State Feedback and State Estimators

“Linear System Theory and Design”, Chapter 8

http://zitompul.wordpress.com

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Homework 7: State Estimators

(a) For the same system as discussed in previous slides, design another closed-loop state estimator, with eigenvalues at -3 and -4.

(b) Compare the performance of the estimator in the previous slides and the one you have designed through simulation using Matlab Simulink.

(c) Give some explanations of the comparison results.

Solution of Homework 7: State Estimators

The system is rewritten as:

$$\dot{\underline{\boldsymbol{x}}}(t) = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} \underline{\boldsymbol{x}}(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} \underline{\boldsymbol{x}}(t)$$

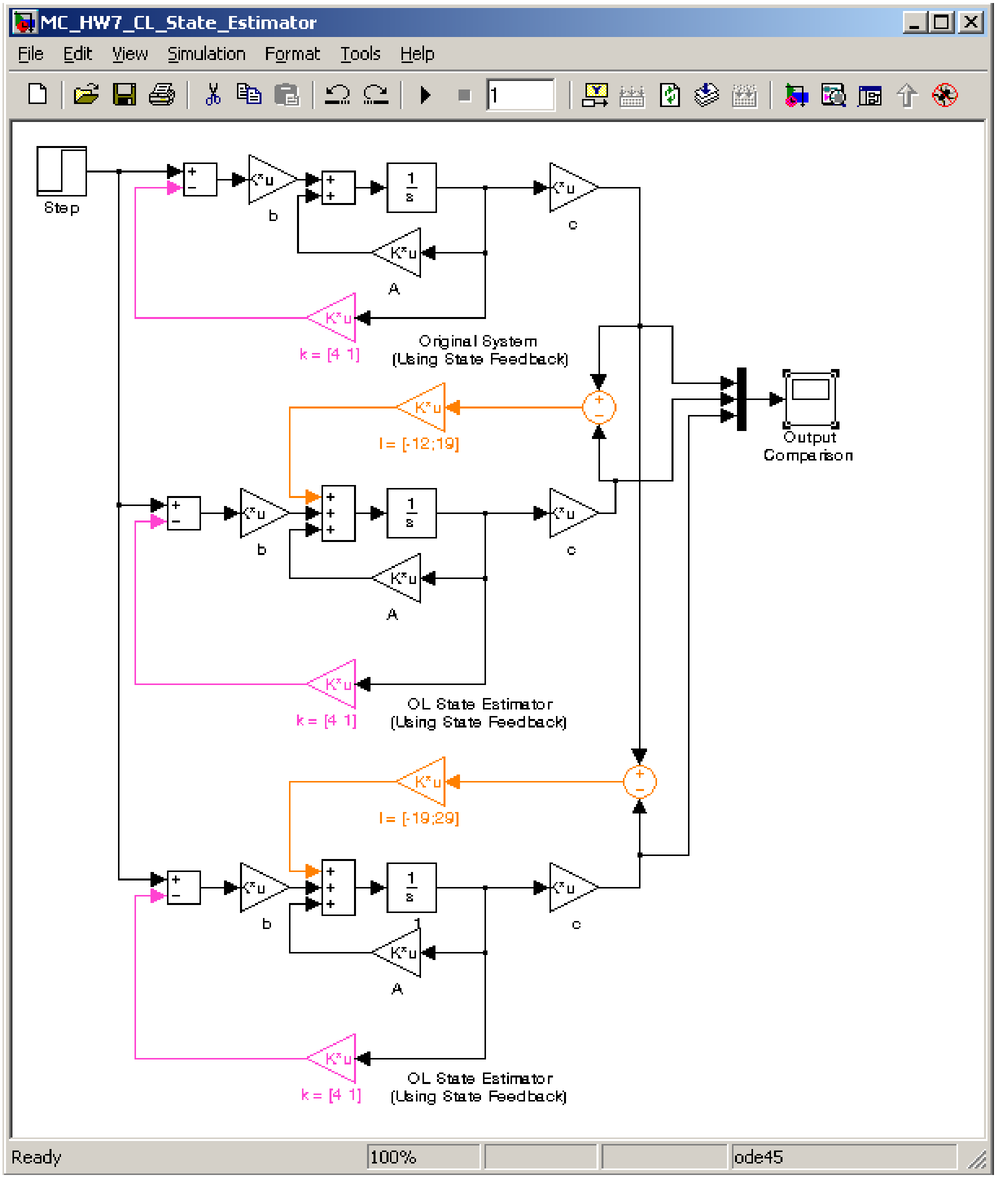
(a) Design another closed-loop state estimator, with eigenvalues at –3 and –4.

