Chapter 2: Motion and Recombination of Electrons and Holes

2.1 Thermal Motion

$$\text{Average electron kinetic energy} = \frac{\text{total kinetic energy}}{\text{number of electrons}}$$

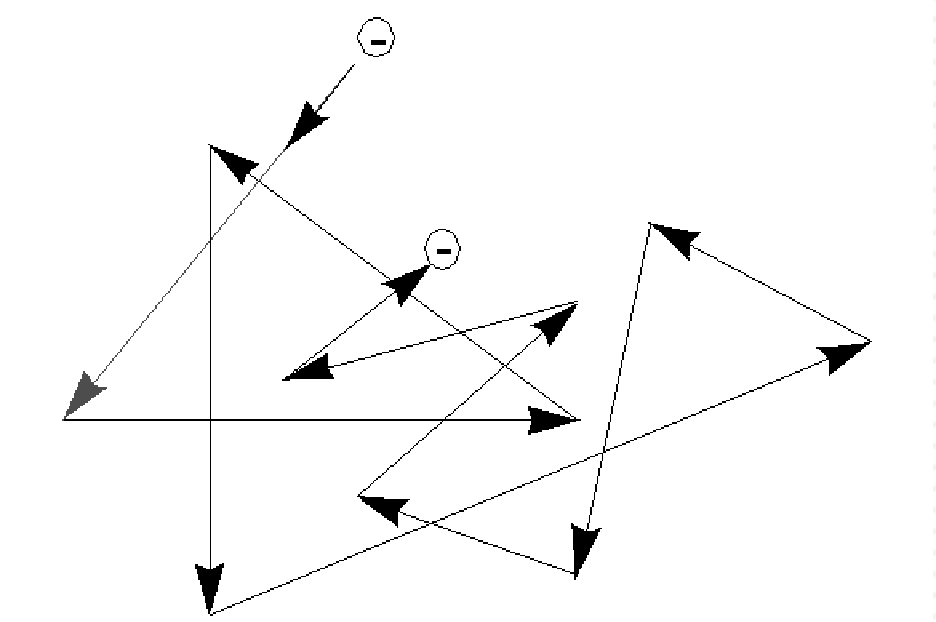
$$= \frac{\int f(E)D(E)(E - E_c)\mathrm{d}E}{\int f(E)D(E)\mathrm{d}E}$$

$$\textit{Average electron or hole kinetic energy} = \frac{3}{2}kT = \frac{1}{2}mv_{th}^2$$

$$v_{th} = \sqrt{\frac{3kT}{m_{eff}}} = \sqrt{\frac{3\times1.38\times10^{-23}\,\mathrm{JK}^{-1}\times300\mathrm{K}}{0.26\times9.1\times10^{-31}\,\mathrm{kg}}}$$

$$= 2.3\times10^{5}\,\mathrm{m/s} = 2.3\times10^{7}\,\mathrm{cm/s}$$

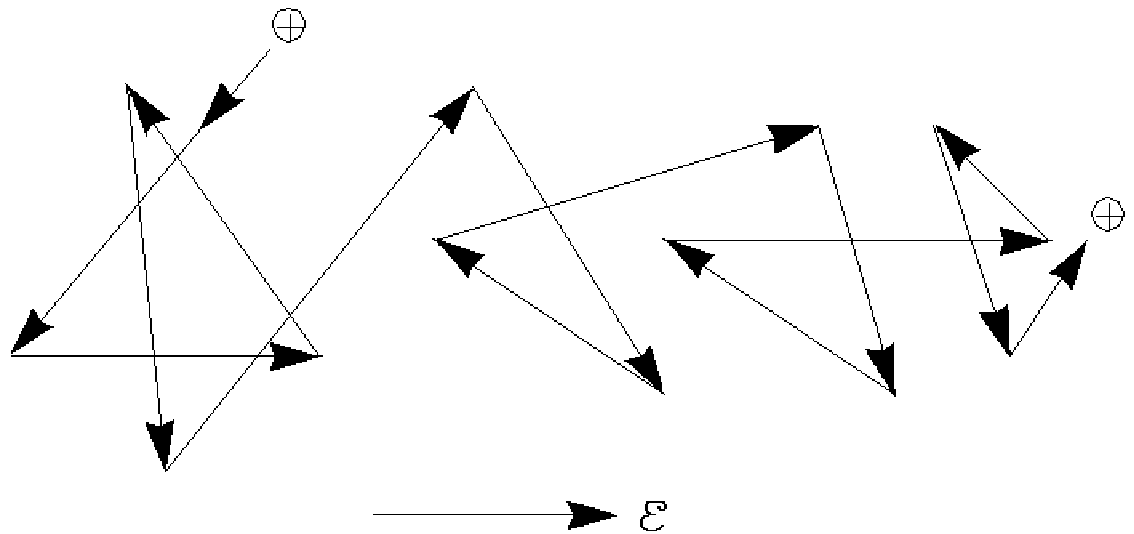
2.1 Thermal Motion



- Zig-zag motion is due to collisions or scattering with imperfections in the crystal.
- Net thermal velocity is zero.
- Mean time between collisions is $\tau_m \sim 0.1\text{ps}$

2.2 *Drift*

2.2.1 Electron and Hole Mobilities



- ***Drift*** is the motion caused by an electric field.

2.2.1 Electron and Hole Mobilities

- The drift momentum gained between collisions is equal to the force, $q\varepsilon$, times the mean free time between collisions τ_{mp}.

$$m_p v = q\varepsilon\tau_{mp}$$

$$v = \frac{q\varepsilon\tau_{mp}}{m_p}$$

$$v = \mu_p \varepsilon$$

$$\mu_p = \frac{q\tau_{mp}}{m_p}$$

$$v = -\mu_n \varepsilon$$

$$\mu_n = \frac{q\tau_{mn}}{m_n}$$

- μ_p is the hole mobility and μ_n is the electron mobility

2.2.1 Electron and Hole Mobilities

$$v = \mu \varepsilon \; ; \quad \mu \text{ has the dimensions of } v/\varepsilon \left[\frac{\text{cm/s}}{\text{V/cm}} = \frac{\text{cm}^2}{\text{V}\cdot\text{s}}\right].$$

Electron and hole mobilities of selected semiconductors

	Si	Ge	GaAs	InAs
μ_n (cm^2/V·s)	1400	3900	8500	30000
μ_p (cm^2/V·s)	470	1900	400	500

Based on the above table alone, which semiconductor and which carriers (electrons or holes) are attractive for applications in high-speed devices?

