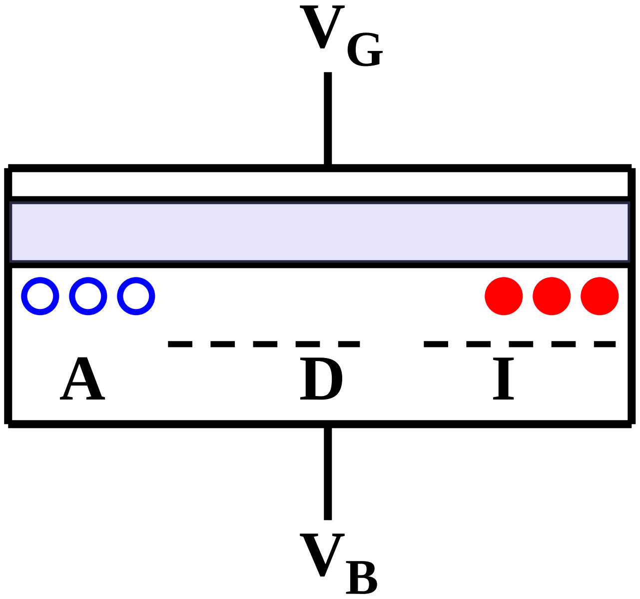
MOS Transistor I

- **Goals**
  - **Understand the basic MOSFET operation**
  - **Learn the components of the threshold voltages**
  - **Be able to handle body effect**
  - **Be able to calculate drain currents for MOSFET**
  - **Be able to extract basic MOSFET static parameters from I-V plots**

MOS Transistor Basics
Two Terminal Structure

Two terminal structure (p-substrate): The MOS capacitor



- **Important derived parameters. With $V_G = V_B = 0$:**
  - **ϕ_F – Buck Fermi Potential** (Substrate)
  - **ϕ_S – Surface Potential** (Substrate)

MOS Transistor Basics
Two Terminal Structure (Continued)

- V_{FB} – **Flat Band Voltage** (applied external voltage to G-B to flatten bands of substrate – equal to built-in potential difference of MOS – equal to **work function difference** ϕ_{GB} between the substrate (channel) and gate.

Operation

With $V_G<0$, $V_B=0$, Accumulation – Holes accumulate at substrate-oxide interface due to attraction of negative bias

With $V_G>0$, but small, $V_B=0$, Depletion – Holes repelled from substrate-oxide interface due to positive bias leaving negatively charged fixed acceptors ions behind. The result is a region below the interface that is depleted of mobile carriers.

Depletion region thickness

$$x_d = \sqrt{\frac{2\varepsilon_{Si}\left|\phi_s - \phi_F\right|}{q N_A}}$$

MOS Transistor Basics
Two Terminal Structure (Continued)

- **Depletion region charge density**

$$Q = -q N_A x_d = -\sqrt{2 q N_A \varepsilon_{Si} \left|\phi_s - \phi_F\right|}$$

Note that this density is per unit of area.

With $V_G>0$ and larger, $V_B=0$, Inversion – A n-type **inversion layer** forms, a condition known as **surface inversion.** The surface is inverted when the density of electrons at the surface equals the density of holes in the bulk. This implies that ϕ_s has the same magnitude but opposite sign to ϕ_F. At the point depletion depth fixed and the maximum depletion region depth is at $\phi_s = -\phi_F$. This depth is:

$$x_{dm} = \sqrt{\frac{2\varepsilon_{Si}\left|2\phi_F\right|}{q N_A}}$$

MOS Transistor Basics
Two Terminal Structure (Continued)

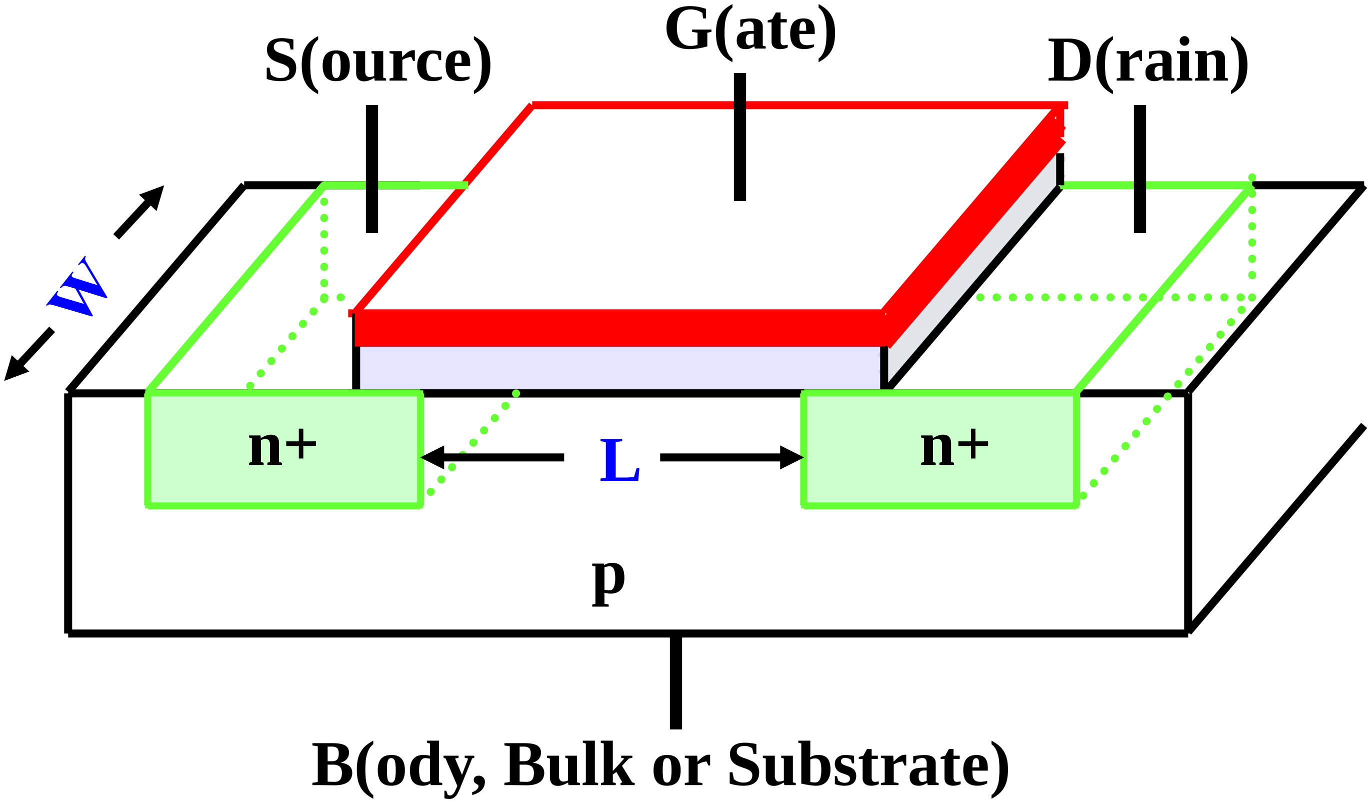
The corresponding **depletion charge density** (per unit area) **at surface inversion** is

$$Q_0 = -qN_A x_d = -\sqrt{2qN_A \varepsilon_{Si} \left| -2\phi_F \right|}$$

The inversion phenomena is the mechanism that forms the n-channel. The depletion depth and the depletion region charge are critical in determining properties of MOSFET.

