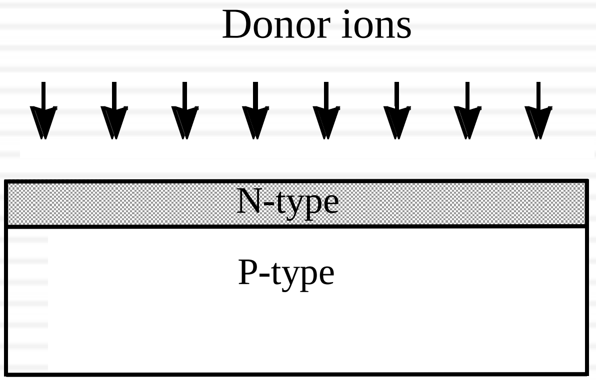
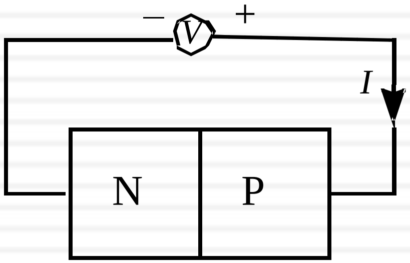
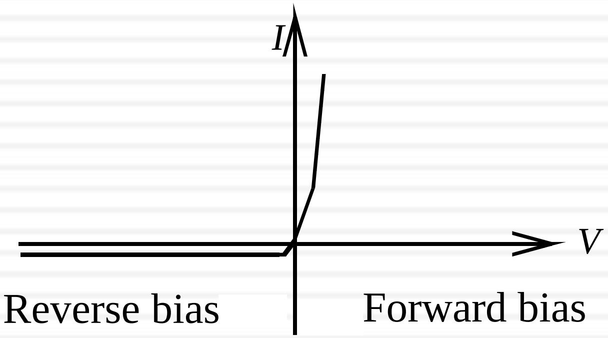
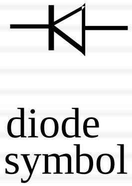
Chapter 4: PN and Metal-Semiconductor Junctions

4.1 Building Blocks of the PN Junction Theory



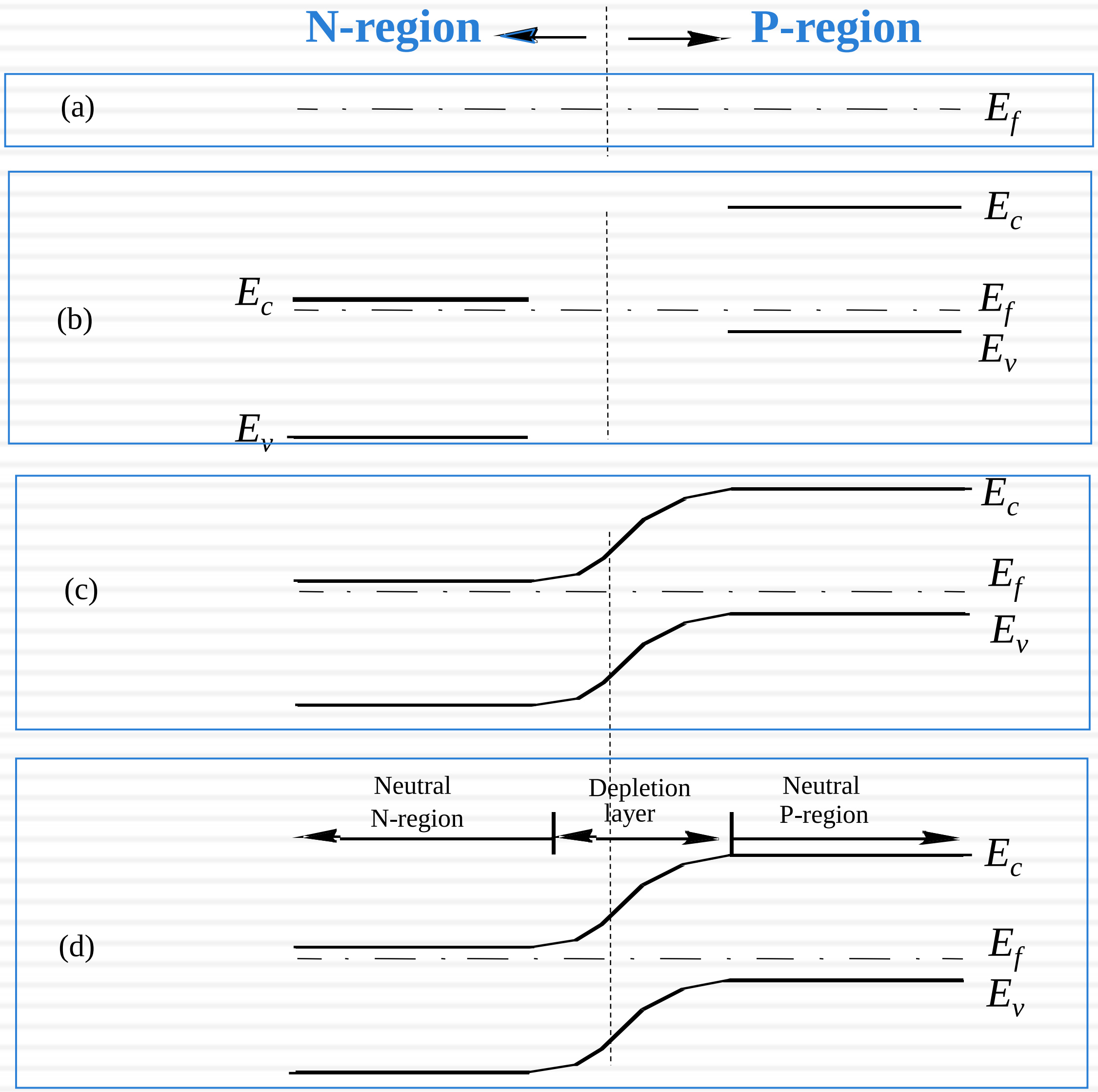






PN junction is present in perhaps every semiconductor device.

4.1.1 Energy Band Diagram of a PN Junction



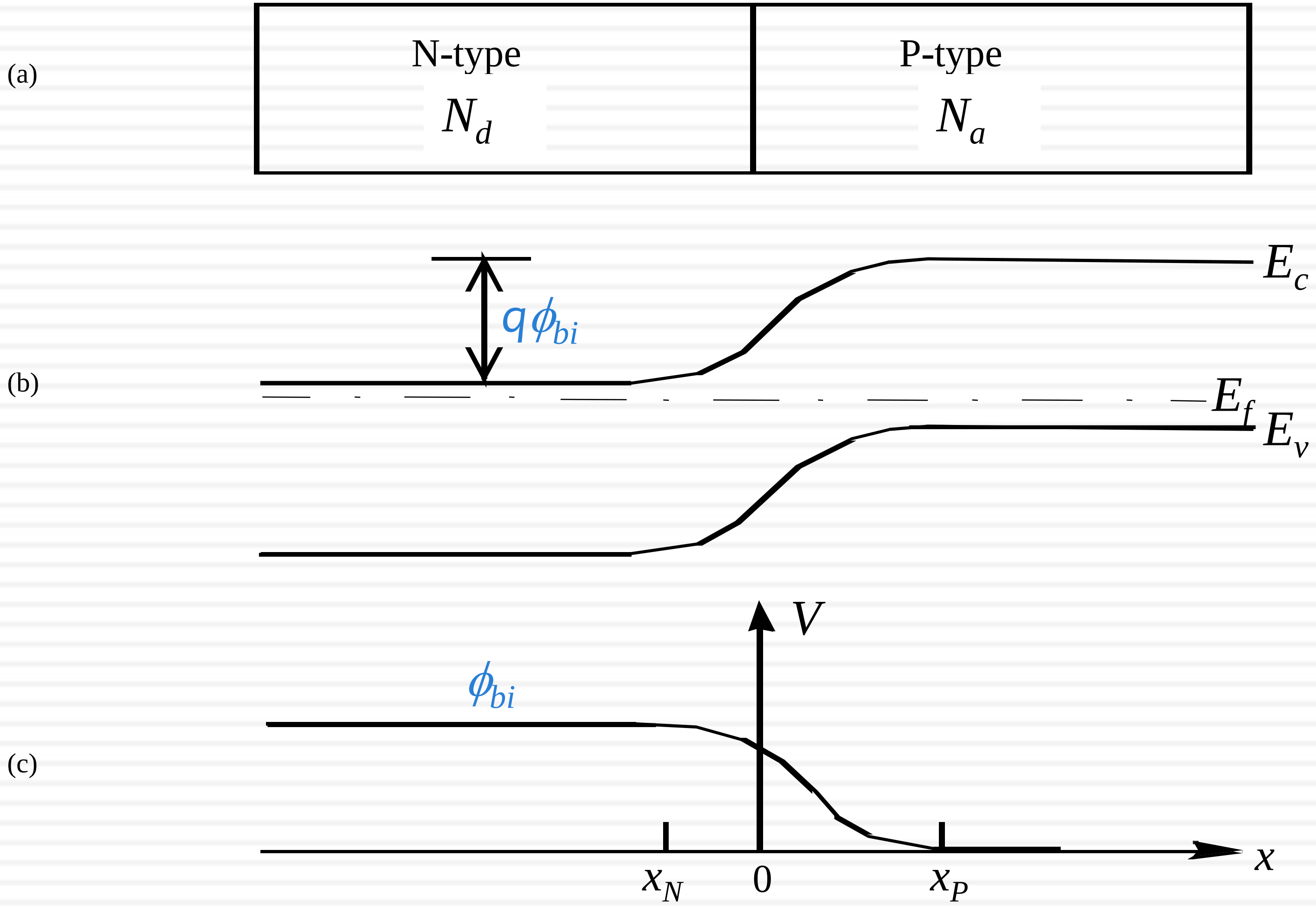
E_f is constant at equilibrium

E_c and E_v are known relative to E_f

E_c and E_v are smooth, the exact shape to be determined.

A depletion layer exists at the PN junction where $n \approx 0$ and $p \approx 0$.

4.1.2 Built-in Potential



Can the built-in potential be measured with a voltmeter?

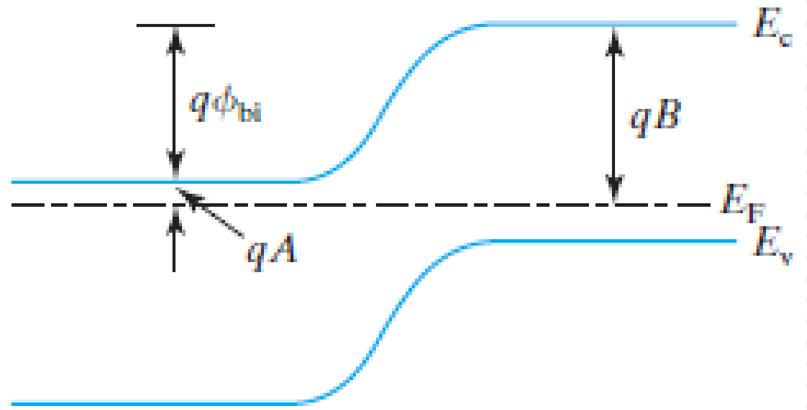
4.1.2 Built-in Potential

N-region $$n = N_d = N_c e^{-qA/kT} \Rightarrow A = \frac{kT}{q} \ln \frac{N_c}{N_d}$$

P-region $$n = \frac{n_i^2}{N_a} = N_c e^{-qB/kT} \Rightarrow B = \frac{kT}{q} \ln \frac{N_c N_a}{n_i^2}$$

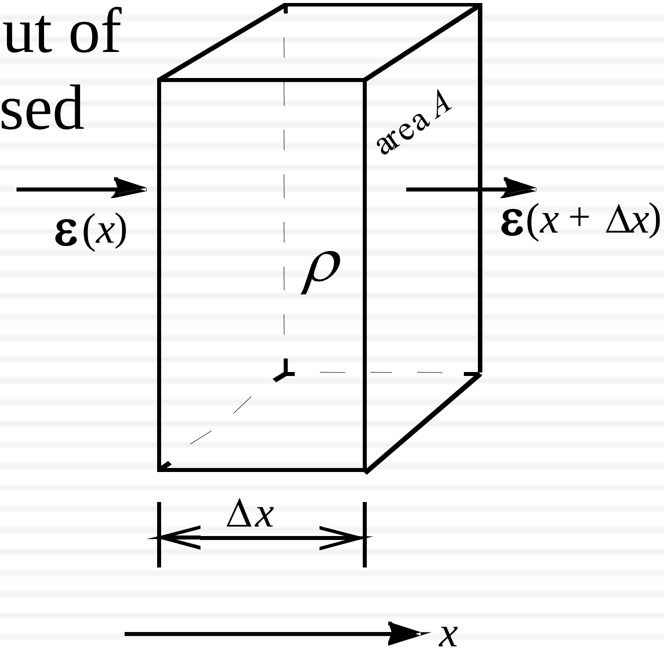
$$\phi_{bi} = B - A = \frac{kT}{q}\left(\ln \frac{N_c N_a}{n_i^2} - \ln \frac{N_c}{N_d} \right)$$

$$\boxed{\phi_{bi} = \frac{kT}{q} \ln \frac{N_d N_a}{n_i^2}}$$



4.1.3 Poisson's Equation

Gauss's Law: The total of the electric flux out of a closed surface is equal to the charge enclosed divided by the permittivity.



$$\varepsilon_s \mathcal{E}(x+\Delta x)A - \varepsilon_s \mathcal{E}(x)A = \rho \Delta x A$$

ε_s: permittivity (~$12\varepsilon_o$ for Si)

ρ: charge density (C/cm^3)

$$\frac{\mathcal{E}(x+\Delta x) - \mathcal{E}(x)}{\Delta x} = \frac{\rho}{\varepsilon_s}$$

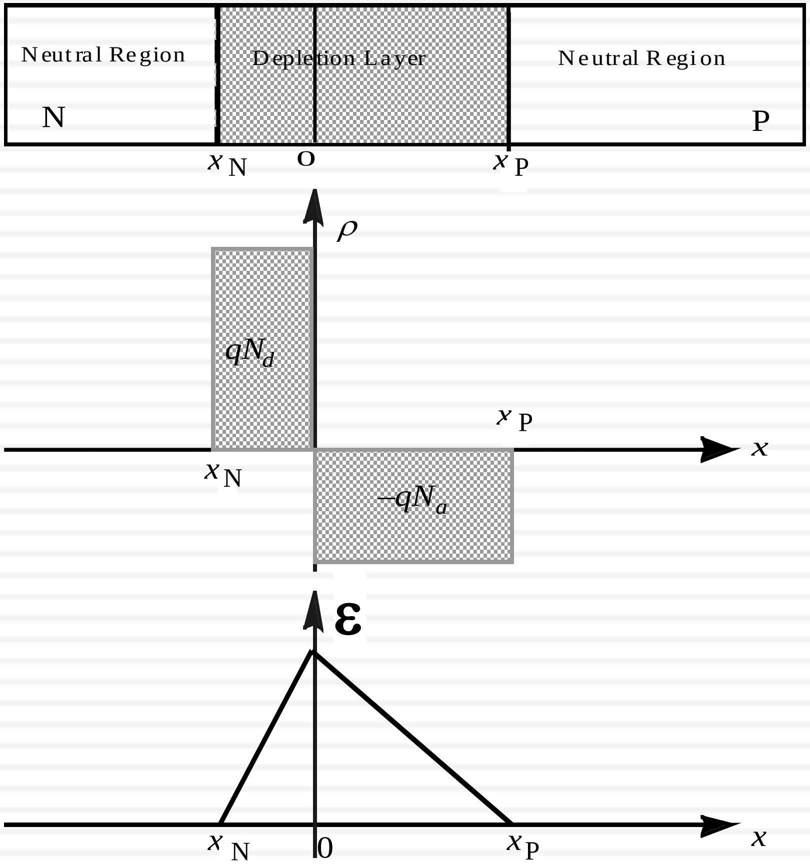
$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{\varepsilon_s}$$

$$\boxed{\frac{d^2V}{dx^2} = -\frac{d\mathcal{E}}{dx} = -\frac{\rho}{\varepsilon_s}}$$

Poisson's equation

4.2 Depletion-Layer Model

4.2.1 Field and Potential in the Depletion Layer



On the ***P-side*** of the depletion layer, $\rho = -qN_a$

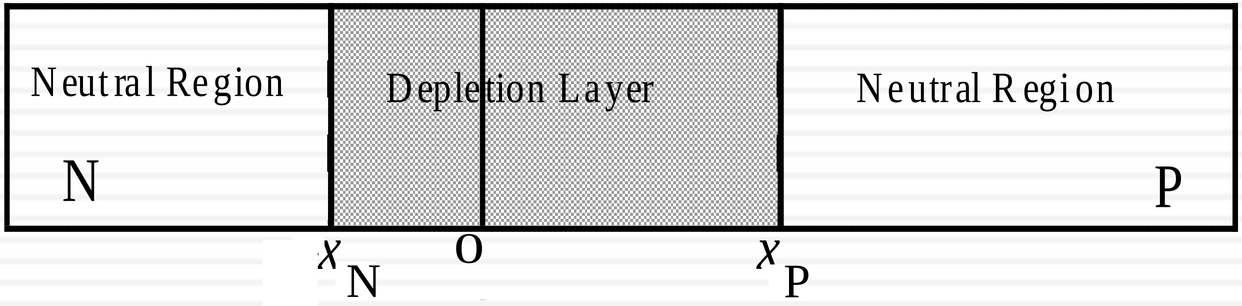
$$\frac{d\boldsymbol{\varepsilon}}{dx} = -\frac{qN_a}{\varepsilon_s}$$

$$\boldsymbol{\varepsilon}(x) = -\frac{qN_a}{\varepsilon_s}x + C_1 = \boxed{\frac{qN_a}{\varepsilon_s}(x_P - x)}$$

On the ***N-side***, $\rho = qN_d$

$$\boxed{\boldsymbol{\varepsilon}(x) = \frac{qN_d}{\varepsilon_s}(x - x_N)}$$

4.2.1 Field and Potential in the Depletion Layer



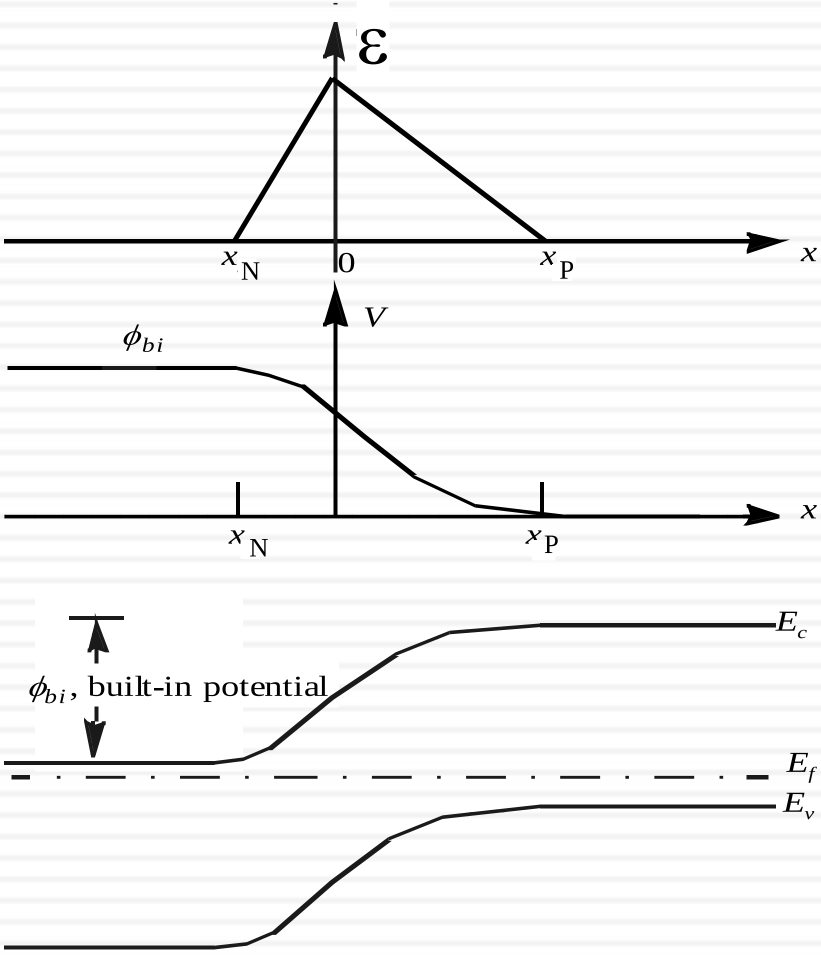
The electric field is continuous at $x = 0$.

$$\boxed{N_a\,|x_P| = N_d|x_N|}$$

Which side of the junction is depleted more?