Drift Velocity, Mean Free Time, Mean Free Path

EXAMPLE: *Given μ_p = 470 cm²/V·s, what is the hole drift velocity at ε = 10³ V/cm? What is τ_{mp} and what is the distance traveled between collisions (called the* ***mean free path****)? Hint: When in doubt, use the MKS system of units.*

Solution: $v = \mu_p \varepsilon = 470 \text{ cm}^2/\text{V·s} \times 10^3 \text{ V/cm} = 4.7\times 10^5 \text{ cm/s}$

$$\tau_{mp} = \mu_p m_p/q = 470 \text{ cm}^2/\text{V·s} \times 0.39 \times 9.1\times10^{-31} \text{ kg}/1.6\times10^{-19} \text{ C}$$

$$= 0.047 \text{ m}^2/\text{V·s} \times 2.2\times10^{-12} \text{ kg/C} = 1\times10^{-13}\text{s} = 0.1 \text{ ps}$$

mean free path $= \tau_{mh} v_{th} \sim 1\times 10^{-13} \text{ s} \times 2.2\times10^{7} \text{ cm/s}$

$$= 2.2\times10^{-6} \text{ cm} = 220 \text{ Å} = 22 \text{ nm}$$

This is smaller than the typical dimensions of devices, but getting close.

2.2.2 Mechanisms of Carrier Scattering

There are two main causes of carrier scattering:

1. Phonon Scattering
2. Ionized-Impurity (Coulombic) Scattering

Phonon scattering mobility decreases when temperature rises:

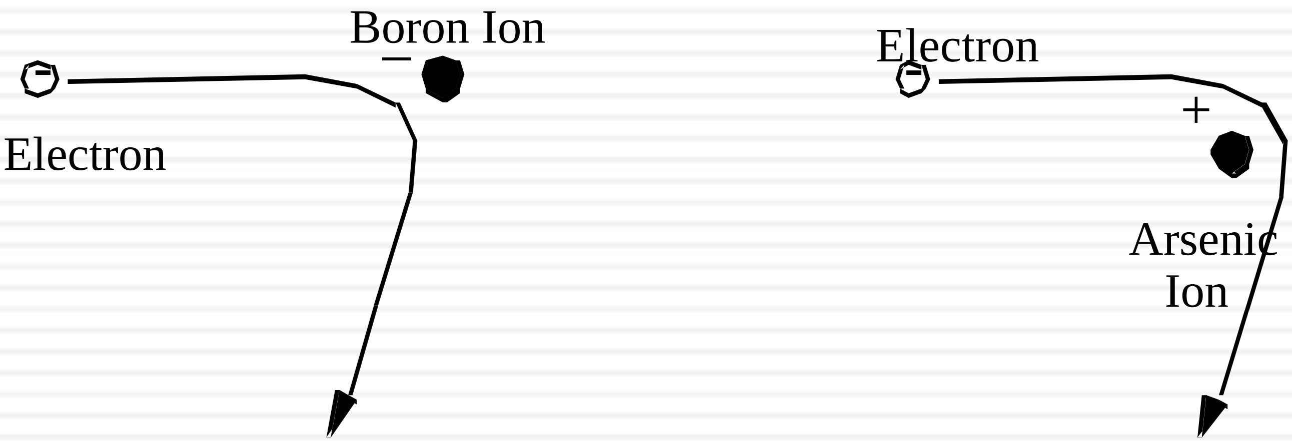
$$\mu_{phonon} \propto \tau_{phonon} \propto \frac{1}{phonon\ \ density \times carrier\ \ thermal\ \ velocity} \propto \frac{1}{T \times T^{1/2}} \propto T^{-3/2}$$