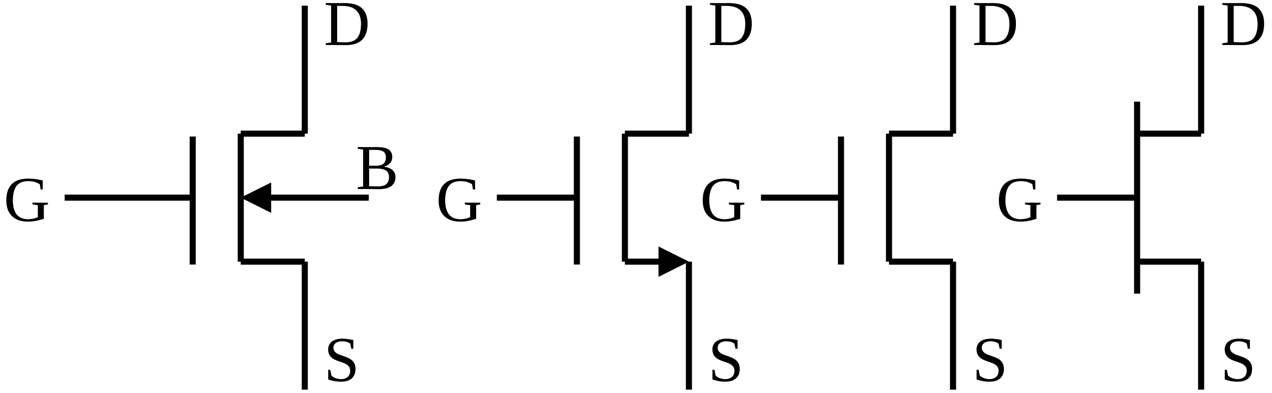
MOS Transistor Basics
Four Terminal Structure

- p-Substrate
- The MOS n-channel transistor structure:

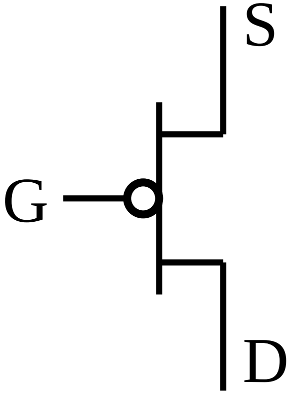


MOS Transistor Basics
Four Terminal Structure (Continued)

- **Symbols: n-channel - p-substrate; p-channel – n-substrate**



N-channel (for P-channel, reverse arrow or add bubbles)

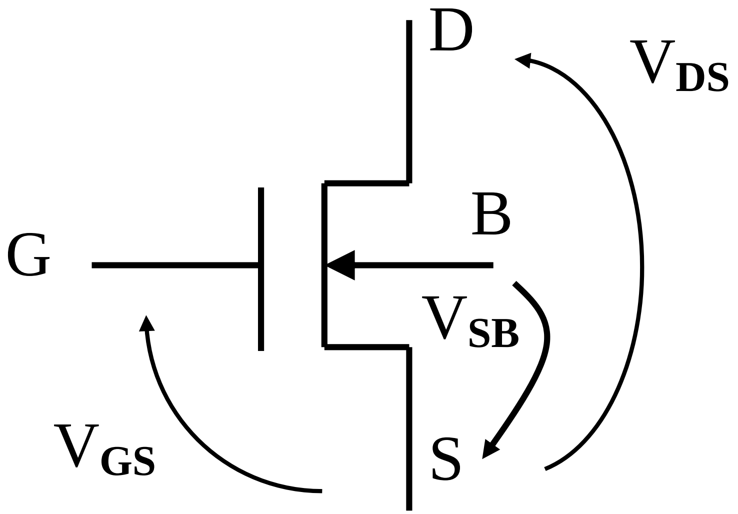


P-channel

- **Enhancement mode:** no conducting channel exists at $V_{GS} = 0$
- **Depletion mode:** a conducting channel exists at $V_{GS} = 0$

MOS Transistor Basics
Four Terminal Structure (Continued)

- **Source and drain identification**



Threshold Voltage Components

- Consider the prior 3-D drawing: Set $V_S=0$, $V_{DS}=0$, and $V_{SB}=0$.
  - Increase V_{GS} until the channel is inverted. Then a conducting channel is formed and the depletion region thickness (depth) is maximum as is the surface potential.
  - The value of V_{GS} needed to cause surface inversion (channel creation) is the ***threshold voltage*** V_{T0}. The 0 refers to $V_{SB}=0$.
  - $V_{GS}< V_{T0}$: no channel implies no current flow possible. With $V_{GS}> V_{T0}$, existence the channel implies possible current flow.

Threshold Voltage Components

1) Φ_{GC} work function difference between gate and channel material which is the built-in voltage that must be offset by voltage applied to flatten the bands at the surface.
2) **A**pply voltage to achieve surface inversion $-2\phi_F$

Threshold Voltage Components (Cont.)

3) **A**dditional voltage must be applied to offset the depletion region charge due to the acceptor ions. At inversion, this charge with **$V_{SB}=0$** **is $Q_{B0}= Q_0$.**

For V_{SB} non-zero,

$$Q = -\sqrt{2q\,N_A\,\varepsilon_{Si}\left|-2\phi_F + V_{SB}\right|}$$

The voltage required to offset the depletion region charge is defined by **$-Q_B/C_{ox}$** where **$C_{ox} = \varepsilon_{ox}/t_{ox}$** with **$t_{ox}$**, the oxide thickness, and **C_{ox}**, the gate oxide capacitance per unit area.

4) **T**he final component is a fixed positive charge density that appears at the interface between the oxide and the substrate, **Q_{ox}**. The voltage to offset this charge is:

$$\frac{-Q_{ox}}{C_{ox}}$$

Threshold Voltage Components (Cont.)

- These components together give:

$$V_T = \Phi_{GC} - 2\phi_F - \frac{Q_B}{C_{ox}} - \frac{Q_{ox}}{C_{ox}}$$

- For $\boldsymbol{V_{SB}=0}$, $\boldsymbol{V_{T0}}$ has $\boldsymbol{Q_B}$ replaced by $\boldsymbol{Q_{B0}}$. This gives a relationship between $\boldsymbol{V_T}$ and $\boldsymbol{V_{T0}}$ which is:

$$V_T = V_{T0} - \frac{Q_B - Q_{B0}}{C_{ox}}$$

- Thus the actual threshold voltage $\boldsymbol{V_T}$ differs from $\boldsymbol{V_{T0}}$ by the term given. Going back to the definition of $\boldsymbol{Q_B}$, this term is equal to:

$$+\gamma\left(\sqrt{\left|-2\phi_F + V_{SB}\right|} - \sqrt{\left|2\phi_F\right|}\right)$$

- In which γ is the **substrate-bias (or body effect) coefficient**.