A one-sided junction is called a ***N^+P junction*** or ***P^+N junction***

4.2.1 Field and Potential in the Depletion Layer



On the P-side,

$$V(x) = \frac{qN_a}{2\varepsilon_s}(x_P - x)^2$$

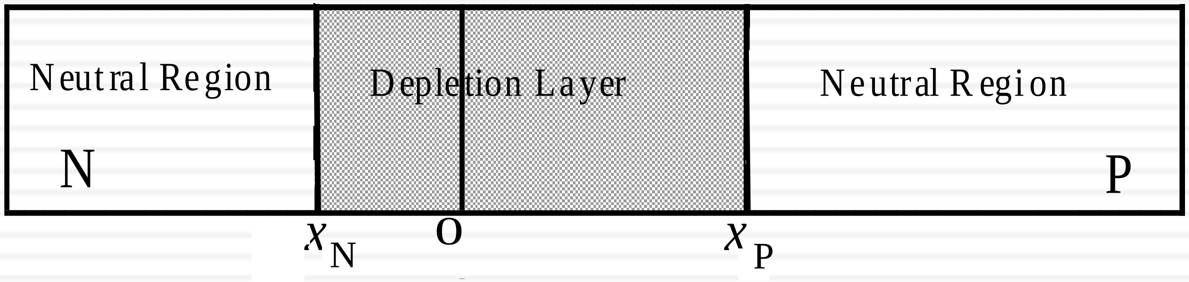
Arbitrarily choose the voltage at $x = x_P$ as $V = 0$.

On the N-side,

$$V(x) = D - \frac{qN_d}{2\varepsilon_s}(x - x_N)^2$$

$$= \phi_{bi} - \frac{qN_d}{2\varepsilon_s}(x - x_N)^2$$

4.2.2 Depletion-Layer Width



V is continuous at $x = 0$ →

$$x_P - x_N = W_{dep} = \sqrt{\frac{2\varepsilon_s\phi_{bi}}{q}\left(\frac{1}{N_a}+\frac{1}{N_d}\right)}$$

If $N_a >> N_d$, as in a P$^+$N junction,

$$\boxed{W_{dep} = \sqrt{\frac{2\varepsilon_s\phi_{bi}}{qN_d}} \approx |x_N|}$$

$$\boxed{|x_P| = |x_N| N_d / N_a \cong 0}$$

What about a N$^+$P junction?

$$W_{dep} = \sqrt{2\varepsilon_s\phi_{bi}/qN} \quad \text{where} \quad \frac{1}{N} = \frac{1}{N_d} + \frac{1}{N_a} \approx \frac{1}{\text{lighter dopant density}}$$

EXAMPLE: *A P^+N junction has $N_a=10^{20}$ cm^{-3} and $N_d=10^{17}$cm^{-3}. What is a) its built in potential, b)W_{dep} , c)x_N , and d) x_P ?*

Solution:

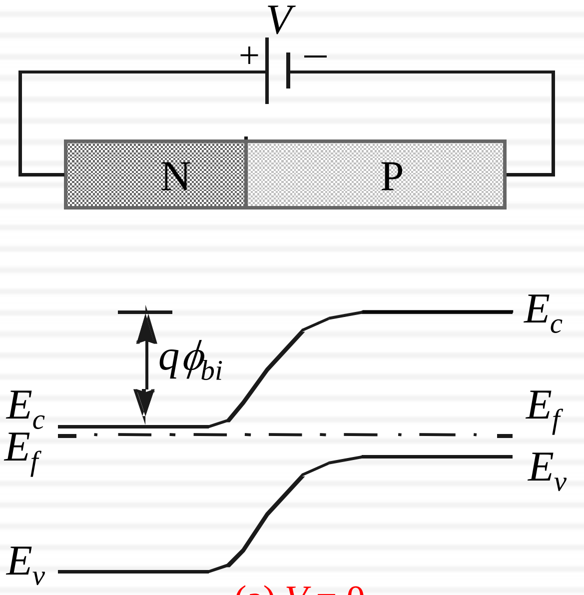
a)
$$\phi_{bi}=\frac{kT}{q}\ln\frac{N_dN_a}{n_i^2}=0.026\text{V}\ln\frac{10^{20}\times10^{17}\,\text{cm}^{-6}}{10^{20}\,\text{cm}^{-6}}\approx 1\,\text{V}$$

b)
$$W_{dep}\approx\sqrt{\frac{2\varepsilon_s\phi_{bi}}{qN_d}}=\left(\frac{2\times12\times8.85\times10^{-14}\times1}{1.6\times10^{-19}\times10^{17}}\right)^{1/2}=0.12\,\mu\text{m}$$

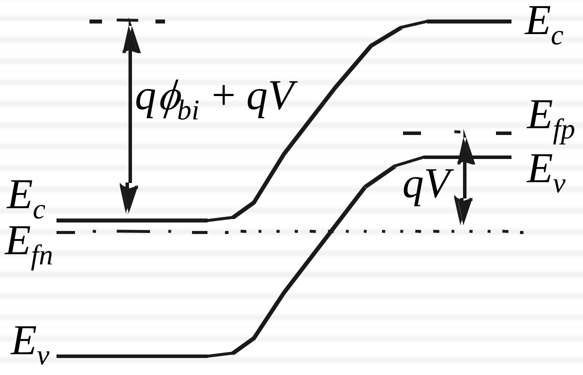
c) $|x_N|\approx W_{dep}=0.12\,\mu\text{m}$

d)$|x_P|=|x_N|N_d/N_a=1.2\times10^{-4}\mu\text{m}=1.2\,\text{Å}\approx0$

4.3 Reverse-Biased PN Junction



(a) $V = 0$



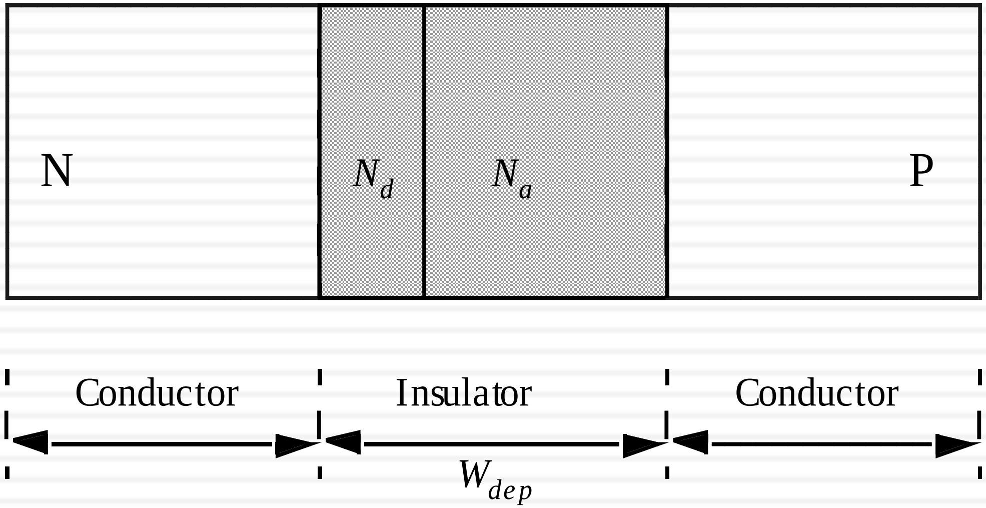
(b) reverse-biased

$$\boxed{W_{dep} = \sqrt{\frac{2\varepsilon_s(\phi_{bi} + |V_r|)}{qN}} = \sqrt{\frac{2\varepsilon_s \cdot potential\,barrier}{qN}}}$$

$$\frac{1}{N} = \frac{1}{N_d} + \frac{1}{N_a} \approx \frac{1}{lighter\;\; dopant\;\; density}$$

- ***Does the depletion layer widen or shrink with increasing reverse bias?***

4.4 Capacitance-Voltage Characteristics



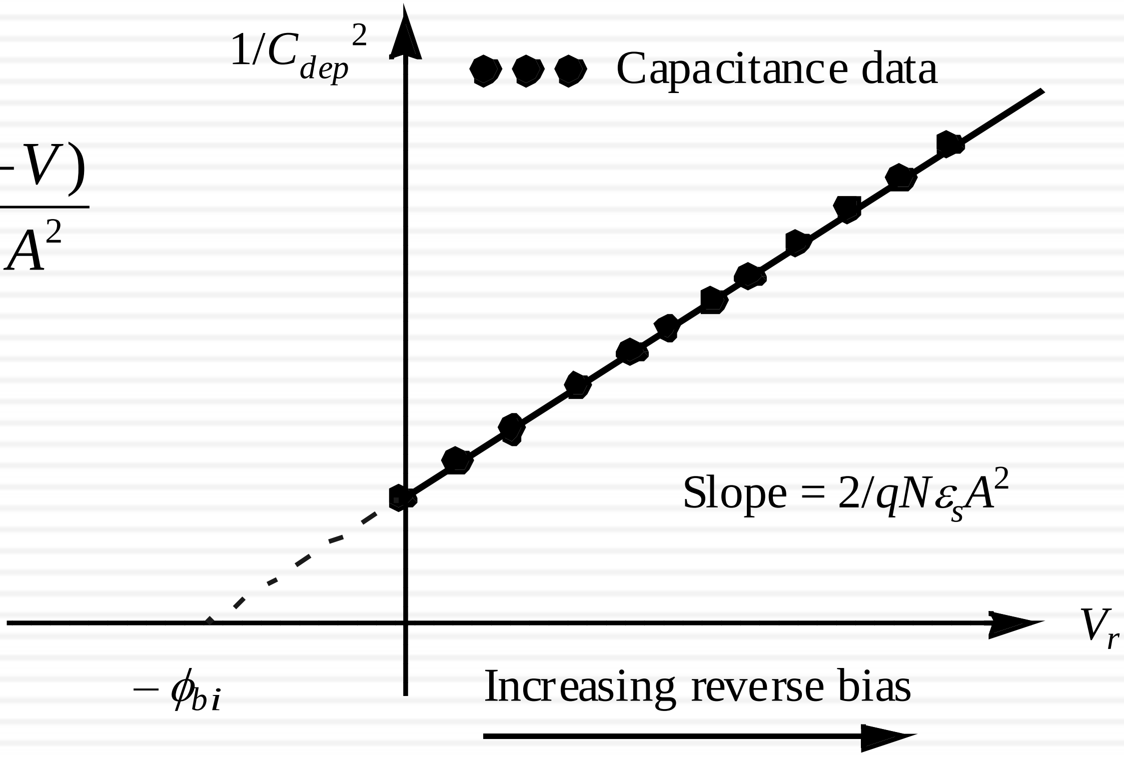
Reverse biased PN junction is a capacitor.

$$C_{dep} = A\frac{\varepsilon_s}{W_{dep}}$$

- Is C_{dep} a good thing?
- How to minimize junction capacitance?

4.4 Capacitance-Voltage Characteristics

$$\frac{1}{C_{dep}^{2}} = \frac{W_{dep}^{2}}{A^2 \varepsilon_s^{2}} = \frac{2(\phi_{bi} + V)}{qN\varepsilon_S A^2}$$



- From this C-V data can N_a and N_d be determined?

EXAMPLE: *If the slope of the line in the previous slide is* $2\text{x}10^{23}\ F^{-2}\ V^{-1}$, *the intercept is 0.84V, and A is 1* μm^2, *find the lighter and heavier doping concentrations* N_l *and* N_h.

Solution:

$$N_l = 2/(slope \times q\varepsilon_s A^2)$$

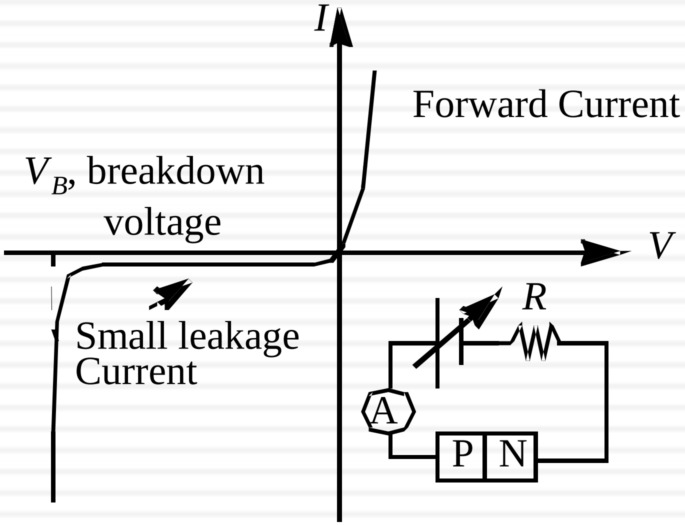
$$= 2/(2\times10^{23}\times1.6\times10^{-19}\times12\times8.85\times10^{-14}\times10^{-8}\,\text{cm}^2)$$

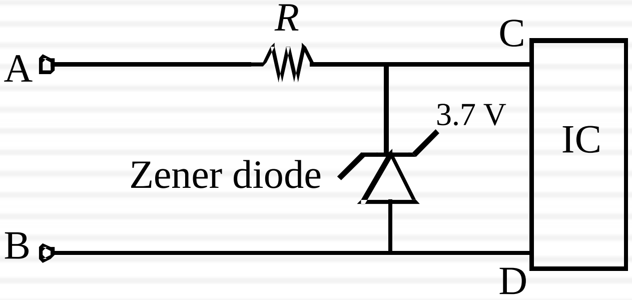
$$= 6\times10^{15}\ \text{cm}^{-3}$$

$$\phi_{bi} = \frac{kT}{q}\ln\frac{N_h N_l}{n_i^{\,2}} \quad\Rightarrow\quad N_h = \frac{n_i^{\,2}}{N_l}e^{\frac{q\phi_{bi}}{kT}} = \frac{10^{20}}{6\times10^{15}}e^{\frac{0.84}{0.026}} = 1.8\times10^{18}\ \text{cm}^{-3}$$

- *Is this an accurate way to determine* N_l ? N_h ?

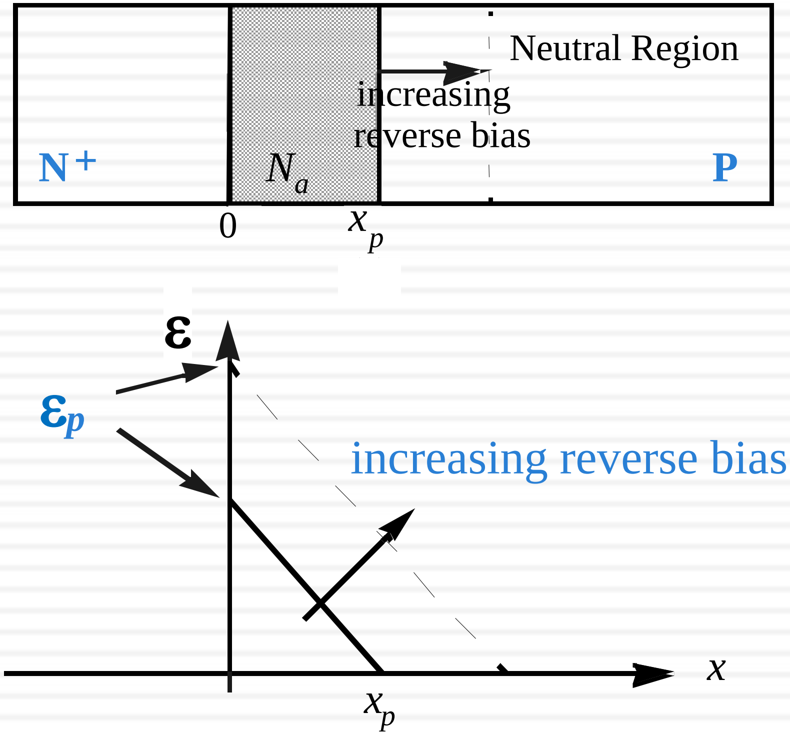
4.5 Junction Breakdown





A ***Zener diode*** is designed to operate in the breakdown mode.

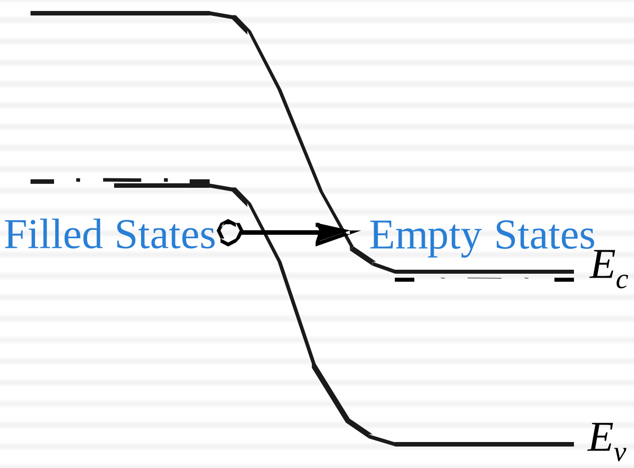
4.5.1 Peak Electric Field

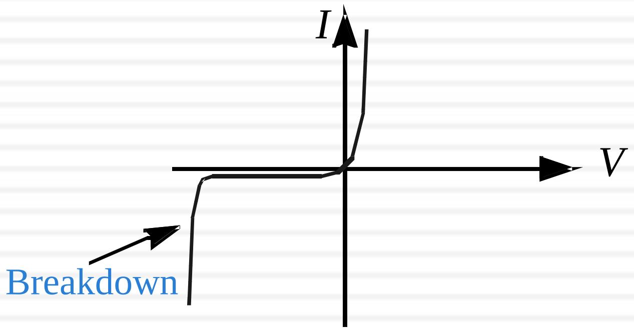


$$\boldsymbol{\varepsilon}_p = \boldsymbol{\varepsilon}(0) = \left[\frac{2qN}{\varepsilon_s}(\phi_{bi} + |V_r|)\right]^{1/2}$$

$$V_B = \frac{\varepsilon_s \boldsymbol{\varepsilon}_{crit}^{2}}{2qN} - \phi_{bi}$$

4.5.2 Tunneling Breakdown



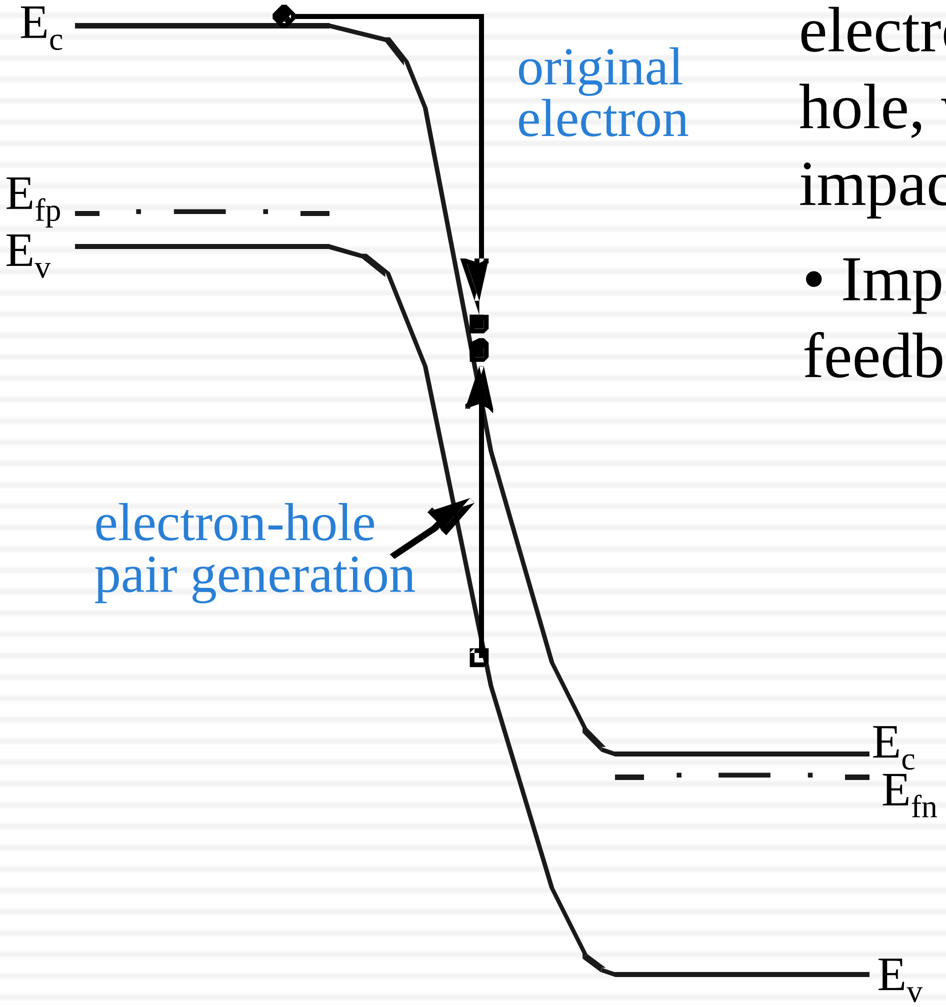


Dominant if both sides of a junction are very heavily doped.