$\mu = q\tau/m$

$\propto T$

$v_{th} \propto T^{1/2}$

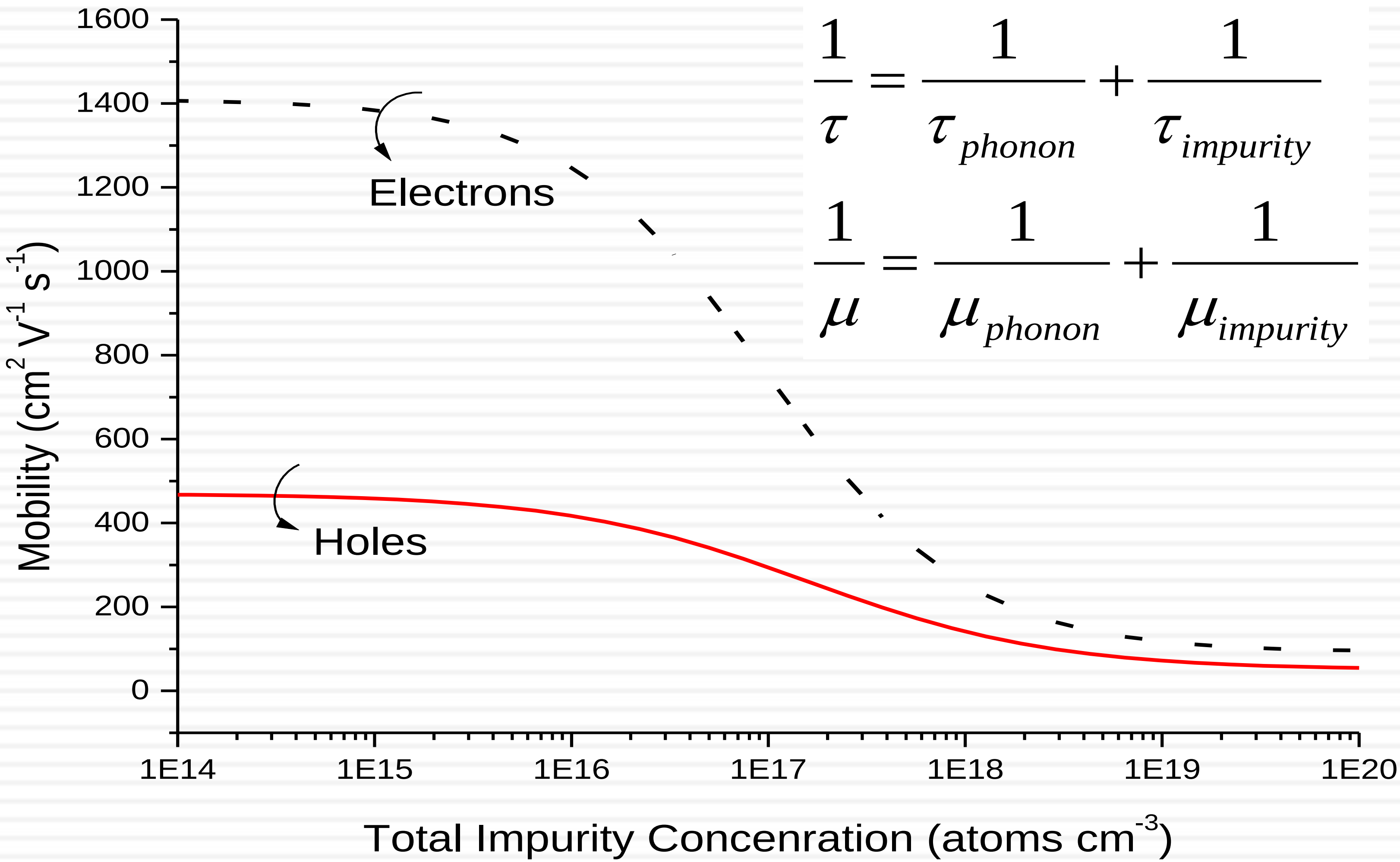
Impurity (Dopant)-Ion Scattering or Coulombic Scattering



There is less change in the direction of travel if the electron zips by the ion at a higher speed.

$$\mu_{impurity} \propto \frac{v_{th}^{3}}{N_a + N_d} \propto \frac{T^{3/2}}{N_a + N_d}$$

Total Mobility

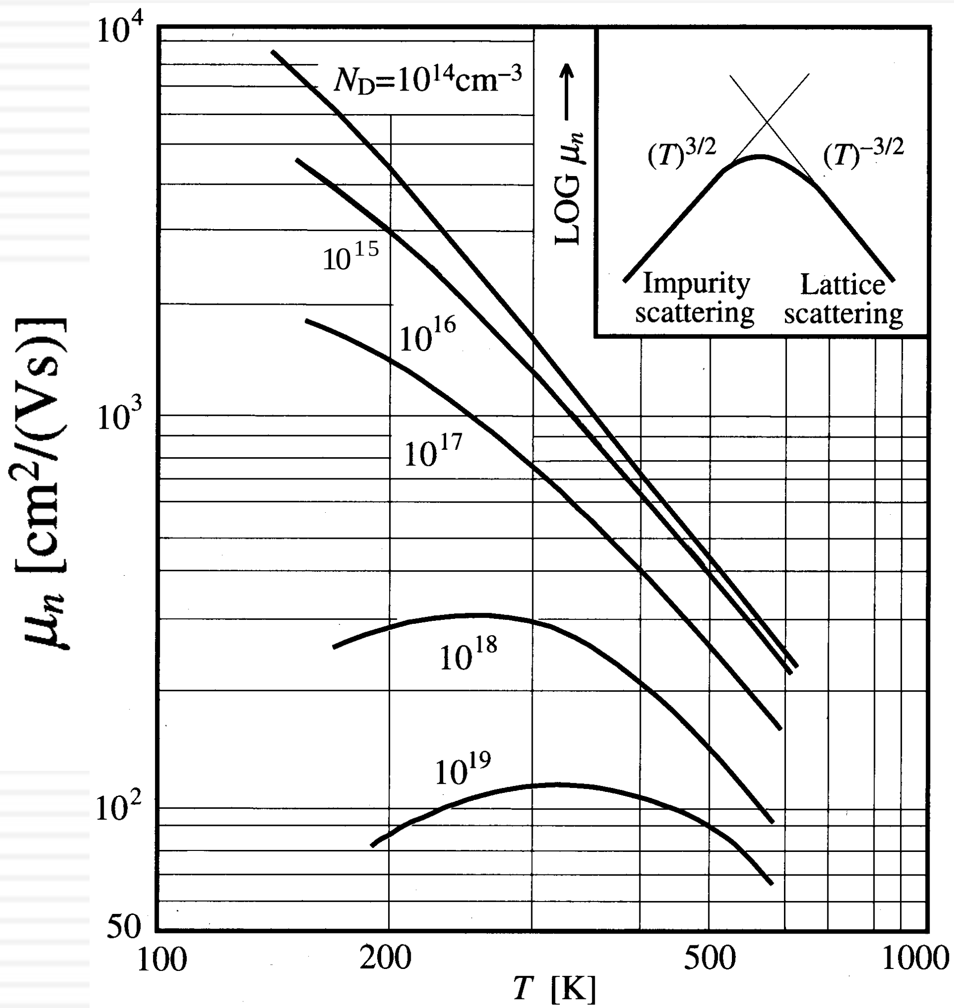


$$\frac{1}{\tau} = \frac{1}{\tau_{phonon}} + \frac{1}{\tau_{impurity}}$$

$$\frac{1}{\mu} = \frac{1}{\mu_{phonon}} + \frac{1}{\mu_{impurity}}$$

$N_a + N_d$ (cm^{-3})

