# State Feedback and State Estimators

## “Linear System Theory and Design”, Chapter 8

**http://zitompul.wordpress.com**

**2 0 1 4**

# Homework 4

Again, consider a SISO system with the state equations:

$$\dot{\underline{\mathbf{x}}}(t) = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \underline{\mathbf{x}}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 2 \end{bmatrix} \underline{\mathbf{x}}(t)$$

a. If the state feedback in the form of:

$$u(t) = r(t) - \begin{bmatrix} k_1 & k_2 \end{bmatrix} \underline{\mathbf{x}}(t)$$

is implemented to the system and it is wished that the poles of the system will be –3 and –4, determine the value of $k_1$ and $k_2$.

b. Find the transfer function of the system and again, check the location of the poles of the transfer function.

# Solution of Homework 4

a. With the state feedback:

$$u(t) = r(t) - \begin{bmatrix} k_1 & k_2 \end{bmatrix} \underline{\boldsymbol{x}}(t)$$

The state equations become:

$$\begin{aligned}
\dot{\underline{\boldsymbol{x}}}(t) &= \left(\underline{\boldsymbol{A}} - \underline{\boldsymbol{b}}\underline{\boldsymbol{k}}\right)\underline{\boldsymbol{x}}(t) + \underline{\boldsymbol{b}}r(t) \\
&= \left(\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{matrix} k_1 & k_2 \end{matrix}\right)\underline{\boldsymbol{x}}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t) \\
&= \left(\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ k_1 & k_2 \end{bmatrix}\right)\underline{\boldsymbol{x}}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t) \\
&= \left(\begin{bmatrix} 1 & 3 \\ 3 - k_1 & 1 - k_2 \end{bmatrix}\right)\underline{\boldsymbol{x}}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t)
\end{aligned}$$

# Solution of Homework 4

The characteristic equation is:

$$a(s) = \det(s\underline{\boldsymbol{I}} - (\underline{\boldsymbol{A}} - \underline{\boldsymbol{b}}\underline{\boldsymbol{k}}))$$
$$= \det\left(\begin{bmatrix} s-1 & -3 \\ -(3-k_1) & s-(1-k_2) \end{bmatrix}\right)$$
$$= (s-1)(s-1+k_2) - (-3)(-(3-k_1))$$
$$= s^2 + (k_2-2)s + (3k_1 - k_2 - 8)$$

The wished poles are –3 and –4, corresponding with the wished characteristic equation of:

$$\breve{a}(s) = (s+3)(s+4)$$
$$= s^2 + 7s + 12$$

Comparing $a(s)$ and $\breve{a}(s)$, we obtain:

$$(k_2 - 2) \equiv 7 \quad \Rightarrow k_2 = \underline{\underline{9}}$$
$$(3k_1 - k_2 - 8) \equiv 12 \quad \Rightarrow k_1 = 29/3 = \underline{\underline{9.67}}$$

# Solution of Homework 4

b. The transfer function of the system can be found as:

$$G(s)=\frac{Y(s)}{R(s)}=\underline{c}(s\underline{I}-(\underline{A}-\underline{b}\underline{k}))^{-1}\underline{b}$$

$$=\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}s-1 & -3\\ -(3-k_1) & s-(1-k_2)\end{bmatrix}^{-1}\begin{bmatrix}0\\1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}s-1 & -3\\ 6.67 & s+8\end{bmatrix}^{-1}\begin{bmatrix}0\\1\end{bmatrix}$$