$$J = G\, e^{-H/\varepsilon_p}$$

$$\boldsymbol{\varepsilon}_p = \boldsymbol{\varepsilon}_{crit} \approx 10^6 \text{ V/cm}$$

4.5.3 Avalanche Breakdown



- *impact ionization*: an energetic electron generating electron and hole, which can also cause impact ionization.
- Impact ionization + positive feedback→*avalanche breakdown*

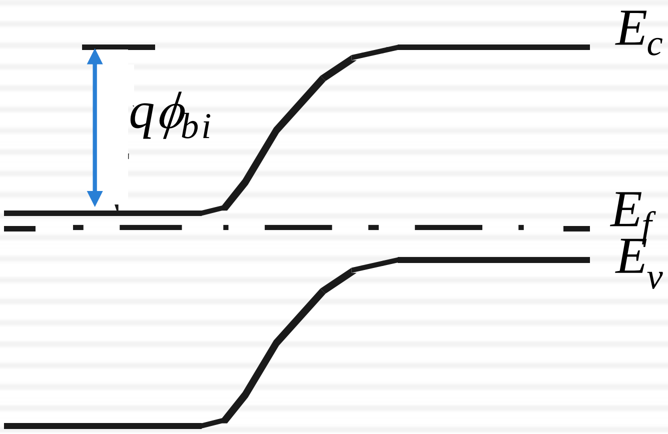
$$V_B = \frac{\varepsilon_s \boldsymbol{\varepsilon}_{crit}^{2}}{2qN}$$

$$\boxed{V_B \propto \frac{1}{N}} = \frac{1}{N_a} + \frac{1}{N_d}$$

4.6 Forward Bias – Carrier Injection

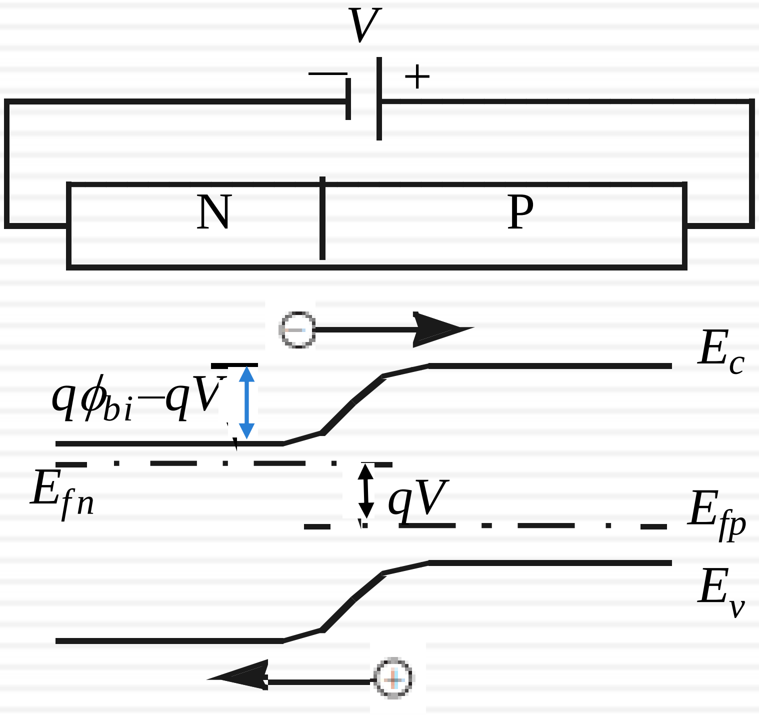
V=0

I=0



Drift and diffusion cancel out

Forward biased

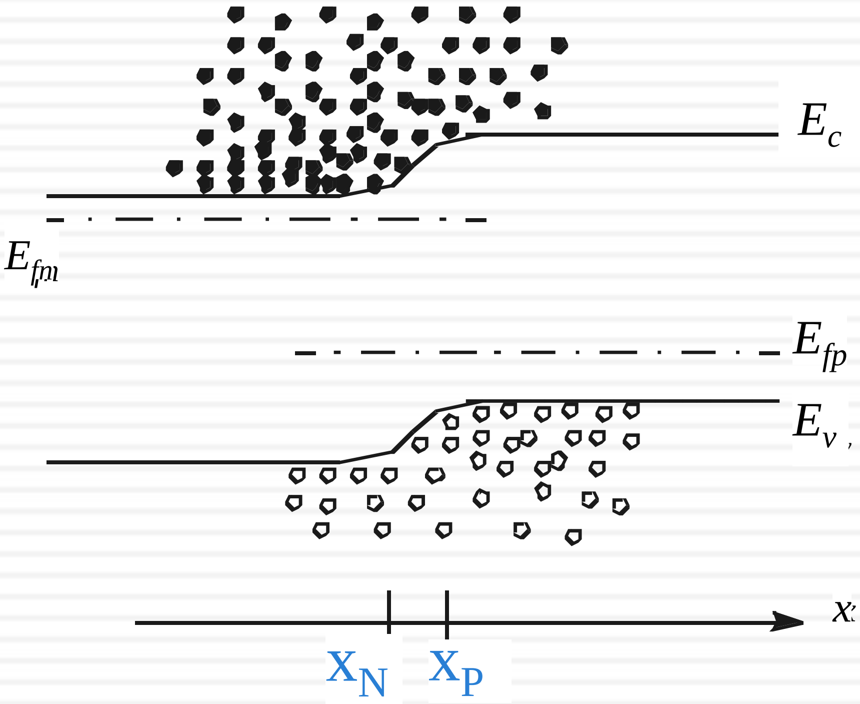


Minority carrier injection

4.6 Forward Bias – Quasi-equilibrium Boundary Condition

$$n(x_P) = N_c e^{-(E_c - E_{fn})/kT} = N_c e^{-(E_c - E_{fp})/kT} e^{(E_{fn} - E_{fp})/kT}$$

$$= n_{P0} e^{(E_{fn} - E_{fp})/kT} = n_{P0} e^{qV/kT}$$



- The minority carrier densities are raised by $e^{qV/kT}$
- Which side gets more carrier injection ?

4.6 Carrier Injection Under Forward Bias–Quasi-equilibrium Boundary Condition

$$n(x_P) = n_{P0}e^{qV/kT} = \frac{n_i^2}{N_a}e^{qV/kT}$$

$$p(x_P) = p_{N0}e^{qV/kT} = \frac{n_i^2}{N_d}e^{qV/kT}$$

$$n'(x_P) \equiv n(x_P) - n_{P0} = n_{P0}(e^{qV/kT} - 1)$$

$$p'(x_N) \equiv p(x_N) - p_{N0} = p_{N0}(e^{qV/kT} - 1)$$

EXAMPLE: Carrier Injection

A PN junction has $N_a=10^{19}\text{cm}^{-3}$ and $N_d=10^{16}\text{cm}^{-3}$. The applied voltage is 0.6 V.

Question: *What are the minority carrier concentrations at the depletion-region edges?*

Solution:

$$n(x_P) = n_{P0}e^{qV/kT} = 10 \times e^{0.6/0.026} = 10^{11}\ \text{cm}^{-3}$$

$$p(x_N) = p_{N0}e^{qV/kT} = 10^{4} \times e^{0.6/0.026} = 10^{14}\ \text{cm}^{-3}$$

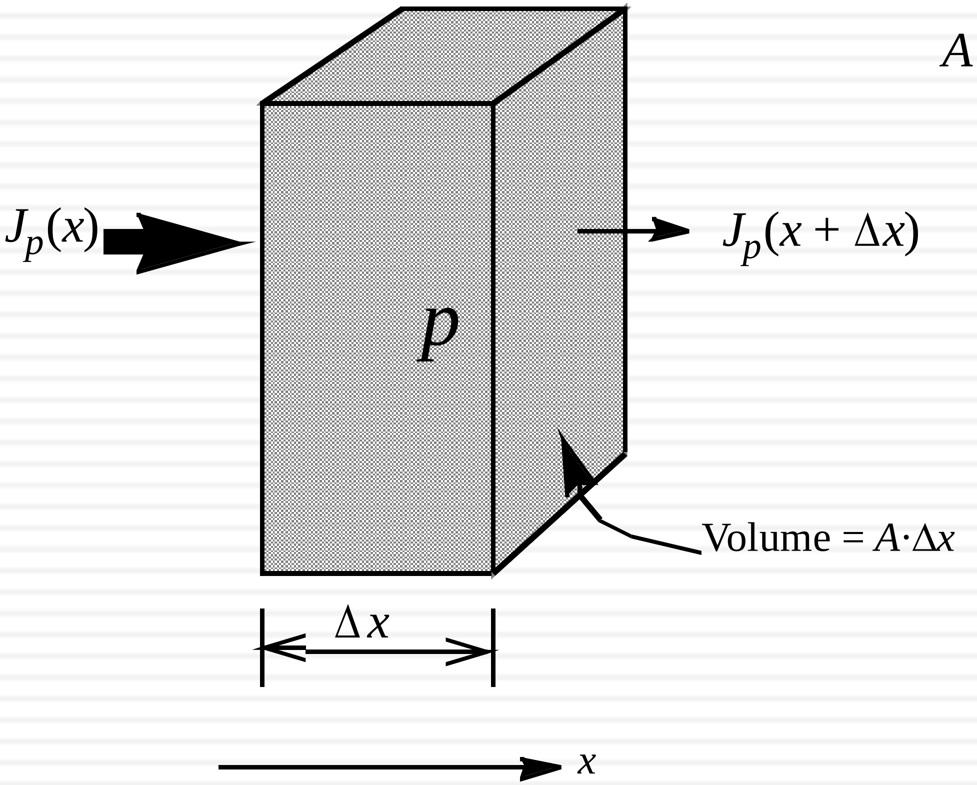
Question: *What are the excess minority carrier concentrations?*

Solution:

$$n'(x_P) = n(x_P) - n_{P0} = 10^{11} - 10 = 10^{11}\ \text{cm}^{-3}$$

$$p'(x_N) = p(x_N) - p_{N0} = 10^{14} - 10^{4} = 10^{14}\ \text{cm}^{-3}$$

4.7 Current Continuity Equation



$$A\cdot\frac{J_p(x)}{q} = A\cdot\frac{J_p(x+\Delta x)}{q} + A\cdot\Delta x\cdot\frac{p'}{\tau}$$

$$-\frac{J_p(x+\Delta x) - J_p(x)}{\Delta x} = q\frac{p'}{\tau}$$

$$-\frac{dJ_p}{dx} = q\frac{p'}{\tau}$$

4.7 Current Continuity Equation

$$-\frac{dJ_p}{dx} = q\frac{p'}{\tau}$$

Minority drift current is negligible;

$\therefore$ $J_p = -qD_p dp/dx$

$$qD_p \frac{d^2 p}{dx^2} = q\frac{p'}{\tau_p}$$

$$\boxed{\frac{d^2 p'}{dx^2} = \frac{p'}{D_p \tau_p} = \frac{p'}{L_p^{\,2}}}$$

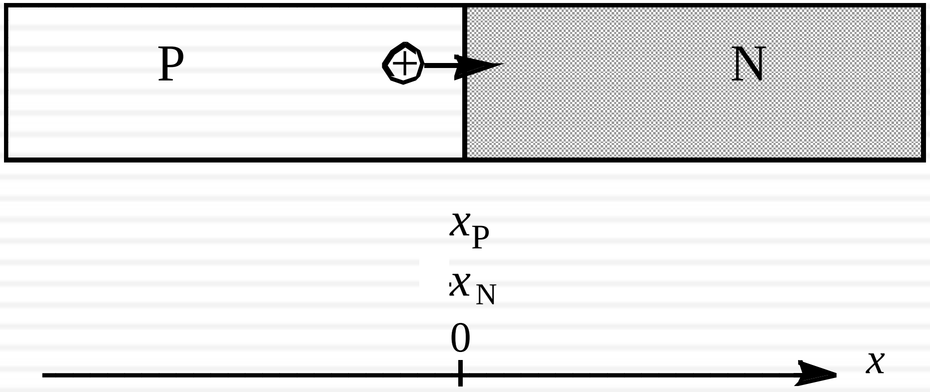
$$\boxed{\frac{d^2 n'}{dx^2} = \frac{n'}{L_n^{\,2}}}$$

L_p and L_n are the diffusion lengths

$$\boxed{L_p \equiv \sqrt{D_p \tau_p}}$$

$$\boxed{L_n \equiv \sqrt{D_n \tau_n}}$$

4.8 Forward Biased Junction-- Excess Carriers



$$\frac{d^2 p'}{dx^2} = \frac{p'}{L_p{}^2}$$

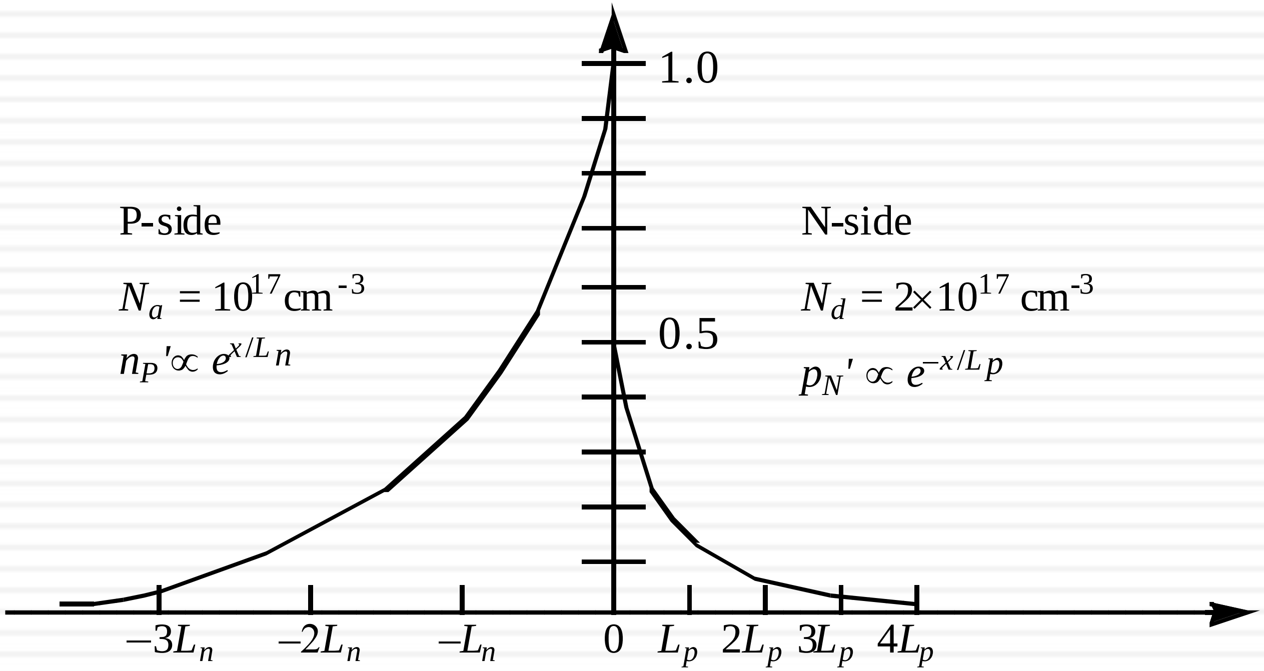
$$p'(\infty) = 0$$

$$p'(x_N) = p_{N0}(e^{qV/kT} - 1)$$

$$p'(x) = Ae^{x/L_p} + Be^{-x/L_p}$$

$$p'(x) = p_{N0}(e^{qV/kT} - 1)e^{-(x-x_N)/L_p}, \;\; x > x_N$$

4.8 Excess Carrier Distributions



$$p'(x) = p_{N0}(e^{qV/kT} - 1)e^{-(x-x_N)/L_p}, \quad x > x_N$$

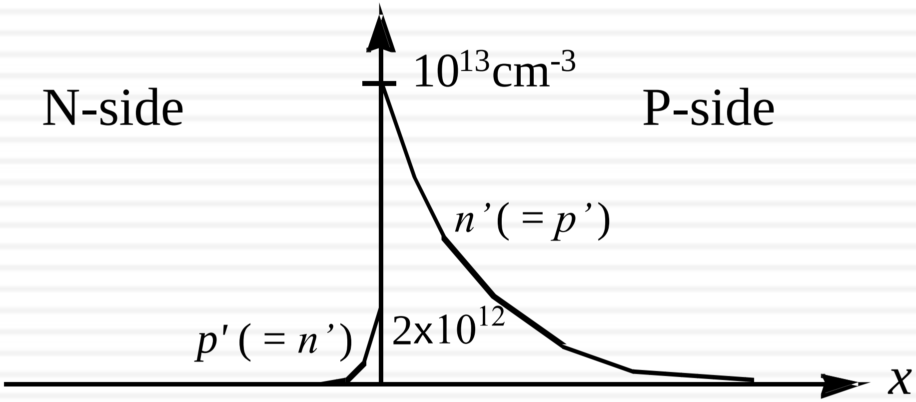
$$n'(x) = n_{P0}(e^{qV/kT} - 1)e^{(x-x_P)/L_n}, \quad x < x_P$$

EXAMPLE: Carrier Distribution in Forward-biased PN Diode

N-type	P-type
$N_d = 5\times10^{17}\ \text{cm}^{-3}$	$N_a = 10^{17}\ \text{cm}^{-3}$
$D_p = 12\ \text{cm}^2/\text{s}$	$D_n = 36.4\ \text{cm}^2/\text{s}$
$\tau_p = 1\ \mu\text{s}$	$\tau_n = 2\ \mu\text{s}$

- *Sketch n'(x) on the P-side.*

$$n'(x_P) = n_{P0}(e^{qV/kT} - 1) = \frac{n_i^2}{N_a}(e^{qV/kT} - 1) = \frac{10^{20}}{10^{17}} e^{0.6/0.026} = 10^{13}\ \text{cm}^{-3}$$



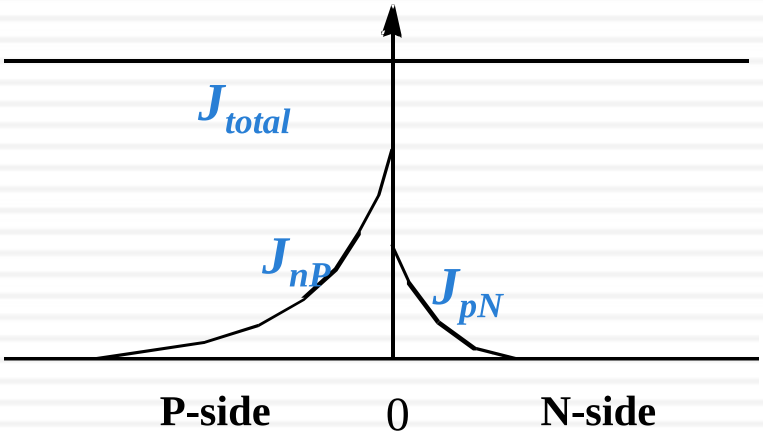
EXAMPLE: Carrier Distribution in Forward-biased PN Diode