Threshold Voltage Components (Cont.)

$$\gamma = \frac{\sqrt{2q\,N_A\,\varepsilon_{Si}}}{C_{ox}}$$

- The final expression for $\boldsymbol{V_{T0}}$ and $\boldsymbol{V_T}$ are

$$V_{T0} = \Phi_{GC} - 2\phi_F - \frac{Q_{B0}}{C_{ox}} - \frac{Q_{ox}}{C_{ox}}$$

and

$$V_T = V_{T0} + \gamma\left(\sqrt{\left|-2\phi_F + V_{SB}\right|} - \sqrt{\left|2\phi_F\right|}\right)$$

  - The threshold voltage depends on the source-to-bulk voltage which is clearly separated out. The component is referred to as ***body effect***. If the source to body voltage $\boldsymbol{V_{SB}}$ is non-zero, the corrective term must be applied to $\boldsymbol{V_{T0}}$.

Threshold Voltage Components (Cont.)

Those parameters in the V_T equation are signed. The following table gives their signs for nMOS and pMOS transistor.

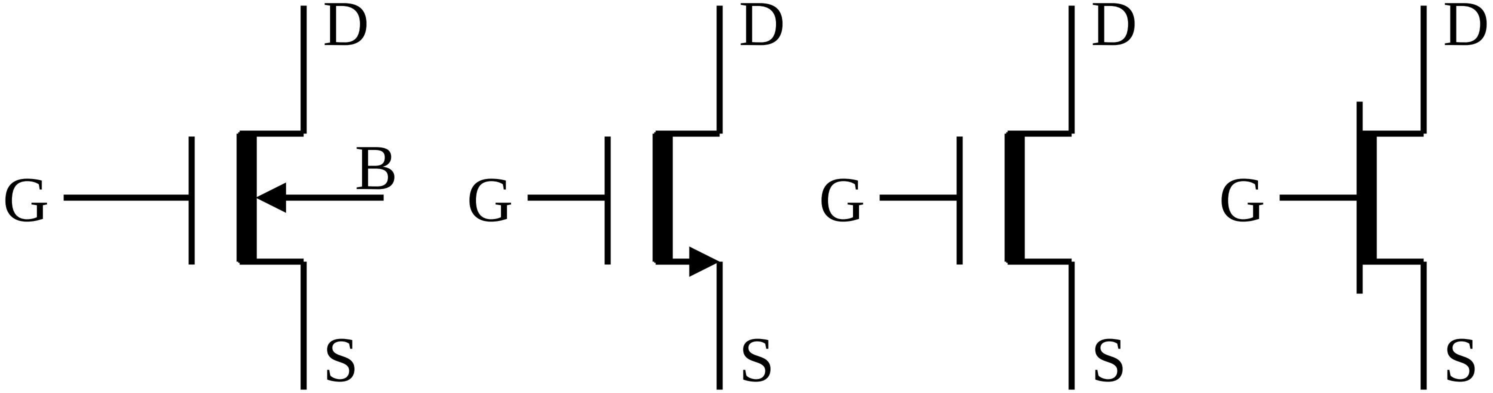
Parameter	nMOS	pMOS
ϕ_F	−	+
Q_B, Q_{B0}	−	+
γ	+	−
V_{SB}	+	−

- For real designs, the threshold voltage, due to variation in oxide thickness, impurity concentrations, etc., V_{T0} and γ should be measured from the actual process.

Threshold Voltage Adjustment by Ion Implant

- **Depletion mode nMOS**

  A channel implanted with donors can be present for $V_{GS}<0$. For this nMOS $V_T<0$. Its symbols are as follows:

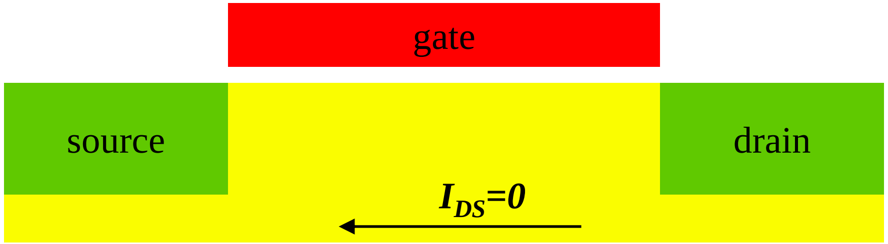


MOSFET Modes of Operation
Cutoff

- **Assume n-channel MOSFET and $V_{SB}=0$**

Cutoff Mode: $0 \le V_{GS} < V_{T0}$

- The channel region is depleted and no current can flow

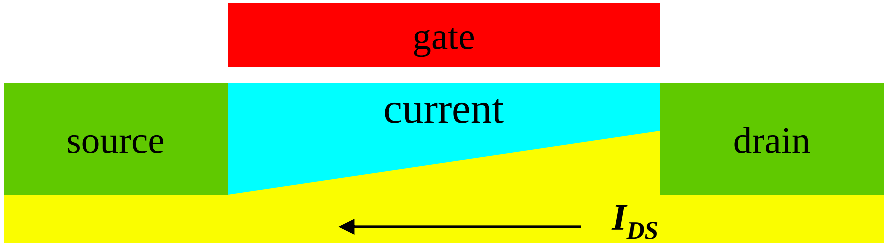


$V_{GS} < V_{T0}$

MOSFET Modes of Operation
Linear

Linear (Active, Triode) Mode: $V_{GS} \geq V_{T0},\ 0 \leq V_{DS} \leq V_{D(SAT)}$

- Inversion has occurred; a channel has formed
- For $V_{DS} > 0$, a current proportional to V_{DS} flows from source to drain
- Behaves like a voltage-controlled resistance

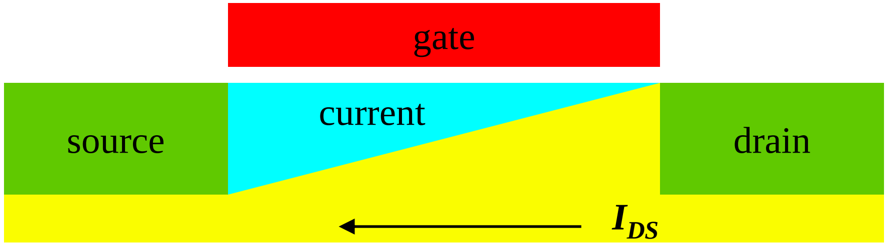


$$V_{DS} < V_{GS} - V_{T0}$$

MOSFET Modes of Operation
Pinch-Off

Pinch-Off Point (Edge of Saturation) : $V_{GS} \geq V_{T0}$, $V_{DS} = V_{D(SAT)}$

- Channel just reaches the drain
- Channel is reduced to zero inversion charge at the drain
- Drifting of electrons through the depletion region between the channel and drain has begun