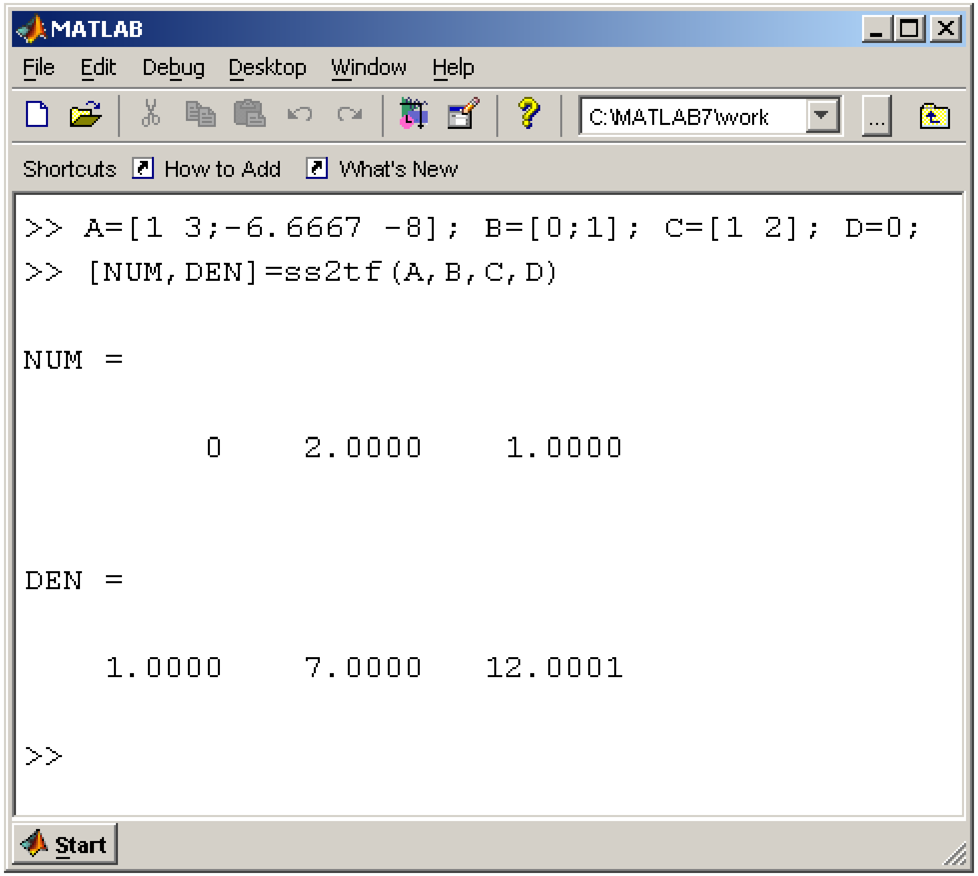
$$=\frac{\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}s+8 & 3\\ -6.67 & s-1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}}{s^2+7s+12}$$

$$=\frac{2s+1}{s^2+7s+12}$$

$$=\frac{2s+1}{(s+3)(s+4)}$$

# Solution of Homework 4

Using Matlab, the following function can be utilized:



# State Feedback

- Consider the *n*-dimensional single-variable state space equations:

$$\dot{\underline{x}}(t) = \underline{A}\underline{x}(t) + \underline{b}u(t)$$
$$y(t) = \underline{c}\underline{x}(t)$$

- For this SISO system, if the pair ($\underline{A}$,$\underline{b}$) is ***controllable***, there exists a nonsingular transformation matrix $\underline{Q}$ such that:

$$\underline{x}(t) = \underline{Q}\underline{z}(t)$$

and:

$$\dot{\underline{z}}(t) = \hat{\underline{A}}\underline{z}(t) + \hat{\underline{b}}u(t)$$
$$y(t) = \hat{\underline{c}}\underline{z}(t)$$

- **State space in Frobenius Form**

with the matrices $\hat{\underline{A}}$ and $\hat{\underline{b}}$ given by:

$$\hat{\underline{A}} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & & \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix}, \quad \hat{\underline{b}} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

# State Feedback

- For the original system, the coefficients $a_i$ are the coefficients of the characteristic equation of $\underline{\boldsymbol{A}}$, that is:

$$a(s) = \det(s\underline{\boldsymbol{I}} - \underline{\boldsymbol{A}}) = s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$$

- Meanwhile, the state feedback for the transformed system is given by:

$$u(t) = r(t) - \underline{\hat{\boldsymbol{k}}}\,\underline{\boldsymbol{z}}(t)$$

with:

$$\underline{\hat{\boldsymbol{k}}} = \begin{bmatrix} \hat{k}_1 & \hat{k}_2 & \cdots & \hat{k}_n \end{bmatrix}$$