Temperature Effect on Mobility



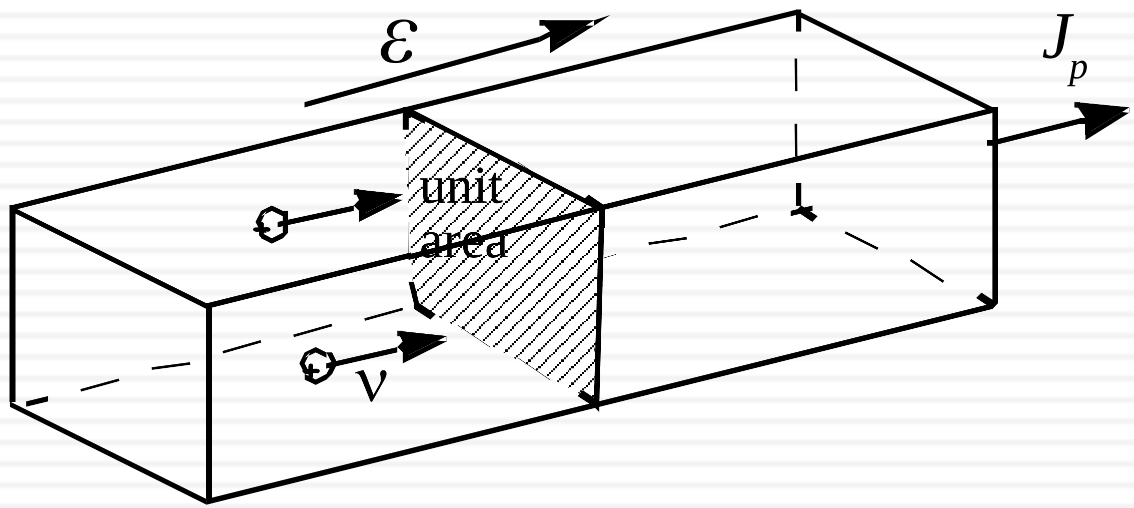
Question:

What N_d will make $d\mu_n/dT = 0$ at room temperature?

Velocity Saturation

- When the kinetic energy of a carrier exceeds a critical value, it generates an optical photon and loses the kinetic energy.
- Therefore, the kinetic energy is capped at large ε, and the velocity does not rise above a saturation velocity, v_{sat}.
- ***Velocity saturation*** has a deleterious effect on device speed as shown in Ch. 6.

2.2.3 Drift Current and Conductivity



Hole current density $J_p = qpv$ A/cm² or C/cm²·sec

EXAMPLE: If $p = 10^{15}$cm^{-3} and $v = 10^4$ cm/s, then

$$J_p = 1.6\times10^{-19}\text{C} \times 10^{15}\text{cm}^{-3} \times 10^4\text{cm/s}$$

$$= 1.6\ \text{C/s}\cdot\text{cm}^2 = 1.6\ \text{A/cm}^2$$

2.2.3 Drift Current and Conductivity

$$J_{p,drift} = qpv = qp\mu\,\varepsilon$$

$$J_{n,drift} = -qnv = qn\mu\,\varepsilon$$

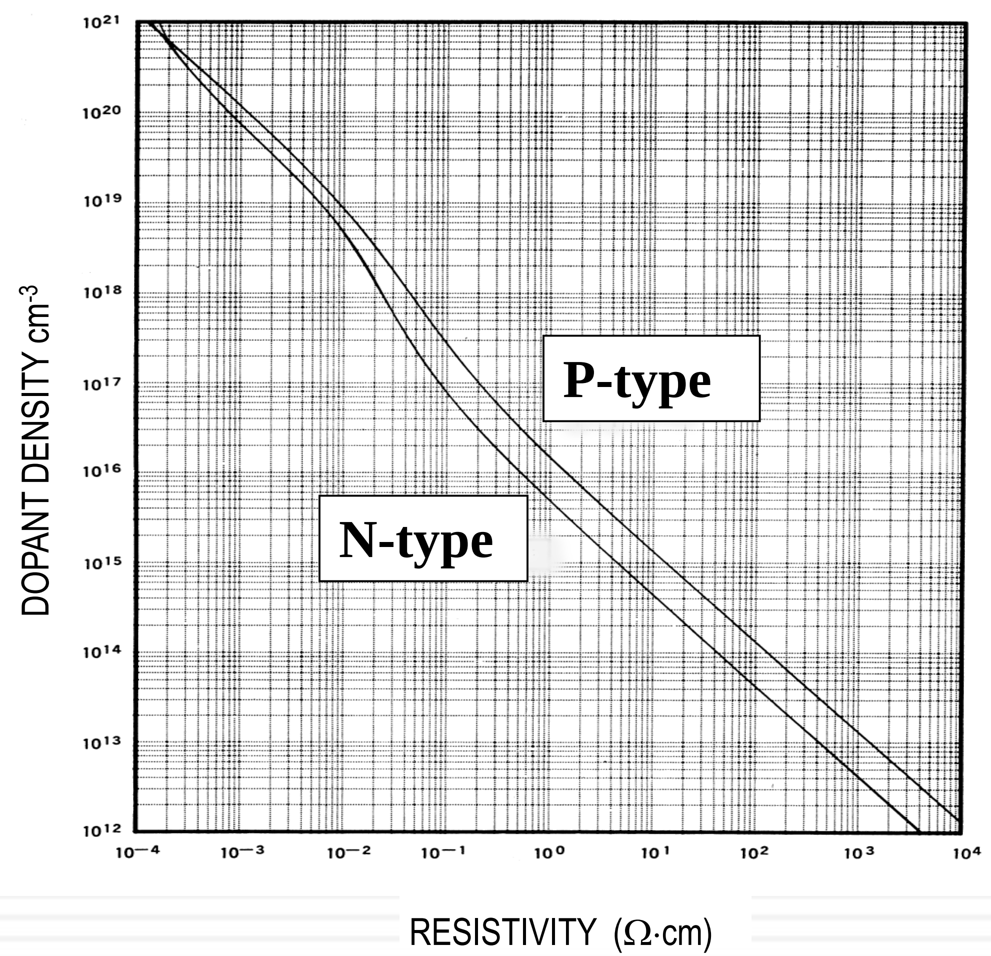
$$J_{drift} = J_{n,drift} + J_{p,drift} = \sigma\varepsilon = (qn\mu_n + qp\mu_p)\,\varepsilon$$

$\therefore$ ***conductivity*** *(1/ohm-cm) of a semiconductor is*

$$\sigma = qn\mu_n + qp\mu_p$$

$1/\sigma$ = is resistivity (ohm-cm)

Relationship between Resistivity and Dopant Density



$$\rho = 1/\sigma$$

EXAMPLE: Temperature Dependence of Resistance

(a) What is the resistivity (ρ) of silicon doped with 10^{17}cm^{-3} of arsenic?