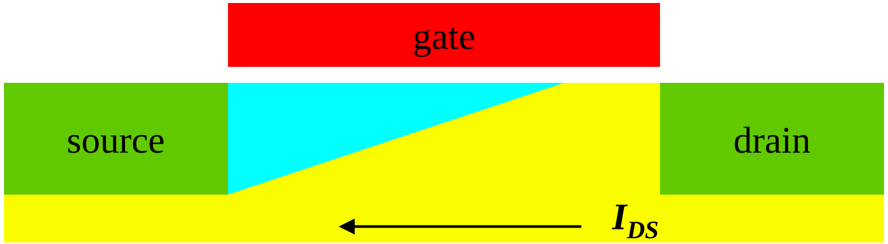


$$V_{DS} = V_{GS} - V_{T0}$$

MOSFET Modes of Operation
Saturation

Saturation Mode: $V_{GS} \geq V_{T0},\ V_{DS} \geq V_{D(SAT)}$

- Channel ends before reaching the drain
- Electrons drift, usually reaching the drift velocity limit, across the depletion region to the drain
- Drift due to high E-field produced by the potential $V_{DS}-V_{D(SAT)}$ between the drain and the end of the channel



$$V_{DS} > V_{GS} - V_{T0}$$

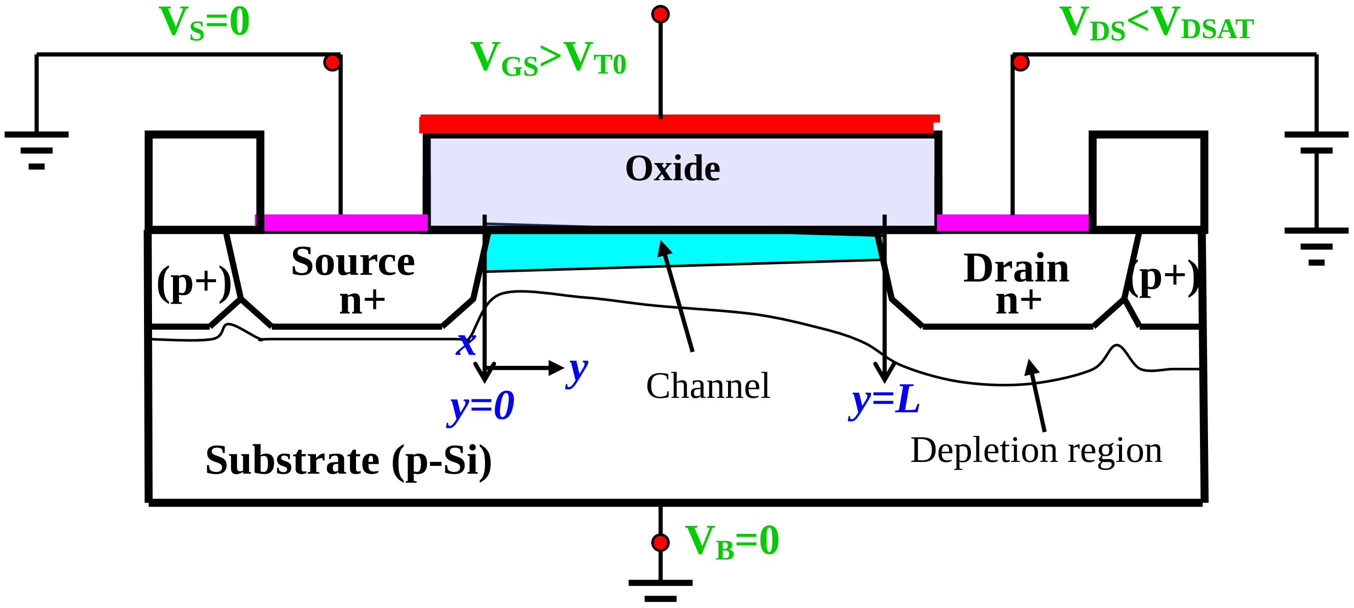
MOSFET I-V Characteristics
Gradual Channel Approximation

- **Preliminaries**
  - Gradual channel approximation will reduce the analysis to a one-dimensional current flow problem.
  - Assumption
    - $V_{SB}=0$
    - V_{T0} is constant along the entire channel
    - E_y dominates E_x ⇒ Only need to consider the current-flow in the y-dimension
- **Cutoff Mode: $0 \le V_{GS} < V_{T0}$**
  - $I_{DS(cutoff)} = 0$

Gradual Channel Approximation

Linear Mode

- **Linear Mode: $V_{GS} \geq V_{T0}$, $0 \leq V_{DS} \leq V_{D(SAT)}$ => $V_{DS} – V_{GS} < V_{T0}$**
  - The channel reaches to the drain.
  - $V_c(y)$: Channel voltage with respect to the source at position y
  - Boundary Conditions: $V_c(y=0)=V_s=0$; $V_c(y=L)=V_{DS}$



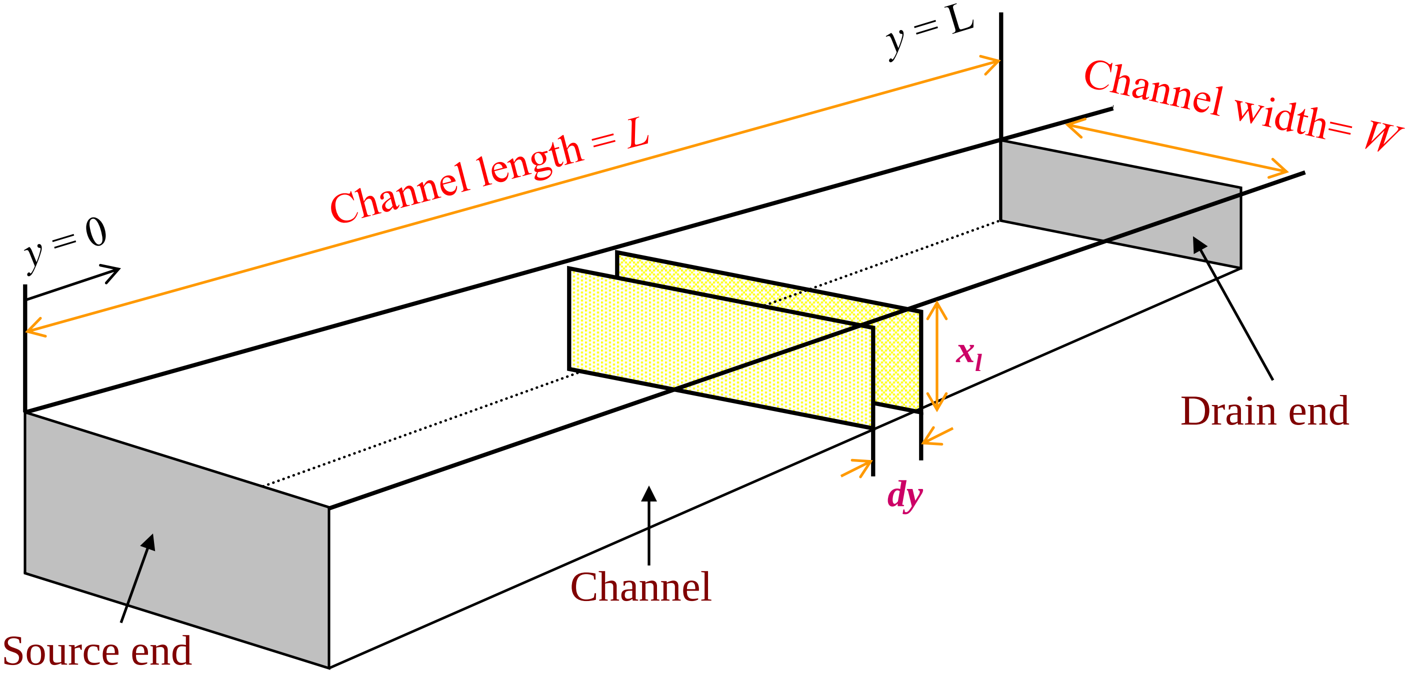
Gradual Channel Approximation

Linear Mode (Cont.)

