Alexandria University
Faculty of Engineering
Division of Communications & Electronics

EE 392: Control Systems & Their Components

Lab 2: DC Motor Speed: Frequency Domain Methods for Controller Design

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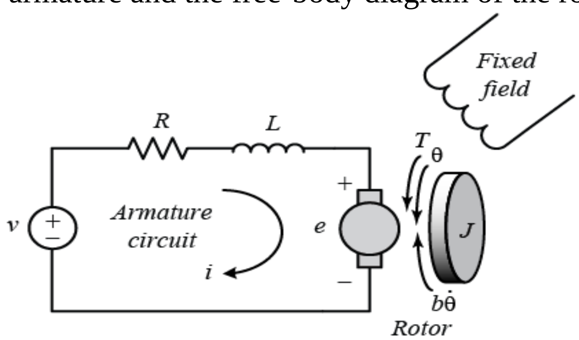
Introduction

A common actuator in control systems is the DC motor. It directly provides rotary motion and, can provide translational motion after some modifications.

The electric circuit of the armature and the free-body diagram of the rotor are shown in the following figure:



We will assume that the input of the system is the voltage source (V) applied to the motor's armature, while the output is the rotational speed of the shaft d(theta)/dt.

The torque generated by a DC motor is proportional to the armature current and the strength of the magnetic field. Here, we assume that the magnetic field is constant and, therefore, that the motor torque is proportional to only the armature current i by a constant factor Kt.

$$T = Kt * i$$

The back emf, e, is proportional to the angular velocity of the shaft by a constant factor Ke.

$$e = Ke * \frac{d\theta}{dt}$$

The motor torque and back emf constants were found to be equal, that is, Kt = Ke; therefore, we will use K to represent both the motor torque constant and the back emf constant.

The resistive torque is assumed to be directly proportional to the angular velocity (b * d(theta)/dt) and acts in opposite direction.

Building The Model

(1) Integrating the acceleration to give velocity: $\int \frac{d^2\theta}{dt^2}\,dt = \frac{d\theta}{dt}$

(2) Integrating the rate of change of the armature current: $\int \frac{di}{dt}\,dt = i$

(3) Apply Newton's law: $J\frac{d^2\theta}{dt^2} = \sum T$

$$J\frac{d^2\theta}{dt^2} = T - b\frac{d\theta}{dt} \Longrightarrow \frac{d^2\theta}{dt^2} = \frac{1}{J}(K_t i - b\frac{d\theta}{dt})$$

where,

J: moment of inertia of the rotor (Kg.m^2).
b: motor viscous friction constant (N.m.s).

(4) Apply Kirchoff's law: $L\frac{di}{dt} = -Ri + V - e \Longrightarrow \frac{di}{dt} = \frac{1}{L}(-Ri + V - K_e\frac{d\theta}{dt})$

where,

L: electric inductance (H).
R: electric resistance (Ohm).

(5) Using Laplace transform on the system equations:

$$s(Js+b)\Theta(s) = KI(s)$$

$$(Ls+R)I(s) = V(s) - Ks\Theta(s)$$

After substitution and simplification, the transfer function becomes:

$$P(s) = \frac{\dot{\Theta}(s)}{V(s)} = \frac{K}{(Js+b)(Ls+R)+K^2} \qquad [\frac{rad/sec}{V}]$$

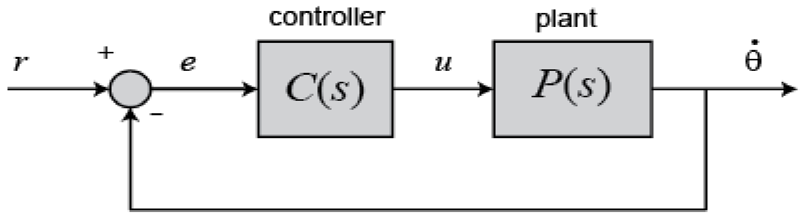
Alternative Modeling Method (Simscape)

The blocks in the Simscape library represent actual physical components; therefore, complex multi-domain models can be built without the need to build mathematical equations from physical principles.
We can build the model by opening a new Simulink model and insert the following blocks to represent the electrical and mechanical elements of the DC motor:

- Resistor, Inductor and Rotational Electromechanical Converter blocks from the Simscape/Foundation Library/Electrical/Electrical Elements library.
- Rotational Damper and Inertia blocks from the Simscape/Foundation Library/Mechanical/Rotational Elements library.
- Four Connection Port blocks from the Simscape/Utilities library.

Connect and label the blocks to model the DC motor circuit.

Frequency-based Controller (Lag Compensator)



The main idea of frequency-based design is to use the Bode plot of the open-loop transfer function to estimate the closed-loop response. Adding a controller to the system changes the open-loop Bode plot, thereby changing the closed-loop response. It is our goal to design the controller to shape the open-loop Bode plot in such a way that the closed-loop system behaves in a desired manner.

Steps:

1- Plot the step response of the original motor system system.
We can find out that the steady state speed is about 0.1 rad/sec when 1 volt is applied to the system (while reference speed is required to be 1 rad/sec).
Moreover, the motor takes (settling time) about 3 seconds to reach its steady state speed.

i.e The behavior if the open-loop system is unsatisfying. We need to add a lag compensator to reach the desired performance.

2- Plot the Bode diagram (A plot of the transfer function (magnitude and phase) versus frequency) of the original motor system.
From the Bode diagram we will notice that we need to increase a proportional gain to increase the gain while still achieving enough phase margin for system stability.

3- Adding a lag compensator.
Lag compensators can be used to adjust frequency response by adding equal numbers of poles and zeroes to a system. Those added singularities may possibly be manipulated to give better stability, better performance and general improvement.

The general form of the transfer function of the lag compensator is:

$$C(s) = C \ \frac{s+z}{s+p}$$

- The gain C is determined according to the requirements of the steady-state error.
- The compensator adds to the system one zero at –z and one pole at –p (where, z > p).
- The positions of the zero and pole are determined according to the new phase margin required to achieve system stability.

Lab Requirements

You are required to:

1- Model the DC motor using MATLAB, Simulink and Simscape.

The parameters needed are:

J = 0.01
b = 0.1
K = 0.01
R = 1
L = 0.5

2- Model the frequency domain controller (lag compensator) using MATLAB and Simulink.

C = 44
z = 1
p = 0.01

3- plot the Bode diagram for:
   - The original motor system.
   - The lag compensator.
   - The closed-loop system (with feedback control).

4- Generate step response for the original motor system and for the closed-loop system (with feedback control) modeled in MATLAB and Simulink. Use a 1-rad/sec step reference.

Useful Matlab Function

tf , bode, step , feedback