Chapter 1: Electrons and Holes in Semiconductors

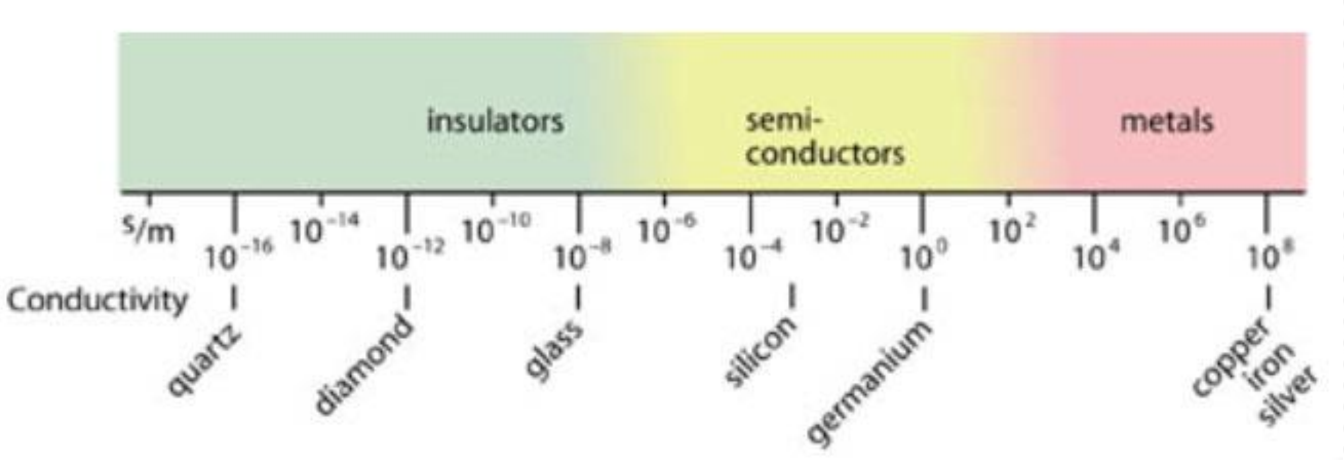
- Course slides are prepared with the aid of the following materials:
  - The lecture slides accompanying the main textbook "Hu, Chenming. *Modern semiconductor devices for integrated circuits*. Prentice Hall, 2010."
  http://www.eecs.berkeley.edu/~hu/Book-Chapters-and-Lecture-Slides-download.html
  - The lecture slides accompanying of the Semiconductor Device Physics course offered by Dr.-Ing. Erwin Sitompul, President University, Indonesia.
  http://zitompul.wordpress.com/1-ee-lectures/2-semiconductor-device-physics/

Chapter Objectives

- Provides the basic concepts and terminology for understanding semiconductors.
- Understand conduction and valence energy band, and how bandgap is formed.
- Understand carriers (electrons and holes), and doping in semiconductor
- Use the density of states and Fermi-Dirac statistics to calculate the carrier concentration

What is a Semiconductor?

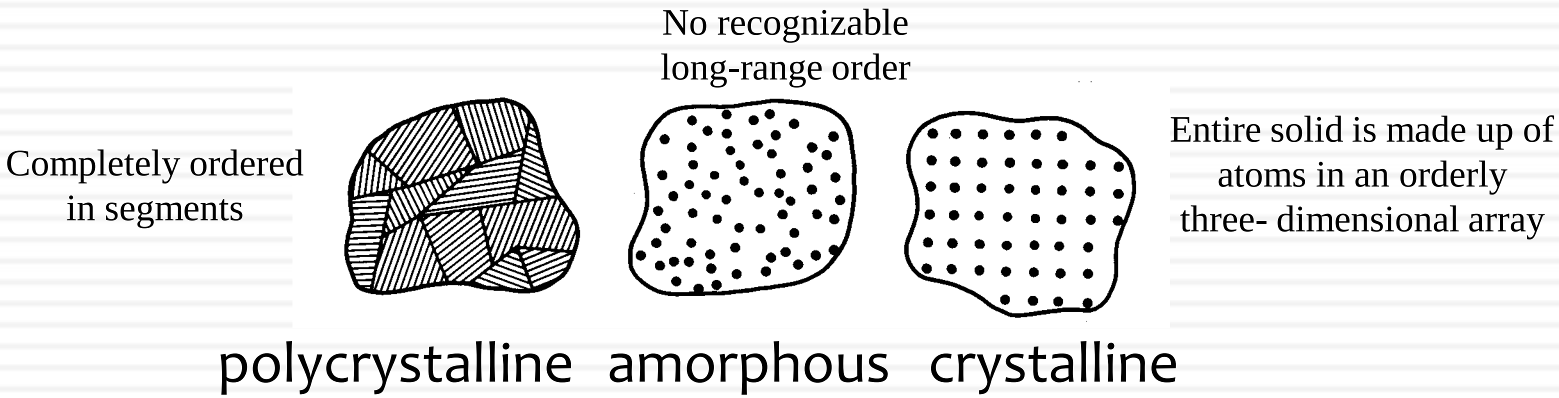
- The conductivity (and at the same time the resistivity) of semiconductors lie between that of conductors and insulators.



- Low resistivity → "conductor"
- High resistivity → "insulator"
- Intermediate resistivity → "semiconductor"

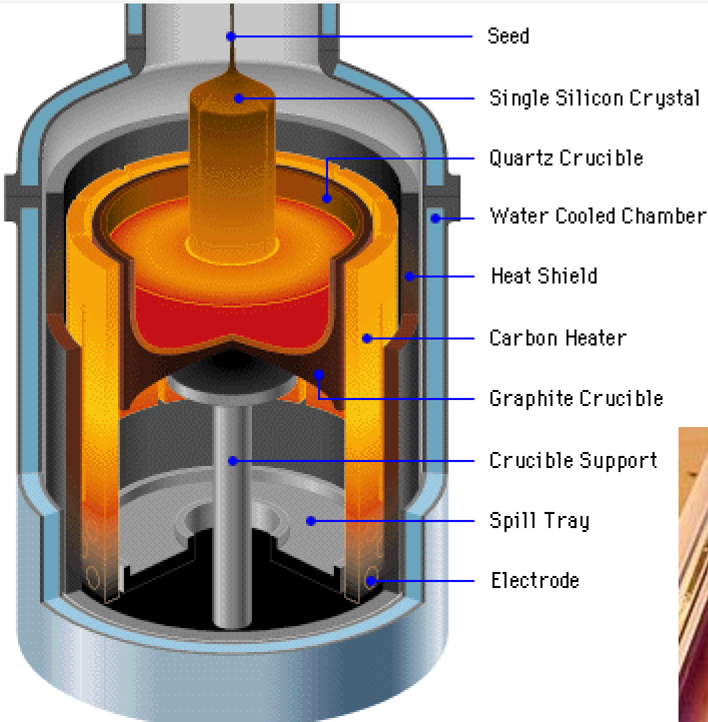
What is a Semiconductor?

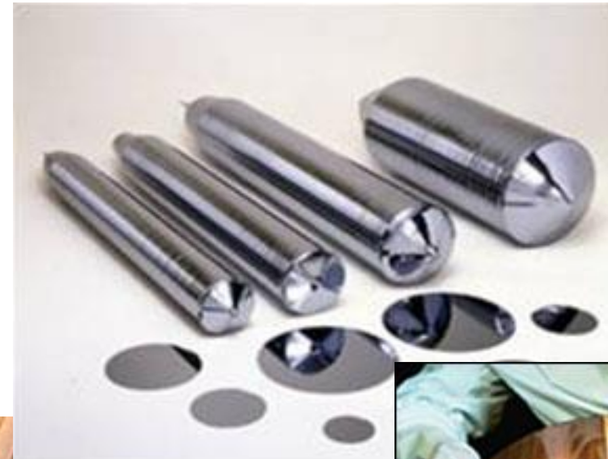
- Semiconductors are some of the purest solid materials in existence, because any trace of impurity atoms called “dopants” can change the electrical properties of semiconductors drastically.

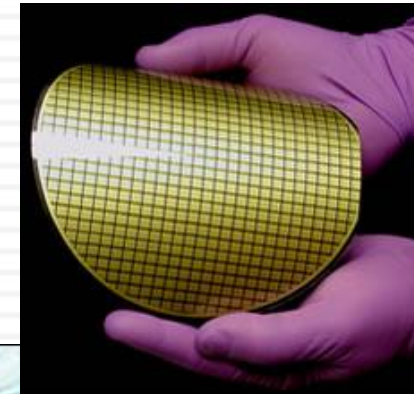


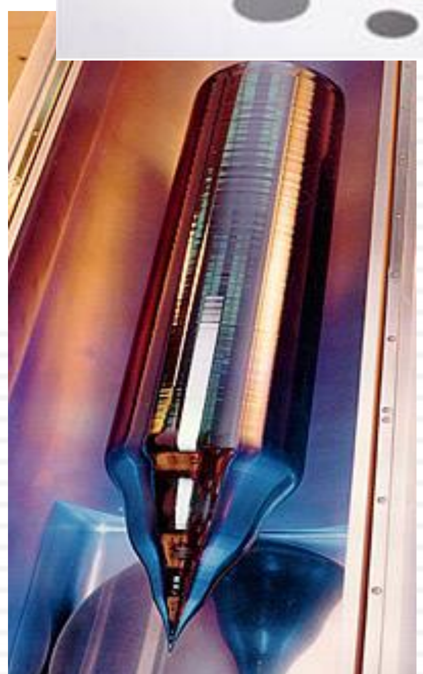
- Most devices fabricated today employ crystalline semiconductors.

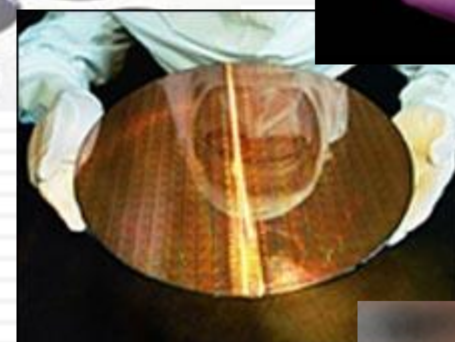
Crystal Growth Until Device Fabrication

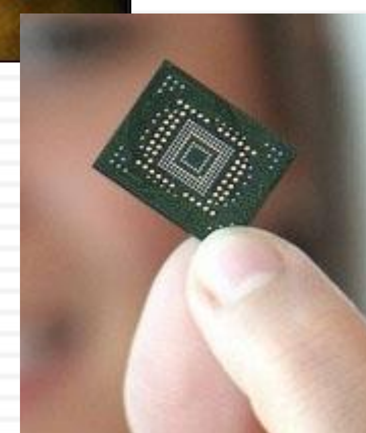










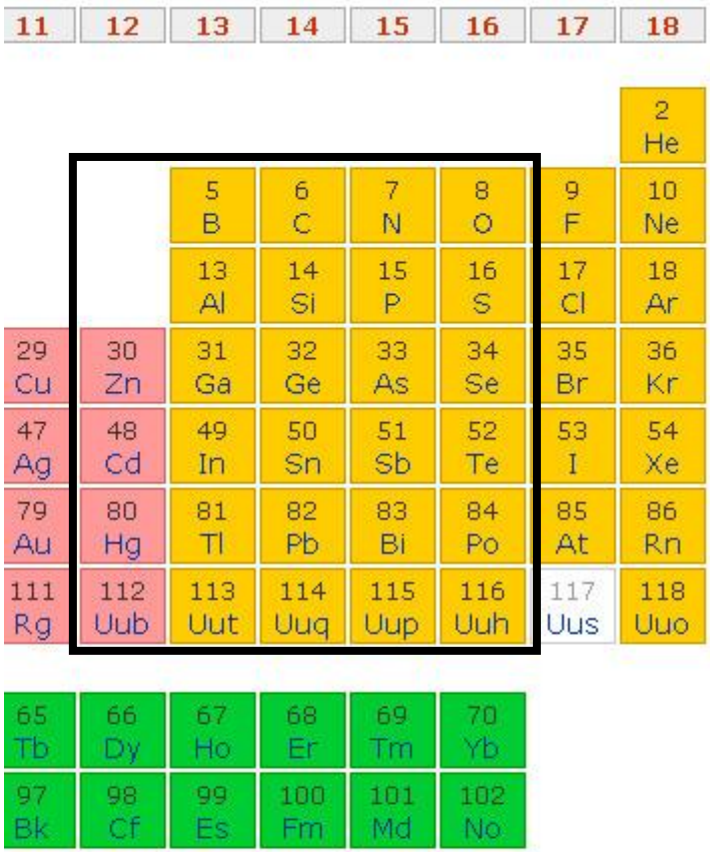


Semiconductor Materials

Elemental: Si, Ge, C

Compound:
- IV-IV SiC
- III-V GaAs, GaN
- II-VI CdSe

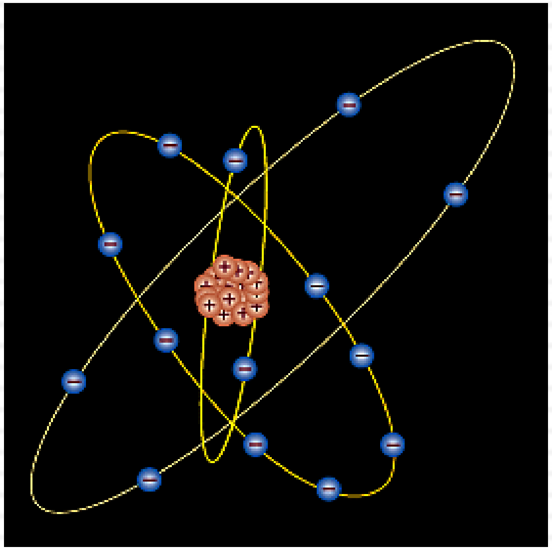
Alloy:
- $Si_{1-x}Ge_x$
- $Al_xGa_{1-x}As$