- $Q_I\,(y)$: the mobile electron charge density in the surface inversion layer.

$$Q_I\,(y)=-C_{ox}\cdot[V_{GS}-V_C(y)-V_{T0}]$$

- The differential resistance (dR) of the channels can represented in terms of the mobile electron charge ($Q_I\,(y)$) in the surface inversion layer, and the **electron surface mobility μ_n** (about ½ of the bulk electron mobility)



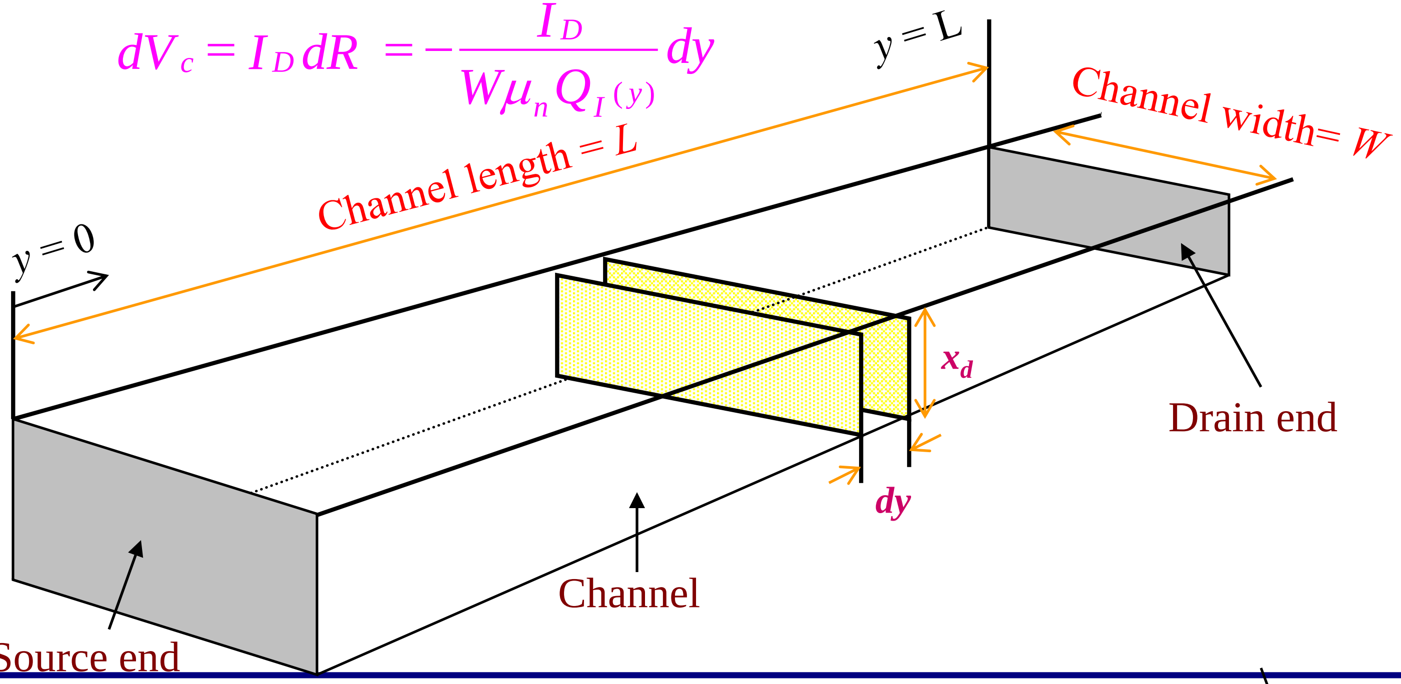
Gradual Channel Approximation

Linear Mode (Cont.)

- The differential resistance (*dR*) of the channels can represented in terms of the mobile electron charge ($Q_I(y)$) in the surface inversion layer, and the **electron surface mobility μ_n** (about ½ of the bulk electron mobility)

$$dR = -\frac{dy}{\sigma_n A_n} = -\frac{dy}{q\mu_n N_A W x_{d}(y)} = -\frac{dy}{W\mu_n Q_I(y)}$$

$$dV_c = I_D\, dR = -\frac{I_D}{W\mu_n Q_I(y)}\, dy$$



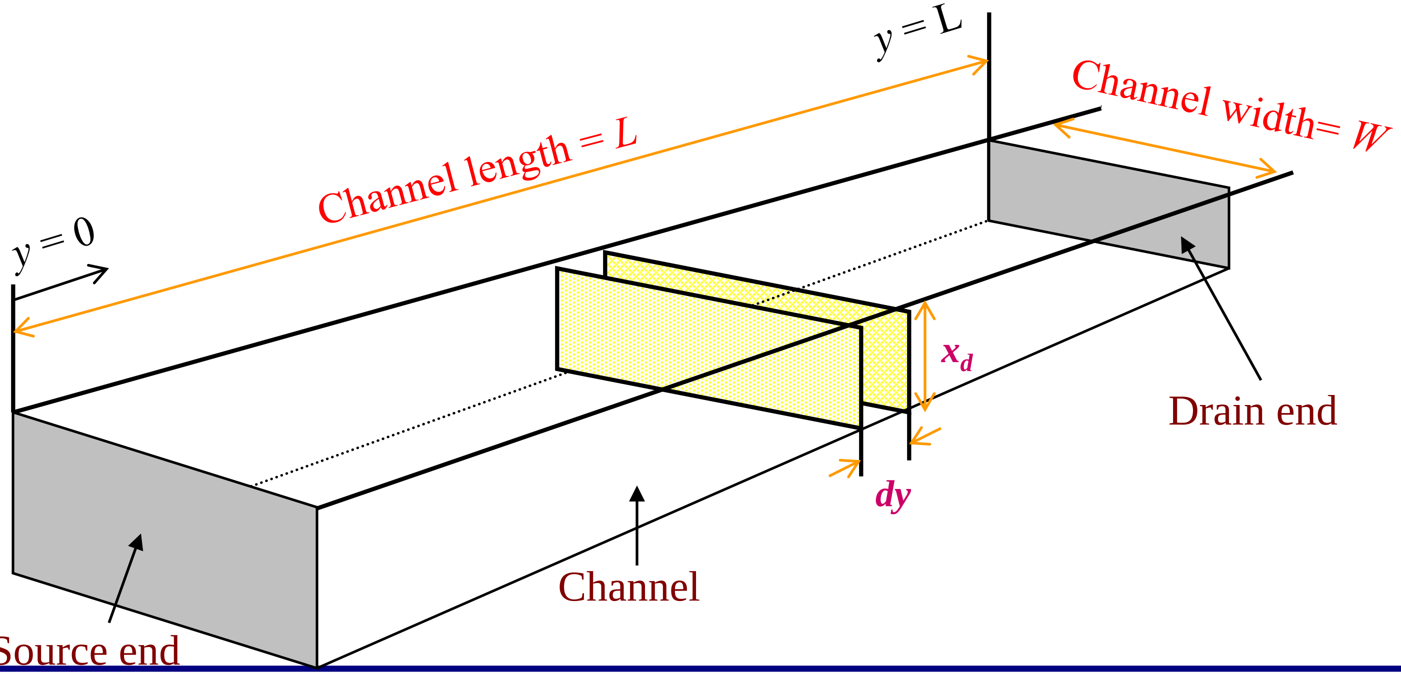
Gradual Channel Approximation

Linear Mode (Cont.)