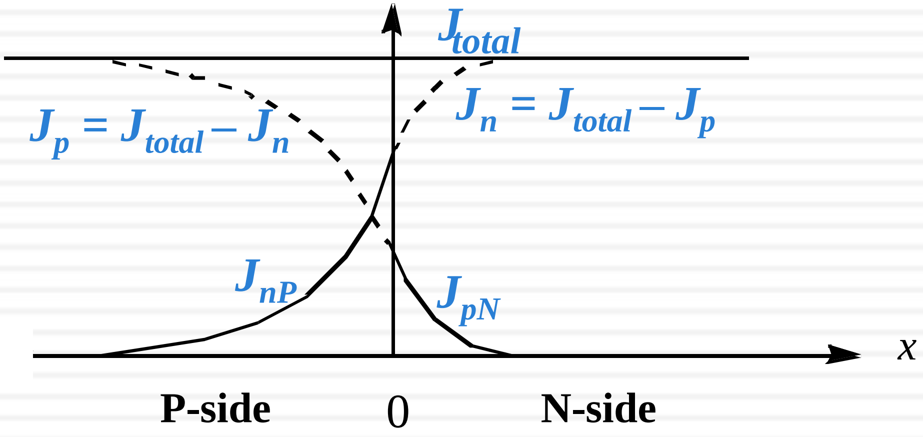
- *How does* L_n *compare with a typical device size?*

$$L_n = \sqrt{D_n \tau_n} = \sqrt{36 \times 2 \times 10^{-6}} = 85 \ \mu\text{m}$$

- *What is p'(x) on the P- side?*

4.9 PN Diode I-V Characteristics





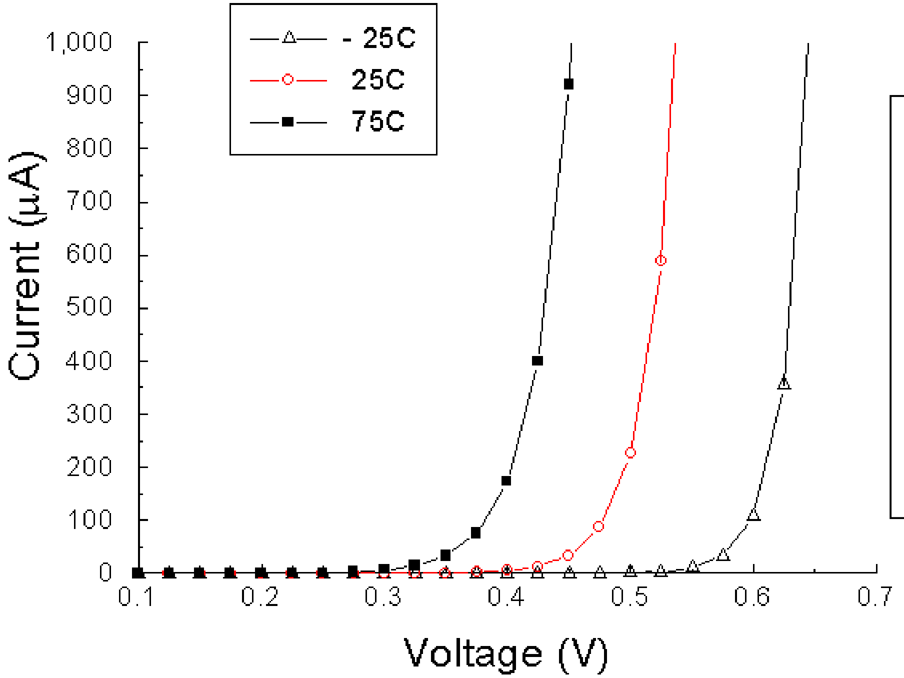
$$J_{pN} = -qD_p \frac{dp'(x)}{dx} = q\frac{D_p}{L_p} p_{N0}(e^{qV/kT}-1)e^{-(x-x_N)/L_p}$$

$$J_{nP} = qD_n \frac{dn'(x)}{dx} = q\frac{D_n}{L_n} n_{P0}(e^{qV/kT}-1)e^{(x-x_P)/L_n}$$

$$\text{Total current} = J_{pN}(x_N) + J_{nP}(x_P) = \left(q\frac{D_p}{L_p} p_{N0} + q\frac{D_n}{L_n} n_{P0} \right)(e^{qV/kT}-1)$$

$$= J \text{ at all } x$$

The PN Junction as a Temperature Sensor

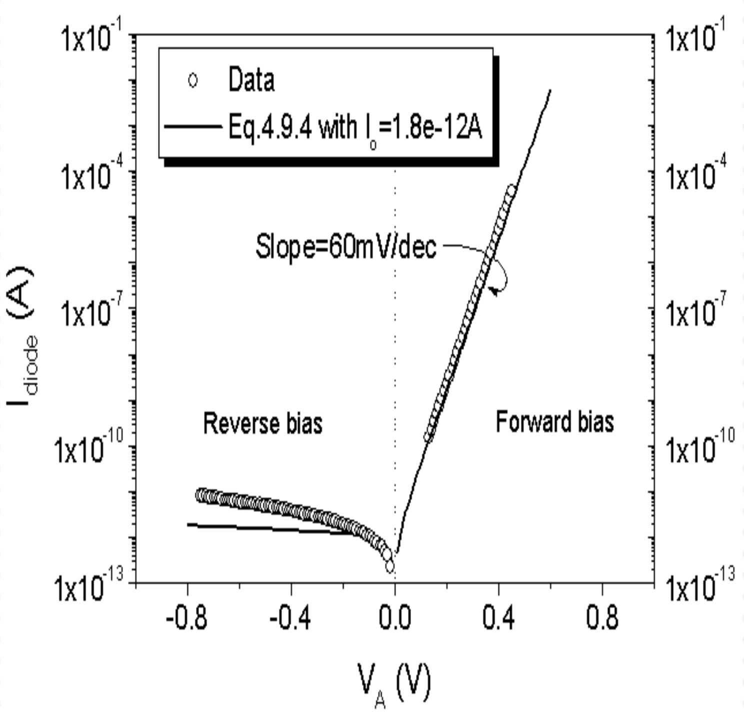


$$I = I_0(e^{qV/kT} - 1)$$

$$I_0 = Aqn_i^{\,2}\left(\frac{D_p}{L_pN_d} + \frac{D_n}{L_nN_a}\right)$$

What causes the IV curves to shift to lower V at higher T ?

4.9.1 Contributions from the Depletion Region



$$n \approx p \approx n_i e^{qV/2kT}$$

Net recombination (generation) rate:

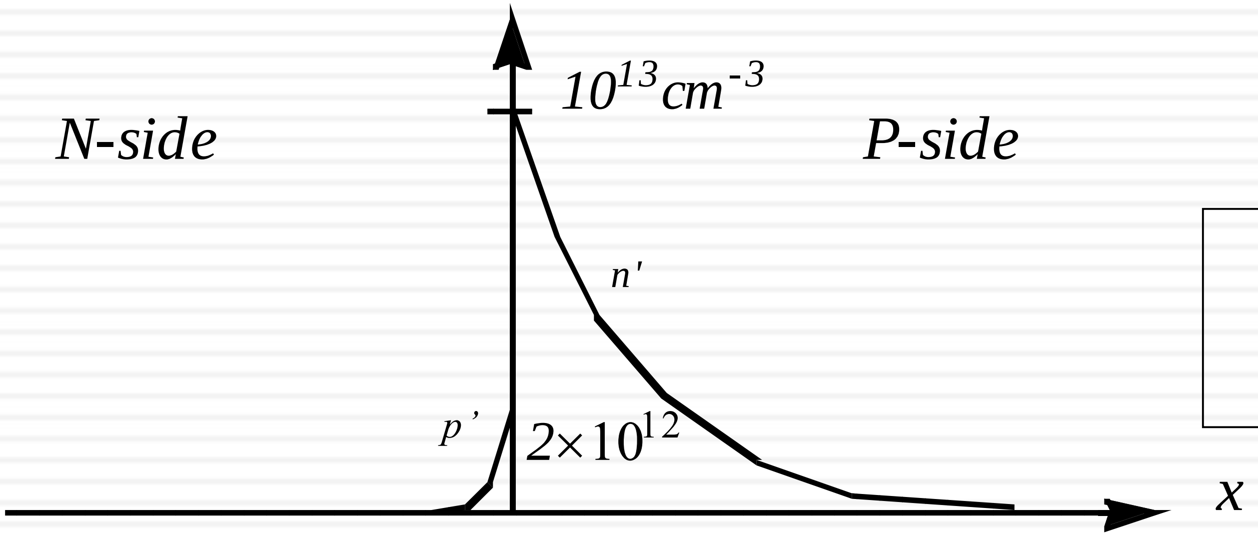
$$\frac{n_i}{\tau_{dep}}(e^{qV/2kT}-1)$$

$$I = I_0(e^{qV/kT}-1) + \underbrace{A\frac{qn_iW_{dep}}{\tau_{dep}}(e^{qV/2kT}-1)}_{\textbf{Space-Charge Region (SCR) current}}$$

$$I_{leakage} = I_0 + A\frac{qn_iW_{dep}}{\tau_{dep}}$$

Under forward bias, SCR current is an extra current with a slope 120mV/decade

4.10 Charge Storage



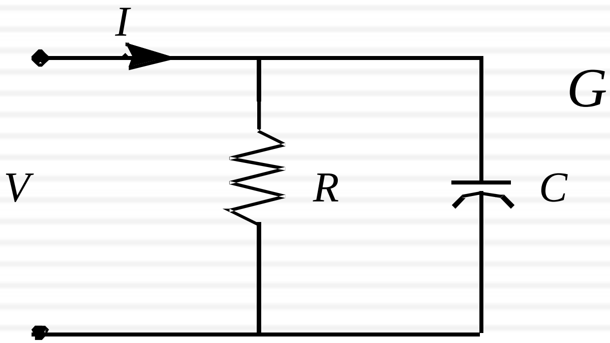
$$Q \propto I$$

$$I = Q/\tau_s$$

$$Q = I\tau_s$$

What is the relationship between τ_s (charge-storage time) and τ (carrier lifetime)?

4.11 Small-signal Model of the Diode



$$G \equiv \frac{1}{R} = \frac{dI}{dV} = \frac{d}{dV} I_0 (e^{qV/kT} - 1) \approx \frac{d}{dV} I_0 e^{qV/kT}$$

$$= \frac{q}{kT} I_0 (e^{qV/kT}) = I_{DC} / \frac{kT}{q}$$

What is G at 300K and I_{DC} = 1 mA?

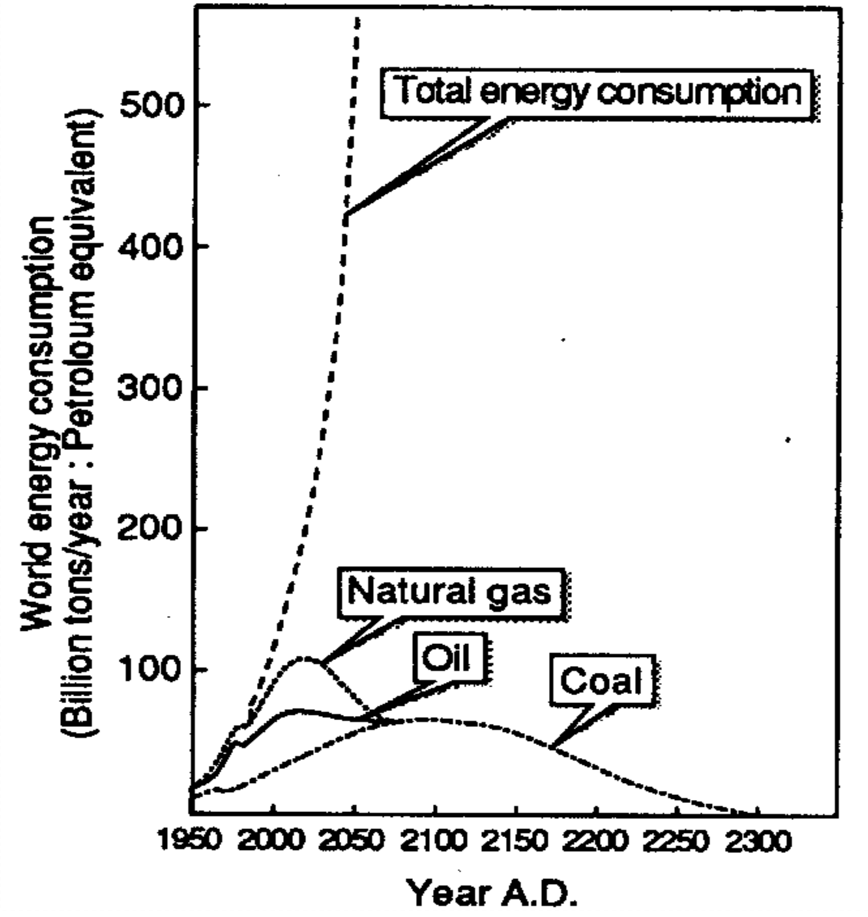
Diffusion Capacitance:

$$C = \frac{dQ}{dV} = \tau_s \frac{dI}{dV} = \tau_s G = \tau_s I_{DC} / \frac{kT}{q}$$

Which is larger, diffusion or depletion capacitance?

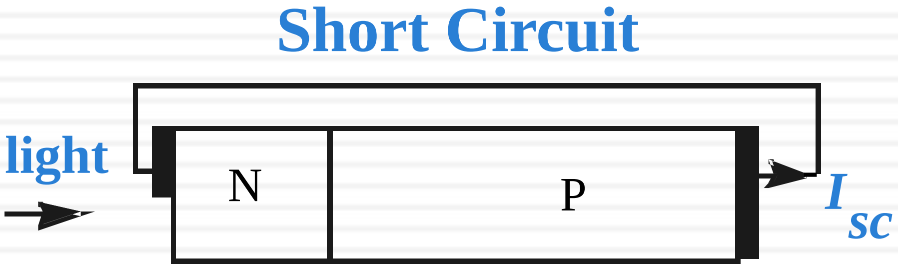
Part II: Application to Optoelectronic Devices

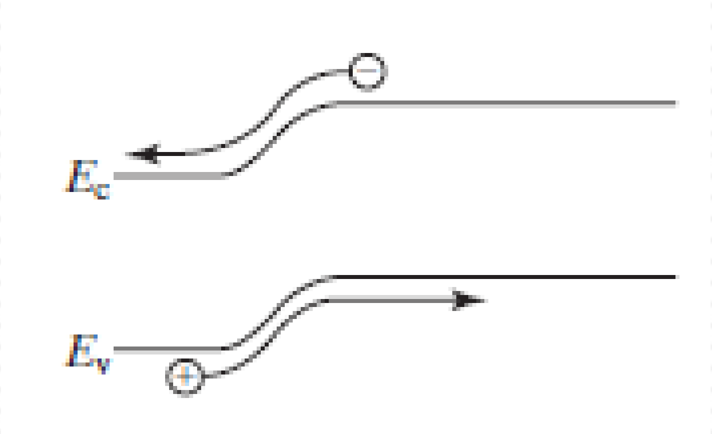
4.12 Solar Cells

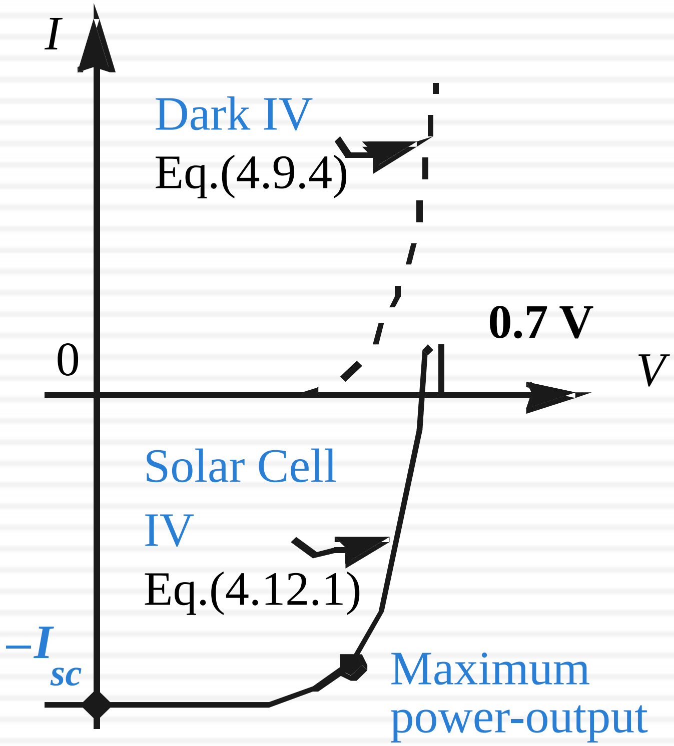


- ***Solar Cells*** is also known as ***photovoltaic cells***.
- Converts sunlight to electricity with 10-30% conversion efficiency.
- 1 m^2 solar cell generate about 150 W peak or 25 W continuous power.
- Low cost and high efficiency are needed for wide deployment.

4.12.1 Solar Cell Basics



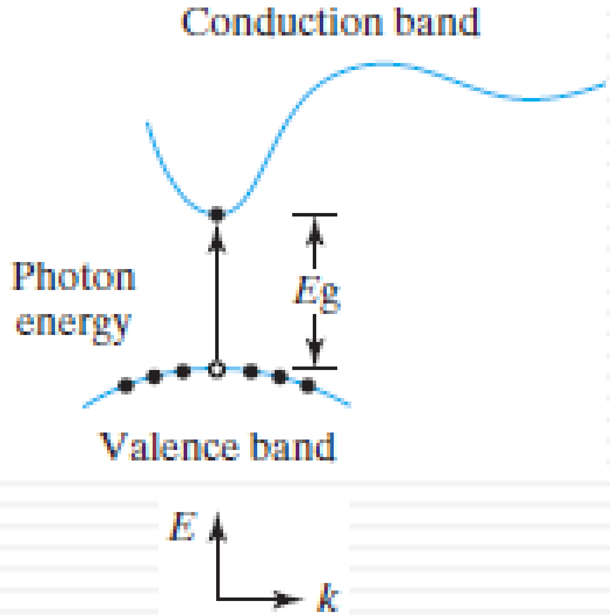




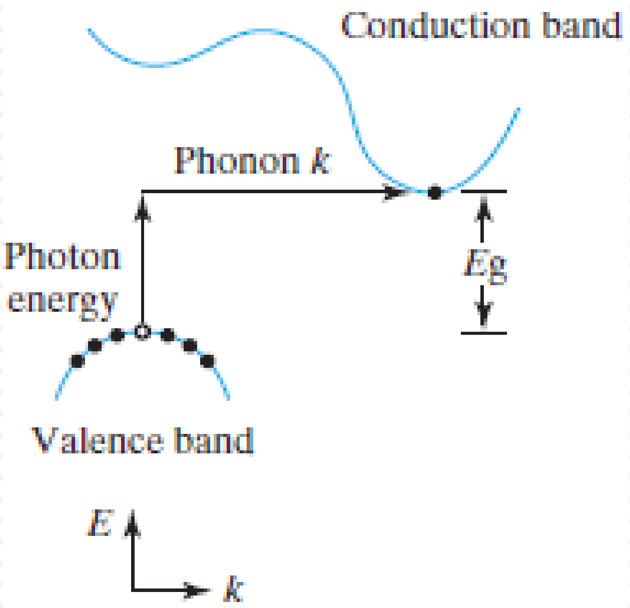
$$I = I_0(e^{qV/kT} - 1) - I_{sc}$$

Direct-Gap and Indirect-Gap Semiconductors

- Electrons have both particle and wave properties.
- An electron has energy E and wave vector k.