$$\begin{aligned} \alpha(s) &= \det(s\underline{\boldsymbol{I}} - \underline{\boldsymbol{A}} + \underline{\boldsymbol{l}}\underline{\boldsymbol{c}}) \\ &= \det\left(\begin{bmatrix} s-2+l_1 & -1+l_1 \\ 1+l_2 & s-1+l_2 \end{bmatrix}\right) \\ &= s^2 + (l_1 + l_2 - 3)s + (-2l_1 - l_2 + 3) \\ &\equiv (s+3)(s+4) \end{aligned}$$

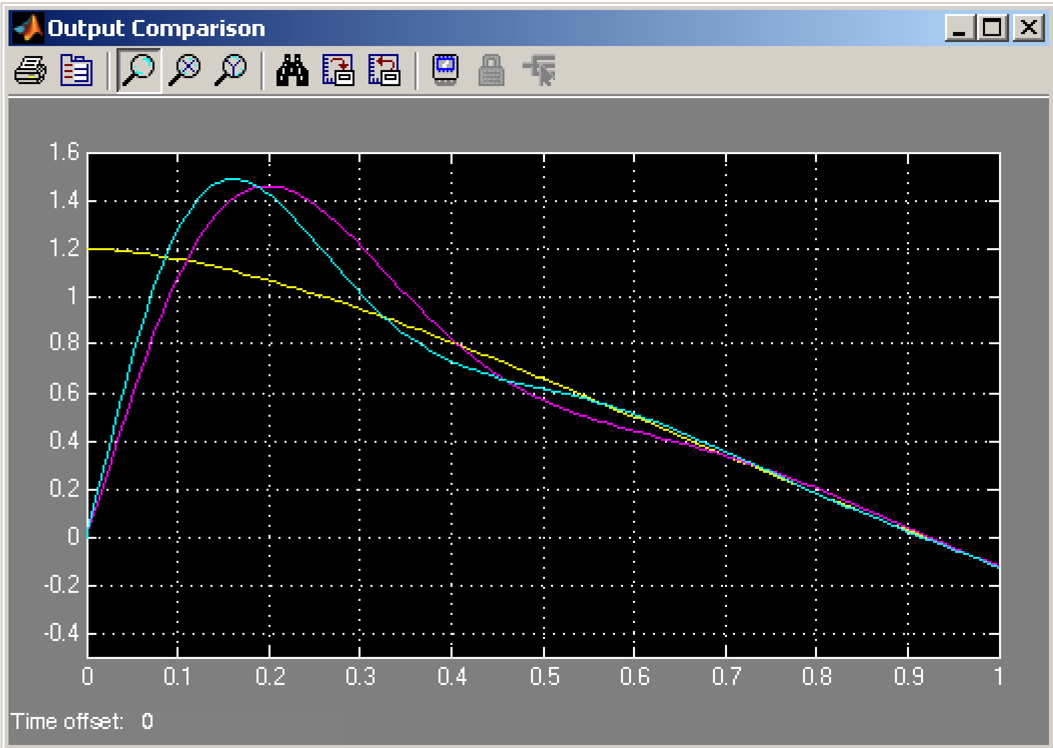
$$\begin{aligned} l_1 + l_2 - 3 &= 7 \\ -2l_1 - l_2 + 3 &= 12 \end{aligned} \qquad \underline{\boldsymbol{l}} = \begin{bmatrix} -19 \\ 29 \end{bmatrix}$$

Solution of Homework 7: State Estimators

(b) Compare the performance of both closed-loop state estimators.



Solution of Homework 7: State Estimators



: $y(t)$

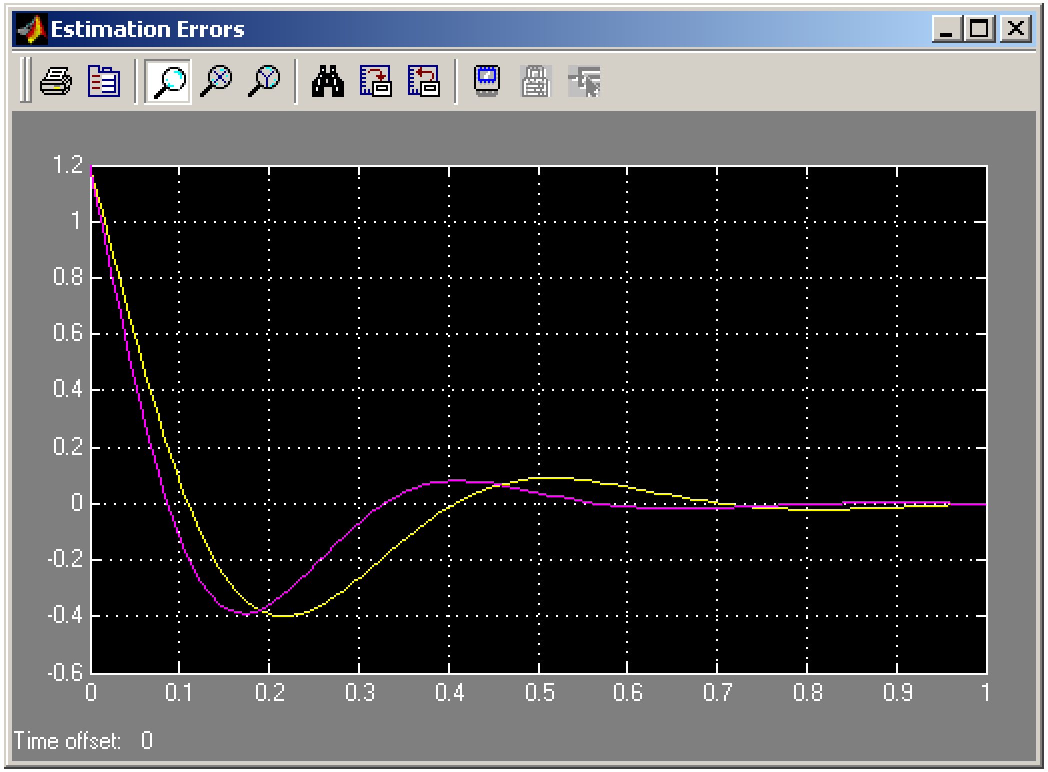
: $\hat{y}(t)$, $\underline{\boldsymbol{l}} = [-12;19]$