(b) What is the resistance (R) of a piece of this silicon material 1μm long and 0.1 μm^2 in cross-sectional area?

Solution:

(a) Using the N-type curve in the previous figure, we find that ρ = 0.084 Ω-cm.

(b) $R = \rho L/A = 0.084\ \Omega\text{-cm} \times 1\ \mu\text{m} / 0.1\ \mu\text{m}^2$

$= 0.084\ \Omega\text{-cm} \times 10^{-4}\ \text{cm}/ 10^{-10}\ \text{cm}^2$

$= 8.4 \times 10^{-4}\ \Omega$

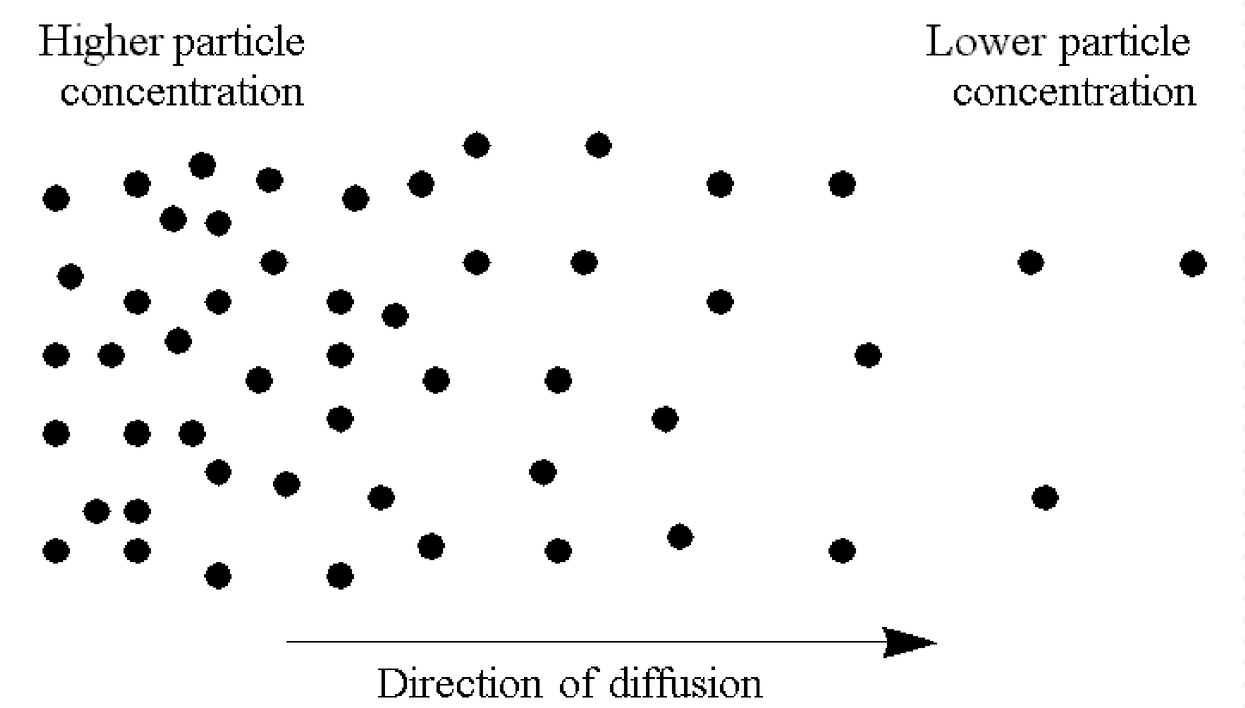
EXAMPLE: Temperature Dependence of Resistance

By what factor will R increase or decrease from T=300 K to T=400 K?

Solution: *The temperature dependent factor in σ (and therefore ρ) is μ_n. From the mobility vs. temperature curve for 10^{17}cm^{-3}, we find that μ_n decreases from 770 at 300K to 400 at 400K. As a result, R* ***increases*** *by*