- Substituting $u(t)$ into the transformed system:

$$\underline{\dot{\boldsymbol{z}}}(t) = \underline{\hat{\boldsymbol{A}}}\,\underline{\boldsymbol{z}}(t) + \underline{\hat{\boldsymbol{b}}}\left(r(t) - \underline{\hat{\boldsymbol{k}}}\,\underline{\boldsymbol{z}}(t)\right)$$

$$\underline{\dot{\boldsymbol{z}}}(t) = \left(\underline{\hat{\boldsymbol{A}}} - \underline{\hat{\boldsymbol{b}}}\,\underline{\hat{\boldsymbol{k}}}\right)\underline{\boldsymbol{z}}(t) + \underline{\hat{\boldsymbol{b}}}\,r(t)$$

# State Feedback

$$\dot{\underline{z}}(t) = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & & \\ 0 & 0 & 0 & \cdots & 1 \\ -(a_0 + \hat{k}_1) & -(a_1 + \hat{k}_2) & -(a_2 + \hat{k}_3) & \cdots & -(a_{n-1} + \hat{k}_n) \end{bmatrix} \underline{z}(t) + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} r(t)$$

- The characteristic equation of the closed-loop system is:

$$a(s) = s^n + (a_{n-1} + \hat{k}_n)s^{n-1} + (a_{n-2} + \hat{k}_{n-1})s^{n-2} + \cdots + (a_1 + \hat{k}_2)s + (a_0 + \hat{k}_1)$$

- If the desired closed-loop poles are specified by $p_1$, $p_2$, …,$p_n$ then:

$$\breve{a}(s) = (s - p_1)(s - p_2)\cdots(s - p_n) = s^n + \breve{a}_{n-1}s^{n-1} + \cdots + \breve{a}_1 s + \breve{a}_0$$

# State Feedback

- By comparing the coefficients of the previous two polynomials, it is clear, that in order to obtain the desired characteristic equation, the feedback gain must satisfy:

$$a_0 + \hat{k}_1 = \breve{a}_0 \quad \longrightarrow \quad \hat{k}_1 = \breve{a}_0 - a_0$$

$$a_1 + \hat{k}_2 = \breve{a}_1 \quad \longrightarrow \quad \hat{k}_2 = \breve{a}_1 - a_1$$

$$\vdots \qquad\qquad\qquad\qquad \vdots$$

$$a_{n-1} + \hat{k}_n = \breve{a}_{n-1} \quad \longrightarrow \quad \hat{k}_n = \breve{a}_{n-1} - a_{n-1}$$

# Example 1: State Feedback

Consider the following system given in Frobenius Form

$$\underline{\dot{z}}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -5 & -10 \end{bmatrix} \underline{z}(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

It is required that the closed-loop system has the eigenvalues located at $s=-1\pm j$ and $s=-5$.

Find the feedback gain vector $\hat{\underline{k}}$.

$$\begin{aligned} \breve{a}(s) &= (s+(1-j))\cdot(s+(1+j))\cdot(s+5) \\ &= ((s+1)-j)\cdot((s+1)+j)\cdot(s+5) \\ &= ((s+1)^2+1)\cdot(s+5) \\ &= (s^2+2s+2)\cdot(s+5) \\ &= s^3+\underbrace{7}_{\breve{a}_2}s^2+\underbrace{12}_{\breve{a}_1}s+\underbrace{10}_{\breve{a}_0} \end{aligned}$$

From the state equations we can find out that:

$$a_0 = 2 \longrightarrow \hat{k}_1 = \breve{a}_0 - a_0 = 8$$

$$a_1 = 5 \longrightarrow \hat{k}_2 = \breve{a}_1 - a_1 = 7$$

$$a_2 = 10 \longrightarrow \hat{k}_3 = \breve{a}_2 - a_2 = -3$$

$$\underline{\hat{k}} = \underline{\underline{\begin{bmatrix} 8 & 7 & -3 \end{bmatrix}}}$$