: $\hat{y}(t)$, $\underline{\boldsymbol{l}} = [-19;29]$

- **What is the difference?**

Solution of Homework 7: State Estimators

(c) Give some explanations of the comparison results.



: $y(t) - \hat{y}(t)$, $\underline{l} = [-12;19]$

: $y(t) - \hat{y}(t)$, $\underline{l} = [-19;29]$

- **If the eigenvalues of a closed-loop estimator lie further to the left of imaginary axis, then the estimation error decays faster**
- **A faster estimator requires larger estimator gain, larger energy, more sensitive to disturbance**

Transformation to Observer Form

- The calculation of state estimator gain $\underline{\boldsymbol{l}}$ can only be done for ***observable*** SISO systems.
- The procedure presented previously can be performed easily if the system is written in Observer Form.
- The original system needs to be transformed using a nonsingular transformation matrix $\underline{\boldsymbol{R}}$.

Transformation to Observer Form

- If the observability matrix of the open-loop system is given by:

$$\underline{\mathcal{O}} = \begin{bmatrix} \underline{C} \\ \hdashline \underline{C}\,\underline{A} \\ \hdashline \underline{C}\,\underline{A}^2 \\ \hdashline \vdots \\ \hdashline \underline{C}\,\underline{A}^{n-1} \end{bmatrix}$$

It can be shown that the required transformation matrix $\underline{\boldsymbol{R}}^{-1}$ to transform a given system to an Observer Form is given by:

$$\underline{\boldsymbol{R}}^{-1} = \underline{\mathcal{T}}\,\underline{\mathcal{O}}$$

$$\underline{\boldsymbol{R}}^{-1} = \underbrace{\begin{bmatrix} a_1 & a_2 & a_3 & \cdots & a_{n-1} & 1 \\ a_2 & a_3 & \cdots & & 1 & 0 \\ a_3 & & & \iddots & & \vdots \\ \vdots & & \iddots & & & \\ a_{n-1} & 1 & 0 & \cdots & & 0 \\ 1 & 0 & 0 & \cdots & & 0 \end{bmatrix}}_{\underline{\mathcal{T}}} \cdot \underbrace{\begin{bmatrix} \underline{C} \\ \hdashline \underline{C}\,\underline{A} \\ \hdashline \underline{C}\,\underline{A}^2 \\ \hdashline \vdots \\ \hdashline \underline{C}\,\underline{A}^{n-1} \end{bmatrix}}_{\underline{\mathcal{O}}}$$

Transformation to Observer Form

- With the pair ($\underline{A}$,$\underline{c}$) being observable, the transformation follows the equation:

$$\underline{x}(t) = \underline{R}\,\underline{z}(t)$$

so that:

$$\begin{bmatrix} \dot z_1(t) \\ \dot z_2(t) \\ \vdots \\ \dot z_n(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & \cdots & -a_0 \\ 1 & 0 & \cdots & -a_1 \\ \vdots & \ddots & & \vdots \\ 0 & 0 & 1 & -a_{n-1} \end{bmatrix}}_{\hat{\underline{A}}} \begin{bmatrix} z_1(t) \\ z_2(t) \\ \vdots \\ z_n(t) \end{bmatrix} + \underbrace{\begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_{n-1} \end{bmatrix}}_{\hat{\underline{b}}} u(t)$$

$$y(t) = \underbrace{\begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix}}_{\hat{\underline{c}}} \begin{bmatrix} z_1(t) \\ z_2(t) \\ \vdots \\ z_n(t) \end{bmatrix}$$

$$\boxed{\begin{aligned} \hat{\underline{A}} &= \underline{R}^{-1}\underline{A}\,\underline{R} \\ \hat{\underline{b}} &= \underline{R}^{-1}\underline{b} \\ \hat{\underline{c}} &= \underline{c}\,\underline{R} \end{aligned}}$$