- Integrating the Ohm's Law equality between the differential voltage in the channel and the differential resistance times the drain current,

$$\int_0^L I_D\,dy = -W\mu_n \int_0^{V_{DS}} Q_{I}(y)\,dV_c$$

$$I_D L = W\mu_n C_{ox} \int_0^{V_{DS}} \left(V_{GS} - V_c - V_{T0}\right) dV_c$$



Gradual Channel Approximation

Linear Mode (Cont.)

- Finally, the drain current is

$$I_{D(lin)} = \frac{\mu_n C_{ox}}{2} \frac{W}{L} \left[2(V_{GS} - V_{T0}) V_{DS} - V_{DS}^2 \right]$$

- To simplify the equation, we define

$$\kappa = \kappa' \frac{W}{L} = \mu_n C_{ox} \frac{W}{L}$$

κ': the *process transconductance parameter*

κ: the *device transconductance parameter*

Gradual Channel Approximation
Pinch-Off, Saturation

Pinch-Off Point (Edge of Saturation) : $V_{GS} \geq V_{T0}, V_{DS} = V_{D(SAT)}$

- Channel just reaches the drain but is reduced to zero inversion charge at the drain
- Electrons drift through the depletion region between the channel and drain

Saturation Mode: $V_{GS} \geq V_{T0}, V_{DS} \geq V_{GS} - V_{T0}$

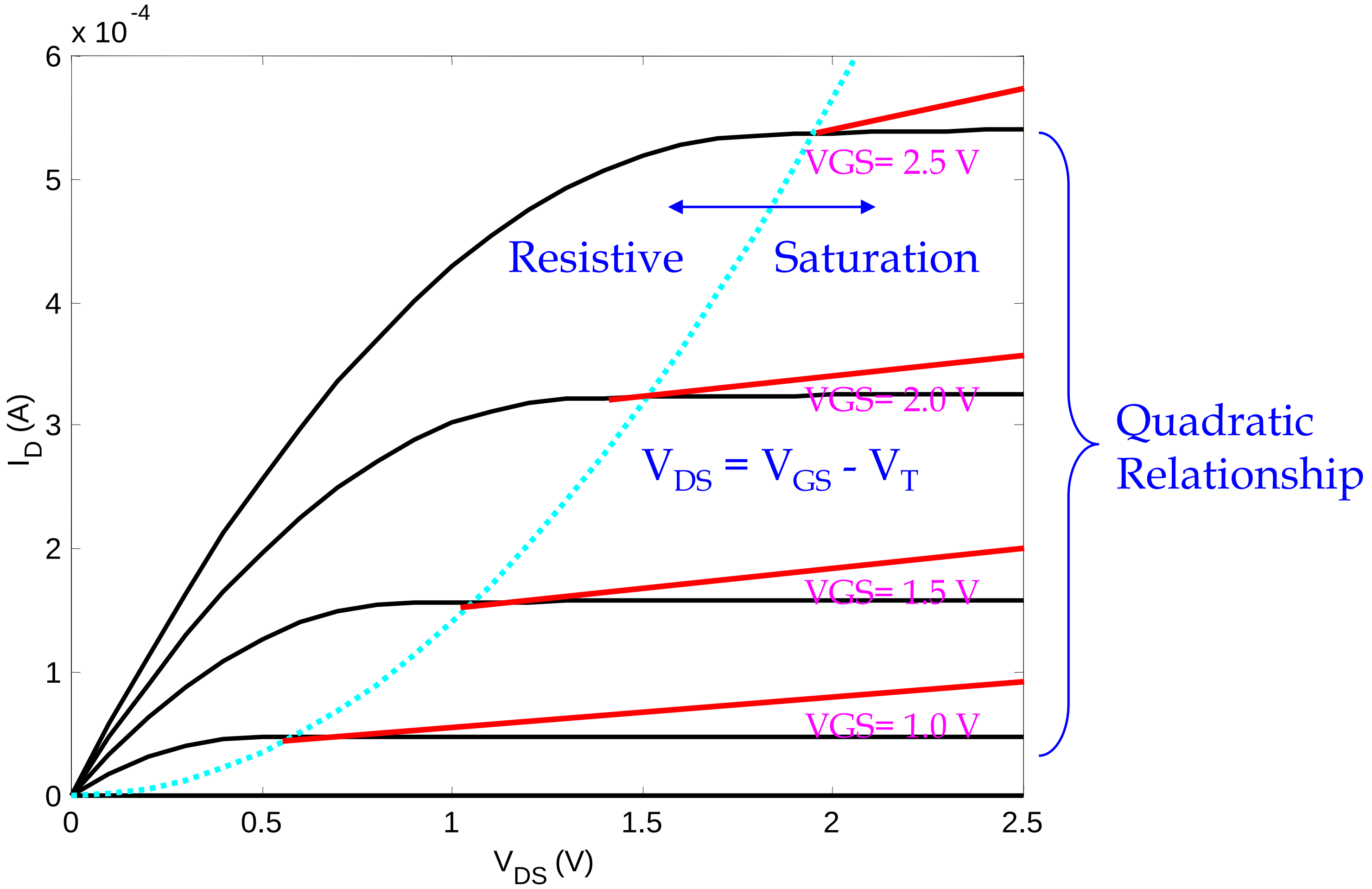
- In pinch-off voltage from the channel end to the source is $V_{D(SAT)} = V_{GS} - V_{T0}$. Substituting this for V_{DS} in the equation for I_D gives:

$$I_{D(SAT)} = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_{T0})^2$$

MOSFET I-V Characteristics

I-V Plots, Channel Length Modulation

- Saturation equation yields curves independent of $\boldsymbol{V_{DS}}$. Not sure! So we consider the effect of channel length modulation.



