**Alexandria University**

**Faculty of Engineering**

***Division of Communications & Electronics***

# EE391: Control Systems and Components

# Sheet 2: State-Space Solutions and Realizations

1. An oscillation can be generated by

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \mathbf{x}$$

Show that its solution is

$$\mathbf{x}(t) = \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix} \mathbf{x}(0)$$

2. Use two different methods to find the unit-step response of

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$
$$y = [2 \quad 3]\mathbf{x}$$

3. Find the companion-form and modal form equivalent equations of

$$\dot{\mathbf{x}} = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & -2 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u$$
$$y = [1 \quad -1 \quad 0]\mathbf{x}$$

4. Consider

$$\dot{\mathbf{x}} = \begin{bmatrix} \lambda & 0 \\ 0 & \bar{\lambda} \end{bmatrix} \mathbf{x} + \begin{bmatrix} b_1 \\ \bar{b}_1 \end{bmatrix} u \qquad y = [c_1 \quad \bar{c}_1]\mathbf{x}$$

Where the overbar denotes complex conjugate. Verify that the equation can be transformed into

$$\dot{\bar{\mathbf{x}}} = \bar{\mathbf{A}}\bar{\mathbf{x}} + \bar{\mathbf{b}}u \qquad y = \bar{\mathbf{c}}\bar{\mathbf{x}}$$

with

$$\bar{\mathbf{A}} = \begin{bmatrix} 0 & 1 \\ -\lambda\bar{\lambda} & \lambda + \bar{\lambda} \end{bmatrix} \qquad \bar{\mathbf{b}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \bar{\mathbf{c}}_1 = [-2\mathrm{Re}(\bar{\lambda} b_1 c_1) \quad 2\mathrm{Re}(b_1 c_1)]$$

by using the transformation $x = Q\bar{x}$ with

$$\mathbf{Q}_1 = \begin{bmatrix} -\bar{\lambda} b_1 & b_1 \\ -\lambda \bar{b}_1 & \bar{b}_1 \end{bmatrix}$$

5. Are the two sets of state equations equivalent?

$$\dot{\mathbf{x}} = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{x} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u \qquad y = [1 \ -1 \ 0]\mathbf{x}$$

and

$$\dot{\mathbf{x}} = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix} \mathbf{x} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u \qquad y = [1 \ -1 \ 0]\mathbf{x}$$

6. Show that

$$\partial \mathbf{\Phi}(t_0, t)/\partial t = -\mathbf{\Phi}(t_0, t)\mathbf{A}(t).$$

7. Verify that

$$\mathbf{X}(t) = e^{\mathbf{A}t}\mathbf{C}e^{\mathbf{B}t}$$

is the solution of

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{X}\mathbf{B} \qquad \mathbf{X}(0) = \mathbf{C}$$