Transformation to Observer Form

- The characteristic equation of the transformed system can easily be found as:

$$a(s) = \det(s\underline{\boldsymbol{I}} - \underline{\hat{\boldsymbol{A}}}) = s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$$

- After connecting the closed-loop state estimator, its output can be written as:

$$\dot{\underline{\hat{\boldsymbol{z}}}}(t) = \left(\underline{\hat{\boldsymbol{A}}} - \underline{\hat{\boldsymbol{l}}}\,\underline{\hat{\boldsymbol{c}}}\right)\underline{\hat{\boldsymbol{z}}}(t) + \underline{\hat{\boldsymbol{b}}}u(t) + \underline{\hat{\boldsymbol{l}}}y(t)$$

with:

$$\underline{\hat{\boldsymbol{l}}} = \begin{bmatrix} \hat{l}_1 & \hat{l}_2 & \cdots & \hat{l}_n \end{bmatrix}^{\mathrm{T}}$$

Transformation to Observer Form

- Further matrix operations yield:

$$\dot{\hat{\underline{z}}}(t)=\begin{bmatrix} 0 & 0 & \cdots & -(a_0+\hat{l}_1) \\ 1 & 0 & \cdots & -(a_1+\hat{l}_2) \\ \vdots & \ddots & & \vdots \\ 0 & 0 & 1 & -(a_{n-1}+\hat{l}_n) \end{bmatrix}\hat{\underline{z}}(t)+\hat{\underline{b}}u(t)+\hat{\underline{l}}y(t)$$

- The characteristic equation of the closed-loop estimator is now:

$$a(s)=s^n+(a_{n-1}+\hat{l}_n)s^{n-1}+(a_{n-2}+\hat{l}_{n-1})s^{n-2}+ \\ \cdots+(a_1+\hat{l}_2)s+(a_0+\hat{l}_1)$$

- If the desired poles of the closed-loop estimator are specified by $p_1, p_2, \ldots, p_n$ then:

$$\breve{a}(s)=(s-p_1)(s-p_2)\cdots(s-p_n) \\ =s^n+\breve{a}_{n-1}s^{n-1}+\cdots+\breve{a}_1 s+\breve{a}_0$$

Transformation to Observer Form

- By comparing the coefficients of the previous two polynomials, it is clear, that in order to obtain the desired characteristic equation, the feedback gain must satisfy:

$$a_0 + \hat{l}_1 = \breve{a}_0 \longrightarrow \hat{l}_1 = \breve{a}_0 - a_0$$

$$a_1 + \hat{l}_2 = \breve{a}_1 \longrightarrow \hat{l}_2 = \breve{a}_1 - a_1$$

$$\vdots \qquad\qquad \vdots$$

$$a_{n-1} + \hat{l}_n = \breve{a}_{n-1} \longrightarrow \hat{l}_n = \breve{a}_{n-1} - a_{n-1}$$

Transformation to Observer Form

$$\dot{\underline{x}}(t) = \underline{A}\underline{x}(t) + \underline{b}u(t)$$
$$y(t) = \underline{c}\underline{x}(t)$$

Original System

$$\underline{x}(t) = \underline{R}\underline{z}(t)$$

$$\dot{\underline{z}}(t) = \hat{\underline{A}}\underline{z}(t) + \hat{\underline{b}}u(t)$$
$$y(t) = \hat{\underline{c}}\underline{z}(t)$$

Equivalent System in Observer Form

Calculate the state estimator gain for the transformed system

$$\hat{\underline{l}}$$

$$\underline{z}(t) = \underline{R}^{-1}\underline{x}(t)$$

The feedback gain for the original system is obtained

$$\underline{l} = \underline{R}\hat{\underline{l}}$$

Example: Transformation to Observer Form

Let us go back to the last example.

$$\underline{\dot{\boldsymbol{x}}}(t) = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} \underline{\boldsymbol{x}}(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} \underline{\boldsymbol{x}}(t)$$

Previously in part (b), the desired closed-loop state estimator for the system should have eigenvalues at $-2 \pm j2$.

Redo part (b), now by using the transformation to the Observer Form.