$$\frac{770}{400} = 1.93$$

2.3 Diffusion Current

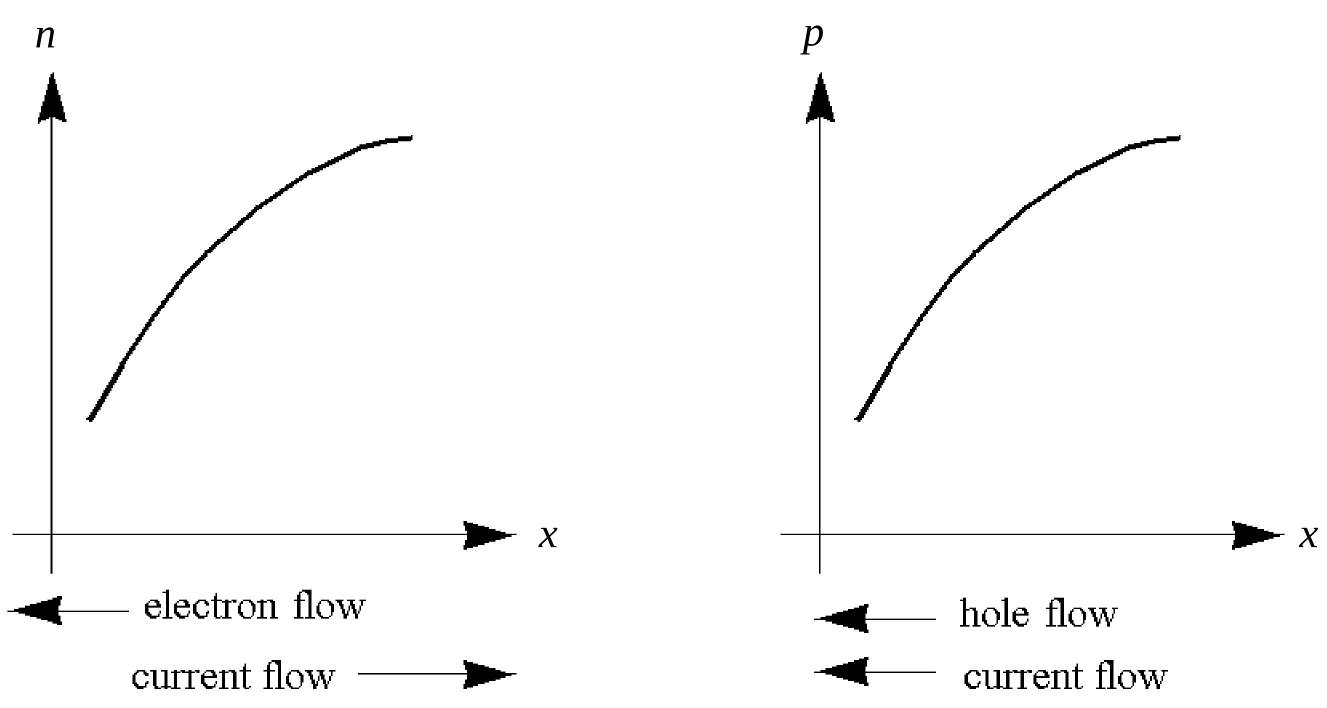


Particles diffuse from a higher-concentration location to a lower-concentration location.

2.3 Diffusion Current

$$J_{n,diffusion} = qD_n \frac{dn}{dx} \qquad J_{p,diffusion} = -qD_p \frac{dp}{dx}$$

D is called the diffusion constant. Signs explained:



Total Current – Review of Four Current Components

$$J_{TOTAL} = J_n + J_p$$

$$J_n = J_{n,drift} + J_{n,diffusion} = qn\mu_n \varepsilon + qD_n \frac{dn}{dx}$$

$$J_p = J_{p,drift} + J_{p,diffusion} = qp\mu_p \varepsilon - qD_p \frac{dp}{dx}$$

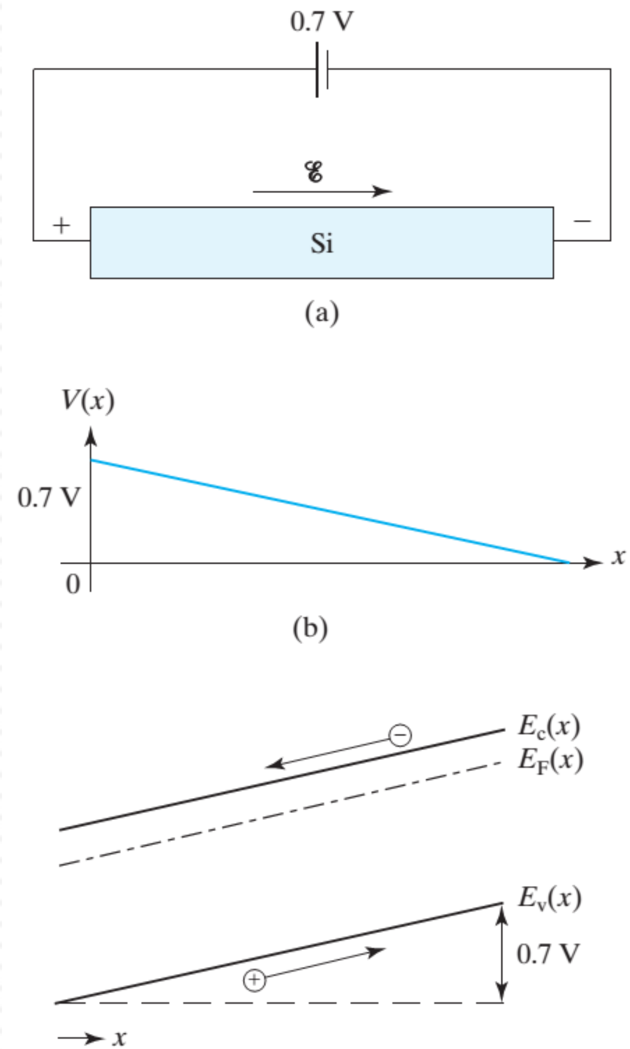
2.4 Relation Between the Energy Diagram and V, Ɛ

- When a voltage is applied across a piece of semiconductor as shown in Figure, it alters the band diagram
- S positive voltage raises the potential energy of a positive charge and lowers the energy of a negative charge

E_c and E_v vary in the opposite direction from the voltage. That is, E_c and E_v are higher where the voltage is lower.