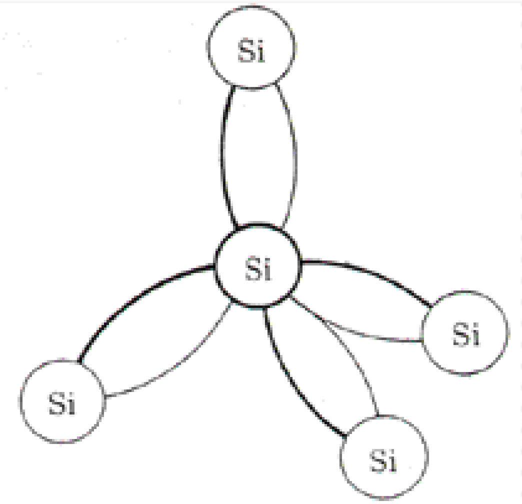


As : Arsenic
Cd : Cadmium
Se : Selenium
Ga : Gallium

The Silicon Atom

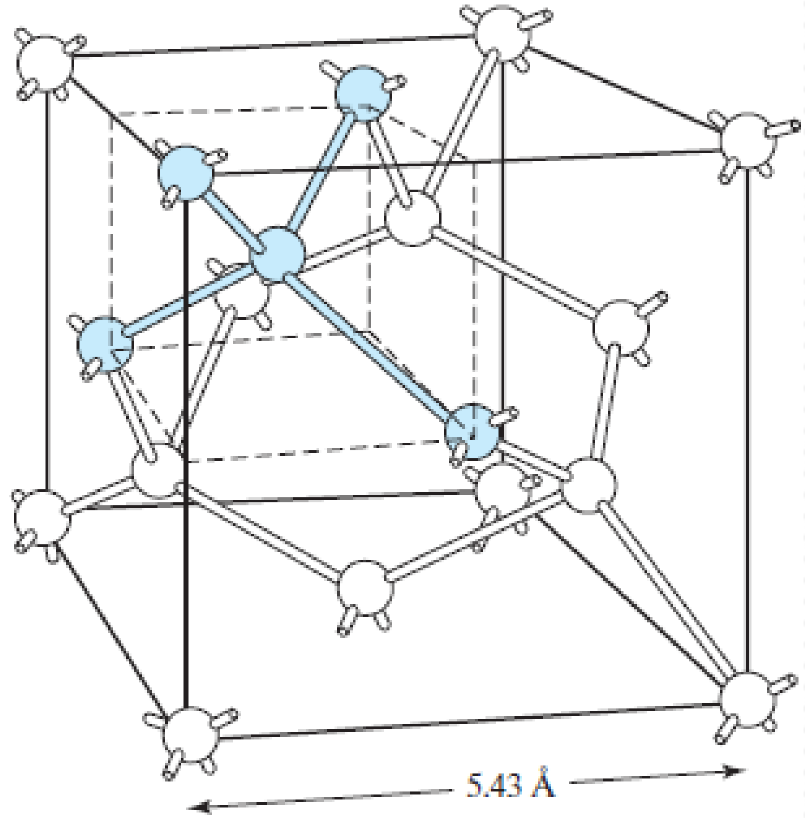
- 14 electrons occupying the first 3 energy levels:
  - 1s, 2s, 2p orbitals are filled by 10 electrons.
  - 3s, 3p orbitals filled by 4 electrons.
- To minimize the overall energy, the 3s and 3p orbitals hybridize to form four tetrahedral 3sp orbital.
- Each has one electron and is capable of forming a bond with a neighboring atom.





1.1 Silicon Crystal Structure

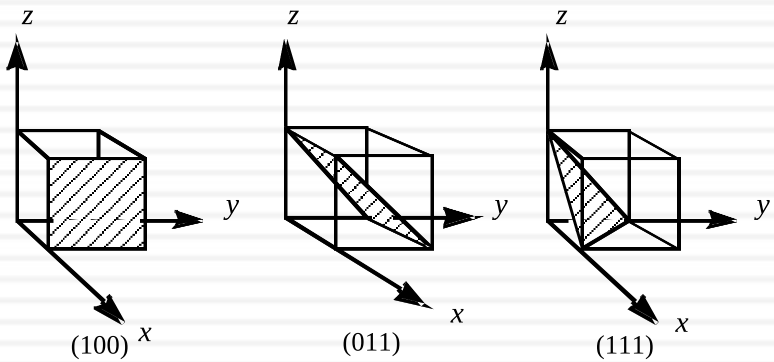
- ***Unit cell*** of silicon crystal is cubic.
- ***Each Si atom has 4 nearest neighbors.***
- **Each cell contains:**
  8 corner atoms
  6 face atoms
  4 interior atoms

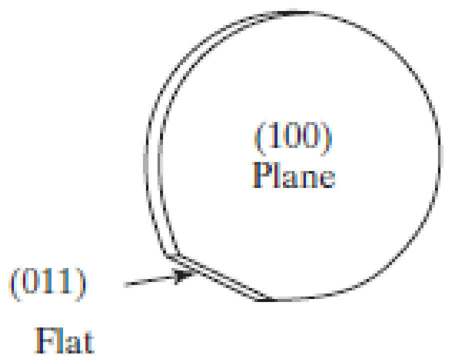


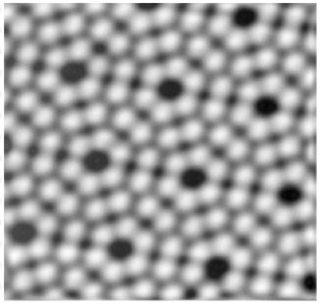
- ***Exercise***

How Many Silicon Atoms per cm^{-3}?

Silicon Wafers and Crystal Planes (Miller Indices)





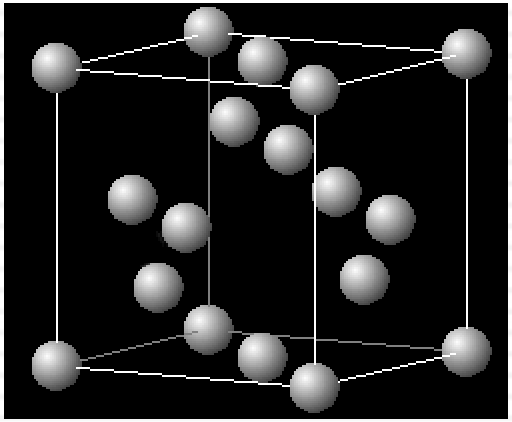


Si (111) plane

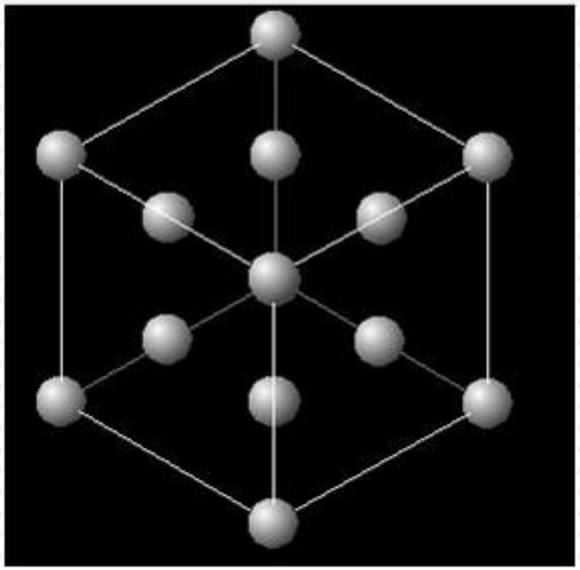
- The standard notation for crystal planes is based on the cubic unit cell.
- Silicon wafers are usually cut along the (100) plane with a flat or notch to help orient the wafer during IC fabrication.

Crystallographic Planes of Si

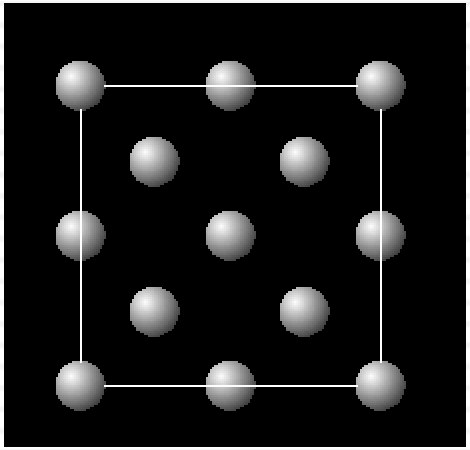
Unit cell:



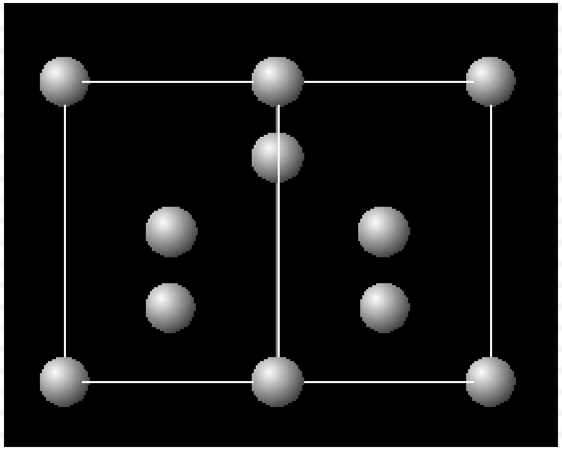
View in <111> direction



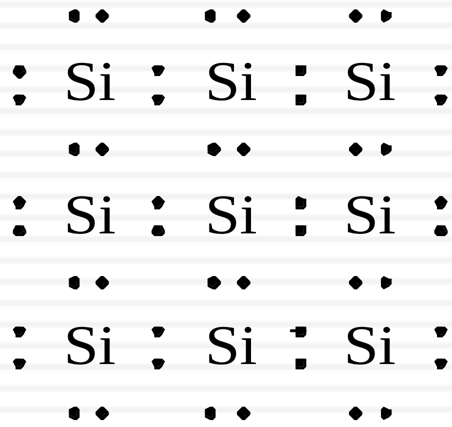
View in <100> direction



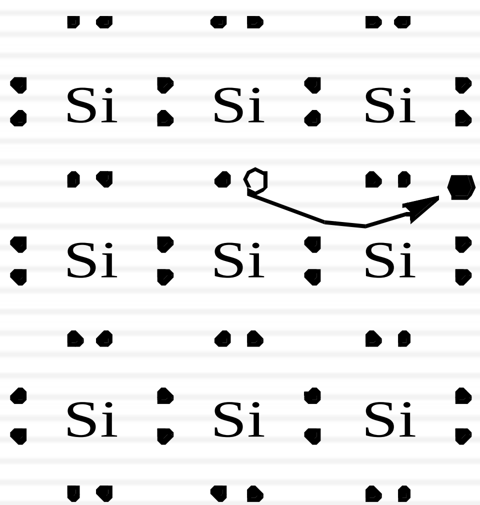
View in <110> direction

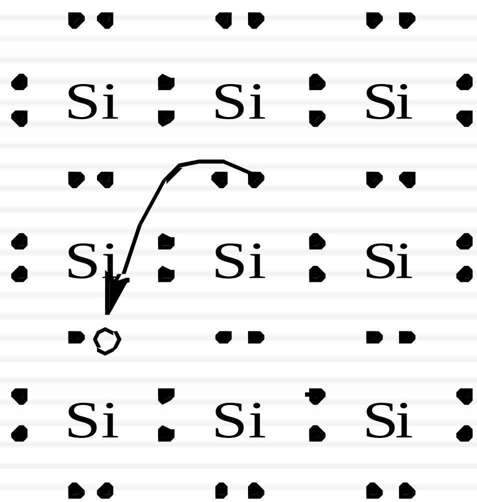


1.2 Bond Model of Electrons and Holes



- Silicon crystal in a two-dimensional representation.