MOSFET I-V Characteristics

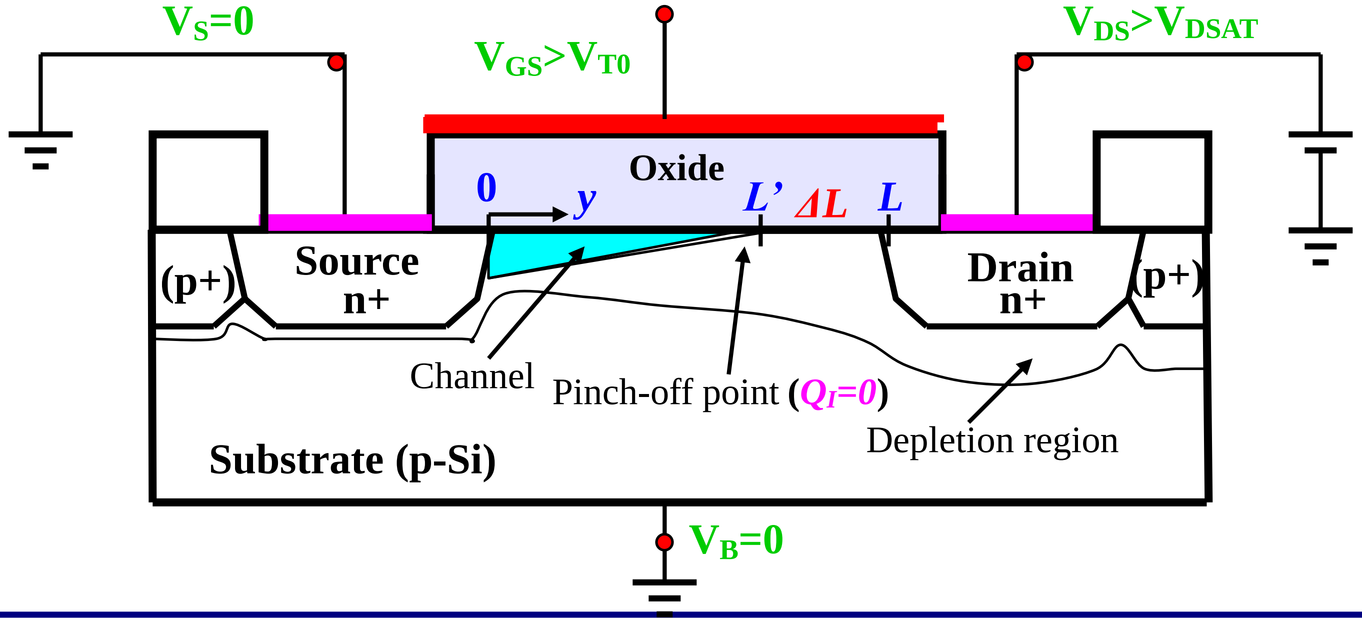
Channel Length Modulation

Channel Length Modulation

- With pinch-off the channel at the point *y* such that $V_c(y)=V_{GS} - V_{T0}$, The effective channel length is equal to $L' = L - \Delta L$

  ΔL is the length of channel segment over which $Q_I=0$.
- Place L' in the $I_{D(SAT)}$ equation:

$$I_{D(SAT)} = \frac{\mu_n C_{ox}}{2}\frac{W}{L'}(V_{GS}-V_{T0})^2$$



MOSFET I-V Characteristics
Channel Length Modulation

ΔL increases with an increase in V_{DS}. We can use

$$\frac{1}{L'}=\frac{1}{L-\Delta L}=\frac{1}{L}\frac{1}{\frac{L-\Delta L}{L}}=\frac{1}{L}\frac{1}{1-\frac{\Delta L}{L}}=\frac{1}{L}\frac{1}{1-\lambda V_{DS}}=\frac{1}{L}(1+\lambda V_{DS})$$

λ: channel length modulation coefficient

$I_{D(SAT)}$ can be rewritten as

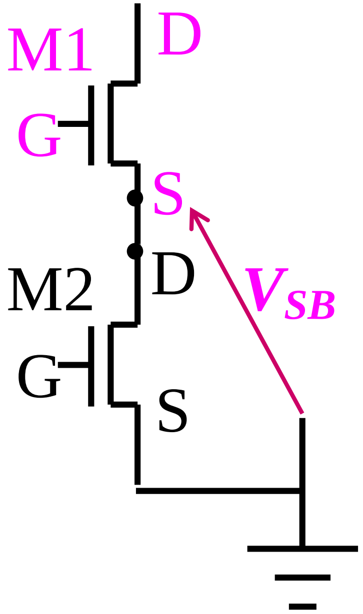
$$I_{D(SAT)}=\frac{\mu_n C_{ox}}{2}\frac{W}{L}(V_{GS}-V_{T0})^2(1+\lambda V_{DS})$$

- The above form produces a discontinuity of current at $V_{DS}=V_{GS}-V_{T0}$. We can include the term in $I_{D(lin)}$ with little error since λ is typically less than 0.1. We will usually ignore λ in manual calculations.

MOSFET I-V Characteristics

Substrate Bias Effect

- So far, $V_{SB}=0$ and thus V_{T0} used in the equations.
- Clearly not always true – must consider body effect
- Two MOSFETs in series:



$V_{SB(M1)} = V_{DS(M2)} \neq 0$. Thus, V_{T0} in the ***M1*** equation is replaced by $V_T = V_{T(V_{SB})}$ as developed in the threshold voltage section.

MOSFET I-V Characteristics

Substrate Bias Effect (Cont.)

The general form of I_D can be written as

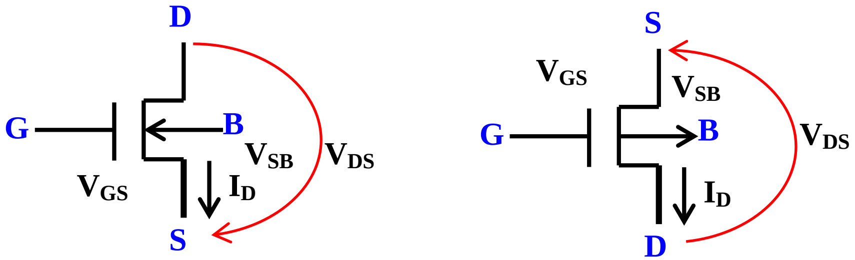
$$I_D = f\,(V_{GS}, V_{DS}, V_{SB})$$

which due to the body effect term is non-linear and more difficult to handle in manual calculations

MOSFET I-V Characteristics

Summary of Analytical Equations

- The voltage directions and relationships for the three modes of pMOS are in contrast to those of nMOS.



nMOS		
Mode	I_D	**Voltage Range**
Cut-off	0	$V_{GS}<V_T$
Linear	$(\mu_n C_{ox}/2)(W/L)[2(V_{GS}-V_T)V_{DS}-V_{DS}^2]$	$V_{GS}\geq V_T,\ \ V_{DS}< V_{GS}-V_T$
Saturation	$(\mu_n C_{ox}/2)(W/L)(V_{GS}-V_T)^2(1+\lambda V_{DS})$	$V_{GS}\geq V_T,\ \ V_{DS}\geq V_{GS}-V_T$
pMOS		
Cut-off	0	$V_{GS}>V_T$
Linear	$(\mu_n C_{ox}/2)(W/L)[2(V_{GS}-V_T)V_{DS}-V_{DS}^2]$	$V_{GS}\leq V_T,\ \ V_{DS}> V_{GS}-V_T$
Saturation	$(\mu_n C_{ox}/2)(W/L)(V_{GS}-V_T)^2(1+\lambda V_{DS})$	$V_{GS}\leq V_T,\ \ V_{DS}\leq V_{GS}-V_T$

