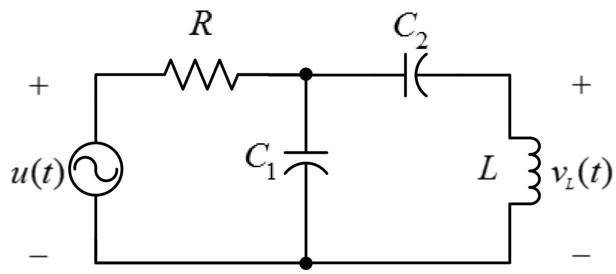
Alexandria University

Faculty of Engineering

Electrical Engineering Department

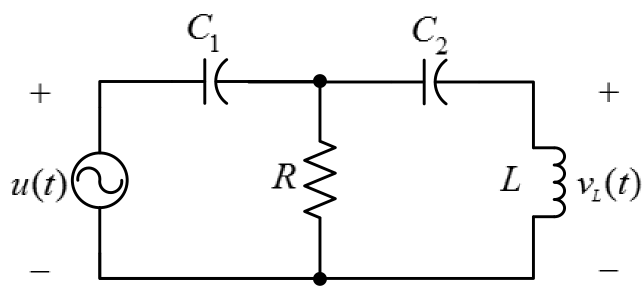
EE391 Control Systems and Components
Modern Control problems

1. Derive the state space representation of the following electric circuit:



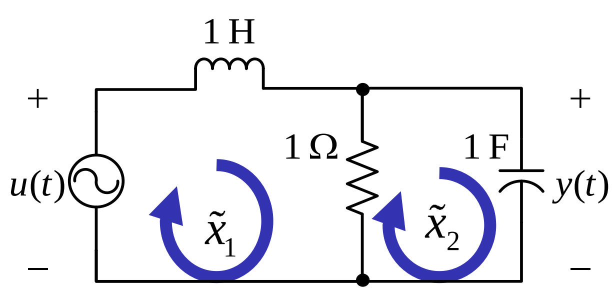
Input variables (Input voltage $u(t)$), Output variables (Inductor voltage $v_L(t)$)

2. Derive the state space representation of the following electric circuit:



Input variables (Input voltage $u(t)$), Output variables (Inductor voltage $v_L(t)$)

3. Prove that for the same system, with different definition of state variables, we can obtain a state space in the form of:

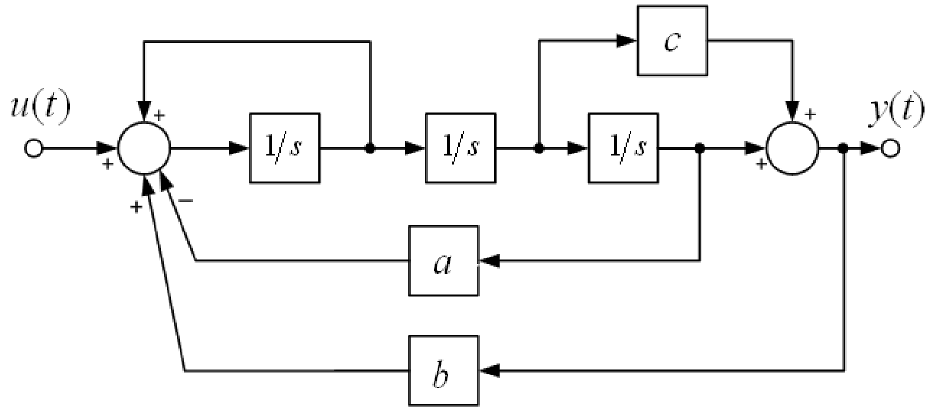


$$\begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix}$$

State variables are: $\tilde{x}_1$: current of left loop, $\tilde{x}_2$: current of left loop

4. *Derive a state-space description for the following diagram*



5. *Find the state-space realizations of the following transfer function in Frobenius Form, Observer Form, and Canonical Form.*

$$G(s) = \frac{Y(s)}{U(s)} = \frac{s+2}{s^3 + 8s^2 + 19s + 12}$$

6. *Perform a step by step transformation (by calculation of transfer matrix) from the following state-space equations to result the corresponding transfer function.*

$$\dot{\underline{\mathbf{x}}}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -4 & -2 \end{bmatrix} \cdot \underline{\mathbf{x}}(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot u(t)$$

$$y(t) = \begin{bmatrix} 5 & 1 & 0 \end{bmatrix} \cdot \underline{\mathbf{x}}(t)$$

7. *Find the state-space realizations of the following transfer function in Frobenius Form, Observer Form, and Canonical Form.*

$$G(s) = \frac{Y(s)}{U(s)} = \frac{s+2}{s^3 + 8s^2 + 19s + 12}$$

8. consider a SISO system with the state equations:

$$\dot{\underline{\mathbf{x}}}(t) = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \underline{\mathbf{x}}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 2 \end{bmatrix} \underline{\mathbf{x}}(t)$$

   a. If the state feedback in the form of:

$$u(t) = r(t) - \begin{bmatrix} k_1 & k_2 \end{bmatrix} \underline{\mathbf{x}}(t)$$

   is implemented to the system and it is wished that the poles of the system will be –3 and –4, determine the value of k_1 and k_2.

   b. Find the transfer function of the system and again, check the location of the poles of the transfer function.