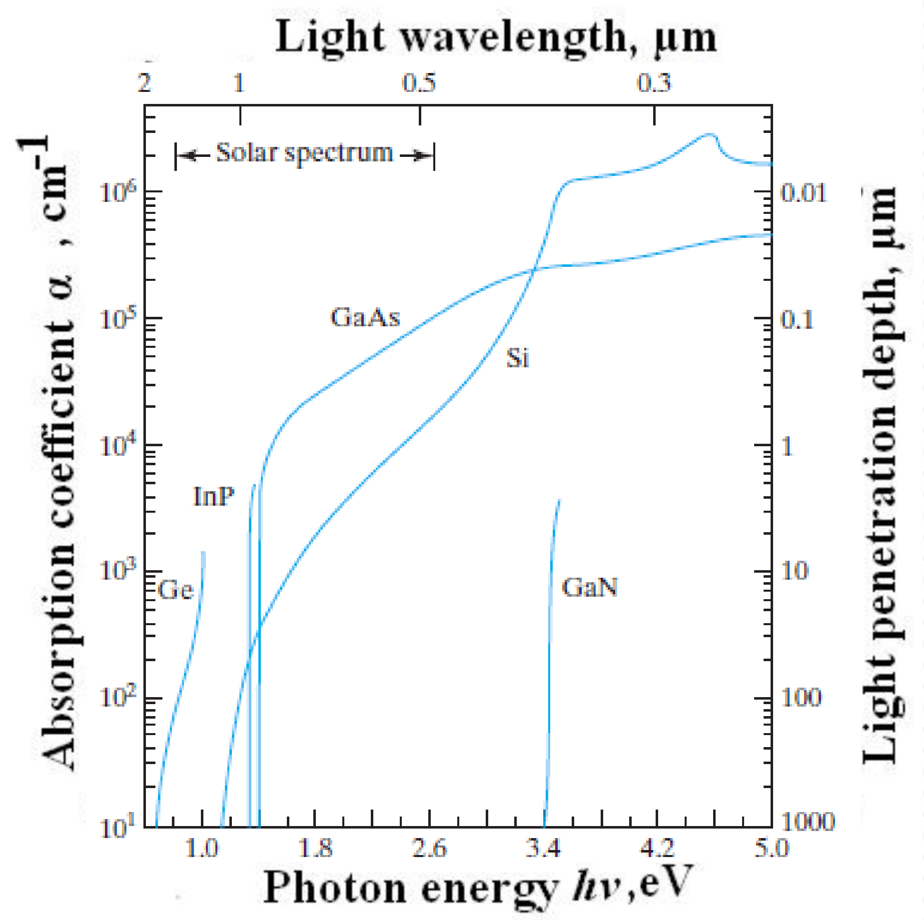


direct-gap semiconductor



indirect-gap semiconductor

4.12.2 Light Absorption



$$\text{Light intensity}(x) \propto e^{-\alpha x}$$

***α(1/cm)*: absorption coefficient**

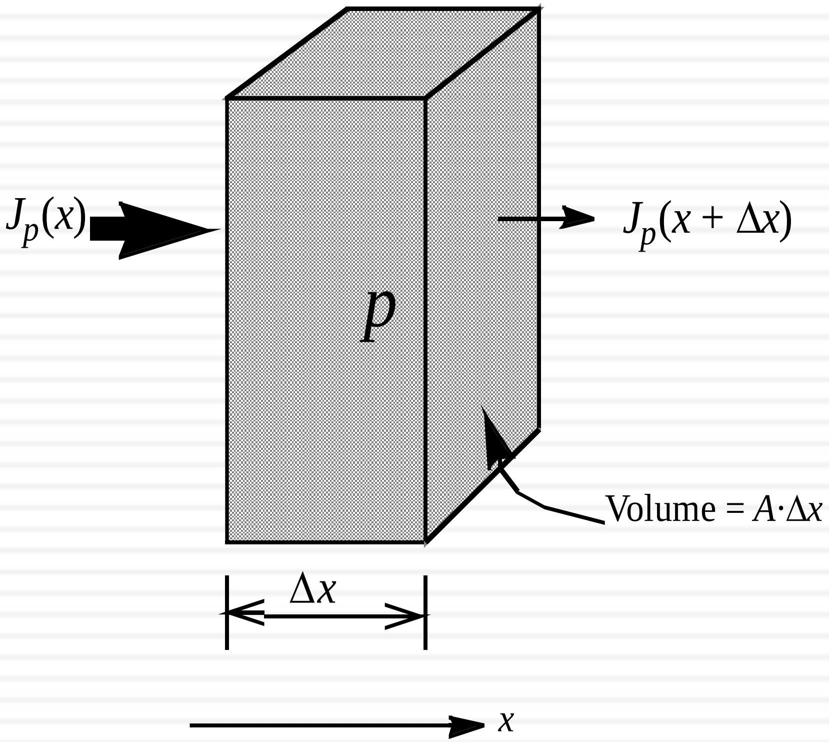
1/*α* : light penetration depth

$$\text{Photon Energy (eV)} = \frac{hc}{\lambda}$$

$$= \frac{1.24}{\lambda}(\mu m)$$

A thinner layer of direct-gap semiconductor can absorb most of solar radiation than indirect-gap semiconductor. But Si…

4.12.3 Short-Circuit Current and Open-Circuit Voltage



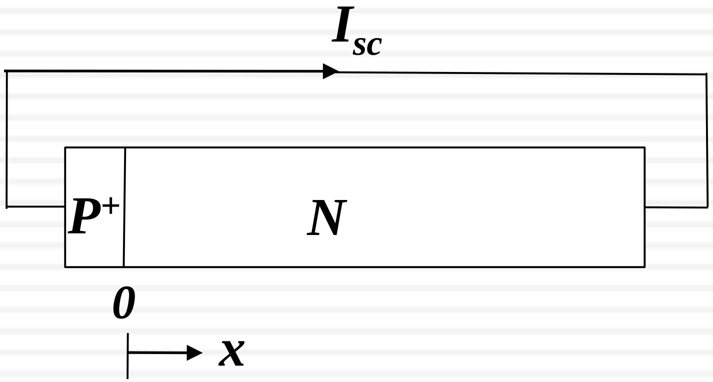
If light shines on the N-type semiconductor and generates holes (and electrons) at the rate of G $s^{-1}cm^{-3}$,

$$\frac{d^2p'}{dx^2} = \frac{p'}{L_p^2} - \frac{G}{D_p}$$

If the sample is uniform (no PN junction), $d^2p'/dx^2 = 0 \rightarrow p' = GL_p^2/D_p = G\tau_p$

Solar Cell Short-Circuit Current, I_{sc}

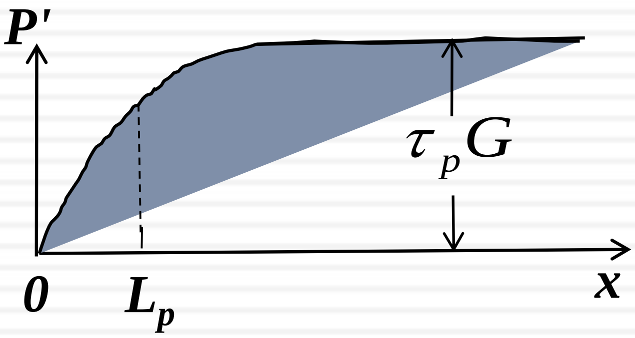
Assume very thin P+ layer and carrier generation in N region only.



$$p'(\infty) = L_p^2 \frac{G}{D_p} = \tau_p G$$

$$p'(0) = 0$$

$$p'(x) = \tau_p G(1 - e^{-x/L_p})$$



$$J_p = -qD_p \frac{dp'(x)}{dx} = q\frac{D_p}{L_p}\tau_p G e^{-x/L_p}$$

$$I_{sc} = AJ_p(0) = AqL_pG$$

G is really not uniform. L_p needs be larger than the light penetration depth to collect most of the generated carriers.

Open-Circuit Voltage

- Total current is I_{SC} plus the PV diode (dark) current:

$$I = Aq\frac{n_i^2}{N_d}\frac{D_p}{L_p}(e^{qV/kT} - 1) - AqL_pG$$

- Solve for the open-circuit voltage (V_{oc}) by setting $I=0$

(assuming $e^{qV_{oc}/kT} >> 1$)

$$0 = \frac{n_i^2}{N_d}\frac{D_p}{L_p}e^{qV_{oc}/kT} - L_pG$$

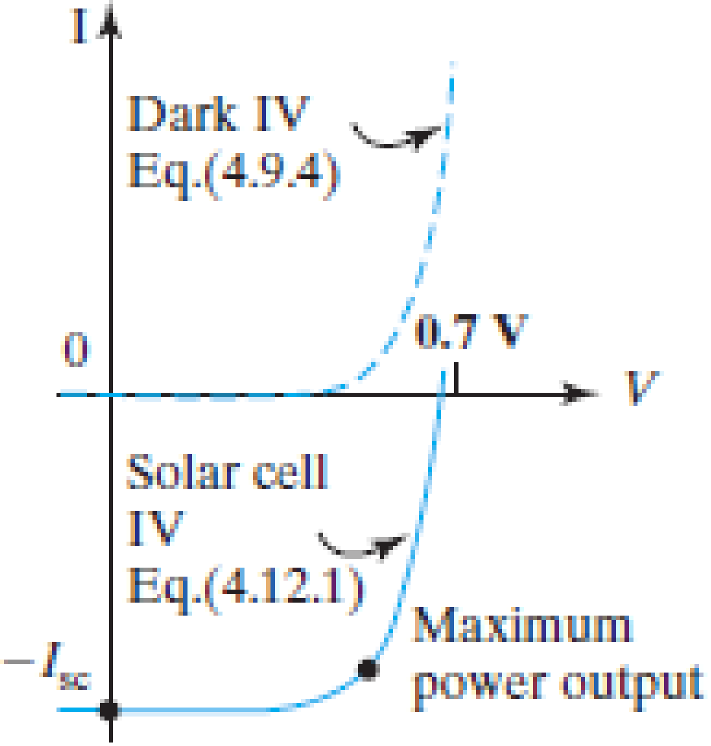
$$V_{oc} = \frac{kT}{q}\ln(\tau_p G N_d / n_i^2)$$

How to raise V_{oc} ?

4.12.4 Output Power

A particular operating point on the solar cell I-V curve maximizes the output power (I ×V).

$$\text{Output Power} = I_{sc} \times V_{oc} \times FF$$

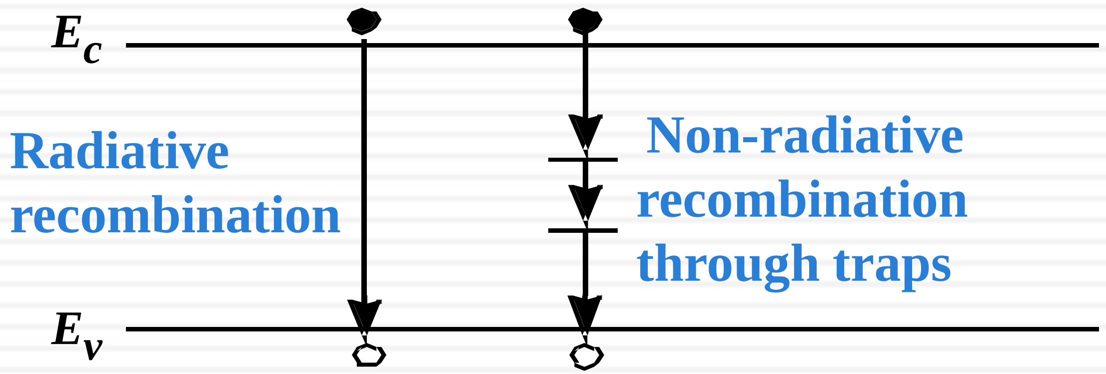


- Si solar cell with 15-20% efficiency dominates the market now
- Theoretically, the highest efficiency (~24%) can be obtained with 1.9eV >Eg>1.2eV. Larger Eg lead to too low Isc (low light absorption); smaller Eg leads to too low Voc.
- ***Tandem solar cells*** gets 35% efficiency using large ***and*** small Eg materials tailored to the short and long wavelength solar light.

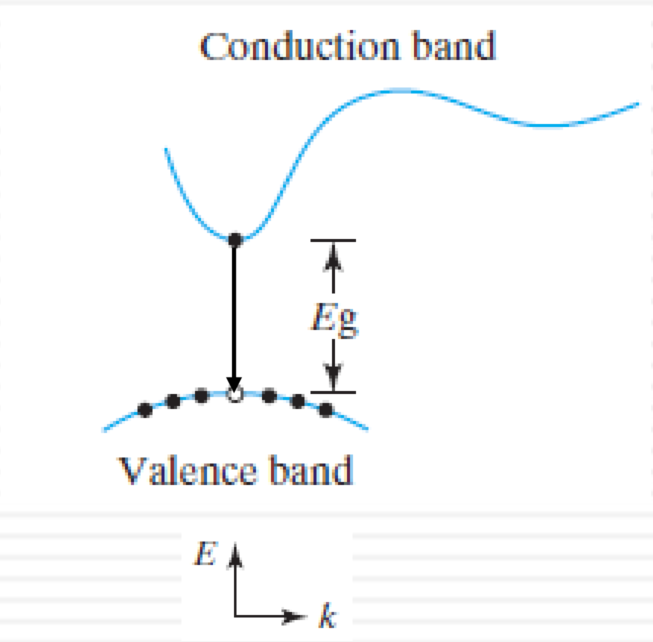
4.13 Light Emitting Diodes and Solid-State Lighting

Light emitting diodes (LEDs)

- LEDs are made of compound semiconductors such as InP and GaN.
- Light is emitted when electron and hole undergo ***radiative recombination***.

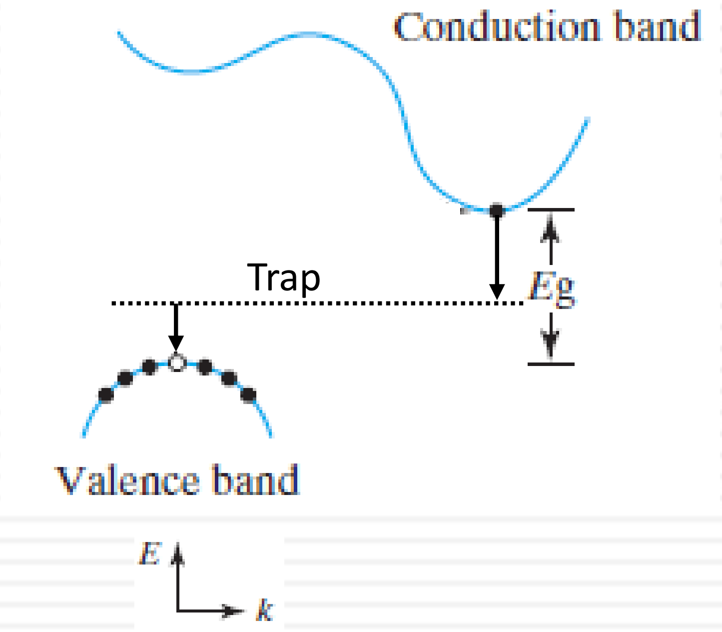


Direct and Indirect Band Gap



Direct band gap
Example: GaAs

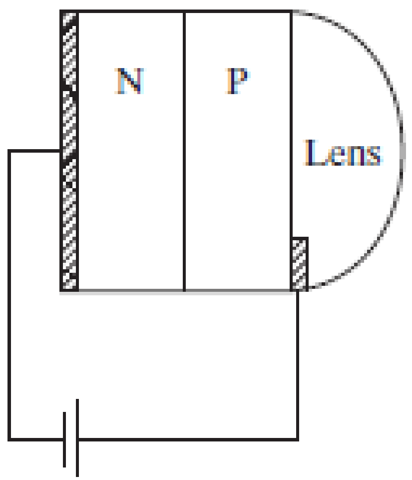
Direct recombination is efficient as k conservation is satisfied.

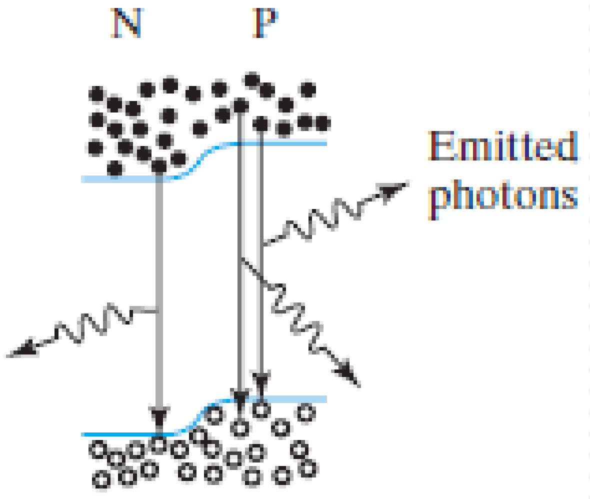


Indirect band gap
Example: Si

Direct recombination is rare as k conservation is not satisfied

4.13.1 LED Materials and Structure





$$\text{LED wavelength}\,(\mu\text{m}) = \frac{1.24}{\text{photon energy}} \approx \frac{1.24}{E_g(eV)}$$

4.13.1 LED Materials and Structure

	E_g(eV)	**Wavelength (µm)**	**Color**	**Lattice constant (Å)**
InAs	**0.36**	**3.44**		**6.05**
InN	**0.65**	**1.91**	**infrared**	**3.45**
InP	**1.36**	**0.92**	↑	**5.87**
GaAs	**1.42**	**0.87**	Red Yellow	**5.66**
GaP	**2.26**	**0.55**	Green Blue	**5.46**
AlP	**3.39**	**0.51**		**5.45**
GaN	**2.45**	**0.37**	↓	**3.19**
AlN	**6.20**	**0.20**	**UV**	**3.11**

Light-emitting diode materials

compound semiconductors

binary semiconductors:

- Ex: GaAs, efficient emitter

ternary semiconductor :

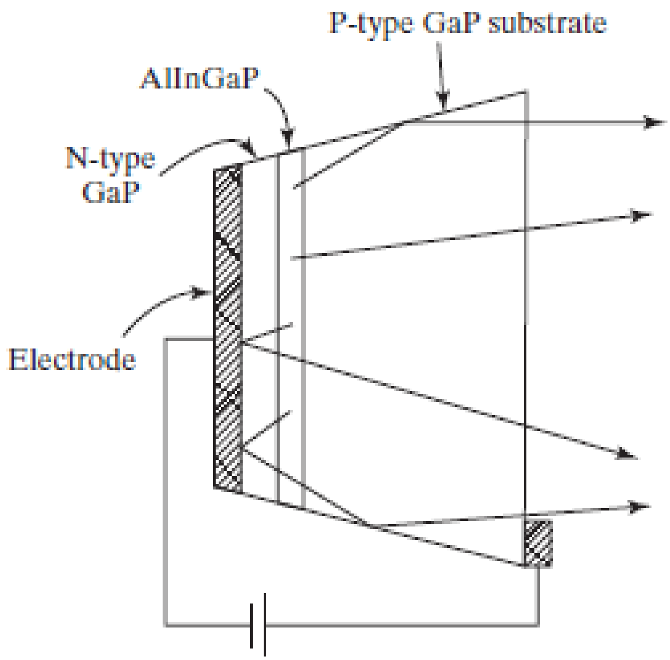
- Ex: $GaAs_{1-x}P_x$, tunable E_g (to vary the color)

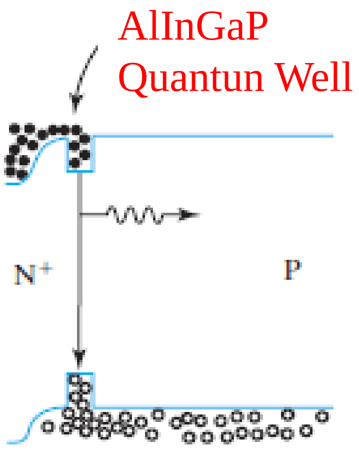
quaternary semiconductors:

- Ex: AlInGaP , tunable E_g and lattice constant (for growing high quality epitaxial films on inexpensive substrates)

Common LEDs

Spectral range	**Material System**	**Substrate**	**Example Applications**
Infrared	**InGaAsP**	**InP**	**Optical communication**
Infrared -Red	**GaAsP**	**GaAs**	**Indicator lamps. Remote control**
Red-Yellow	**AlInGaP**	**GaA or GaP**	**Optical communication. High-brightness traffic signal lights**
Green-Blue	**InGaN**	**GaN or sapphire**	**High brightness signal lights. Video billboards**
Blue-UV	**AlInGaN**	**GaN or sapphire**	**Solid-state lighting**
Red-Blue	**Organic semicon-ductors**	**glass**	**Displays**





4.13.2 Solid-State Lighting

luminosity (lumen, lm): a measure of visible light energy normalized to the sensitivity of the human eye at different wavelengths

Incandescent lamp	Compact fluorescent lamp	Tube fluorescent lamp	**White LED**	Theoretical limit at peak of eye sensitivity (λ=555nm)	Theoretical limit (white light)
17	**60**	**50-100**	**90-?**	**683**	**~340**

Luminous efficacy of lamps in lumen/watt

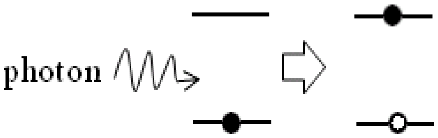
Organic Light Emitting Diodes (OLED) :
has lower efficacy than nitride or aluminide based compound semiconductor LEDs.

