Alexandria University
Faculty of Engineering
Electrical Engineering Department

ECE 336: Semiconductor Devices
Sheet 1

1.
   a. Under equilibrium condition, what is the probability of an electron state being occupied if it is located at the Fermi level?
   b. If E_F is positioned at E_c, determine the probability of finding electrons in states at E_c + kT. (A numerical answer is required.)
   c. The probability of a state being filled at E_c + kT is equal to the probability of a state being empty at E_c+ 3 kT. Where is the Fermi level located?

2.
   a. What is the probability of an electron state being filled if it is located at the Fermi level?
   b. If the probability that a state being filled at the conduction band edge (E_c) is precisely equal to the probability that a state is empty at the valence band edge (E_v), where is the Fermi level located?
   c. The Maxwell–Boltzmann distribution is often used to approximate the Fermi-Dirac distribution function. On the same set of axes, sketch both distributions as a function of $(E - E_F)/kT$. Consider only positive values of $E - E_F$. For what range of $(E- E_F)/kT$ is the Maxwell–Boltzmann approximation accurate to within 10%?

3. Show that the probability of an energy state being occupied ΔE above the Fermi level is the same as the probability of a state being empty ΔE below the Fermi level.

$$f(E_F + \Delta E) = 1 - f(E_F - \Delta E)$$

4.
   a. Sketch the Fermi–Dirac distribution f(E)at room temperature (300 K) and at a lower temperature such as 150 K.(Qualitative hand drawing.)
   b. The state distribution in a system is given in Fig. 1–27, where each circle represents two electron states (one is spin-up; one is spin-down). Each electron state can be occupied by one electron. There is no state below E_{min}. The Fermi level at 0 K is given in Fig. 1–27. How many electrons are there in the system?

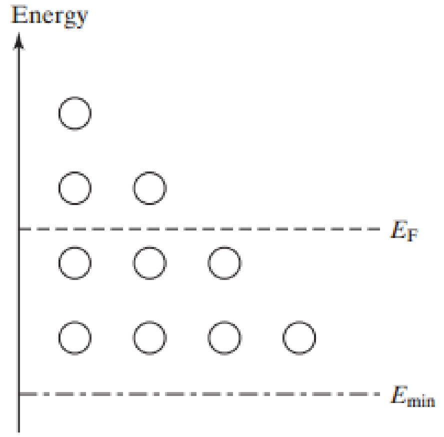

FIGURE 1–27

5. The carrier distributions in the conduction and valence bands were noted to peak at energies close to the band edges. (Refer to carrier distribution in Fig. 1–20.) Using Boltzmann approximation, show that the energy at which the carrier distribution peaks is $E_c + kT/2$ and $E_v – kT/2$ for the conduction and valence bands, respectively.

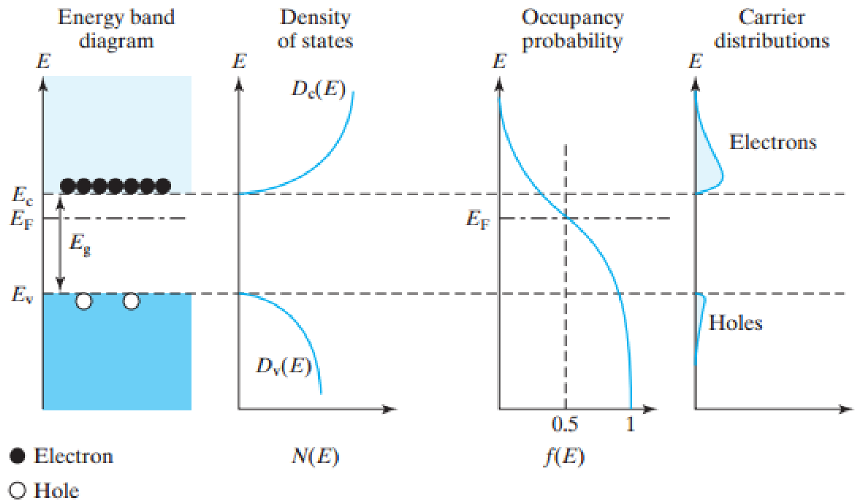

FIGURE 1–20 Schematic band diagram, density of states, Fermi–Dirac distribution, and carrier distributions versus energy.