# Example 2: State Feedback

Given

$$\underline{\boldsymbol{A}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \underline{\boldsymbol{b}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Design the feedback gain $\underline{\boldsymbol{k}}$ so that the eigenvalues of $\underline{\boldsymbol{A}} - \underline{\boldsymbol{b}}\,\underline{\boldsymbol{k}}$ are at $-1$ and $-1 \pm j1$.

$$a(s) = s^3 - s^2 - 1$$
$$\breve{a}(s) = s^3 + 3s^2 + 4s + 2$$
$$\underline{\boldsymbol{k}} = \begin{bmatrix} 3 & 4 & 4 \end{bmatrix}$$

# Transformation to Frobenius Form

- By performing the procedure presented previously, we are be able to place the poles of a controllable SISO system in any location so easily.
- **The condition**: The system is written in Frobenius Form.
- In order to be able to apply this procedure to any controllable SISO systems easily, we need to transform the systems to Frobenius Form first.
- That means, we need to know the nonsingular transformation matrix $\underline{\boldsymbol{Q}}$.

# Transformation to Frobenius Form

- If the controllability matrix of the open-loop system is given by:

$$\underline{\mathcal{C}} = \left[ \underline{\boldsymbol{b}} \;\vdots\; \underline{\boldsymbol{A}}\underline{\boldsymbol{b}} \;\vdots\; \underline{\boldsymbol{A}}^2\underline{\boldsymbol{b}} \;\vdots\; \cdots \;\vdots\; \underline{\boldsymbol{A}}^{n-1}\underline{\boldsymbol{b}} \right]$$

It can be shown that the required transformation matrix $\underline{\boldsymbol{Q}}$ to transform the system to a Frobenius Form is given by:

$$\underline{\boldsymbol{Q}} = \left[ \underline{\boldsymbol{q}}_1 \;\vdots\; \underline{\boldsymbol{q}}_2 \;\vdots\; \underline{\boldsymbol{q}}_3 \;\vdots\; \cdots \;\vdots\; \underline{\boldsymbol{q}}_n \right]$$

where

$$\begin{aligned}
\underline{\boldsymbol{q}}_n &= \underline{\boldsymbol{b}} \\
\underline{\boldsymbol{q}}_{n-1} &= \underline{\boldsymbol{A}}\underline{\boldsymbol{q}}_n + a_{n-1}\underline{\boldsymbol{q}}_n = \underline{\boldsymbol{A}}\underline{\boldsymbol{b}} + a_{n-1}\underline{\boldsymbol{b}} \\
\underline{\boldsymbol{q}}_{n-2} &= \underline{\boldsymbol{A}}\underline{\boldsymbol{q}}_{n-1} + a_{n-2}\underline{\boldsymbol{q}}_n = \underline{\boldsymbol{A}}^2\underline{\boldsymbol{b}} + a_{n-1}\underline{\boldsymbol{A}}\underline{\boldsymbol{b}} + a_{n-2}\underline{\boldsymbol{b}} \\
&\;\;\vdots \qquad\qquad\qquad\qquad \vdots \\
\underline{\boldsymbol{q}}_1 &= \underline{\boldsymbol{A}}\underline{\boldsymbol{q}}_2 + a_1\underline{\boldsymbol{q}}_n = \underline{\boldsymbol{A}}^{n-1}\underline{\boldsymbol{b}} + a_{n-1}\underline{\boldsymbol{A}}^{n-2}\underline{\boldsymbol{b}} + \cdots + a_2\underline{\boldsymbol{A}}\underline{\boldsymbol{b}} + a_1\underline{\boldsymbol{b}}
\end{aligned}$$

- **$a_i$ are the coefficients of the characteristic polynomial $a(s)$ of matrix $\underline{A}$**