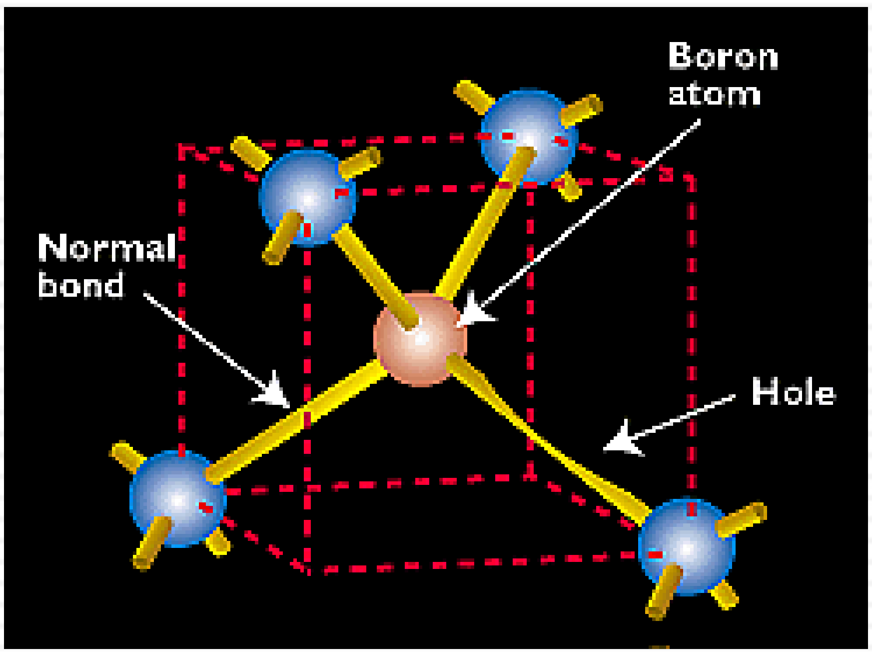


- When an electron breaks loose and becomes a ***conduction electron***, a ***hole*** is also created.

Doping - Manipulation of Carrier Numbers

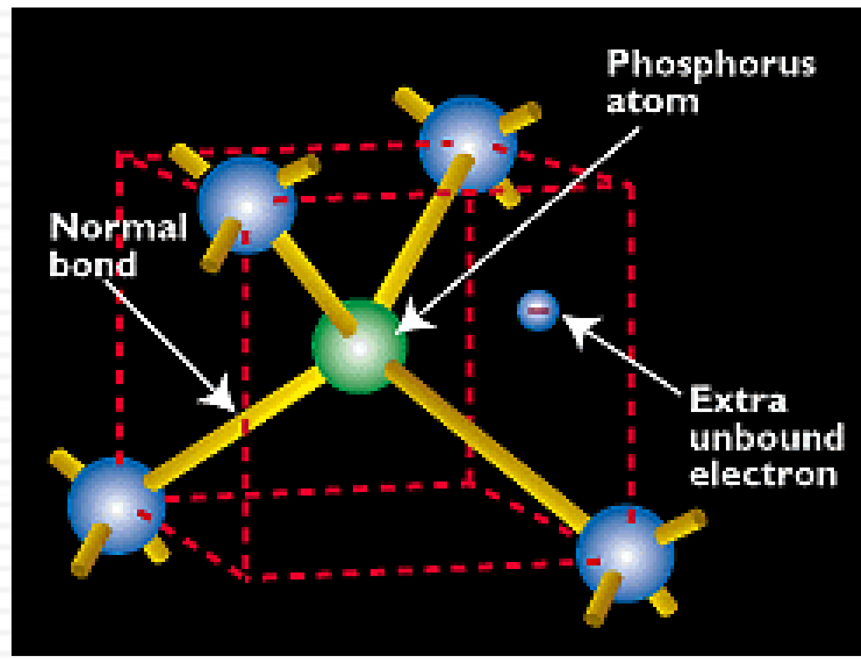
- By substituting an Si atom with a special impurity atom (elements from **Group III** or **Group V**), a hole or conduction electron can be created.

Acceptors: B, Ga, In, Al



Boron, Gallium Indium, Aluminum

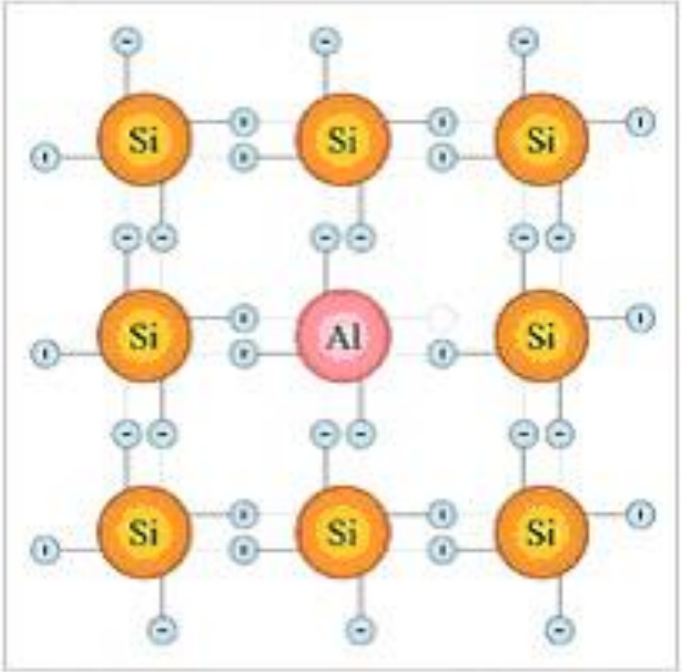
Donors: P, As, Sb



Phosphorus, Arsenic, Antimony

Doping Silicon with Acceptors

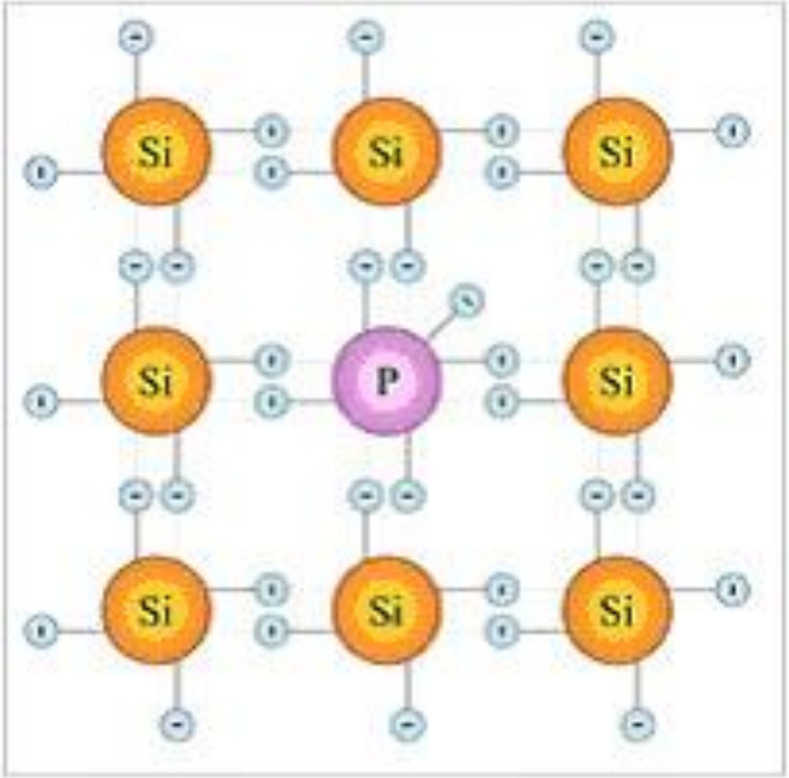
- **Example:** Aluminum atom is doped into the Si crystal.



- The Al atom accepts an electron from a neighboring Si atom, resulting in a missing bonding electron, or “hole”.
- The hole is free to roam around the Si lattice, and as a moving positive charge, the hole carries current.

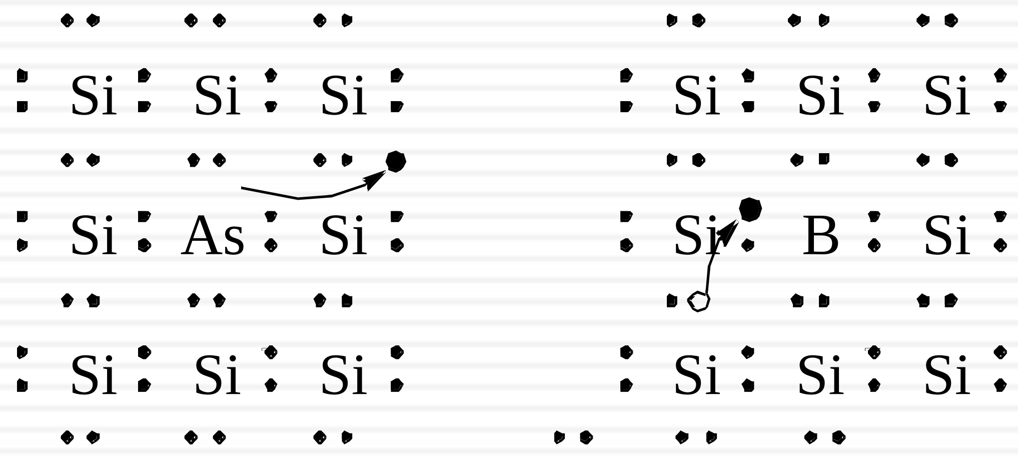
Doping Silicon with Donors

- **Example:** Phosphorus atom is doped into the Si crystal.



- The loosely bounded fifth valence electron of the P atom can "break free" easily and becomes a mobile conducting electron.
- This electron contributes in current conduction.

Dopants in Silicon



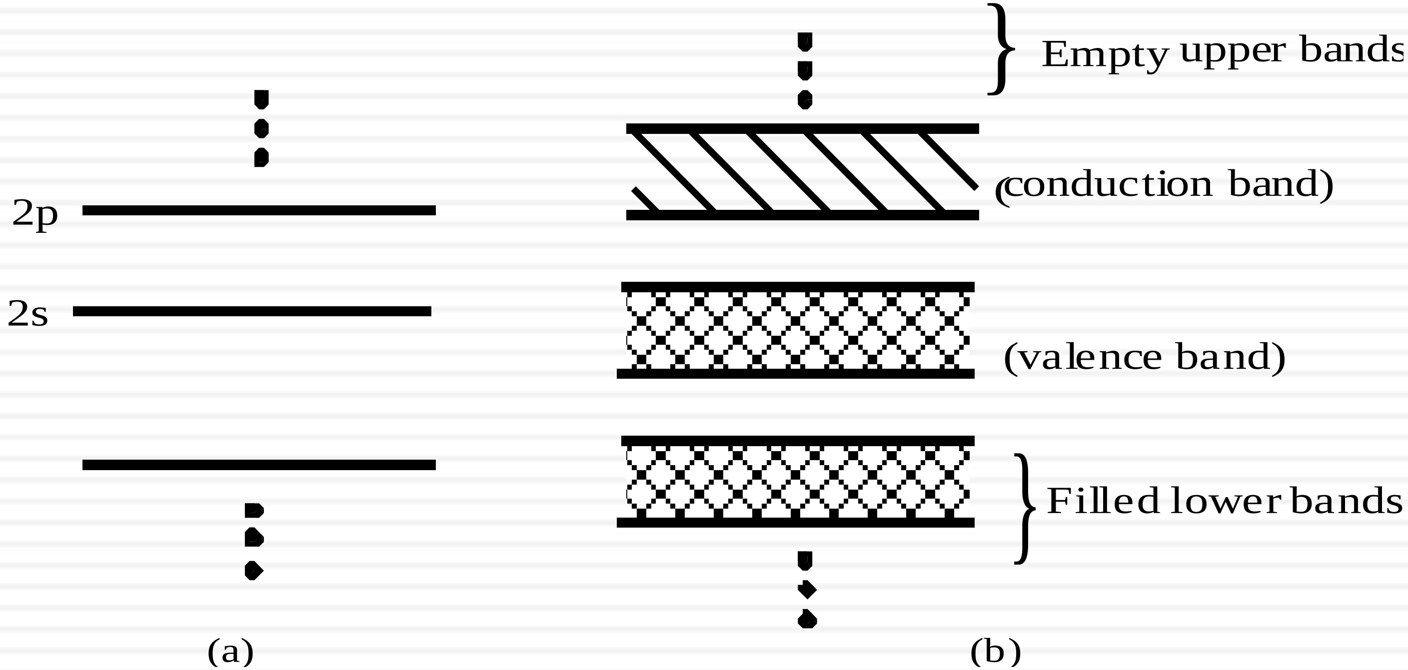
- As, a Group V element, introduces conduction electrons and creates ***N-type silicon,*** and is called a ***donor.***
- B, a Group III element, introduces holes and creates ***P-type silicon***, and is called an ***acceptor***.
- Donors and acceptors are known as dopants. Dopant ionization energy ~50meV (very low).