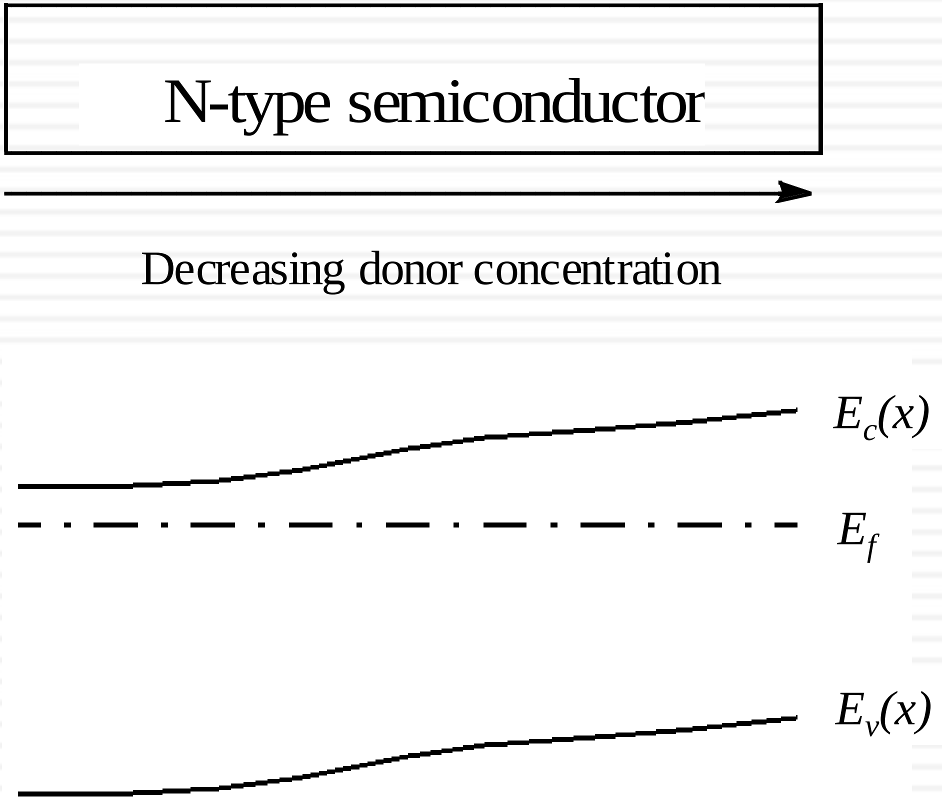
$$E_c(x) = \text{constant} - qV(x)$$

$$\mathcal{E}(x) = -\frac{dV}{dx} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_v}{dx}$$



2.5 Einstein Relationship between D and μ

Consider a piece of non-uniformly doped semiconductor.



$$n = N_c e^{-(E_c - E_f)/kT}$$

$$\frac{dn}{dx} = -\frac{N_c}{kT} e^{-(E_c - E_f)/kT} \frac{dE_c}{dx}$$

$$= -\frac{n}{kT}\frac{dE_c}{dx}$$

$$= -\frac{n}{kT} q\varepsilon$$

2.5 Einstein Relationship between D and μ

$$\frac{dn}{dx} = -\frac{n}{kT} q\mathcal{E}$$

$$J_n = qn\mu_n\mathcal{E} + qD_n\frac{dn}{dx} = 0 \quad \text{at equilibrium.}$$

$$0 = qn\mu_n\mathcal{E} - qn\frac{qD_n}{kT}\mathcal{E}$$

$$\boxed{D_n = \frac{kT}{q}\mu_n}$$

Similarly,

$$\boxed{D_p = \frac{kT}{q}\mu_p}$$

These are known as the Einstein relationship.

EXAMPLE: Diffusion Constant

What is the hole diffusion constant in a piece of silicon with μ_p = 410 cm^2 V^{-1}s^{-1} ?

Solution:

$$D_p = \left(\frac{kT}{q}\right)\mu_p = (26\,\mathrm{mV})\cdot 410\,\mathrm{cm^2V^{-1}s^{-1}} = 11\,\mathrm{cm^2/s}$$

Remember: kT/q = 26 mV at room temperature.

2.6 Electron-Hole Recombination

- The equilibrium carrier concentrations are denoted with n_o and p_o.
- The total electron and hole concentrations can be different from n_o and p_o .
- *The* differences are called the ***excess carrier concentrations*** *n'* and *p'*.

$$n \equiv n_0 + n'$$
$$p \equiv p_0 + p'$$

Charge Neutrality

- Charge neutrality is satisfied at equilibrium ($n' = p' = 0$).
- When a non-zero n' is present, an equal p' may be assumed to be present to maintain charge equality and vice-versa.
- If charge neutrality is not satisfied, the net charge will attract or repel the (majority) carriers through the drift current until neutrality is restored.

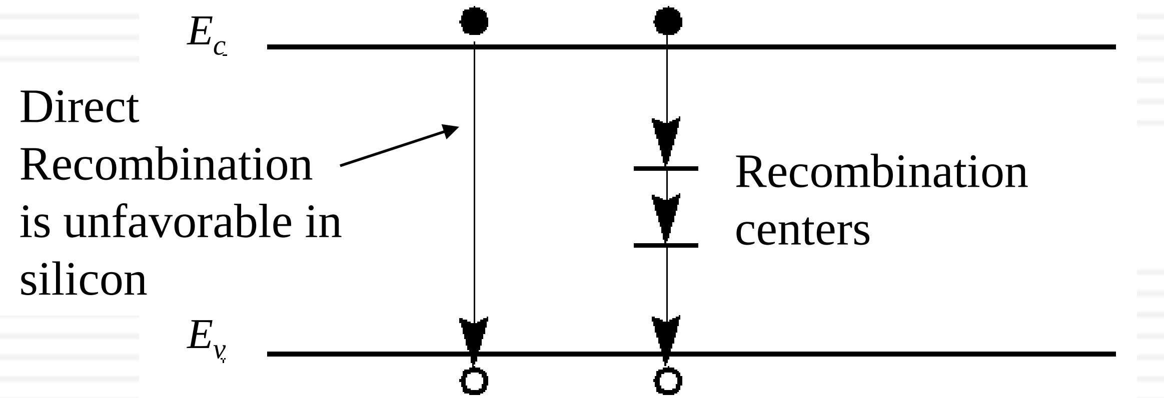
$$n' = p'$$

Recombination Lifetime

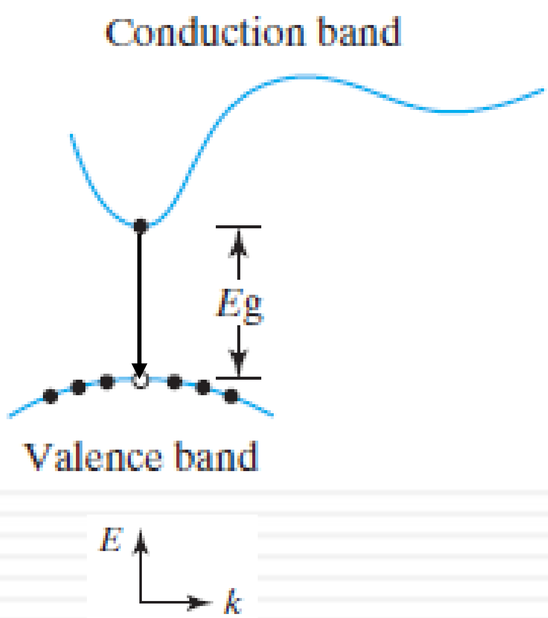
- Assume light generates *n'* and *p'*. If the light is suddenly turned off, *n'* and *p'* decay with time until they become zero.
- The process of decay is called ***recombination.***
- The time constant of decay is the ***recombination time*** or ***carrier lifetime,*** τ.
- Recombination is nature's way of restoring equilibrium ($n' = p' = 0$).