# Transformation to Frobenius Form

- In matrix form, the set of equations can be formulated as:

$$\underline{Q} = \left[ \underline{q}_1 \;\vdots\; \underline{q}_2 \;\vdots\; \underline{q}_3 \;\vdots\; \cdots \;\vdots\; \underline{q}_n \right]$$

$$\underline{Q} = \underbrace{\left[ \underline{b} \;\vdots\; \underline{A}\underline{b} \;\vdots\; \underline{A}^2\underline{b} \;\vdots\; \cdots \;\vdots\; \underline{A}^{n-1}\underline{b} \right]}_{\underline{\mathcal{C}}} \cdot \underbrace{\begin{bmatrix} a_1 & a_2 & a_3 & \cdots & a_{n-1} & 1 \\ a_2 & a_3 & \cdots & & 1 & 0 \\ a_3 & & & \iddots & & \vdots \\ \vdots & & \iddots & & & \\ a_{n-1} & 1 & 0 & \cdots & & 0 \\ 1 & 0 & 0 & \cdots & & 0 \end{bmatrix}}_{\underline{\mathcal{T}}}$$

$$\underline{Q} = \underline{\mathcal{C}}\,\underline{\mathcal{T}}$$

- $a_i$ **are the coefficients of the characteristic polynomial** $a(s)$ **of matrix** $\underline{A}$

# Transformation to Frobenius Form

$$\underline{\dot{x}}(t) = \underline{A}\,\underline{x}(t) + \underline{b}u(t)$$
$$y(t) = \underline{c}\,\underline{x}(t)$$

**Original System**

$$\underline{x}(t) = \underline{Q}\,\underline{z}(t)$$

$$\underline{\dot{z}}(t) = \underline{\hat{A}}\,\underline{z}(t) + \underline{\hat{b}}u(t)$$
$$y(t) = \underline{\hat{c}}\,\underline{z}(t)$$

**Equivalent System in Frobenius Form**

Calculate the feedback gain for the transformed system

$$\underline{\hat{k}}$$

$$\underline{z}(t) = \underline{Q}^{-1}\underline{x}(t)$$

The feedback gain for the original system is obtained

$$\underline{k} = \underline{\hat{k}}\,\underline{Q}^{-1}$$

# Example 3: Transformation

Two poles at −1 are wished for the following system:

$$\underline{\dot{\boldsymbol{x}}}(t)=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\underline{\boldsymbol{x}}(t)+\begin{bmatrix}1\\ 0\end{bmatrix}u(t)$$

Calculate the required $\underline{\boldsymbol{k}}$.

- Find the characteristic equation

$$a(s)=\det(s\underline{\boldsymbol{I}}-\underline{\boldsymbol{A}})=\det\left(\begin{bmatrix}s-1 & -1\\ -1 & s-1\end{bmatrix}\right)$$
$$=(s-1)(s-1)-(-1)(-1)$$
$$=s(s-2)$$

- **unstable**

$$=s^2-2s \qquad a_0=0,\quad a_1=-2$$

- Calculate $\underline{\mathcal{C}}$, $\underline{\mathcal{T}}$

$$\underline{\mathcal{C}}=\left[\underline{\boldsymbol{b}}\ \vdots\ \underline{\boldsymbol{A}}\underline{\boldsymbol{b}}\right]=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix},$$

$$\underline{\mathcal{T}}=\begin{bmatrix}a_1 & 1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}-2 & 1\\ 1 & 0\end{bmatrix}$$

# Example 3: Transformation

- Calculate $\underline{Q}$

$$\underline{Q} = \underline{\mathcal{C}}\,\underline{\mathcal{T}} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\underline{Q}^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

- Perform transformation

$$\hat{\underline{A}} = \underline{Q}^{-1}\underline{A}\,\underline{Q} = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$$

$$\hat{\underline{b}} = \underline{Q}^{-1}\underline{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- Calculate $\hat{\underline{k}}$

$$\breve{a}(s) = (s+1)\cdot(s+1)$$
$$= s^2 + 2s + 1 \qquad \breve{a}_0 = 1 \qquad \hat{k}_1 = \breve{a}_0 - a_0 = 1$$
$$\breve{a}_1 = 2 \qquad \hat{k}_2 = \breve{a}_1 - a_1 = 4$$
$$\hat{\underline{k}} = \begin{bmatrix} 1 & 4 \end{bmatrix}$$