Hydrogen: $$E_{ion} = \frac{m_0 q^4}{8e_0^2 h^2} = 13.6 \text{ eV}$$

Carrier-Related Terminology

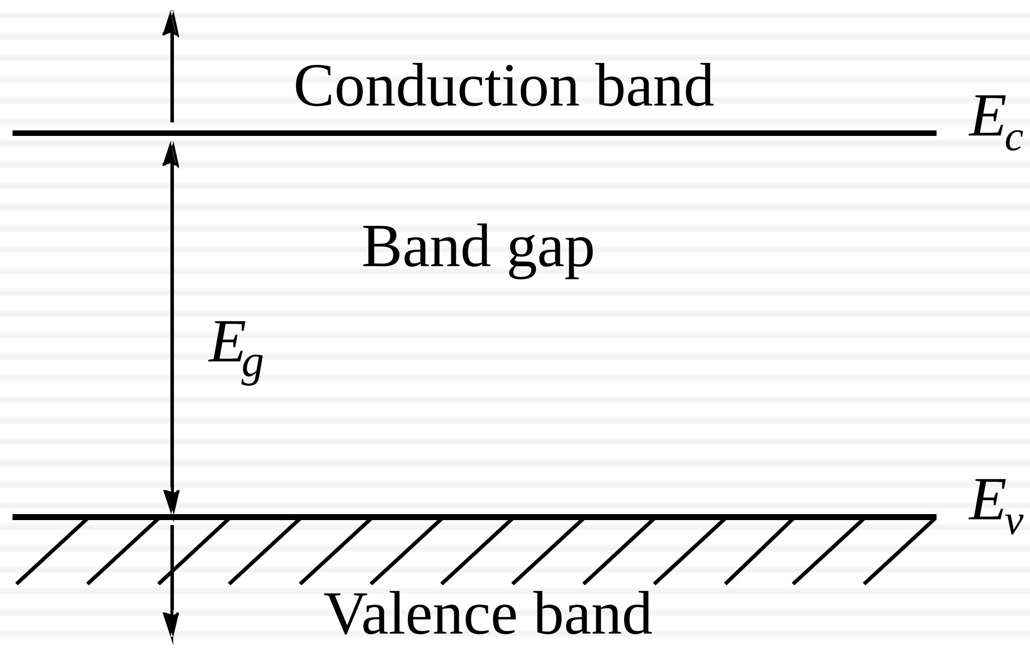
- **Donor**: impurity atom that increases *n* (conducting electron).
  Acceptor: impurity atom that increases *p* (hole).
- ***n*-type material:** contains more electrons than holes.
  ***p*-type material:** contains more holes than electrons.
- **Majority carrier**: the most abundant carrier.
  Minority carrier: the least abundant carrier.
- **Intrinsic semiconductor**: undoped semiconductor $n = p = n_i$.
  Extrinsic semiconductor: doped semiconductor.

1.3 Energy Band Model



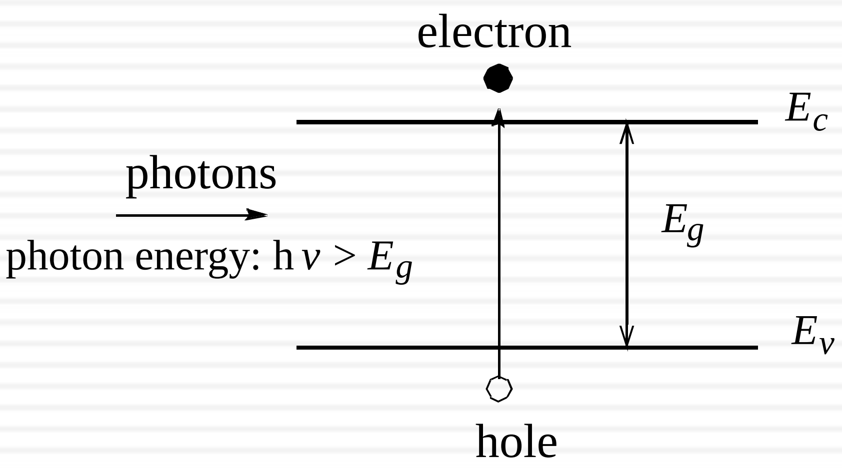
- Energy states of Si atom (a) expand into energy bands of Si crystal (b).
- The lower bands are filled and higher bands are empty in a semiconductor.
- The highest filled band is the ***valence band.***
- The lowest empty band is the ***conduction band***

1.3.1 Energy Band Diagram



- ***Energy band diagram*** shows the bottom edge of conduction band, E_c , and top edge of valence band, E_v .
- E_c and E_v are separated by the ***band gap energy, E_g*** .

Measuring the Band Gap Energy by Light Absorption

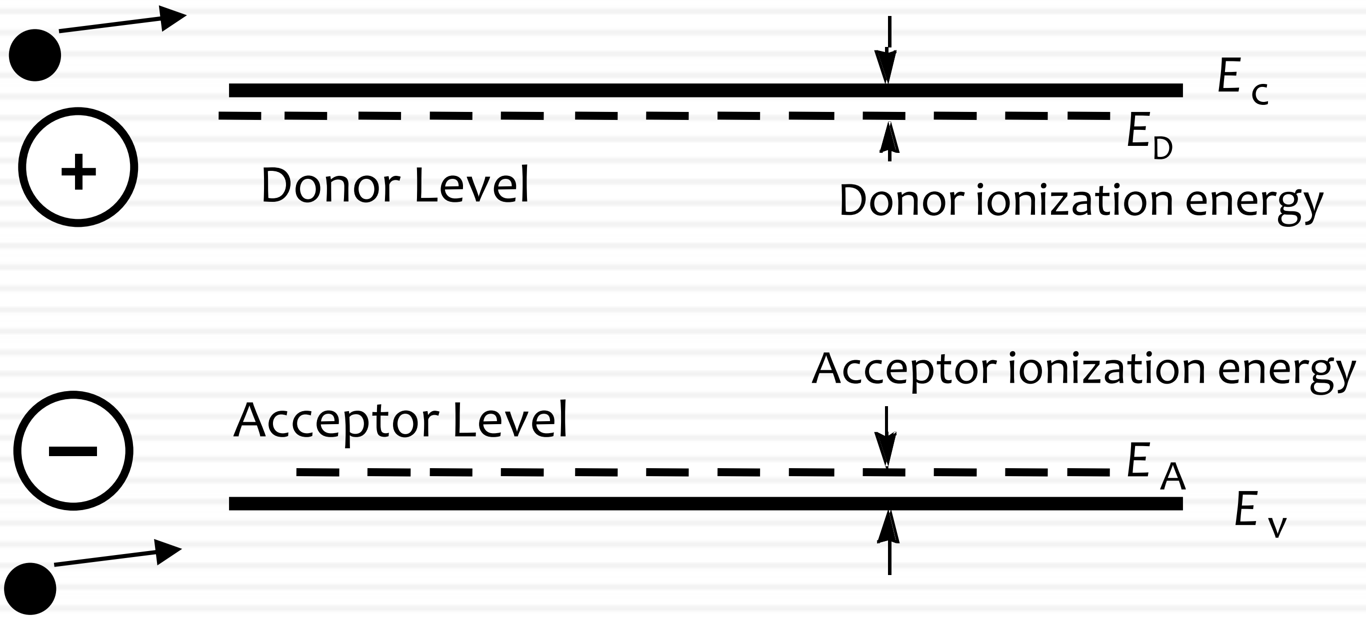


- E_g can be determined from the minimum energy ($h\nu$) of photons that are absorbed by the semiconductor.

Bandgap energies of selected semiconductors

Semi-conductor	InSb	Ge	Si	GaAs	GaP	ZnSe	Diamond
Eg (eV)	0.18	0.67	1.12	1.42	2.25	2.7	6

Donor / Acceptor Levels (Band Model)

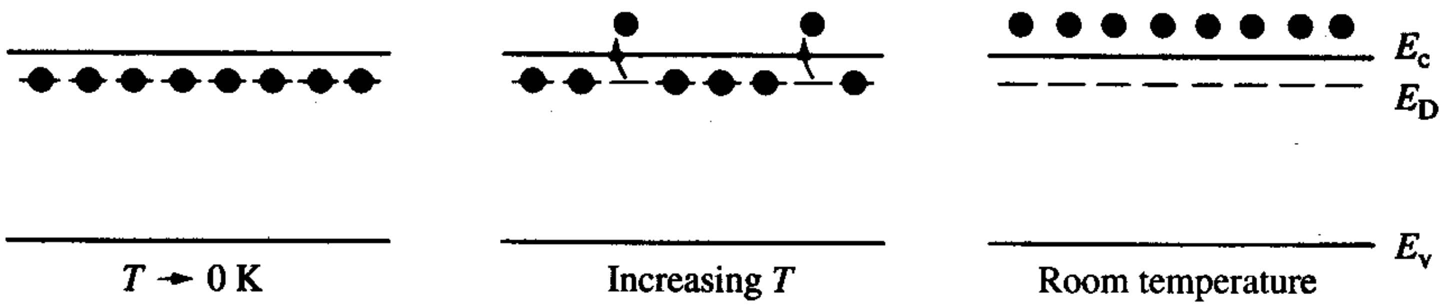


Ionization energy of selected donors and acceptors in Silicon (E_G = 1.12 eV)

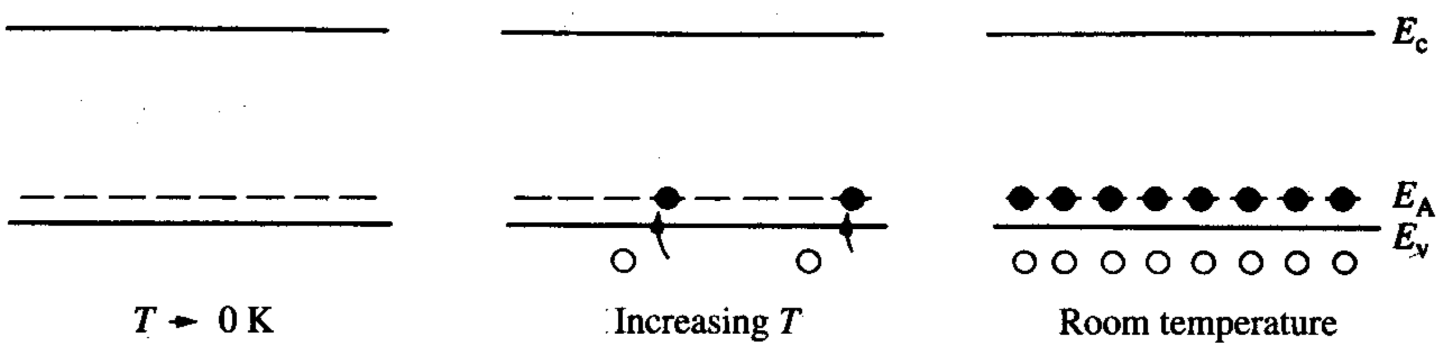
	Donors			Acceptors		
Ionization energy of dopant	Sb	P	As	B	Al	In
$E_C - E_D$ *or* $E_A - E_V$ (meV)	39	45	54	45	67	160

Dopant Ionization (Band Model)