Terms: **luminosity** measured in **lumens**. **luminous efficacy,**

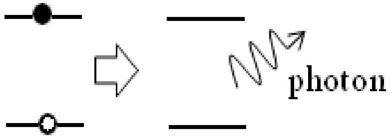
4.14 Diode Lasers

4.14.1 Light Amplification

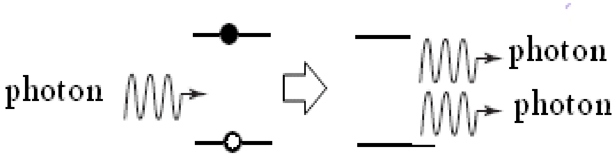
(a) Absorption



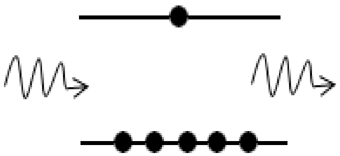
(b) Spontaneous Emission



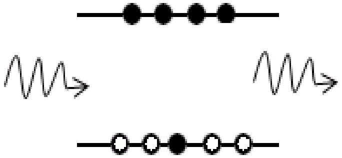
(c) Stimulated Emission



(d) Net Light Absorption



(e) Net Light Amplification



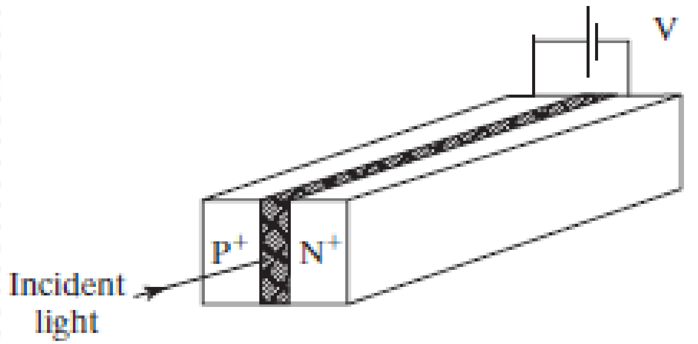
Light amplification requires population inversion: electron occupation probability is larger for higher E states than lower E states.

Stimulated emission: emitted photon has identical frequency and directionality as the stimulating photon; light wave is amplified.

4.14.1 Light Amplification in PN Diode

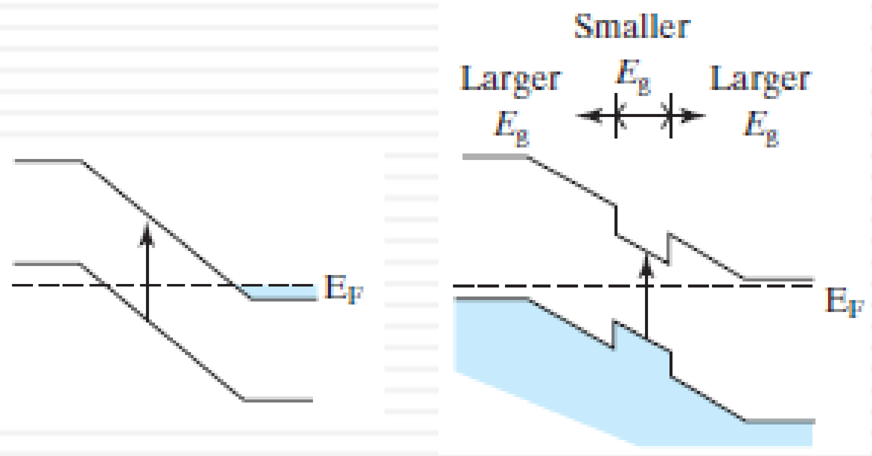
Population inversion is achieved when

$$qV = E_{fn} - E_{fp} > E_g$$

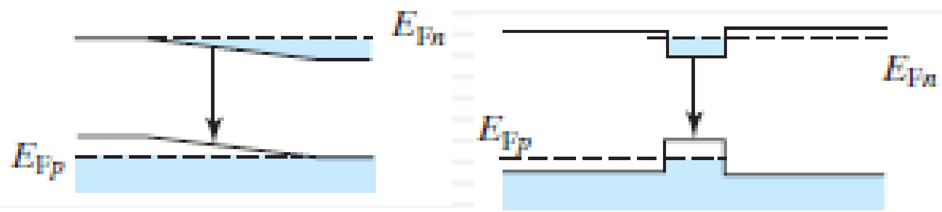


P^+N^+diode

Quantum-well diode

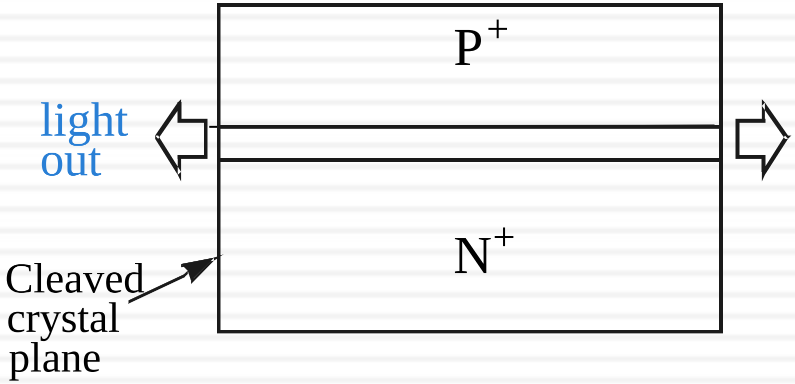


Equilibrium, V=0



Population inversion, qV > Eg

4.14.2 Optical Feedback and Laser



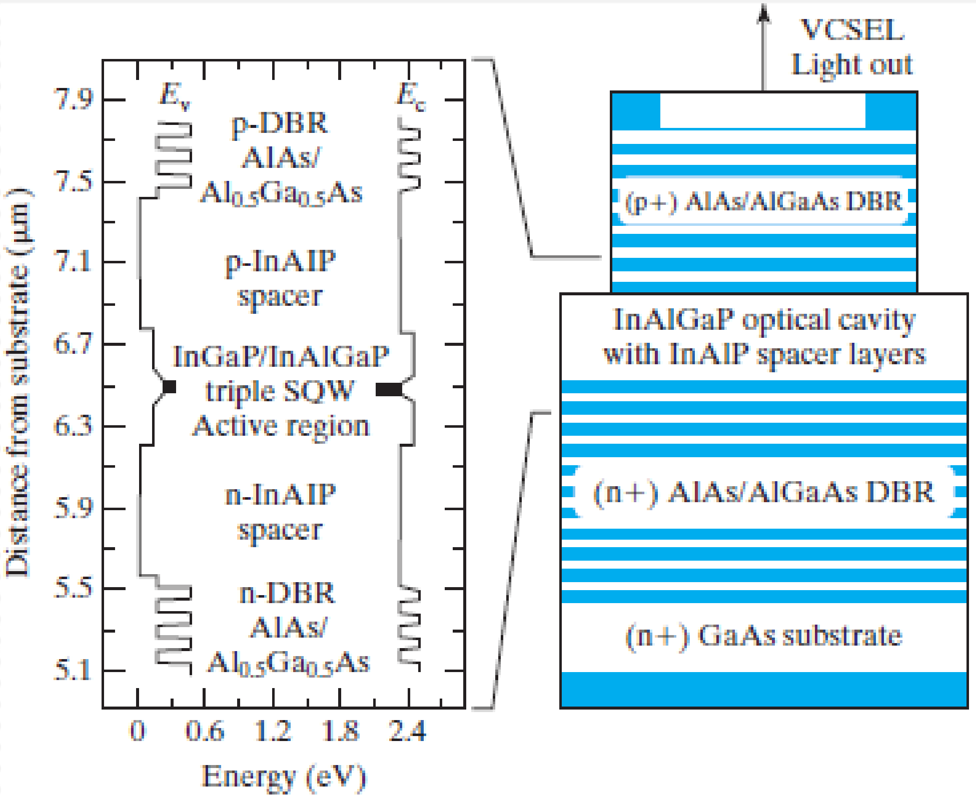
Laser threshold is reached (light intensity grows by feedback) when

$$R_1 \times R_2 \times G \geq 1$$

- **R1, R2**: reflectivities of the two ends
- **G** : light amplification factor (gain) for a round-trip travel of the light through the diode

Light intensity grows until $R_1 \times R_2 \times G = 1$, when the light intensity is just large enough to stimulate carrier recombinations at the same rate the carriers are injected by the diode current.

4.14.2 Optical Feedback and Laser Diode



- ***Distributed Bragg reflector (DBR)*** **reflects light with multi-layers of semiconductors.**
- ***Vertical-cavity surface-emitting laser (VCSEL)*** **is shown on the left.**
- **Quantum-well laser has smaller threshold current because fewer carriers are needed to achieve population inversion in the small volume of the thin small-*Eg* well.**

4.14.3 Laser Applications

Red ***diode lasers***: *CD, DVD reader/writer*

Blue ***diode lasers***: *Blu-ray DVD (higher storage density)*

1.55 μm infrared ***diode lasers:*** *Fiber-optic communication*

4.15 Photodiodes

Photodiodes: Reverse biased PN diode. Detects photo-generated current (similar to Isc of solar cell) for optical communication, DVD reader, etc. Avalanche photodiodes: Photodiodes operating near avalanche breakdown amplifies photocurrent by impact ionization.

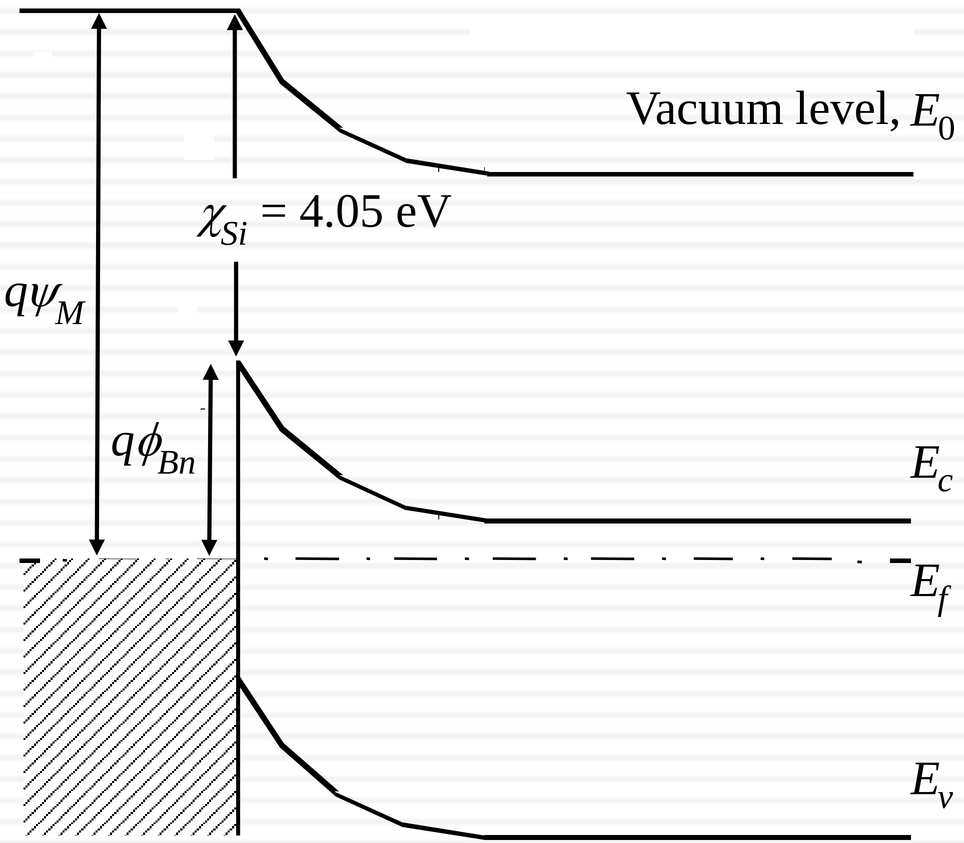
Part III: Metal-Semiconductor Junction

Two kinds of metal-semiconductor contacts:

- *Rectifying* ***Schottky diodes****: metal on lightly doped silicon*

- *Low-resistance* ***ohmic contacts****: metal on heavily doped silicon*

ϕ_{Bn} Increases with Increasing Metal Work Function



ψ_M : Work Function of metal

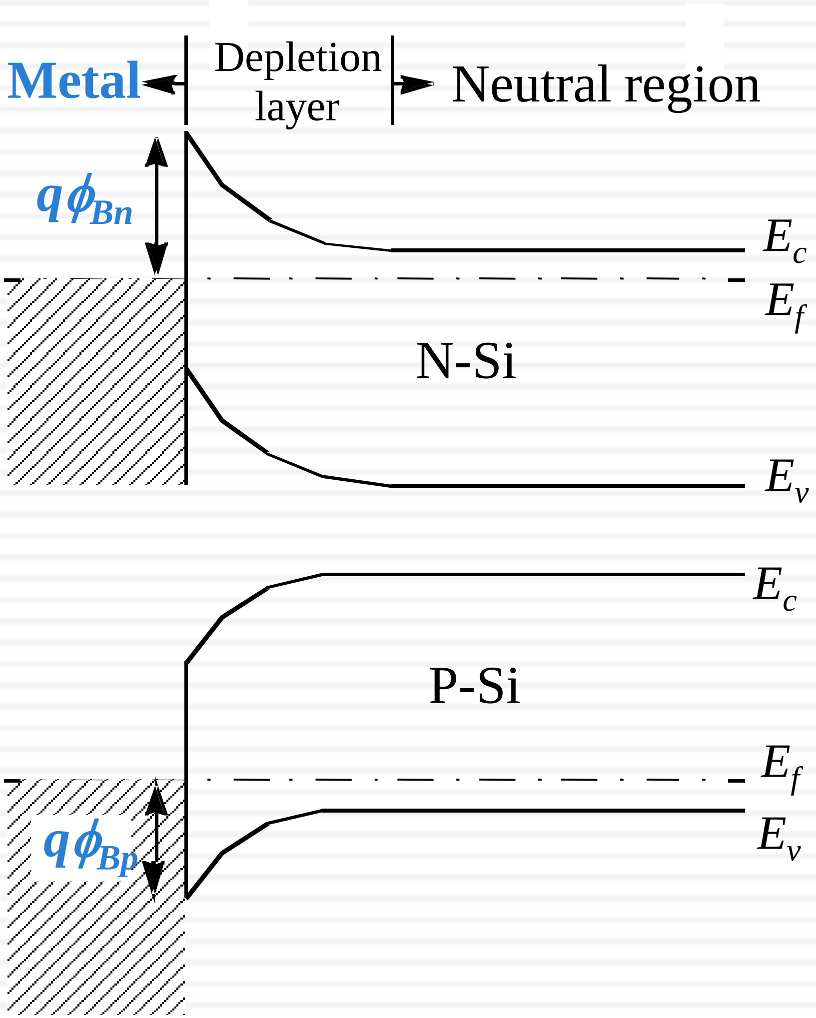
χ_{Si} : Electron Affinity of Si

Theoretically,

$$\phi_{Bn} = \psi_M - \chi_{Si}$$

4.16 Schottky Barriers

Energy Band Diagram of Schottky Contact



- Schottky barrier height, ϕ_B , is a function of the metal material.
- ϕ_B is the most important parameter. The sum of $q\phi_{Bn}$ and $q\phi_{Bp}$ is equal to E_g .

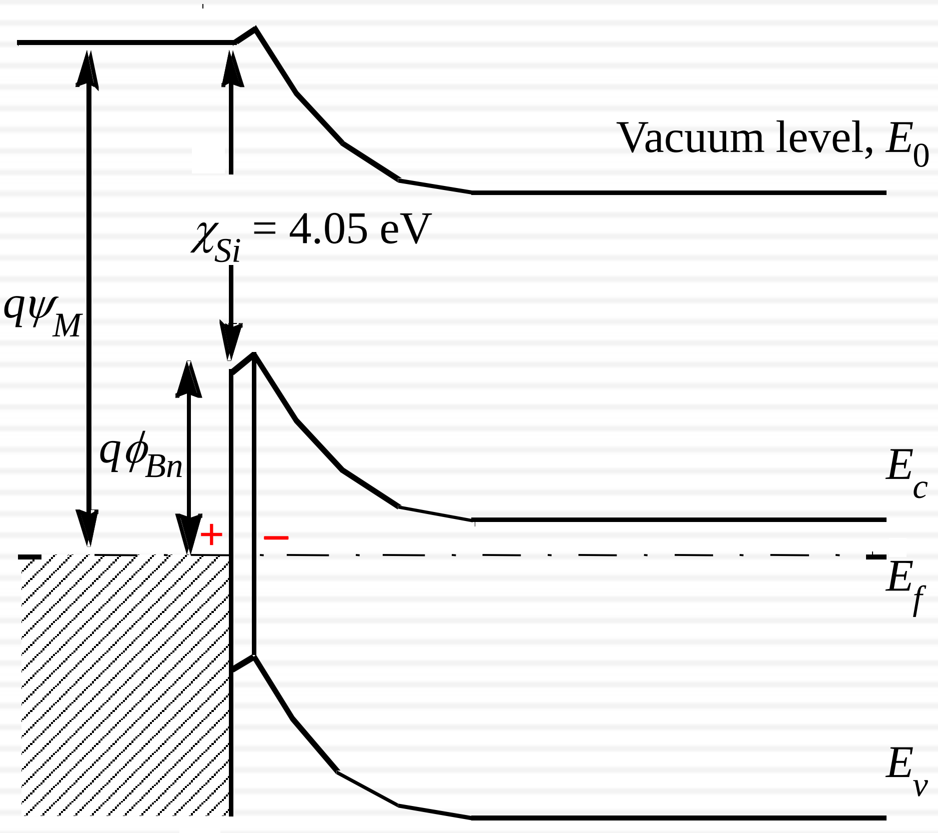
Schottky barrier heights for electrons and holes

Metal	Mg	Ti	Cr	W	Mo	Pd	Au	Pt
ϕ_{Bn} (V)	0.4	0.5	0.61	0.67	0.68	0.77	0.8	0.9
ϕ_{Bp} (V)		0.61	0.5		0.42		0.3	
Work Function ψ_m (V)	3.7	4.3	4.5	4.6	4.6	5.1	5.1	5.7

$$\phi_{Bn} + \phi_{Bp} \approx \mathrm{E_g}$$

ϕ_{Bn} increases with increasing metal work function

Fermi Level Pinning



- A high density of energy states in the bandgap at the metal-semiconductor interface pins E_f to a narrow range and $\boldsymbol{\phi_{Bn}}$ ***is typically 0.4 to 0.9 V***
- ***Question***: What is the typical range of ϕ_{Bp}?