6. For a certain semiconductor, the densities of states in the conduction and valance bands are constants A and B, respectively. Assume non-degeneracy, i.e., E_F is not close to E_c or E_v
   a. Derive expressions for electron and hole concentrations.
   b. If A = 2B, determine the location of the intrinsic Fermi energy (Ei) at 300 K with respect to the mid-bandgap of the semiconductor.
   Hint: These relationships may be useful:

$$\int_0^{\infty} x^{n-1} e^{-x} dx = \Gamma(n) \qquad \text{(Gamma function)}$$

$$\Gamma(2) = \Gamma(1) = 1, \Gamma(3) = 2, \Gamma(4) = 6$$

$$\Gamma(1/2) = \sqrt{\pi},\ \Gamma(3/2) = 1/2\sqrt{\pi},\ \Gamma(5/2) = 1/3\sqrt{\pi}.$$

7. For a certain semiconductor, the densities of states in the conduction and valence bands are: $D_c(E) = A \cdot (E – E_c) \cdot u(E – E_c)$ and $D_v(E) = B \cdot (E_v – E) \cdot u(E_v – E)$, respectively. u(x), the unit step function, is defined as u(x) = 0 if x < 0 and u(x) = 1 if x > 0. Assume non-degeneracy, i.e. not too highly doped. You may find this fact useful:

$$\int_0^{\infty} x e^{-x} dx = 1$$

   a. Derive expressions for electron and hole concentrations as functions of the Fermi energy, E_F.
   b. If A = 2B, compute the intrinsic Fermi energy at 300 K.

8. The Maxwell–Boltzmann distribution function $f(E)=e^{-(E-Ef)/2}$ is often used as an approximation to the Fermi–Dirac function. Use this approximation and the densities of the states in the conduction band $D_c=A(E-E_c)^{1/2}$ to find:
   a. The energy at which one finds the most electrons ($1/cm^3 \cdot eV$).
   b. The conduction-band electron concentration (explain any approximation made).
   c. The ratio of the peak electron concentration at the energy of (a) to the electron concentration at $E= E_c+ 40$ kT (about 1eV above E_c at 300 K). Does this result justify one of the approximations in part(b)?

    d. The average kinetic energy, $E - E_c$ of the electrons.

9.
    a. The electron concentration in a piece of Si at 300 K is $10^5 cm^{-3}$ What is the hole concentration?
    b. A semiconductor is doped with impurity concentrations N_d and N_a such that $N_d - N_a >> n_i$ and all the impurities are ionized. Determine n and p.
    c. In a silicon sample at T = 300 K, the Fermi level is located at 0.26 eV (10 kT) above the intrinsic Fermi level. What are the hole and electron concentrations?
    d. What are the hole and electron concentration at T = 800 K for the sample in part(c), and where approximately is E_F? Comment on your results.

10. Boron atoms are added to a Si film resulting in an impurity density of $4 \times 10^{16} cm^{-3}$.
    a. What is the conductivity type (N-type or P-type) of this film?
    b. What are the equilibrium electron and hole densities at 300 K and 600 K?
    c. Why does the mobile carrier concentration increase at high temperatures?
    d. Where is the Fermi level located if T = 600 K?

11. An N-type sample of silicon has uniform density ($N_d = 10^{19}/cm^{-3}$) of arsenic, and a P-type silicon sample has a uniform density ($N_a = 10^{15}/cm^{-3}$) of boron. For each sample, determine the following:
    a. The temperature at which the intrinsic concentration n_i exceeds the impurity density by factor of 10.
    b. The equilibrium minority-carrier concentrations at 300 K. Assume full ionization of impurities.
    c. The Fermi level relative to the valence–band edge E_v in each material at 300 K.
    d. The electron and hole concentrations and the Fermi level if both types of impurities are present in the same sample.

12. A silicon sample is doped with $N_d = 10^{17} cm^{-3}$ of As atoms.
    a. What are the electron and hole concentrations and the Fermi level position (relative to E_c or E_v) at 300 K? (Assume full ionization of impurities.)
    b. Check the full ionization assumption using the calculated Fermi level, (i.e., find the probability of donor states being occupied by electrons and therefore not ionized.) Assume that the donor level lies 50 meV below the conduction band, (i.e., $E_c - E_D = 50$ meV.)
    c. Repeat (a) and (b) for $N_d = 10^{19}$ cm^{-3}. (Discussion: when the doping concentration is high, donor (or acceptor) band is formed and that allows all dopant atoms to contribute to conduction such that "full ionization" is a good approximation after all).
    d. Repeat (a) and (b) for $N_d = 10^{17} cm^{-3}$ but T = 30K. (This situation is called dopant freeze-out.)

13. Given N-type silicon sample with uniform donor doping of (a) $N_d = 10^{18}/cm^3$, (b) $N_d = 10^{19}/cm^3$, and (c) $N_d = 10^{6/}cm^3$, calculate the Fermi levels at room temperature assuming full ionization for all cases. Check whether the above assumption of full ionization of each case is correct with the calculated Fermi level. When this is not correct, what is the relative position of E_F and E_D? Assume that

$$E_c - E_D = 0.05 \text{ eV}$$