More Parameter Extraction

- Need numerical values for parameters in V_T and I_D equations
- Parameters can be derived from the measured *I-V* characteristics for a given MOSFET process.
- To illustrate, seeking Level 1 Spice model parameters V_{T0}, $\mu_n(\kappa_n)$, γ, and λ
- To obtain V_{T0}, $\mu_n(\kappa_n)$, and γ, we plot $(I_D)^{1/2}$ *vs* $V_{DS} = V_{GS}$ with V_{SB} set to zero and one positive value. MOSFET is in saturation mode (ignoring channel length modulation):

$$\sqrt{I_D} = \sqrt{\frac{\kappa_n}{2}}(V_{GS} - V_{T0})$$

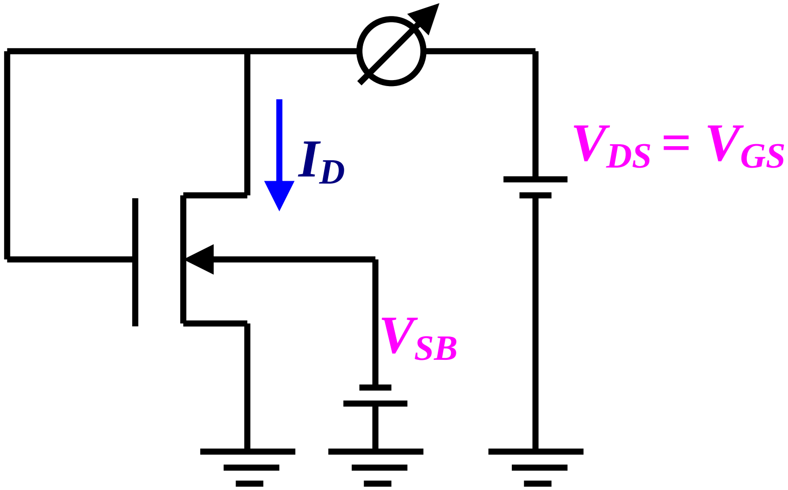
- Note that this (ideally!) gives a linear relationship that will allow us to determine κ_n and V_{TO}.
  - » The slope of the lines is $\sqrt{\kappa_n / 2}$
  - » The intercept of the $V_{SB} = 0$ line with the V_{GS} axis is V_{T0}

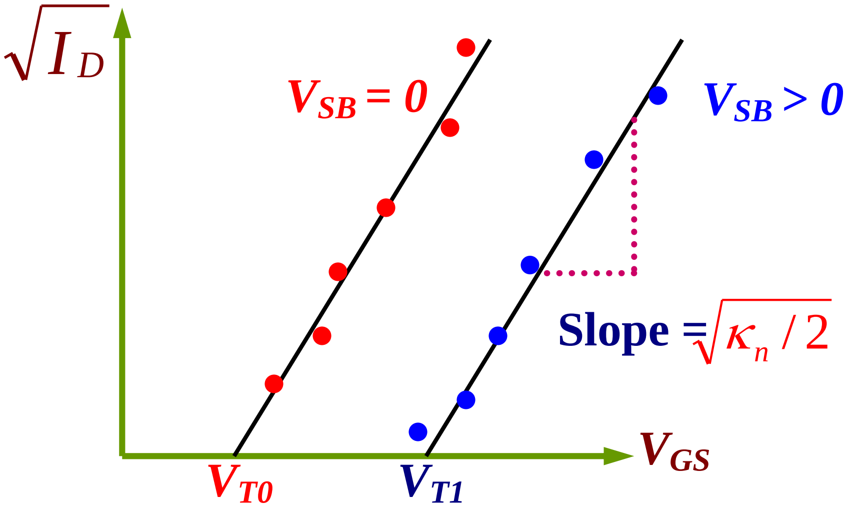
More Parameter Extraction (Cont.)

- Using the intercept of the line for V_{SB} nonzero, the *body effect coefficient* γ can be found

$$\gamma = \frac{V_T(V_{SB}) - V_{T0}}{\sqrt{|2\phi_F| + V_{SB}} - \sqrt{|2\phi_F|}}$$

ϕ_F can be obtained from the substrate acceptor density N_A and other known physical constants





More Parameter Extraction (Cont.)

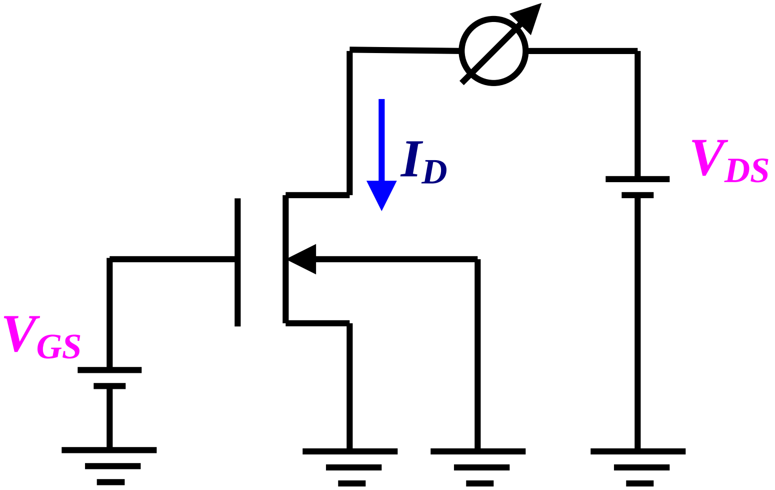
- The ***I*-*V*** curve for $V_{GS} = V_{T0}+1$ can be used to obtain λ.

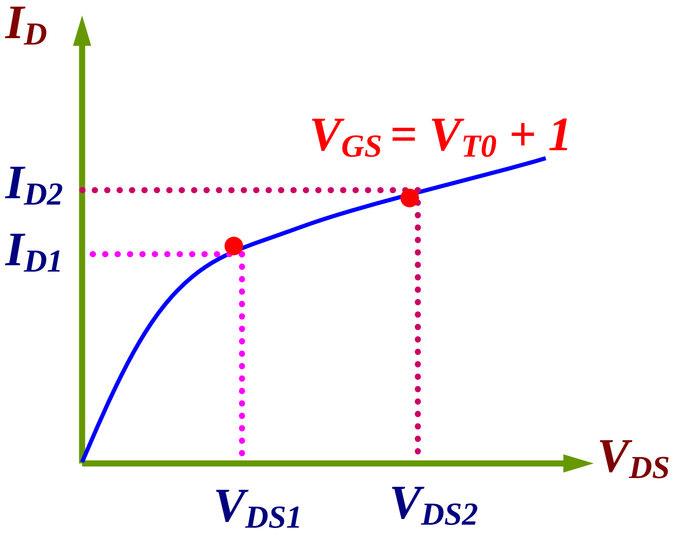
$$I_D(sat) = \kappa_n/2.\ \ (V_{GS}-V_{T0})^2.\ \ (1+\lambda V_{DS}) = \kappa_n/2.\ \ (1+\lambda V_{DS})$$

Therefore

$$\lambda = 2S/\kappa_n$$

where S is the slope of this curve in the saturation region.





More Parameter Extraction (Cont.)

- The Level 1 model is valid only for long devices and is obsolete for most of today's technologies for detail simulation.
- Parameter extraction for more advanced models such as Level 3 or 4 is usually performed by an automatic parameter extraction system that optimizes the combined parameter values for a best non-linear fit to the I-V curves.
- Due to this optimization, derivation of Level 1 model by simply deleting selected parameters from a Level 3 model is invalided. Instead, use the Level 3 model to produce I-V curves and linear curve fitting to extract Level 1 parameters.

Summary

- Basic MOSFET operation
- Components of the threshold voltage
- Threshold voltage and body effect
- Drain currents
- MOSFET static parameter extraction from I-V plots
- All of the above for both nMOS and pMOS.