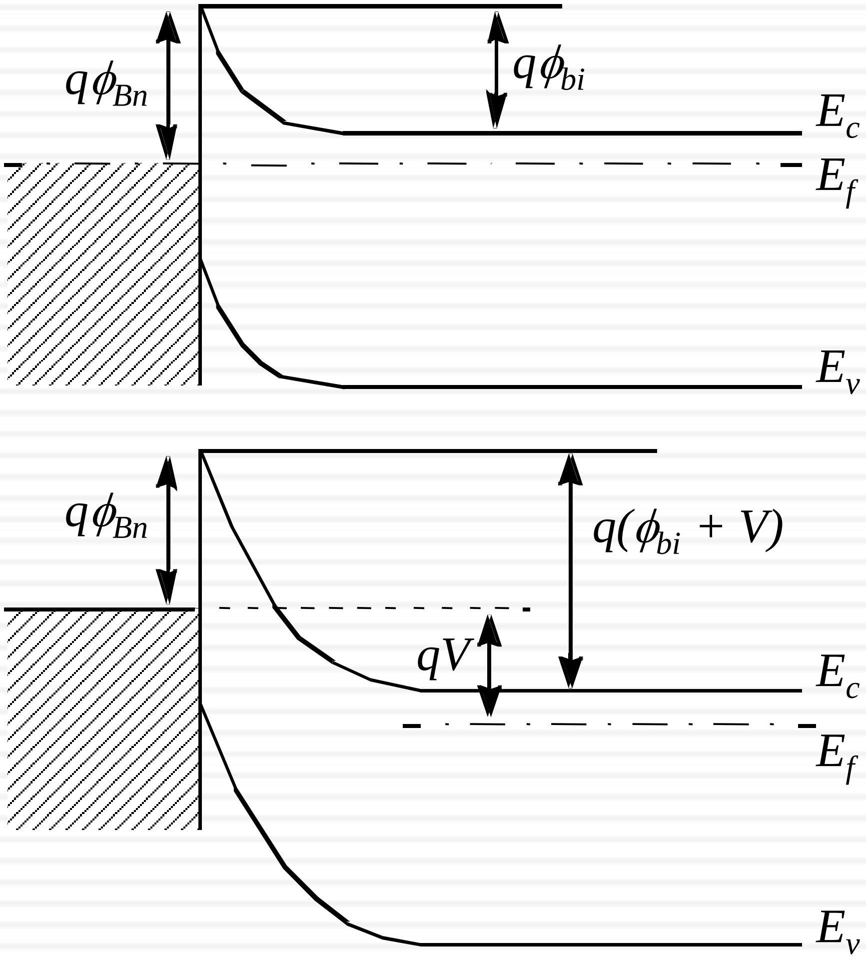
Schottky Contacts of Metal Silicide on Si

Silicide: A silicon and metal compound. It is conductive similar to a metal.

Silicide-Si interfaces are more stable than metal-silicon interfaces. After metal is deposited on Si, an annealing step is applied to form a silicide-Si contact. The term ***metal-silicon contact*** includes and almost always means silicide-Si contacts.

Silicide	$ErSi_{1.7}$	HfSi	$MoSi_2$	$ZrSi_2$	$TiSi_2$	$CoSi_2$	WSi_2	$NiSi_2$	Pd_2Si	PtSi
ϕ_{Bn} (V)	0.28	0.45	0.55	0.55	0.61	0.65	0.67	0.67	0.75	0.87
ϕ_{Bp} (V)			0.55	0.49	0.45	0.45	0.43	0.43	0.35	0.23

Using C-V Data to Determine ϕ_B



$$q\phi_{bi} = q\phi_{Bn} - (E_c - E_f)$$

$$= q\phi_{Bn} - kT \ln \frac{N_c}{N_d}$$

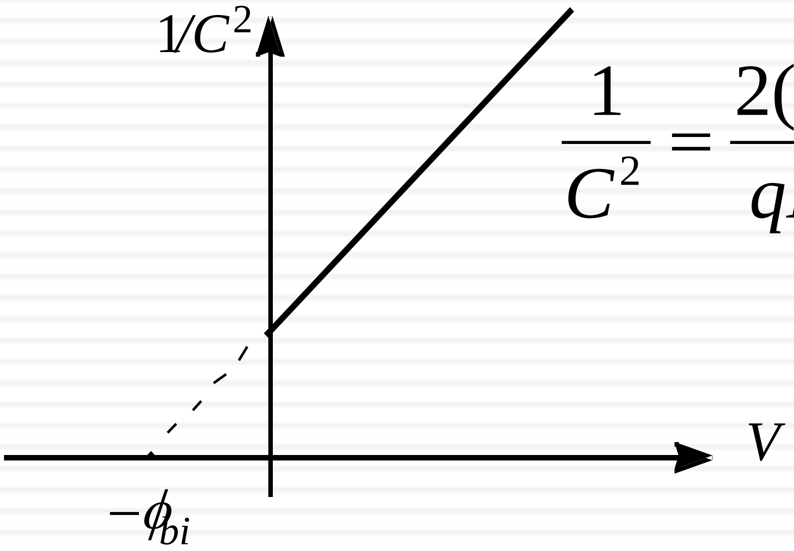
$$W_{dep} = \sqrt{\frac{2\varepsilon_s(\phi_{bi} + V)}{qN_d}}$$

$$C = \frac{\varepsilon_s}{W_{dep}} A$$

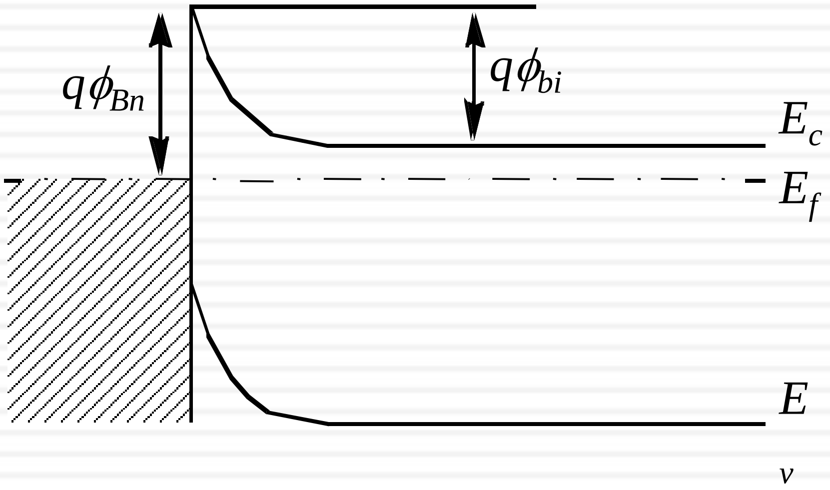
Question:

How should we plot the CV data to extract ϕ_{bi}?

Using CV Data to Determine ϕ_B



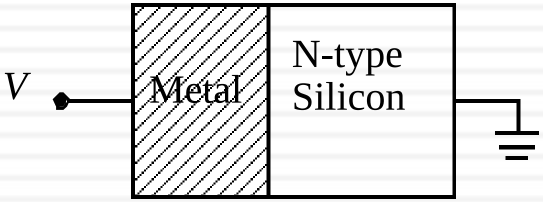
$$\frac{1}{C^2} = \frac{2(\phi_{bi} + V)}{qN_d \varepsilon_s A^2}$$

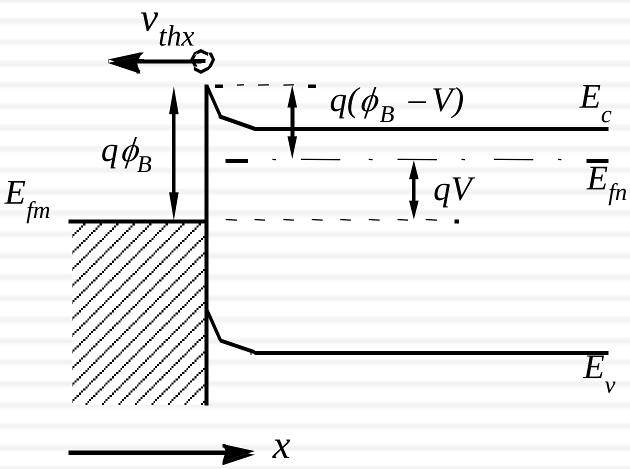


Once ϕ_{bi} is known, ϕ_B can be determined using

$$q\phi_{bi} = q\phi_{Bn} - (E_c - E_f) = q\phi_{Bn} - kT \ln \frac{N_c}{N_d}$$

4.17 Thermionic Emission Theory





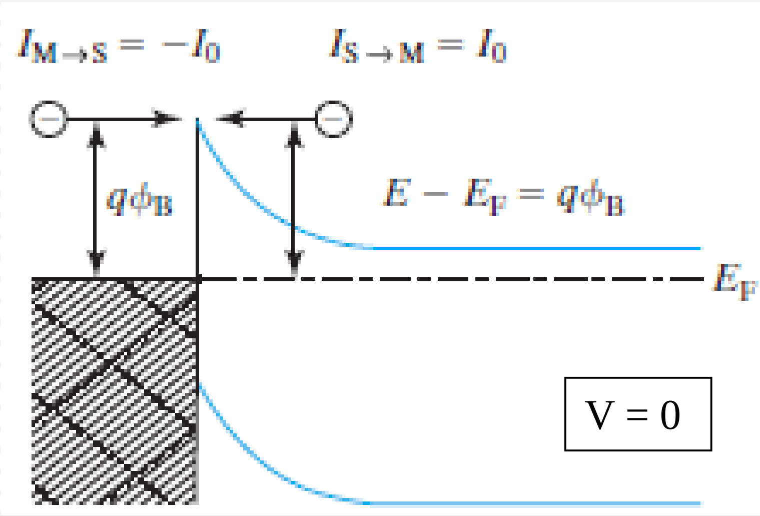
$$n = N_c e^{-q(\phi_B - V)/kT} = 2\left[\frac{2\pi m_n kT}{h^2}\right]^{3/2} e^{-q(\phi_B - V)/kT}$$

$$v_{th} = \sqrt{3kT/m_n} \qquad v_{thx} = -\sqrt{2kT/\pi m_n}$$

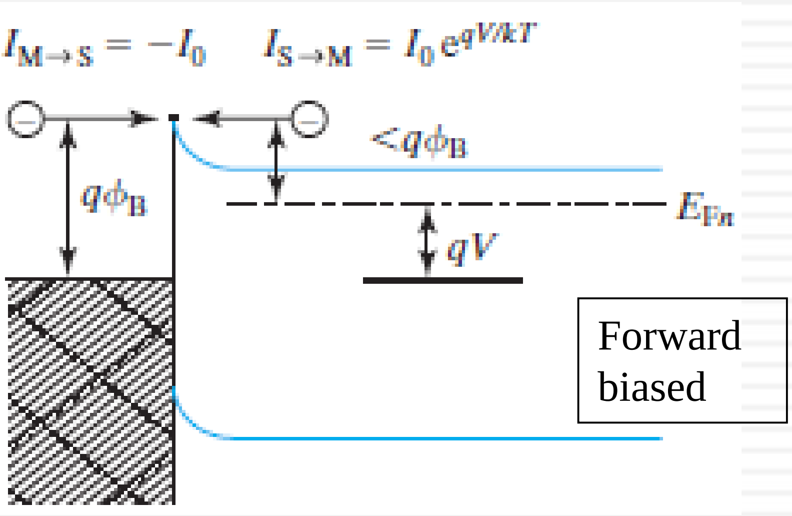
$$J_{S\rightarrow M} = -\frac{1}{2} q n v_{thx} = \frac{4\pi q m_n k^2}{h^3} T^2 e^{-q\phi_B/kT} e^{qV/kT}$$

$$= J_0 e^{qV/kT}, \text{where } J_o \approx 100 e^{-q\phi_B/kT} \ \text{A/cm}^2$$

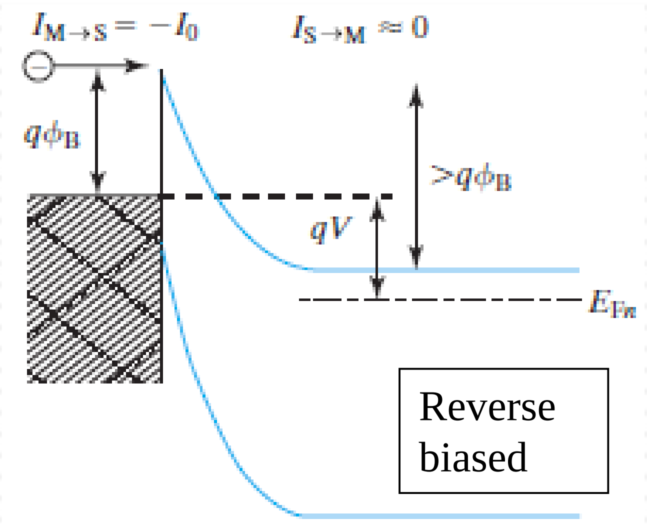
4.18 Schottky Diodes



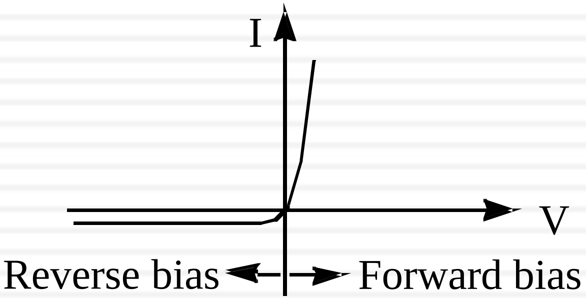
V = 0



Forward biased

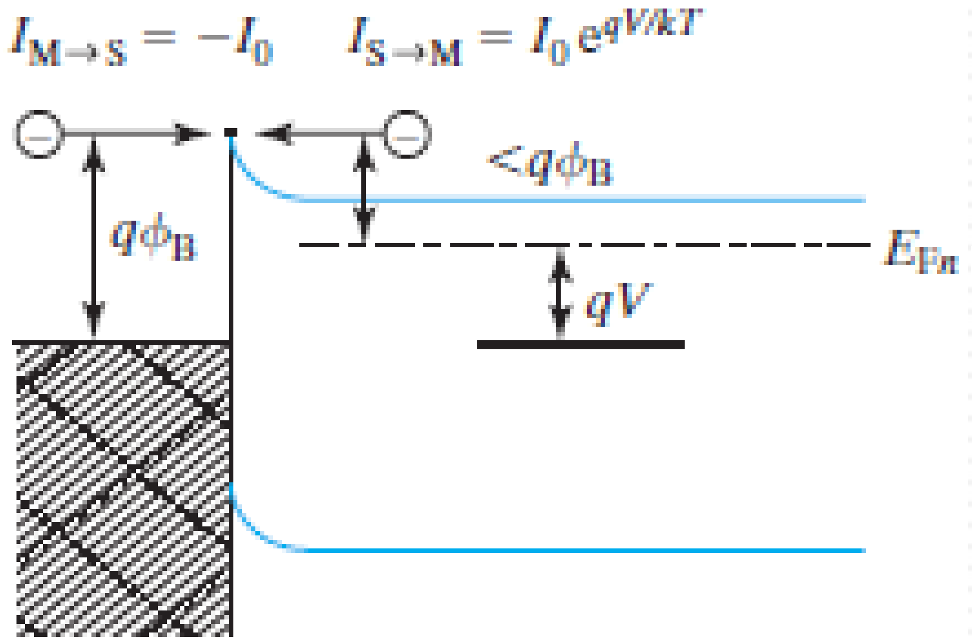


Reverse biased



Reverse bias ← → Forward bias

4.18 Schottky Diodes

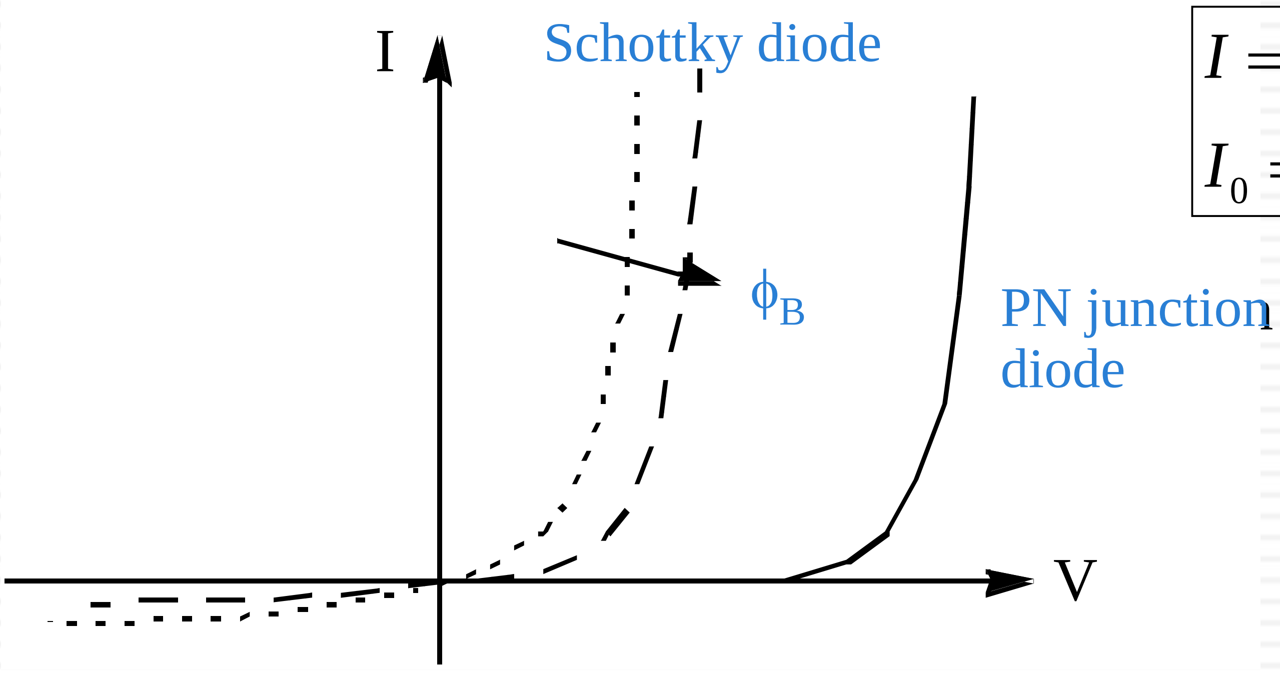


$$I_0 = AKT^2 e^{-q\phi_B/kT}$$

$$K = \frac{4\pi q m_n k^2}{h^3} \approx 100\ \mathrm{A/(cm^2 \cdot K^2)}$$

$$I = I_{S\to M} + I_{M\to S} = I_0 e^{qV/kT} - I_0 = I_0(e^{qV/kT} - 1)$$

4.19 Applications of Schottly Diodes

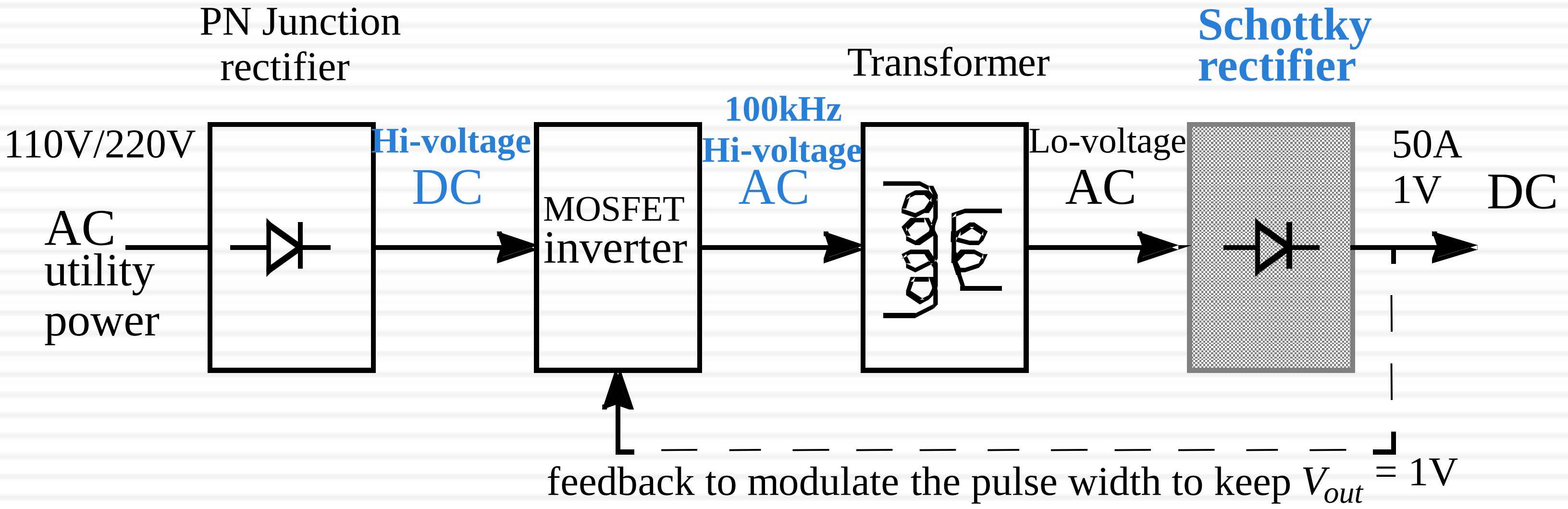


$$I = I_0(e^{qV/kT} - 1)$$

$$I_0 = AKT^2 e^{-q\phi_B/kT}$$

- I_0 of a Schottky diode is 10^3 to 10^8 times larger than a PN junction diode, depending on ϕ_B. A larger I_0 means a smaller forward drop V.
- A Schottky diode is the preferred rectifier in low voltage, high current applications.