- Calculate $\underline{k}$

$$\underline{k} = \hat{\underline{k}}\,\underline{Q}^{-1} = \begin{bmatrix} 1 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 5 \end{bmatrix}$$

- **Check the new characteristic equation**

# Example 4: Transformation

Given

$$\underline{\boldsymbol{A}} = \begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix} \quad \text{and} \quad \underline{\boldsymbol{b}} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Design the feedback gain $\underline{\boldsymbol{k}}$ so that the eigenvalues of $\underline{\boldsymbol{A}} - \underline{\boldsymbol{b}}\,\underline{\boldsymbol{k}}$ are −1, −1, and −1.

$$\underline{\boldsymbol{k}} = \begin{bmatrix} 0.8 & 1.4 & 1.2 \end{bmatrix}$$

# Example 5: Transformation

Referring back to Example 3, where the complete state space equation of the last system is

$$\dot{\underline{x}}(t)=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\underline{x}(t)+\begin{bmatrix}1\\ 0\end{bmatrix}u(t)$$

$$y(t)=\begin{bmatrix}-1.5 & 0.5\end{bmatrix}\underline{x}(t)$$

and the required $\underline{k}$ is already known to be $\underline{k}=[4\ 5]$, calculate the steady state response of the system to unit step reference trajectory.

$$\dot{\underline{x}}(t)=\left(\underline{A}-\underline{b}\underline{k}\right)\underline{x}(t)+\underline{b}r(t)$$

$$=\left(\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}-\begin{bmatrix}1\\ 0\end{bmatrix}\begin{bmatrix}4 & 5\end{bmatrix}\right)\underline{x}(t)+\begin{bmatrix}1\\ 0\end{bmatrix}r(t)$$

$$=\begin{bmatrix}-3 & -4\\ 1 & 1\end{bmatrix}\underline{x}(t)+\begin{bmatrix}1\\ 0\end{bmatrix}r(t)$$

$$y(t)=\begin{bmatrix}-1.5 & 0.5\end{bmatrix}\underline{x}(t)$$

# Example 5: Transformation

$$G(s)=\frac{Y(s)}{R(s)}=\underline{c}(s\underline{I}-(\underline{A}-\underline{b}\underline{k}))^{-1}\underline{b} =\begin{bmatrix}-1.5 & 0.5\end{bmatrix}\begin{bmatrix}s+3 & 4\\ -1 & s-1\end{bmatrix}^{-1}\begin{bmatrix}1\\0\end{bmatrix}$$

$$=\frac{\begin{bmatrix}-1.5 & 0.5\end{bmatrix}\begin{bmatrix}s-1 & -4\\ 1 & s+3\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}}{s^2+2s+1}$$

$$=\frac{-1.5s+2}{s^2+2s+1}$$

Using the Final Value Theorem to find the steady state response,

$$y(\infty)=\lim_{t\to\infty}y(t)=\lim_{s\to 0}\, s\cdot Y(s)=\lim_{s\to 0}\, s\cdot G(s)R(s)$$

$$=\lim_{s\to 0}s\cdot\frac{-1.5s+2}{s^2+2s+1}\cdot\frac{1}{s}\;=\underline{\underline{2}}$$

- **The reference input is unit step 1(*t*), but the output is 2·1(*t*).**
- **There is an offset, a non-decaying difference between set point and output.**

# Tracking a Step Input Reference

- Given a transfer function that relates the set point and the output,

$$G(s)=\frac{Y(s)}{R(s)}=\frac{b_m s^m + b_{m-1}s^{m-1}+\cdots+b_1 s+b_0}{s^n + a_{n-1}s^{n-1}+\cdots+a_1 s+a_0}$$

- The dc-gain of the transfer function is given by:

$$G(0)=\frac{b_0}{a_0}$$

- If $G(0) = 1$, then $r(t)-y(t)=0$ → the tracking is perfect.
- If $G(0) \neq 1$, then $r(t)-y(t)\neq 0$ → tracking error exists.