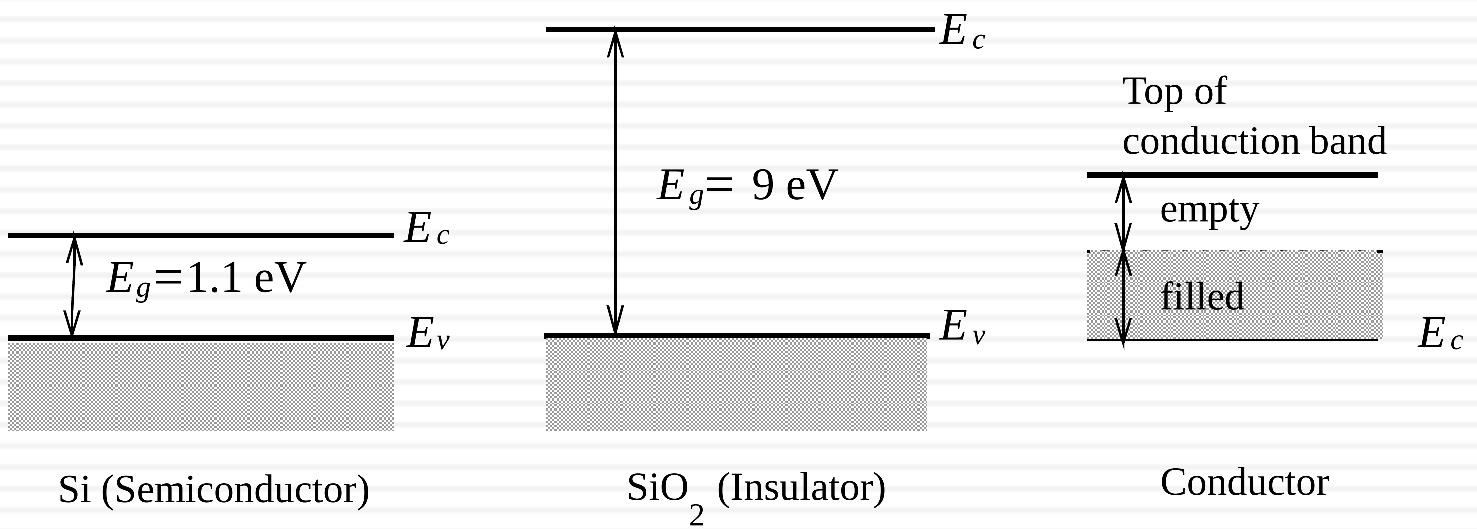
- Donor atoms



- Acceptor atoms

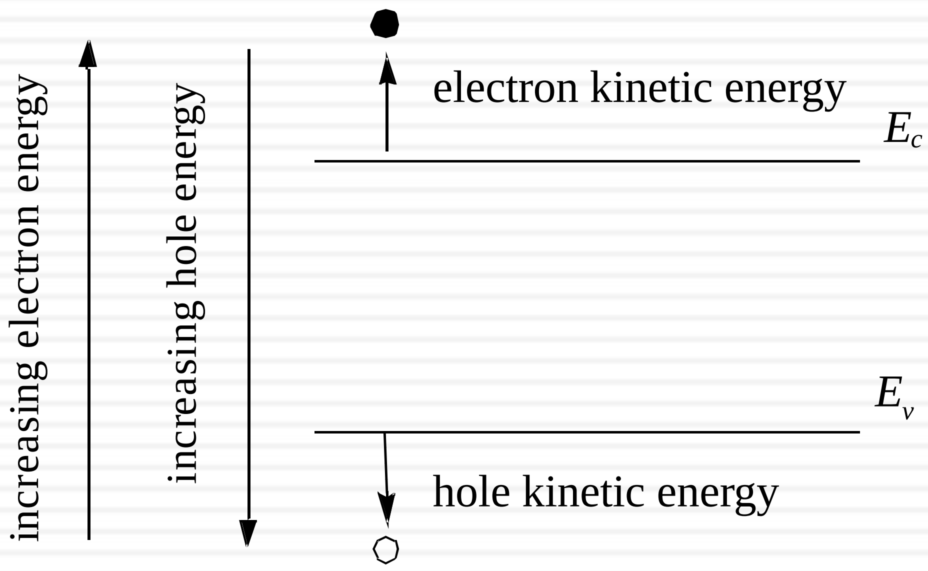


1.4 Semiconductors, Insulators, and Conductors



- Totally filled bands and totally empty bands do not allow current flow. (Just as there is no motion of liquid in a totally filled or totally empty bottle.)
- Metal conduction band is half-filled.
- Semiconductors have lower E_g's than insulators and can be doped.

1.5 Electrons and Holes



- Both electrons and holes tend to seek their lowest energy positions.
- Electrons tend to fall in the energy band diagram.
- Holes float up like bubbles in water

1.5.1 Effective Mass

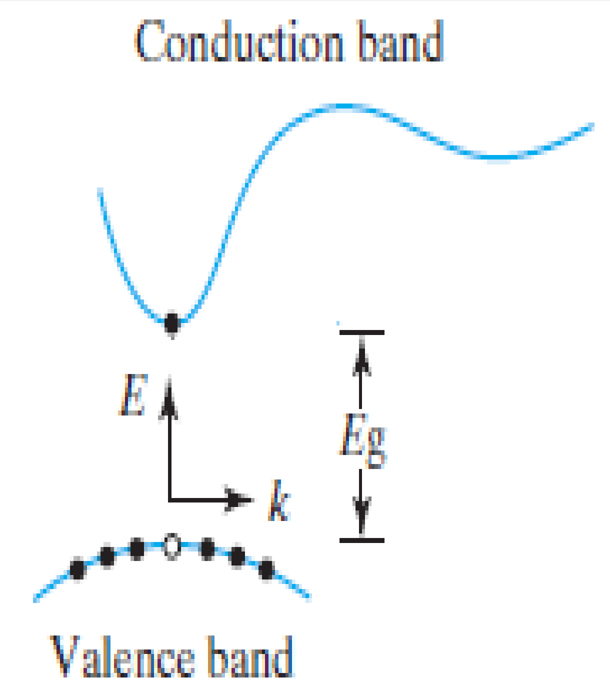
The electron wave function is the solution of the three dimensional Schrodinger wave equation

$$-\frac{\hbar^2}{2\mathrm{m}_0}\nabla^2\psi + V(r)\,\psi = \psi$$

The solution is of the form $\exp(\pm\,\mathbf{k}\ \ \mathbf{r})$

k = wave vector = 2π/electron wavelength

For each k, there is a corresponding E.



$$\text{acceleration} = -\frac{q\varepsilon}{\hbar^2}\frac{d^2E}{dk^2} = \frac{F}{m}$$

$$\text{effective mass} \equiv \frac{\hbar^2}{d^2E/dk^2}$$

1.5.1 Effective Mass

In an electric field, $\mathcal{E}$, an electron or a hole accelerates.

$$a = \frac{-q\mathcal{E}}{m_n}$$ electrons

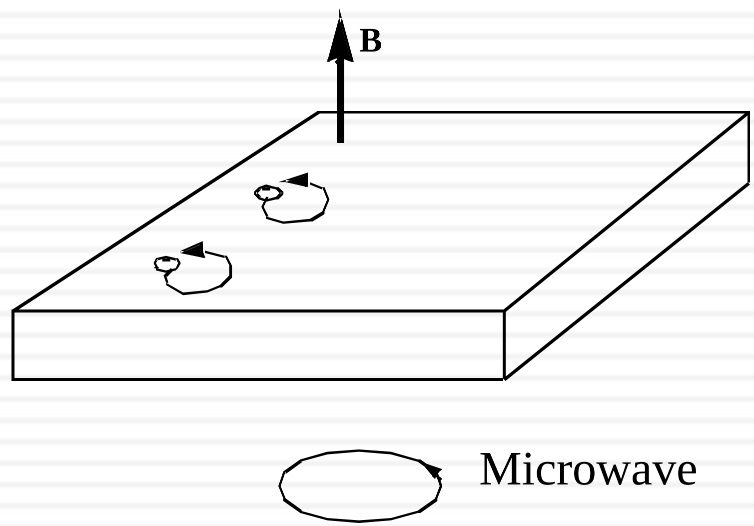
$$a = \frac{q\mathcal{E}}{m_p}$$ holes

Electron and hole effective masses

	Si	Ge	GaAs	InAs	AlAs
m_n/m_0	0.26	0.12	0.068	0.023	2
m_p/m_0	0.39	0.3	0.5	0.3	0.3

1.5.2 How to Measure the Effective Mass

Cyclotron Resonance Technique



Centripetal force = Lorentzian force

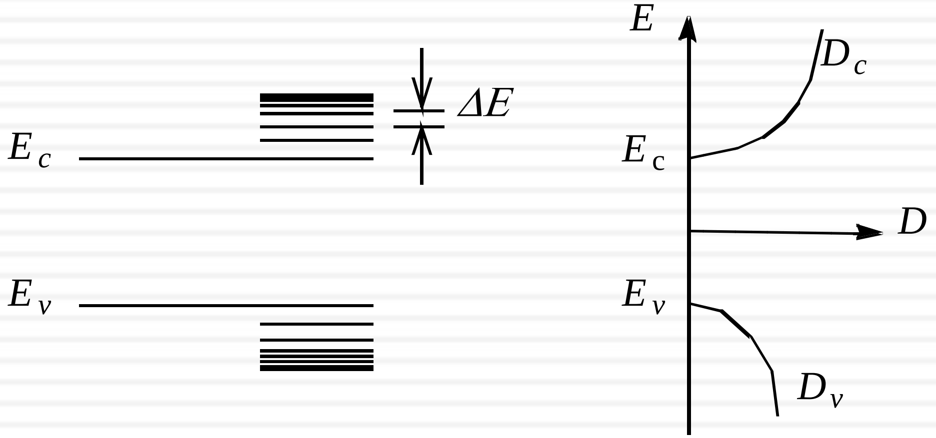
$$\frac{m_n v^2}{r} = qvB$$

$$v = \frac{qBr}{m_n}$$

$$f_{cr} = \frac{v}{2\pi r} = \frac{qB}{2\pi m_n}$$

- f_{cr} is the Cyclotron resonance frequency.
- It is independent of v and r.
- Electrons strongly absorb microwaves of that frequency.
- By measuring f_{cr}, m_n can be found.

1.6 Density of States



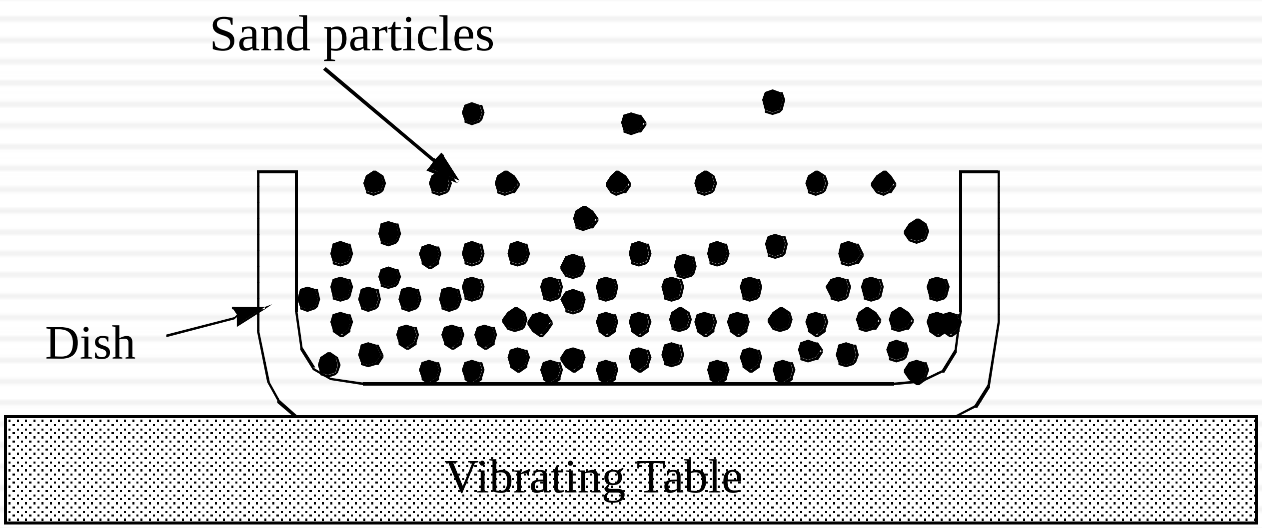
$$D_c(E) \equiv \frac{\text{number of states in } \Delta E}{\Delta E \cdot \text{volume}} \left(\frac{1}{\text{eV} \cdot \text{cm}^3} \right)$$

$$D_c(E) \equiv \frac{8\pi m_n \sqrt{2m_n(E - E_c)}}{h^3}$$

$$D_v(E) \equiv \frac{8\pi m_p \sqrt{2m_p(E_v - E)}}{h^3}$$

1.7 Thermal Equilibrium and the Fermi Function

1.7.1 An Analogy for Thermal Equilibrium

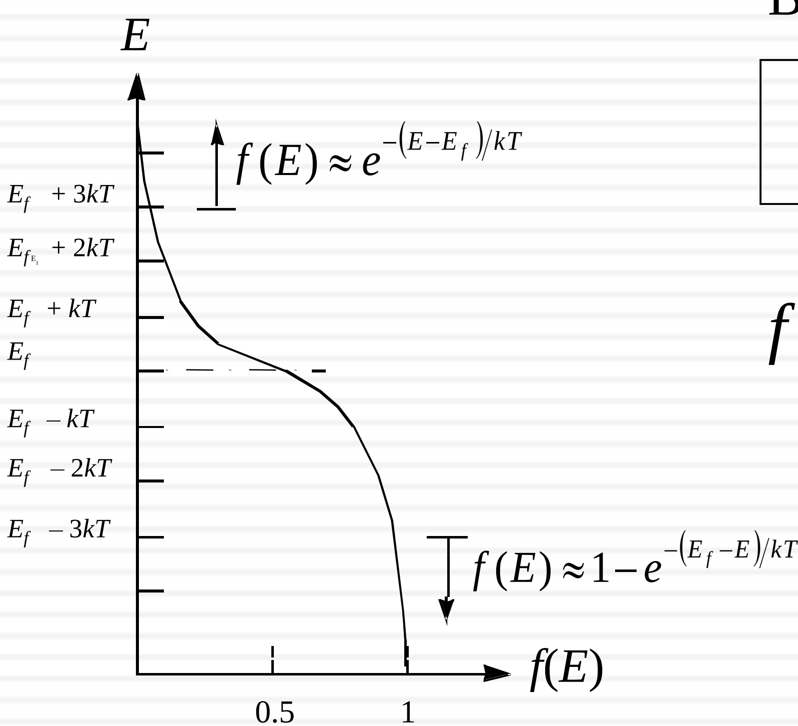


- There is a certain probability for the electrons in the conduction band to occupy high-energy states under the agitation of thermal energy.