Switching Power Supply



feedback to modulate the pulse width to keep $V_{out} = 1V$

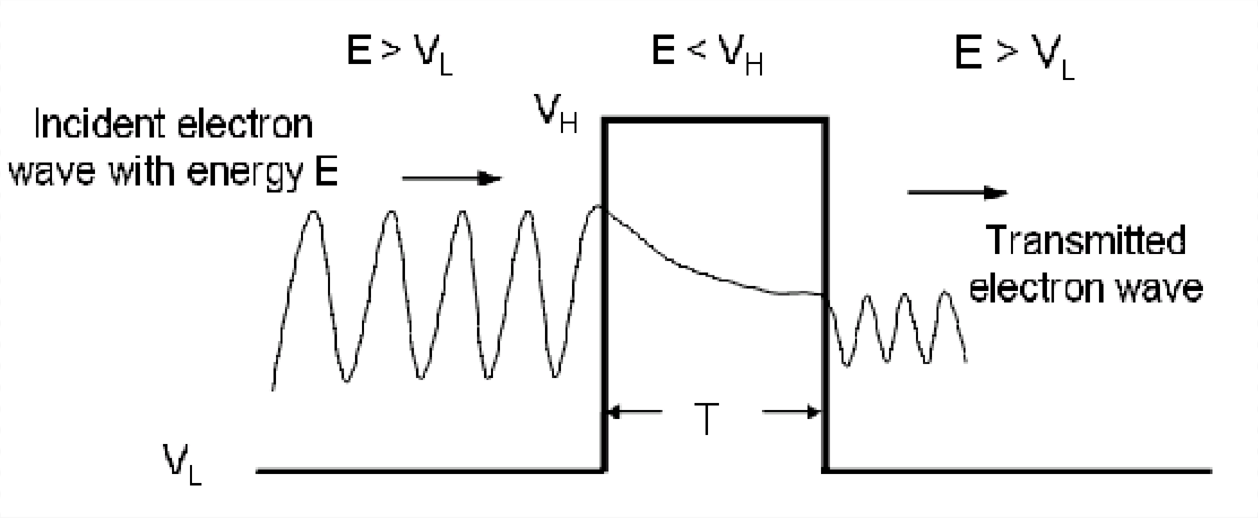
4.19 Applications of Schottky diodes

Question: What sets the lower limit in a Schottky diode’s forward drop?

- ***Synchronous Rectifier***: For an even lower forward drop, replace the diode with a wide-W MOSFET which is not bound by the tradeoff between diode *V* and leakage current.

- There is no minority carrier injection at the Schottky junction. Therefore, Schottky diodes can operate at higher frequencies than PN junction diodes.

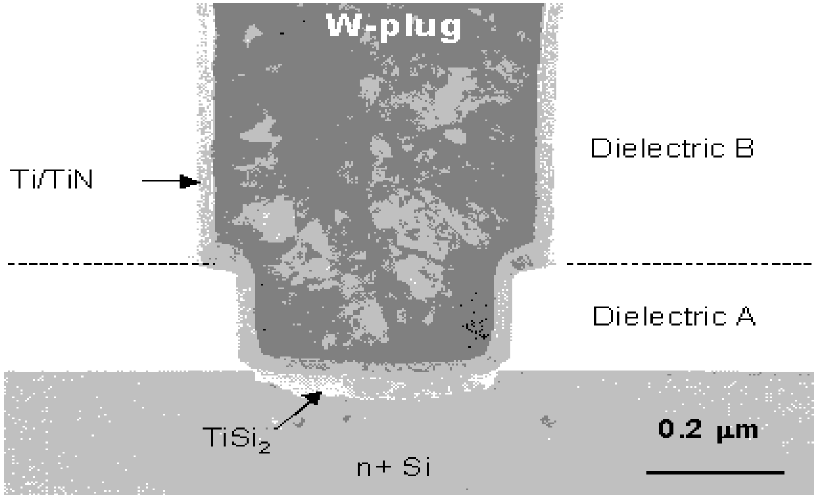
4.20 Quantum Mechanical Tunneling

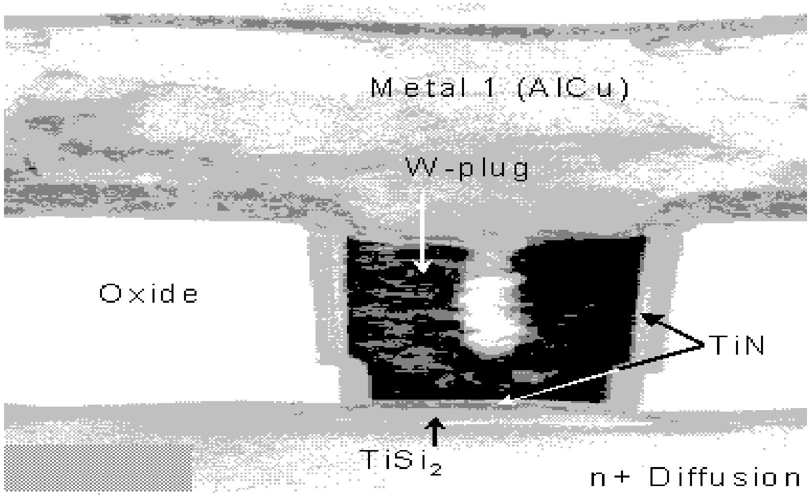


Tunneling probability:

$$P \approx \exp\left(-2T\sqrt{\frac{8\pi^2 m}{h^2}(V_H - E)}\right)$$

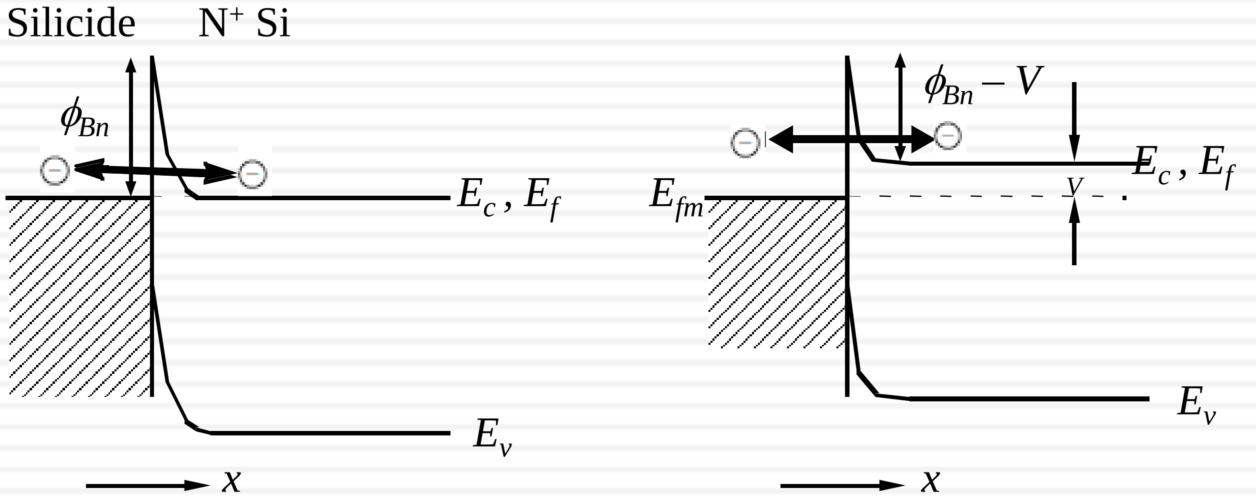
4.21 Ohmic Contacts





4.21 Ohmic Contacts

$$W_{dep} = \sqrt{\frac{2\varepsilon_s \phi_{Bn}}{qN_d}}$$



Tunneling probability:

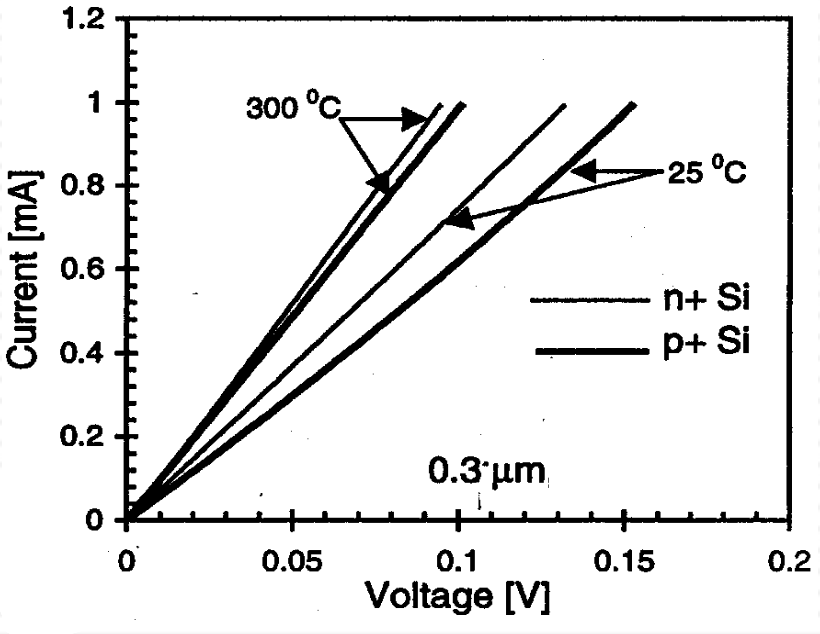
$$P \approx e^{-H\phi_{Bn}/\sqrt{N_d}}$$

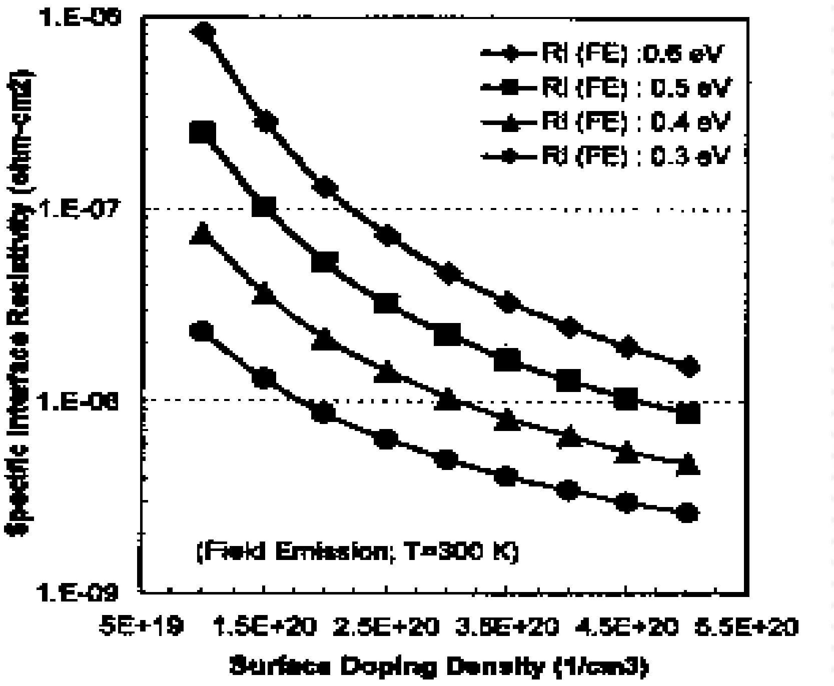
$$T \approx W_{dep}/2 = \sqrt{\varepsilon_s \phi_{Bn}/2qN_d}$$

$$H = \frac{4\pi}{h}\sqrt{\varepsilon_s m_n/q}$$

$$J_{S\rightarrow M} \approx \frac{1}{2} qN_d v_{thx} P = qN_d \sqrt{kT/2\pi m_n}\, e^{-H(\phi_{Bn}-V)/\sqrt{N_d}}$$

4.21 Ohmic Contacts





$$R_c \equiv \left(\frac{dJ_{S\to M}}{dV}\right)^{-1} = \frac{2e^{H\phi_{Bn}/\sqrt{N_d}}}{qv_{thx}H\sqrt{N_d}} \propto e^{H\phi_{Bn}/\sqrt{N_d}} \quad \Omega\cdot\mathrm{cm}^2$$

4.22 Chapter Summary

Part I: PN Junction

$$\boxed{\phi_{bi} = \frac{kT}{q} \ln \frac{N_d N_a}{n_i^2}}$$

The potential barrier increases by 1 V if a 1 V reverse bias is applied

depletion width

$$\boxed{W_{dep} = \sqrt{\frac{2\varepsilon_s \cdot potential\,barrier}{qN}}}$$

junction capacitance

$$\boxed{C_{dep} = A\frac{\varepsilon_s}{W_{dep}}}$$

4.22 Chapter Summary

- Under forward bias, minority carriers are injected across the jucntion.

- The quasi-equilibrium boundary condition of minority carrier densities is:

$$n(x_p) = n_{P0} e^{qV/kT}$$
$$p(x_N) = p_{N0} e^{qV/kT}$$

- Most of the minority carriers are injected into the more lightly doped side.

4.22 Chapter Summary

- Steady-state continuity equation:

$$\frac{d^2p'}{dx^2}=\frac{p'}{D_p\tau_p}=\frac{p'}{L_p^{\,2}}$$

$$L_p\equiv\sqrt{D_p\tau_p}$$

- Minority carriers diffuse outward $\propto e^{-|x|/L_p}$ and $e^{-|x|/L_n}$
- L_p and L_n are the diffusion lengths

$$I=I_0(e^{qV/kT}-1)$$

$$I_0=Aqn_i^{\,2}\left(\frac{D_p}{L_pN_d}+\frac{D_n}{L_nN_a}\right)$$

4.22 Chapter Summary

Charge storage: $\boxed{Q = I\tau_s}$

Diffusion capacitance: $\boxed{C = \tau_s G}$

Diode conductance: $\boxed{G = I_{DC} / \frac{kT}{q}}$

4.22 Chapter Summary

Part II: Optoelectronic Applications

$$\text{Solar cell power} = I_{sc} \times V_{oc} \times FF$$

•~100um Si or <1um direct–gap semiconductor can absorb most of solar photons with energy larger than E_g.

•Carriers generated within diffusion length from the junction can be collected and contribute to the Short Circuit Current I_{sc}.

•Theoretically, the highest efficiency (~24%) can be obtained with 1.9eV $>E_g>$1.2eV. Larger E_g lead to too low I_{sc} (low light absorption); smaller E_g leads to too low Open Circuit VoltageVoc.

•Si cells with ~15% efficiency dominate the market. >2x cost reduction (including package and installation) is required to achieve cost parity with base-load non-renewable electricity.

4.22 Chapter Summary

Part II: Optoelectronic Applications

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4.22 Chapter Summary

Laser Diodes

- Light is amplified under the condition of population inversion – states at higher E have higher probability of occupation than states at lower E.
- Population inversion occurs when diode forward bias $qV > E_g$.
- Optical feedback is provided with cleaved surfaces or distributed Bragg reflectors.
- When the round-trip gain (including loss at reflector) exceeds unity, laser threshold is reached.
- Quantum-well structures significantly reduce the threshold currents.
- Purity of laser light frequency enables long-distance fiber-optic communication. Purity of light direction allows focusing to tiny spots and enables DVD writer/reader and other application.

4.22 Chapter Summary

Part III: Metal-Semiconductor Junction

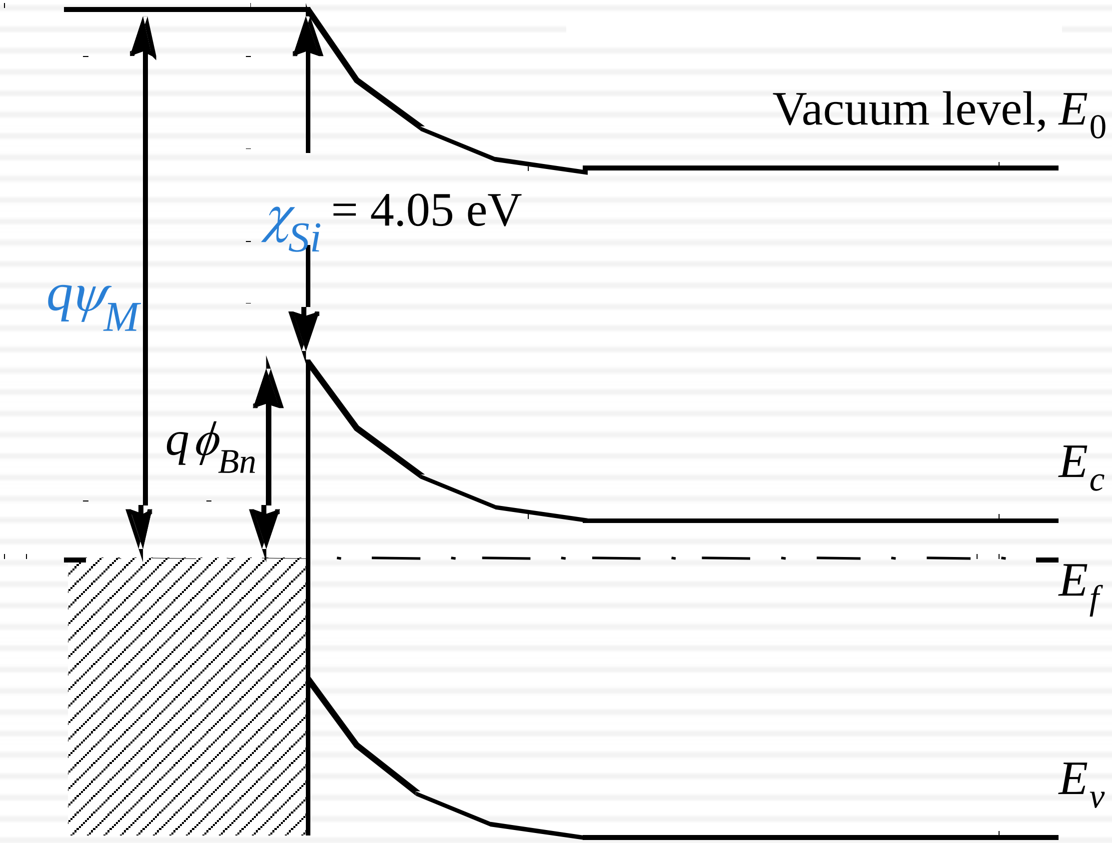
$$I_0 = AKT^2 e^{-q\phi_B / kT}$$

- Schottky diodes have large reverse saturation current, determined by the Schottky barrier height ϕ_B, and therefore lower forward voltage at a given current density.

- Ohmic contacts relies on tunneling. Low resistance contact requires low ϕ_B and higher doping concentration.

$$R_c \propto e^{-(\frac{4\pi}{h}\phi_B \sqrt{\varepsilon_s m_n / qN_d})} \quad \Omega \cdot \text{cm}^2$$

ϕ_{Bn} Increases with Increasing Metal Work Function



Ideally,

$$\phi_{Bn} = \psi_M - \chi_{Si}$$