9. Consider a SISO system with the state equations:

$$\underline{\dot{\mathbf{x}}}(t)=\begin{bmatrix}1 & 2\\ 3 & 1\end{bmatrix}\underline{\mathbf{x}}(t)+\begin{bmatrix}1\\ -1\end{bmatrix}u(t)$$

$$y(t)=\begin{bmatrix}2 & 3\end{bmatrix}\underline{\mathbf{x}}(t)$$

   a. If the state feedback in the form of:

   $$u(t)=r(t)-\begin{bmatrix}k_1 & k_2\end{bmatrix}\underline{\mathbf{x}}(t)$$

   is implemented to the system and it is wished that the damping factor ζ of the system is equal to 0.8 while keeping the system stable. Determine the required value of k_1 and k_2.

   b. Find the transfer function of the system and again, check the location of the poles of the transfer function.

10. A state-space equation of a third-order system is given as:

$$\dot{\underline{\tilde{\mathbf{x}}}}(t)=\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 0\\ 0 & 0 & -3\end{bmatrix}\underline{\tilde{\mathbf{x}}}(t)+\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}u(t)$$

$$y(t)=\begin{bmatrix}6 & -6 & 1\end{bmatrix}\underline{\tilde{\mathbf{x}}}(t)$$

   a. Perform a step-by-step transformation of the given model to Frobenius Form.

   b. Calculate the required feedback gain $\underline{\boldsymbol{k}}$ so that the system may have two conjugate poles at $-2\pm j1$ and -4.

11. A state-space equation of a third-order system is given as:

$$\dot{\underline{\tilde{\mathbf{x}}}}(t)=\begin{bmatrix}-1 & 1 & 0\\ 1 & -2 & 0\\ 0 & 0 & -3\end{bmatrix}\underline{\tilde{\mathbf{x}}}(t)+\begin{bmatrix}1\\ 0\\ 1\end{bmatrix}u(t)$$

$$y(t)=\begin{bmatrix}6 & 6 & -1\end{bmatrix}\underline{\tilde{\mathbf{x}}}(t)$$

   a. Perform a step-by-step transformation of the given model to Frobenius Form.

   b. Calculate the required feedback gain $\underline{\boldsymbol{k}}$ so that the system may have two conjugate poles at $-1\pm j3$ and -2.

12. A system is given in state space form as below:

$$\underline{\dot{\mathbf{x}}}(t)=\begin{bmatrix}2 & 1\\ -1 & 1\end{bmatrix}\underline{\mathbf{x}}(t)+\begin{bmatrix}1\\ 2\end{bmatrix}u(t)$$

$$y(t)=\begin{bmatrix}1 & 1\end{bmatrix}\underline{\mathbf{x}}(t)$$

   a. Find a state feedback gain $\underline{\boldsymbol{k}}$, so that the closed-loop system has -1 and -2 as its eigenvalues.

   b. Design three closed-loop state estimators for the system, with eigenvalues at ($-2\pm j2$), ($-3, -4$), and ($-0.5\pm j1$).

   c. Compare the performance of the three estimators and give some explanations of the comparison results (You can use Matlab for this requirement).

13. Consider the following linear system given by:

$$\underline{\dot{x}}(t)=\begin{bmatrix}-1 & 2 & 0\\ 1 & -3 & 4\\ -1 & 1 & -9\end{bmatrix}\underline{x}(t)+\begin{bmatrix}1\\ 2\\ -1\end{bmatrix}u(t)$$
$$y(t)=\begin{bmatrix}1 & 0 & 1\end{bmatrix}\underline{x}(t)$$

a. Using the transformation to the Observer Form, find the gain vector $\underline{\boldsymbol{l}}$ of the closed-loop state estimator if the desired poles are –3 and –4 ± j2.
b. Recall again the output feedback. In observer form, its effect on the characteristic equation of the system can be calculated much easier. By calculation, prove that the poles of the system cannot be assigned to any arbitrary location by only setting the value of output feedback j.

14. Consider the following linear system given by:

$$\underline{\dot{x}}(t)=\begin{bmatrix}-6 & -11 & -6\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}\underline{x}(t)+\begin{bmatrix}-1\\ 0\\ 3\end{bmatrix}u(t)$$
$$y(t)=\begin{bmatrix}0 & 1 & 1\end{bmatrix}\underline{x}(t)$$

a. Calculate the eigenvalues and eigenvectors of the system. Is it stable or unstable?
b. Using the transformation to the Observer Form, find the gain vector $\underline{\boldsymbol{l}}$ of the closed-loop state estimator if the desired poles are –1 ± j2.5 and –3.

15. It is desired that the following linear system has zero steady state error to a unit step input. Find the solution by using:

$$\underline{\dot{x}}(t)=\begin{bmatrix}-1 & 2 & 0\\ 1 & -3 & 4\\ -1 & 1 & -9\end{bmatrix}\underline{x}(t)+\begin{bmatrix}1\\ 2\\ -1\end{bmatrix}u(t)$$
$$y(t)=\begin{bmatrix}1 & 0 & 1\end{bmatrix}\underline{x}(t)$$

a. **Pre-scaling method**, by calculating the gain E.
b. **Integral control method**, by calculate the gain [k_{int} $\boldsymbol{k}$].
Hint: Assume the additional pole to be –1 and do not move the original poles of the system.