1.7.2 Fermi Function–The Probability of an Energy State Being Occupied by an Electron

$$f(E) = \frac{1}{1+e^{(E-E_f)/kT}}$$

E_f is called the ***Fermi energy*** or the ***Fermi level.***



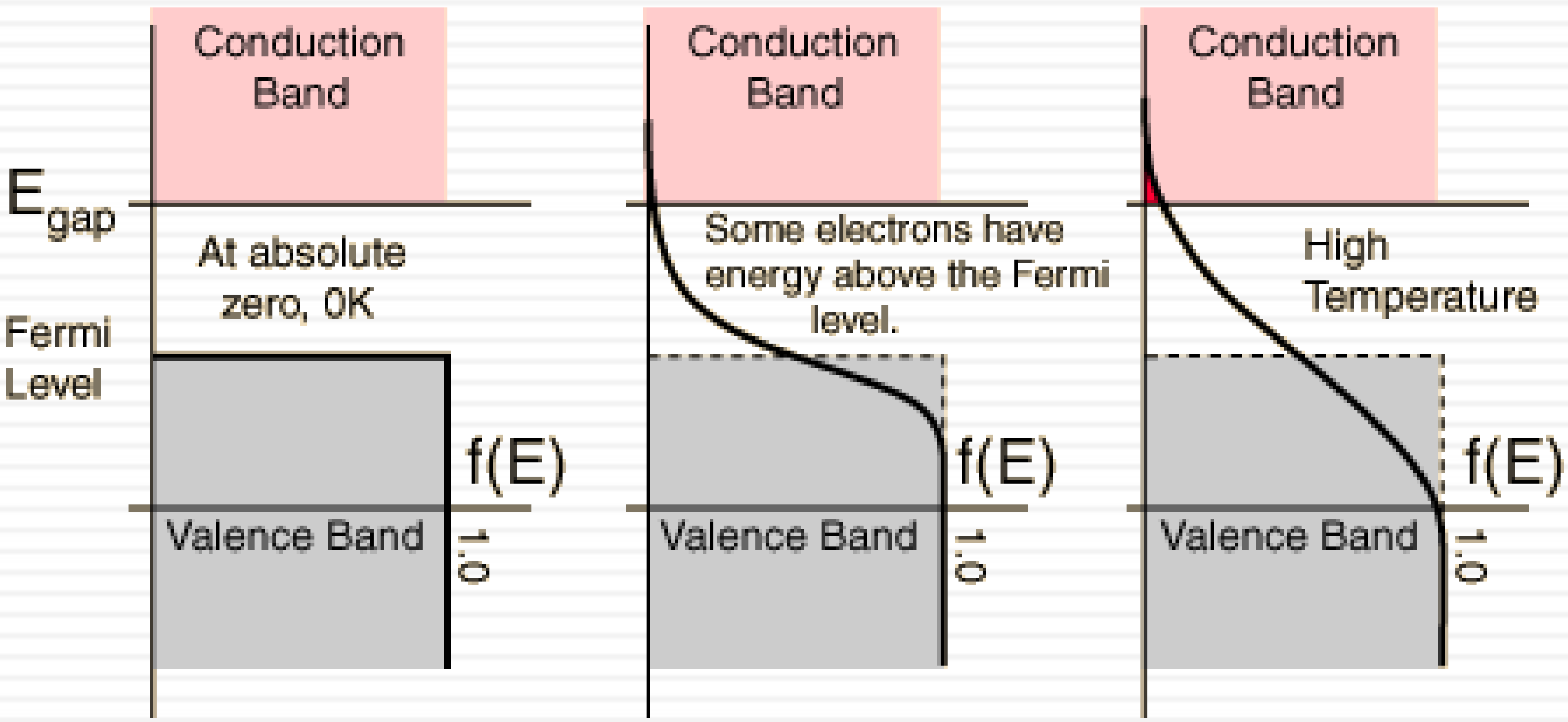
Boltzmann approximation:

$$f(E) \approx e^{-(E-E_f)/kT} \qquad E - E_f >> kT$$

$$f(E) \approx 1 - e^{-(E_f-E)/kT} \qquad E - E_f << -kT$$

Remember: there is only one Fermi-level in a system at equilibrium.

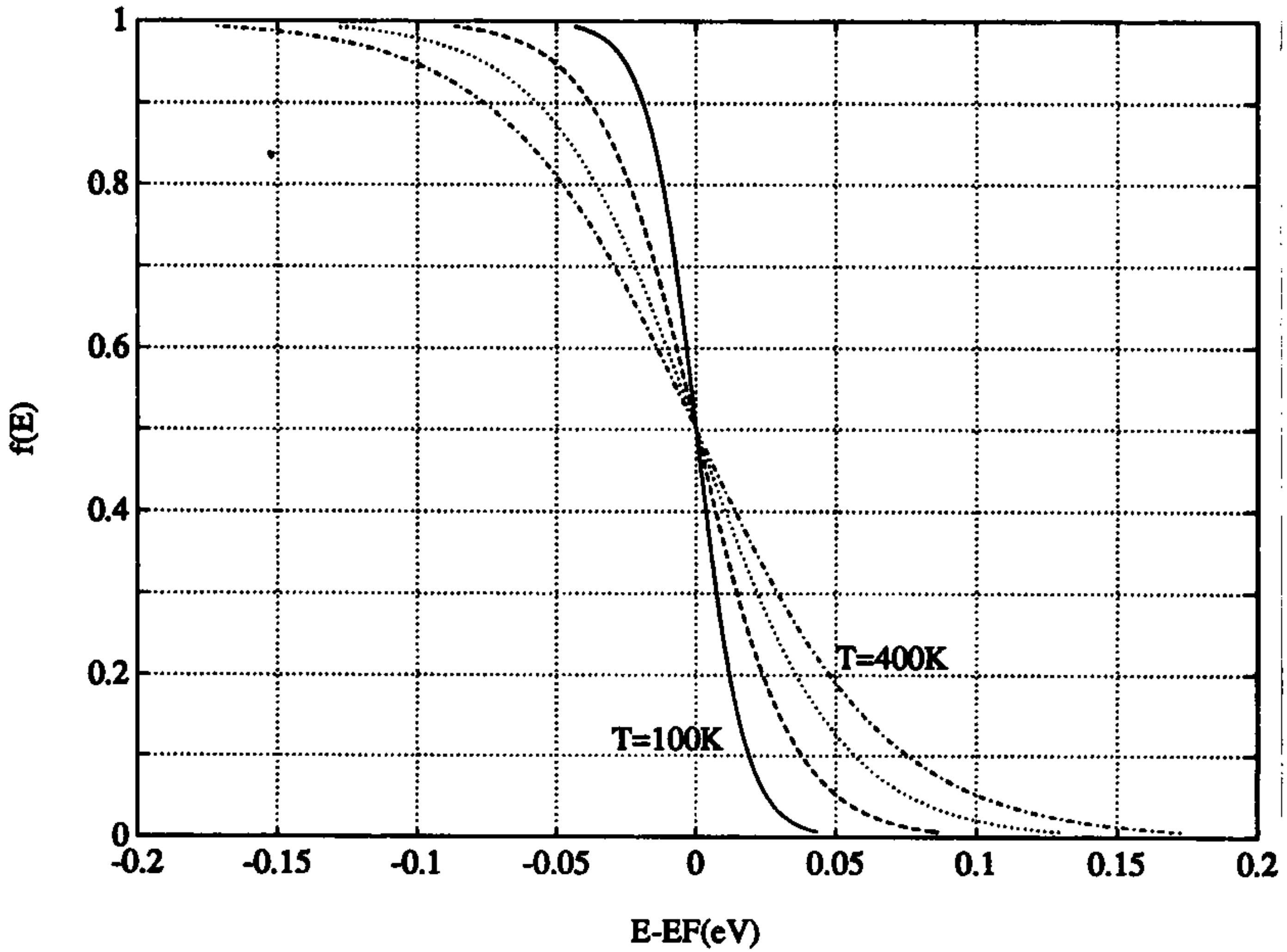
Effect of Temperature on *f(E)*



No electrons can be above the valence band at 0K, since none have energy above the Fermi level and there are no available energy states in the band gap.

At high temperatures, some electrons can reach the conduction band and contribute to electric current.

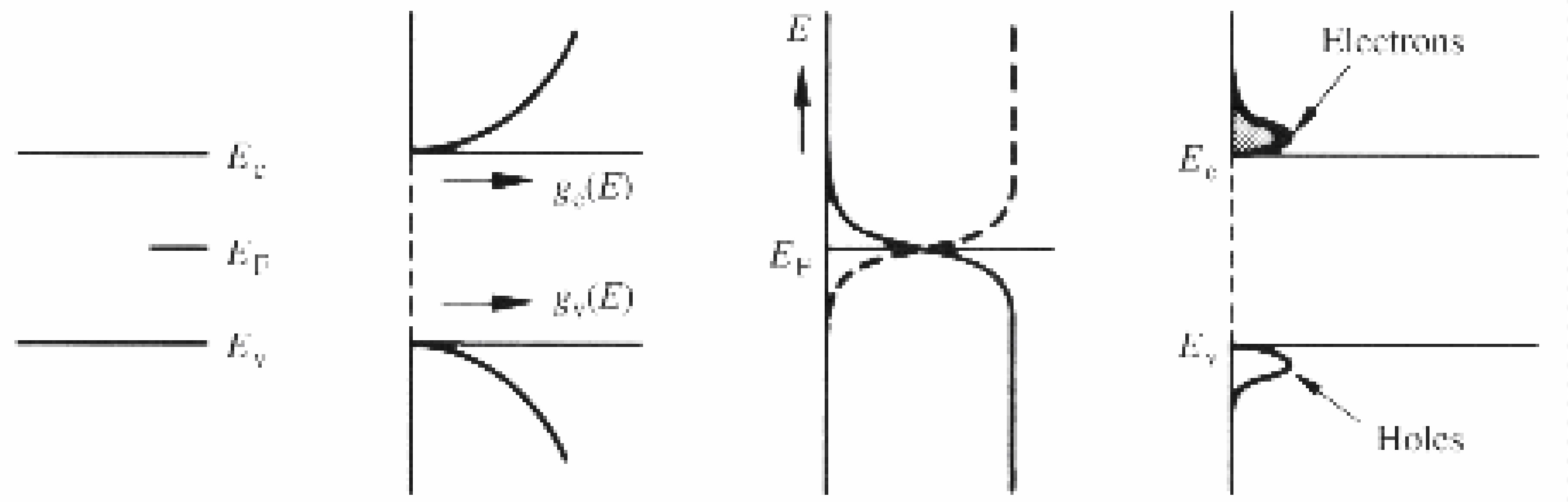
Effect of Temperature on *f*(*E*)



Equilibrium Distribution of Carriers - Intrinsic

- $n(E)$ is obtained by multiplying $g_c(E)$ and $f(E)$,
  $p(E)$ is obtained by multiplying $g_v(E)$ and $1–f(E)$.

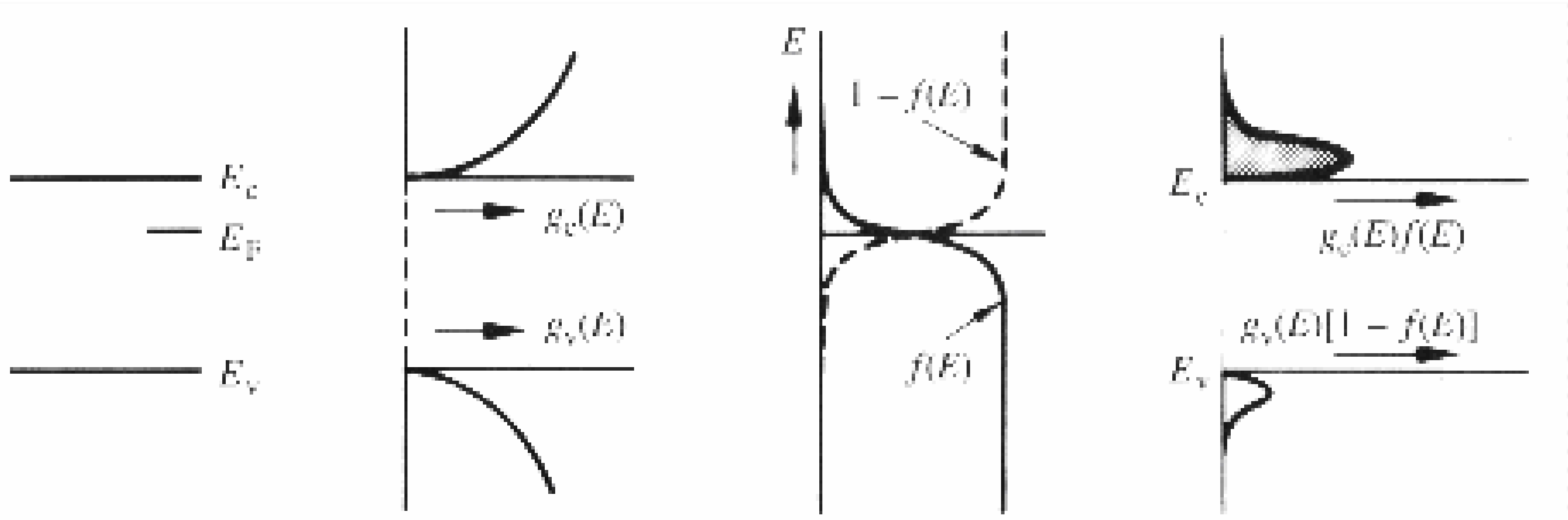
- **Intrinsic semiconductor material**



Energy band diagram | **Density of states** | **Probability of occupancy** | **Carrier distribution**