Example: Transformation to Observer Form

$$a(s) = \det(s\underline{\boldsymbol{I}} - \underline{\boldsymbol{A}})$$
$$= \det\left(\begin{bmatrix} s-2 & -1 \\ 1 & s-1 \end{bmatrix}\right)$$
$$= s^2 \underbrace{-3}_{a_1}s + \underbrace{3}_{a_0}$$

$$\underline{\mathcal{O}} = \begin{bmatrix} \underline{\boldsymbol{C}} \\ \underline{\boldsymbol{C}}\,\underline{\boldsymbol{A}} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\underline{\mathcal{T}} = \begin{bmatrix} a_1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\underline{\boldsymbol{R}}^{-1} = \underline{\mathcal{T}}\,\underline{\mathcal{O}} = \begin{bmatrix} -2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\underline{\boldsymbol{R}} = \begin{bmatrix} -1 & -1 \\ 1 & 2 \end{bmatrix}$$

Checking the Observer Form,

$$\underline{\hat{\boldsymbol{A}}} = \underline{\boldsymbol{R}}^{-1}\underline{\boldsymbol{A}}\,\underline{\boldsymbol{R}} = \begin{bmatrix} 0 & -3 \\ 1 & 3 \end{bmatrix} \checkmark$$

$$\underline{\hat{\boldsymbol{c}}} = \underline{\boldsymbol{c}}\,\underline{\boldsymbol{R}} = \begin{bmatrix} 0 & 1 \end{bmatrix} \checkmark$$

Example: Transformation to Observer Form

The desired characteristic equation of the state observer is:

$$\breve{a}(s) = (s+2+j2)(s+2-j2) = s^2 + \underbrace{4}_{\breve{a}_1}s + \underbrace{8}_{\breve{a}_0}$$

$$\hat{l}_1 = \breve{a}_0 - a_0 = 8 - 3 = 5$$
$$\hat{l}_2 = \breve{a}_1 - a_1 = 4 - (-3) = 7$$

$$\underline{\hat{\boldsymbol{l}}} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

For the transformed system

$$\underline{\boldsymbol{l}} = \underline{\boldsymbol{R}}\underline{\hat{\boldsymbol{l}}} = \underline{\underline{\begin{bmatrix} -12 \\ 19 \end{bmatrix}}}$$

For the original system

Now, if the desired poles are −3 and −4, we can repeat the calculation as follows:

$$\breve{a}(s) = (s+3)(s+4) = s^2 + \underbrace{7}_{\breve{a}_1}s + \underbrace{12}_{\breve{a}_0}$$

$$\hat{l}_1 = \breve{a}_0 - a_0 = 12 - 3 = 9$$
$$\hat{l}_2 = \breve{a}_1 - a_1 = 7 - (-3) = 10$$

$$\underline{\hat{\boldsymbol{l}}} = \begin{bmatrix} 9 \\ 10 \end{bmatrix}$$

For the transformed system

$$\underline{\boldsymbol{l}} = \underline{\boldsymbol{R}}\underline{\hat{\boldsymbol{l}}} = \underline{\underline{\begin{bmatrix} -19 \\ 29 \end{bmatrix}}}$$

For the original system

Feedback of Estimated States

- The estimated states will now be used to change the behavior of the system, through state feedback.

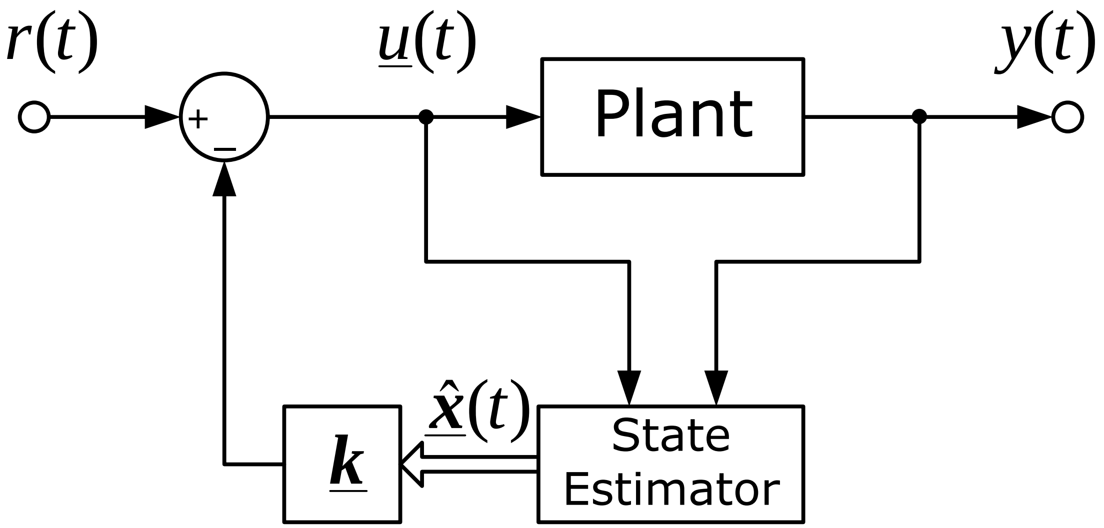
- Consider the *n*-dimensional single-variable state space equations:

$$\dot{\underline{\mathbf{x}}}(t) = \underline{\mathbf{A}}\underline{\mathbf{x}}(t) + \underline{\mathbf{b}}u(t)$$

$$y(t) = \underline{\mathbf{c}}\underline{\mathbf{x}}(t)$$

- If the pair ($\underline{\boldsymbol{A}}$,$\underline{\boldsymbol{b}}$) is ***controllable***, then the state feedback $u(t) = r(t) - \underline{\boldsymbol{kx}}(t)$ can place the eigenvalues of ($\underline{\boldsymbol{A}}-\underline{\boldsymbol{bk}}$) in any desired positions.
- If the state variables are not available for feedback, then a state estimator with arbitrary eigenvalues can be designed for the system, provided that the pair ($\underline{\boldsymbol{A}}$,$\underline{\boldsymbol{c}}$) is ***observable***.

Feedback of Estimated States



"Controller-Estimator Configuration"

Feedback of Estimated States

- We recall again the state equation of the state estimator:

$$\dot{\hat{\underline{\mathbf{x}}}}(t) = (\underline{\mathbf{A}} - \underline{\mathbf{l}}\,\underline{\mathbf{c}})\hat{\underline{\mathbf{x}}}(t) + \underline{\mathbf{b}}u(t) + \underline{\mathbf{l}}y(t)$$

- The rate of how the estimated states $\hat{\underline{\mathbf{x}}}(t)$ approach the actual states $\underline{\mathbf{x}}(t)$ can be adjusted by selecting an appropriate value for matrix $\underline{\mathbf{l}}$.

- Because $\underline{\mathbf{x}}(t)$ is not available in the configuration, it is replaced by $\hat{\underline{\mathbf{x}}}(t)$ for feedback:

$$u(t) = r(t) - \underline{\mathbf{k}}\,\hat{\underline{\mathbf{x}}}(t)$$

Feedback of Estimated States

- Substituting the last equation to the original system and the state estimator, will yield:

$$\dot{\underline{x}}(t) = \underline{A}\underline{x}(t) - \underline{b}\underline{k}\hat{\underline{x}}(t) + \underline{b}r(t)$$

$$\dot{\hat{\underline{x}}}(t) = \left(\underline{A} - \underline{l}\underline{c} - \underline{b}\underline{k}\right)\hat{\underline{x}}(t) + \underline{b}r(t) + \underline{l}y(t)$$

- The two equations above can be combined in a new state space equation in the form of:

$$\begin{bmatrix} \dot{\underline{x}}(t) \\ \dot{\hat{\underline{x}}}(t) \end{bmatrix} = \begin{bmatrix} \underline{A} & -\underline{b}\underline{k} \\ \underline{l}\underline{c} & \underline{A} - \underline{l}\underline{c} - \underline{b}\underline{k} \end{bmatrix} \begin{bmatrix} \underline{x}(t) \\ \hat{\underline{x}}(t) \end{bmatrix} + \begin{bmatrix} \underline{b} \\ \underline{b} \end{bmatrix} r(t)$$

$$y(t) = \begin{bmatrix} \underline{c} & \underline{0} \end{bmatrix} \begin{bmatrix} \underline{x}(t) \\ \hat{\underline{x}}(t) \end{bmatrix}$$

Feedback of Estimated States

- To analyze these closed-loop systems, it is convenient to change the state variables by using the following transformation:

$$\begin{bmatrix} \underline{\mathbf{x}}(t) \\ \underline{\mathbf{e}}(t) \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{x}}(t) \\ \underline{\mathbf{x}}(t) - \underline{\hat{\mathbf{x}}}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} \underline{\mathbf{I}} & \underline{\mathbf{0}} \\ \underline{\mathbf{I}} & -\underline{\mathbf{I}} \end{bmatrix}}_{\underline{\mathbf{P}}} \begin{bmatrix} \underline{\mathbf{x}}(t) \\ \underline{\hat{\mathbf{x}}}(t) \end{bmatrix}$$

$$\underline{\mathbf{P}}, \quad \underline{\mathbf{P}}^{-1} = \underline{\mathbf{P}}$$

- After performing the equivalence transformation, the following state equations can be obtained:

$$\begin{bmatrix} \underline{\dot{\mathbf{x}}}(t) \\ \underline{\dot{\mathbf{e}}}(t) \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{A}} - \underline{\mathbf{b}}\underline{\mathbf{k}} & \underline{\mathbf{b}}\underline{\mathbf{k}} \\ \underline{\mathbf{0}} & \underline{\mathbf{A}} - \underline{\mathbf{l}}\underline{\mathbf{c}} \end{bmatrix} \begin{bmatrix} \underline{\mathbf{x}}(t) \\ \underline{\mathbf{e}}(t) \end{bmatrix} + \begin{bmatrix} \underline{\mathbf{b}} \\ \underline{\mathbf{0}} \end{bmatrix} r(t)$$

$$y(t) = \begin{bmatrix} \underline{\mathbf{c}} & \underline{\mathbf{0}} \end{bmatrix} \begin{bmatrix} \underline{\mathbf{x}}(t) \\ \underline{\mathbf{e}}(t) \end{bmatrix}$$