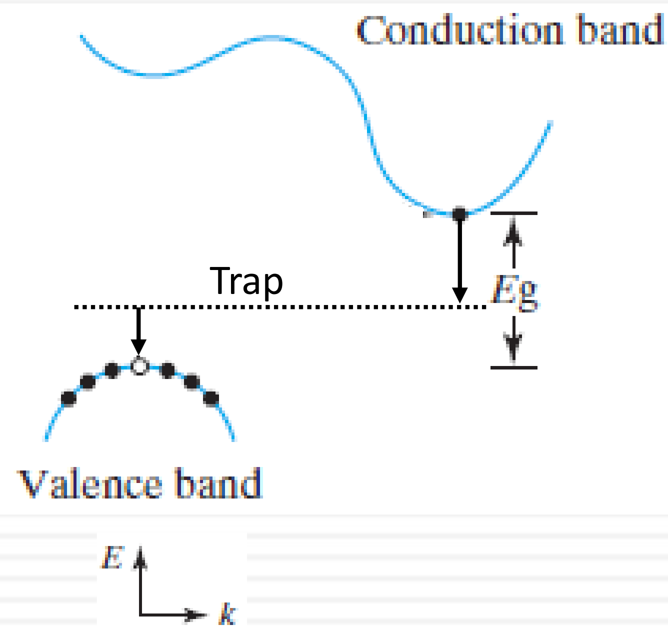
Recombination Lifetime

- τ ranges from 1ns to 1ms in Si and depends on the density of metal impurities (contaminants) such as Au and Pt.
- These ***deep traps*** capture electrons and holes to facilitate recombination and are called ***recombination centers.***



Direct and Indirect Band Gap





Direct band gap
Example: GaAs

Direct recombination is efficient as k conservation is satisfied.

Indirect band gap
Example: Si

Direct recombination is rare as k conservation is not satisfied

Rate of recombination ($s^{-1}cm^{-3}$)

$$\frac{dn'}{dt} = -\frac{n'}{\tau}$$

$$n' = p'$$

$$\frac{dn'}{dt} = -\frac{n'}{\tau} = -\frac{p'}{\tau} = \frac{dp'}{dt}$$

EXAMPLE: Photoconductors

- *A bar of Si is doped with boron at 10^{15}cm^{-3}. It is exposed to light such that electron-hole pairs are generated throughout the volume of the bar at the rate of 10^{20}/s·cm^3. The recombination lifetime is 10 μs. What are (a) p_o, (b) n_o, (c) p', (d) n', (e) p, (f) n, and (g) the np product?*

EXAMPLE: Photoconductors

Solution:

(a) What is p_o?

$p_o = N_a = 10^{15}$ cm^{-3}

(b) What is n_o ?

$n_o = n_i^2/p_o = 10^5$ cm^{-3}

(c) What is p'?

In steady-state, the rate of generation is equal to the rate of recombination.

$$10^{20}/\text{s-cm}^3 = p'/\tau$$

$$\therefore\ p' = 10^{20}/\text{s-cm}^3 \cdot 10^{-5}\text{s} = 10^{15}\ \text{cm}^{-3}$$

EXAMPLE: Photoconductors

(d) What is n'?

$n' = p' = 10^{15}\ cm^{-3}$

(e) What is p?

$p = p_o + p' = 10^{15} cm^{-3} + 10^{15} cm^{-3} = 2\times 10^{15} cm^{-3}$

(f) What is n?

$n = n_o + n' = 10^{5} cm^{-3} + 10^{15} cm^{-3} \sim 10^{15} cm^{-3}$ since $n_o << n'$

(g) What is np?

$np \sim 2\times 10^{15} cm^{-3} \cdot 10^{15} cm^{-3} = 2\times 10^{30}\ cm^{-6} >> n_i^2 = 10^{20}\ cm^{-6}$.

The np product can be very different from n_i^2.

2.7 Thermal Generation

If *n'* is negative, there are fewer electrons than the equilibrium value.

As a result, there is a net rate of ***thermal generation*** at the rate of $|n'|/\tau$.

2.8 Quasi-equilibrium and Quasi-Fermi Levels

- Whenever $n' = p' \neq 0$, $np \neq n_i^2$. We would like to preserve and use the simple relations:

$$n = N_c e^{-(E_c - E_f)/kT}$$

$$p = N_v e^{-(E_f - E_v)/kT}$$

- But these equations lead to $np = n_i^2$. The solution is to introduce two ***quasi-Fermi levels*** E_{fn} and E_{fp} such that

$$n = N_c e^{-(E_c - E_{fn})/kT}$$

$$p = N_v e^{-(E_{fp} - E_v)/kT}$$

Even when electrons and holes are not at equilibrium, *within each group* the carriers can be at equilibrium. Electrons are closely linked to other electrons but only loosely to holes.

EXAMPLE: Quasi-Fermi Levels and Low-Level Injection

Consider a Si sample with N_d=10^{17}cm^{-3} and n'=p'=10^{15}cm^{-3}.

(a) Find E_f.

$n = N_d = 10^{17}\ \text{cm}^{-3} = N_c \exp[-(E_c - E_f)/kT]$

$\therefore\ E_c - E_f = 0.15$ eV. (E_f is below E_c by 0.15 eV.)

*Note: n' and p' are much less than the majority carrier concentration. This condition is called **low-level injection.***