Equilibrium Distribution of Carriers – n-type

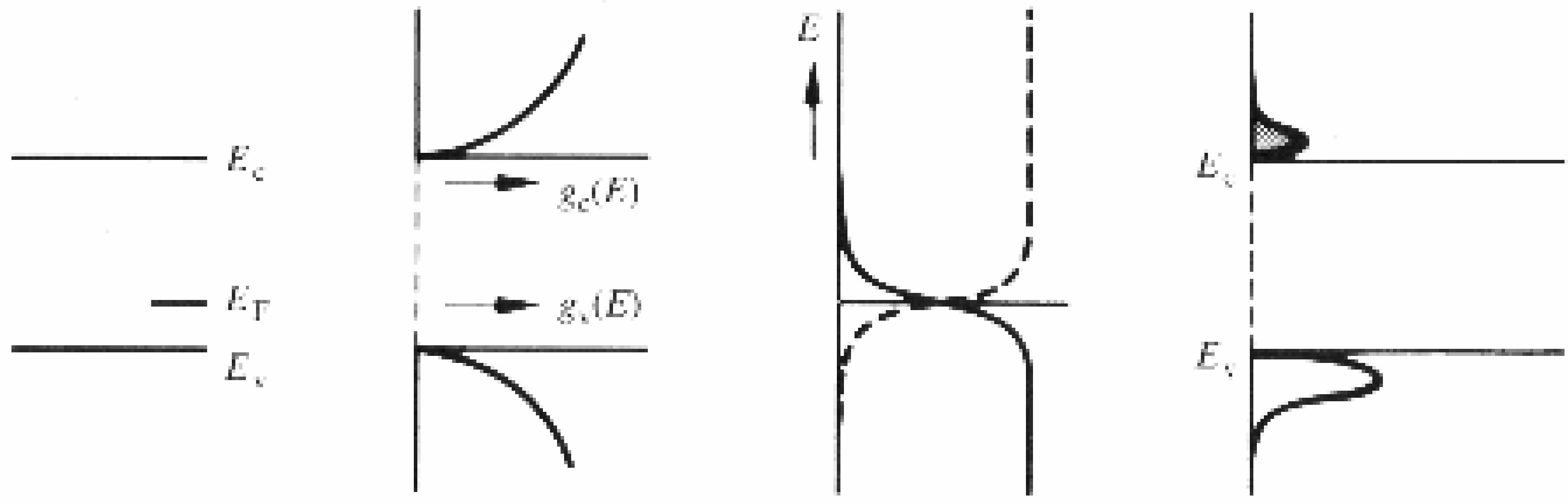
- ***n*-type semiconductor material**



Energy band diagram	Density of States	Probability of occupancy	Carrier distribution

Equilibrium Distribution of Carriers – p-type

- ***p*-type semiconductor material**

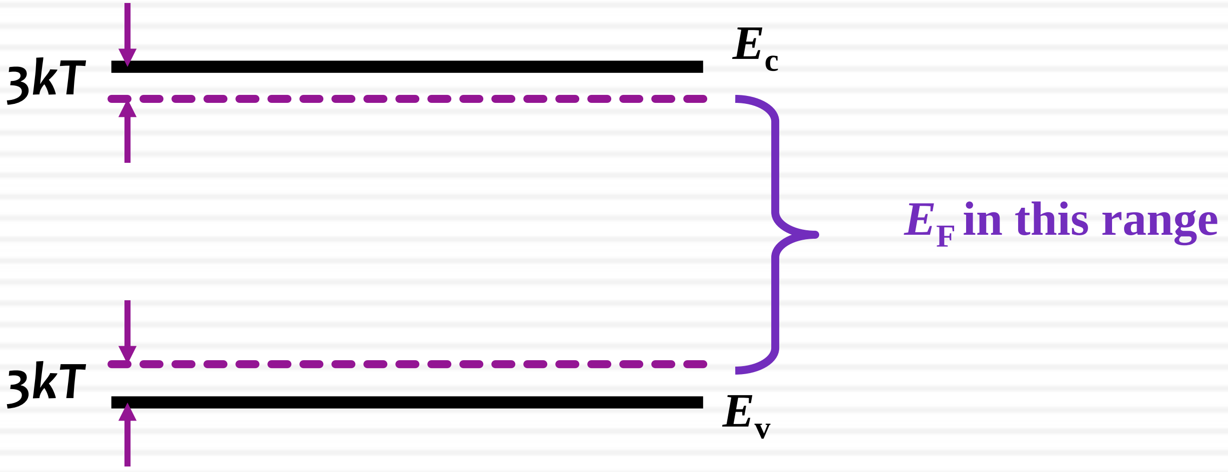


Energy band diagram | **Density of States** | **Probability of occupancy** | **Carrier distribution**

Nondegenerately Doped Semiconductor

- The expressions for *n* and *p* will now be derived in the range where the Boltzmann approximation can be applied:

$$E_v + 3kT \leq E_F \leq E_c - 3kT$$



- The semiconductor is said to be **nondegenerately doped (lightly doped)** in this case.

Degenerately Doped Semiconductor

- If a semiconductor is very heavily doped, the Boltzmann approximation is not valid.
- For Si at T = 300 K,
  $E_c - E_F < 3kT$ if $N_D > 1.6 \times 10^{18}$ cm^{-3}
  $E_F - E_v < 3kT$ if $N_A > 9.1 \times 10^{17}$ cm^{-3}
- The semiconductor is said to be **degenerately doped (heavily doped)** in this case.

  - **N_D = total number of donor atoms/cm^3**
  - **N_A = total number of acceptor atoms/cm^3**

Important Constants

- Electronic charge, $q = 1.6\times10^{-19}$ C
- Permittivity of free space, $\varepsilon_o = 8.854\times10^{-12}$ F/m
- Boltzmann constant, $k = 8.62\times10^{-5}$ eV/K
- Planck constant, $h = 4.14\times10^{-15}$ eV·s, $\hbar = h/2\pi$
- Free electron mass, $m_o = 9.1\times10^{-31}$ kg
- Thermal energy, $kT = 0.02586$ eV (at 300 K)
- Thermal voltage, $kT/q = 0.02586$ V (at 300 K)
- Silicon energy band gap, $E_G = 1.12 eV$
- Intrinsic Si carrier concentration $n_i = 1\times10^{10}\ cm^{-3}$ (at 300 K)

1.8 Electron and Hole Concentrations

1.8.1 Derivation of *n* and *p* from *D*(*E*) and *f*(*E*)

$$n = \int_{E_c}^{\text{top of conduction band}} f(E) D_c(E) dE$$

$$n = \frac{8\pi m_n \sqrt{2m_n}}{h^3} \int_{E_c}^{\infty} \sqrt{E - E_c}\, e^{-(E-E_f)/kT} dE$$

$$= \frac{8\pi m_n \sqrt{2m_n}}{h^3} e^{-(E_c - E_f)/kT} \int_0^{E-Ec} \sqrt{E - E_c}\, e^{-(E-E_c)/kT} d(E - Ec)$$

- Similarly p can be derived

$$p = \int_{E_{\text{bottom}}}^{E_{\text{V}}} D_{\text{v}}(E)\left[1 - f(E)\right] dE$$

Electron and Hole Concentrations

$$n = N_c e^{-(E_c - E_f)/kT}$$

$$N_c \equiv 2\left[\frac{2\pi m_n kT}{h^2}\right]^{3/2}$$

N_c is called the ***effective density of states (of the conduction band)*** **.**

$$p = N_v e^{-(E_f - E_v)/kT}$$

$$N_v \equiv 2\left[\frac{2\pi m_p kT}{h^2}\right]^{3/2}$$

N_v is called the ***effective density of states of the valence band.***

Remember: the closer E_f moves up to N_c, the larger n is; the closer E_f moves down to E_v, the larger p is.

For Si, $N_c = 2.8 \times 10^{19}\ \text{cm}^{-3}$ and $N_v = 1.04 \times 10^{19}\ \text{cm}^{-3}$.

1.8.2 The Fermi Level and Carrier Concentrations

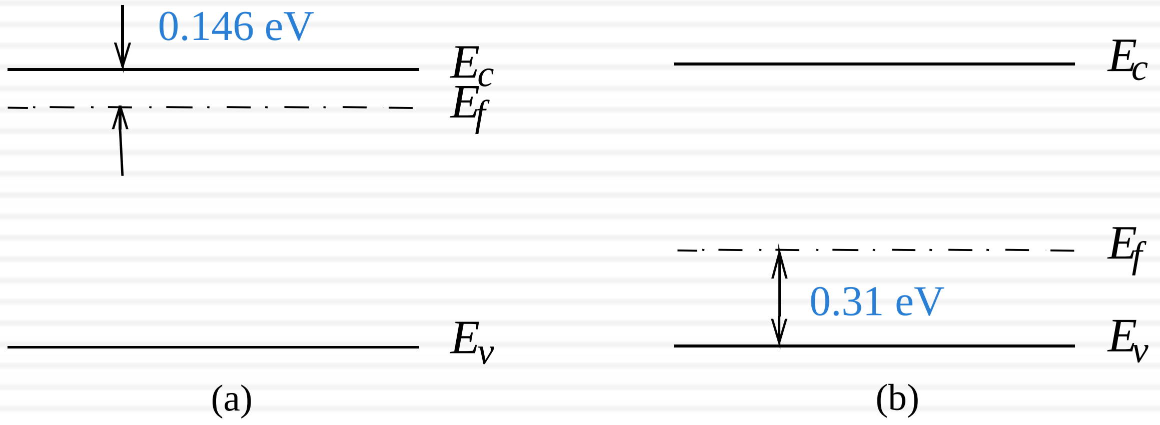
Example: *Where is E_f for $n = 10^{17}$ cm^{-3}? And for $p = 10^{14}$ cm^{-3}?*

Solution: *(a)* $n = N_c e^{-(E_c - E_f)/kT}$

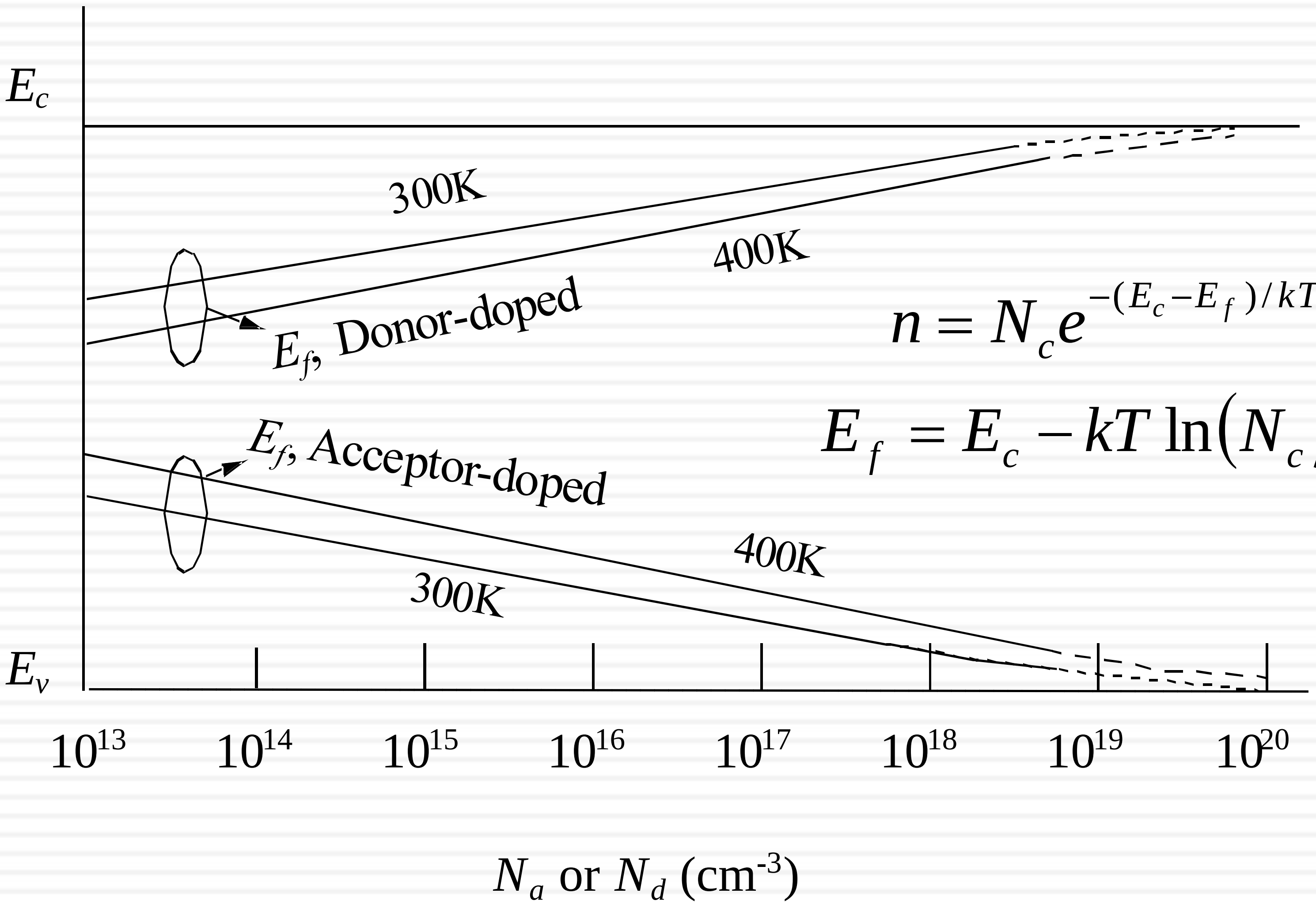
$$E_c - E_f = kT \ln(N_c/n) = 0.026 \ln(2.8\times10^{19}/10^{17}) = 0.146 \text{ eV}$$

(b) For $p = 10^{14}$cm^{-3}, from Eq.(1.8.8),

$$E_f - E_v = kT \ln(N_v/p) = 0.026 \ln(1.04\times10^{19}/10^{14}) = 0.31 \text{ eV}$$



1.8.2 The Fermi Level and Carrier Concentrations



$$n = N_c e^{-(E_c - E_f)/kT}$$

$$E_f = E_c - kT \ln\left(N_c / n\right)$$

1.8.3 The *np* Product and the Intrinsic Carrier Concentration

Multiply $n = N_c e^{-(E_c - E_f)/kT}$ and $p = N_v e^{-(E_f - E_v)/kT}$

$$np = N_c N_v e^{-(E_c - E_v)/kT} = N_c N_v e^{-E_g/kT}$$

$$np = n_i^{\,2}$$

$$n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT}$$

- In an intrinsic (undoped) semiconductor, $n = p = n_i$.
- n_i is the ***intrinsic carrier concentration***, ~10^{10} cm^{-3} for Si.