Feedback of Estimated States

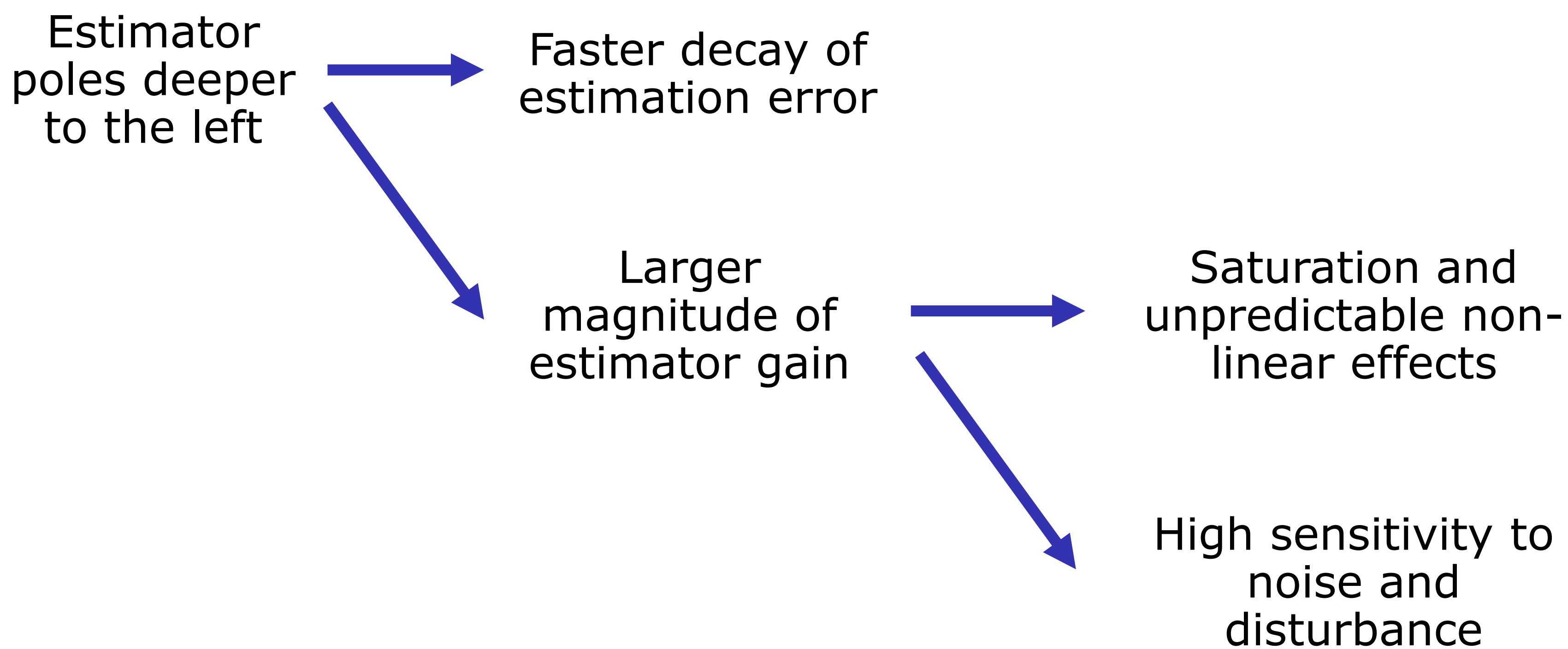
- The eigenvalues of the new system in the "controller-estimator configuration" is the union of those of ($\underline{\boldsymbol{A}}$–$\underline{\boldsymbol{bk}}$) and ($\underline{\boldsymbol{A}}$–$\underline{\boldsymbol{lc}}$).
- This fact means, that the implementation of the state estimator does not affect the eigenvalues of the system with state feedback, and vice versa.
- The design of state feedback and state estimator are separated from each other. This is known as "**separation principle**" or "**separation property**."