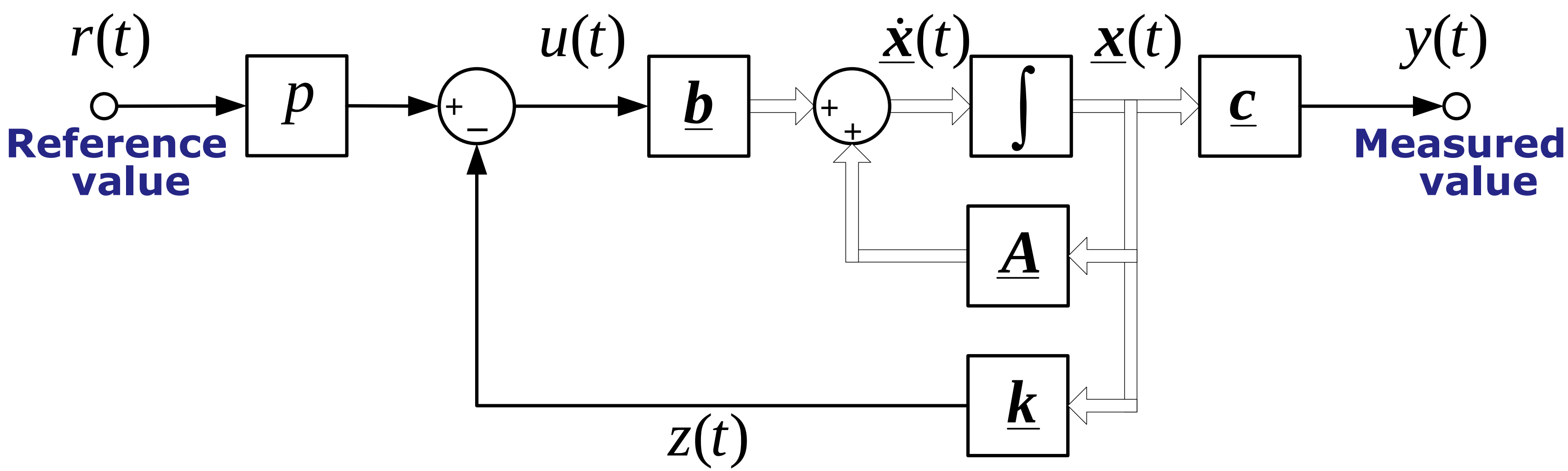
# Tracking a Step Input Reference

- To correct the tracking error, a feedforward gain $p$ needs to be introduced,

$$\hat{G}(s) = \frac{\hat{Y}(s)}{\hat{R}(s)} = p\frac{b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$

with the value of $p$ is chosen to be $p = a_0/b_0$

- By giving the feedforward gain, $\hat{G}(0) = 1$ → the tracking error is corrected.



# Homework 5

A state-space equation of a third-order system is given as:

$$\underline{\dot{\mathbf{x}}}(t)=\begin{bmatrix}-1 & 0 & 0\\ 0 & -2 & 0\\ 0 & 0 & -3\end{bmatrix}\underline{\mathbf{x}}(t)+\begin{bmatrix}1\\1\\1\end{bmatrix}u(t)$$

$$y(t)=\begin{bmatrix}6 & -6 & 1\end{bmatrix}\underline{\mathbf{x}}(t)$$

a. Perform a step-by-step transformation of the given model to Frobenius Form.

b. Calculate the required feedback gain $\underline{\boldsymbol{k}}$ so that the system may have two conjugate poles at $-2\pm j1$ and $-4$.

# Homework 5A

A state-space equation of a third-order system is given as:

$$\underline{\dot{\boldsymbol{x}}}(t) = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \underline{\boldsymbol{x}}(t) + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 6 & 6 & -1 \end{bmatrix} \underline{\boldsymbol{x}}(t)$$

a. Perform a step-by-step transformation of the given model to Frobenius Form.

b. Calculate the required feedback gain $\underline{\boldsymbol{k}}$ so that the system may have two conjugate poles at $-1 \pm j3$ and $-2$.