$$\begin{bmatrix} \dot{\underline{\boldsymbol{x}}}(t) \\ \dot{\underline{\boldsymbol{e}}}(t) \end{bmatrix} = \begin{bmatrix} \underline{\boldsymbol{A}} - \underline{\boldsymbol{b}}\underline{\boldsymbol{k}} & \underline{\boldsymbol{b}}\underline{\boldsymbol{k}} \\ \underline{\boldsymbol{0}} & \underline{\boldsymbol{A}} - \underline{\boldsymbol{l}}\underline{\boldsymbol{c}} \end{bmatrix} \begin{bmatrix} \underline{\boldsymbol{x}}(t) \\ \underline{\boldsymbol{e}}(t) \end{bmatrix} + \begin{bmatrix} \underline{\boldsymbol{b}} \\ \underline{\boldsymbol{0}} \end{bmatrix} r(t)$$

$$y(t) = \begin{bmatrix} \underline{\boldsymbol{c}} & \underline{\boldsymbol{0}} \end{bmatrix} \begin{bmatrix} \underline{\boldsymbol{x}}(t) \\ \underline{\boldsymbol{e}}(t) \end{bmatrix}$$

Feedback of Estimated States

- [Franklin, Powell, Emami-Naeini] recommends that the real parts of the state estimator poles be a **factor of 2 to 6 deeper** in the left-half plane than the real parts of the state feedback poles.



Example 1: Feedback of Estimated States

A system is given in state space form as below:

$$\dot{\underline{x}}(t)=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\underline{x}(t)+\begin{bmatrix}0\\ 1\end{bmatrix}u(t)$$

$$y(t)=\begin{bmatrix}1 & 0\end{bmatrix}\underline{x}(t)$$

Thus, ($\underline{\boldsymbol{A}}$,$\underline{\boldsymbol{b}}$) is controllable and ($\underline{\boldsymbol{A}}$,$\underline{\boldsymbol{c}}$) is observable.

(a) The desired poles for the state feedback is a pair of complex poles with $\omega_n = 2$ and $\zeta = 0.5$. Prove that the required feedback gain is $\underline{\boldsymbol{k}} = [4 \;\; 2]$.

(b) The desired poles for the state observer is a pair of complex poles with $\omega_n = 10$ and $\zeta = 0.5$. Prove that the required feedback gain is $\underline{\boldsymbol{l}} = [10 \;\; 100]^T$.

(c) Write down the equations which govern the controller-estimator configuration.

Example 1: Feedback of Estimated States

(a) & (b) left for exercise.

(c) Write down the equations which govern the controller-estimator configuration.

$$\dot{\hat{\underline{\boldsymbol{x}}}}(t) = (\underline{\boldsymbol{A}} - \underline{\boldsymbol{l}}\underline{\boldsymbol{c}})\hat{\underline{\boldsymbol{x}}}(t) + \underline{\boldsymbol{b}}u(t) + \underline{\boldsymbol{l}}y(t)$$

$$= \left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 10 \\ 100 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} \right) \hat{\underline{\boldsymbol{x}}}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 10 \\ 100 \end{bmatrix} y(t)$$

$$= \begin{bmatrix} -10 & 1 \\ -100 & 0 \end{bmatrix} \hat{\underline{\boldsymbol{x}}}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 10 \\ 100 \end{bmatrix} y(t)$$

$$u(t) = r(t) - \underline{\boldsymbol{k}}\,\hat{\underline{\boldsymbol{x}}}(t)$$

$$= r(t) - \begin{bmatrix} 4 & 2 \end{bmatrix} \hat{\underline{\boldsymbol{x}}}(t)$$

$$\dot{\hat{x}}_1(t) = \hat{x}_2(t) + 10(y(t) - \hat{x}_1(t))$$

$$\dot{\hat{x}}_2(t) = u(t) + 100(y(t) - \hat{x}_1(t))$$

$$u(t) = r(t) - 4\hat{x}_1(t) - 2\hat{x}_2(t)$$

Homework 8

Consider the following linear system given by:

$$\dot{\underline{\boldsymbol{x}}}(t) = \begin{bmatrix} -1 & 2 & 0 \\ 1 & -3 & 4 \\ -1 & 1 & -9 \end{bmatrix} \underline{\boldsymbol{x}}(t) + \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \underline{\boldsymbol{x}}(t)$$

(a) Using the transformation to the Observer Form, find the gain vector $\underline{\boldsymbol{l}}$ of the closed-loop state estimator if the desired poles are −3 and −4 ± j2.

(b) Recall again the output feedback. In observer form, its effect on the characteristic equation of the system can be calculated much easier. By calculation, prove that the poles of the system cannot be assigned to any arbitrary location by only setting the value of output feedback j.

Homework 8A

Consider the following linear system given by:

$$\underline{\dot{\boldsymbol{x}}}(t) = \begin{bmatrix} -6 & -11 & -6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \underline{\boldsymbol{x}}(t) + \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} \underline{\boldsymbol{x}}(t)$$

(a) Calculate the eigenvalues and eigenvectors of the system. Is it stable or unstable?

(b) Using the transformation to the Observer Form, find the gain vector $\underline{\boldsymbol{l}}$ of the closed-loop state estimator if the desired poles are $-1 \pm j2.5$ and -3.