Alternative Expressions: $n(n_i, E_i)$ and $p(n_i, E_i)$

- In an intrinsic semiconductor, $n = p = n_i$ and $E_F = E_i$, where E_i denotes the intrinsic Fermi level.

$$n = N_C e^{(E_F - E_c)/kT}$$

$$n_i = N_C e^{(E_i - E_c)/kT}$$

$$\Rightarrow N_C = n_i e^{-(E_i - E_c)/kT}$$

$$n = n_i e^{-(E_i - E_c)/kT} \cdot e^{(E_F - E_c)/kT}$$

$$n = n_i e^{(E_F - E_i)/kT}$$

$$E_F = E_i + kT \ln\left(\frac{n}{n_i}\right)$$

$$p = N_V e^{(E_v - E_F)/kT}$$

$$p_i = N_V e^{(E_v - E_i)/kT}$$

$$\Rightarrow N_V = n_i e^{-(E_v - E_i)/kT}$$

$$p = n_i e^{-(E_v - E_i)/kT} \cdot e^{(E_v - E_F)/kT}$$

$$p = n_i e^{(E_i - E_F)/kT}$$

$$E_F = E_i - kT \ln\left(\frac{p}{n_i}\right)$$

Intrinsic Fermi Level, E_i

- To find E_F for an intrinsic semiconductor, we use the fact that $n = p$.

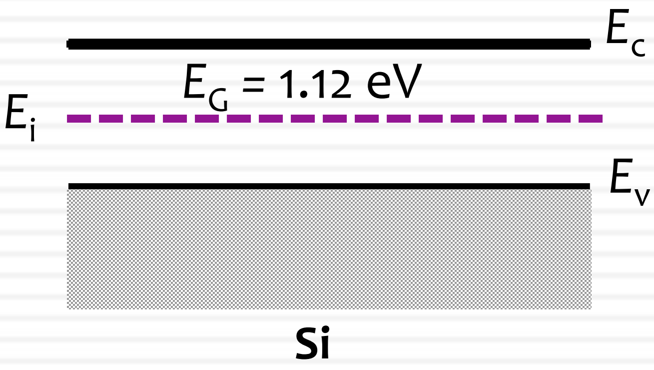
$$N_C e^{(E_i - E_c)/kT} = N_V e^{(E_v - E_i)/kT}$$

$$\Downarrow$$

$$E_i = \frac{E_c + E_v}{2} + \frac{kT}{2} \ln\left(\frac{N_V}{N_C}\right)$$

$$E_i = \frac{E_c + E_v}{2} + \frac{3kT}{4} \ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$E_i \cong \frac{E_c + E_v}{2}$$



- **E_i lies (almost) in the middle between E_c and E_v**

EXAMPLE: Carrier Concentrations

Question: *What is the hole concentration in an N-type semiconductor with 10^{15} cm^{-3} of donors?*

Solution: *n = 10^{15} cm^{-3}.*

$$p = \frac{n_i^2}{n} \approx \frac{10^{20}\,\mathrm{cm}^{-3}}{10^{15}\,\mathrm{cm}^{-3}} = 10^5\,\mathrm{cm}^{-3}$$

After increasing T by 60 ℃, n remains the same at 10^{15} cm^{-3} while p increases by about a factor of 2300 because $n_i^2 \propto e^{-E_g/kT}$

Question: *What is n if p = 10^{17}cm^{-3} in a P-type silicon wafer?*

Solution:

$$n = \frac{n_i^2}{p} \approx \frac{10^{20}\,\mathrm{cm}^{-3}}{10^{17}\,\mathrm{cm}^{-3}} = 10^3\,\mathrm{cm}^{-3}$$

Example: Energy-Band Diagram

- For Silicon at 300 K, where is E_F if $n = 10^{17}$ cm^{-3} ?

Silicon at 300 K, $n_i = 10^{10}$ cm^{-3}

$$E_F = E_i + kT \ln\left(\frac{n}{n_i}\right)$$

$$= 0.56 + 8.62 \cdot 10^{-5} \cdot 300 \cdot \ln\left(\frac{10^{17}}{10^{10}}\right) \text{ eV}$$

$$= 0.56 + 0.417 \text{ eV}$$

$$= \underline{\underline{0.977 \text{ eV}}}$$

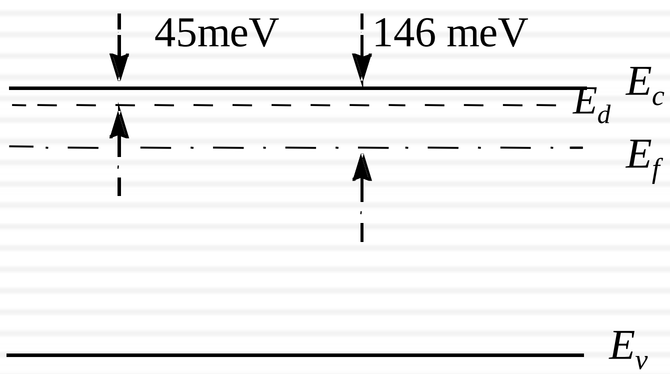
1.9 General Theory of n and p

EXAMPLE: Complete ionization of the dopant atoms

$N_d = 10^{17}$ cm^{-3}. What fraction of the donors are not ionized?

Solution: *First assume that all the donors* ***are*** *ionized.*

$$n = N_d = 10^{17}\,\text{cm}^{-3} \Rightarrow E_f = E_c - 146\,\text{meV}$$



Probability of not being ionized $\approx \dfrac{1}{1+\frac{1}{2}e^{(E_d-E_f)/kT}} = \dfrac{1}{1+\frac{1}{2}e^{((146-45)\text{meV})/26\text{meV}}} = 0.04$

Therefore, it is reasonable to assume complete ionization, i.e., $n = N_d$.

1.9 General Theory of n and p (2)

Charge neutrality: $n + N_a = p + N_d$

$$np = n_i^2$$

$$p = \frac{N_a - N_d}{2} + \left[\left(\frac{N_a - N_d}{2} \right)^2 + n_i^2 \right]^{1/2}$$

$$n = \frac{N_d - N_a}{2} + \left[\left(\frac{N_d - N_a}{2} \right)^2 + n_i^2 \right]^{1/2}$$

1.9 General Theory of on n and p (3)

I. $N_d - N_a >> n_i$ (i.e., N-type)

$$n = N_d - N_a$$
$$p = n_i^2 / n$$

If $N_d >> N_a$, $n = N_d$ and $p = n_i^2 / N_d$

II. $N_a - N_d >> n_i$ (i.e., P-type)

$$p = N_a - N_d$$
$$n = n_i^2 / p$$

If $N_a >> N_d$, $p = N_a$ and $n = n_i^2 / N_a$

EXAMPLE: Dopant Compensation

What are n and p in Si with (a) $N_d = 6\times10^{16}$ cm^{-3} and $N_a = 2\times10^{16}$ cm^{-3} and (b) additional 6×10^{16} cm^{-3} of N_a?

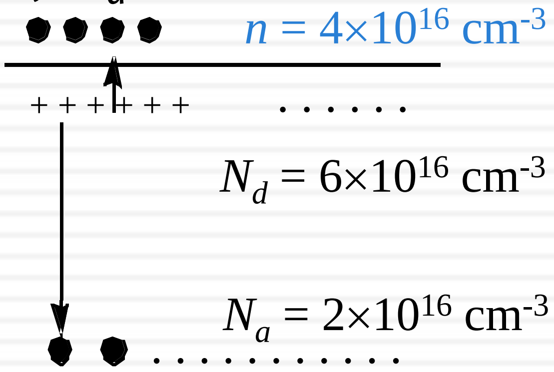
(a) $n = N_d - N_a = 4\times10^{16}\,\text{cm}^{-3}$

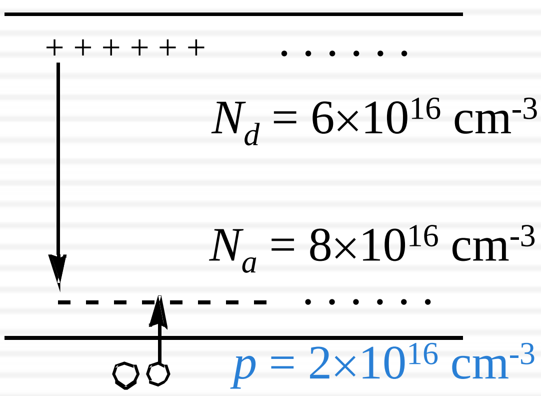
$$p = n_i^2 / n = 10^{20} / 4\times10^{16} = 2.5\times10^{3}\,\text{cm}^{-3}$$

(b) $N_a = 2\times10^{16} + 6\times10^{16} = 8\times10^{16}$ cm^{-3} > N_d

$$p = N_a - N_d = 8\times10^{16} - 6\times10^{16} = 2\times10^{16}\,\text{cm}^{-3}$$

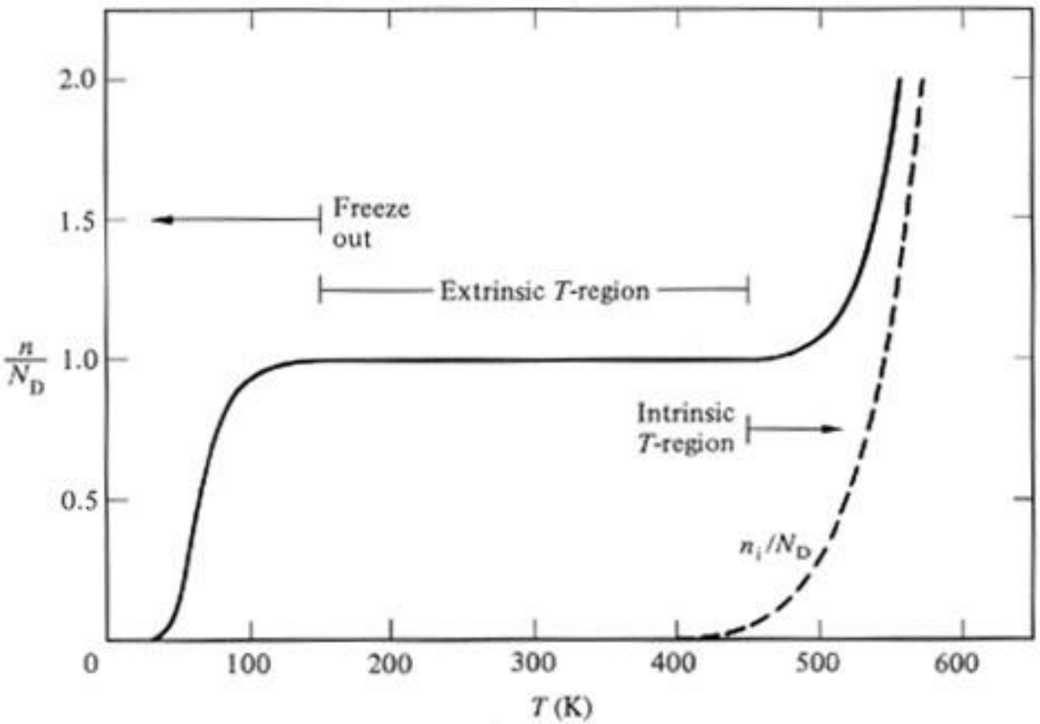
$$n = n_i^2 / p = 10^{20} / 2\times10^{16} = 5\times10^{3}\,\text{cm}^{-3}$$





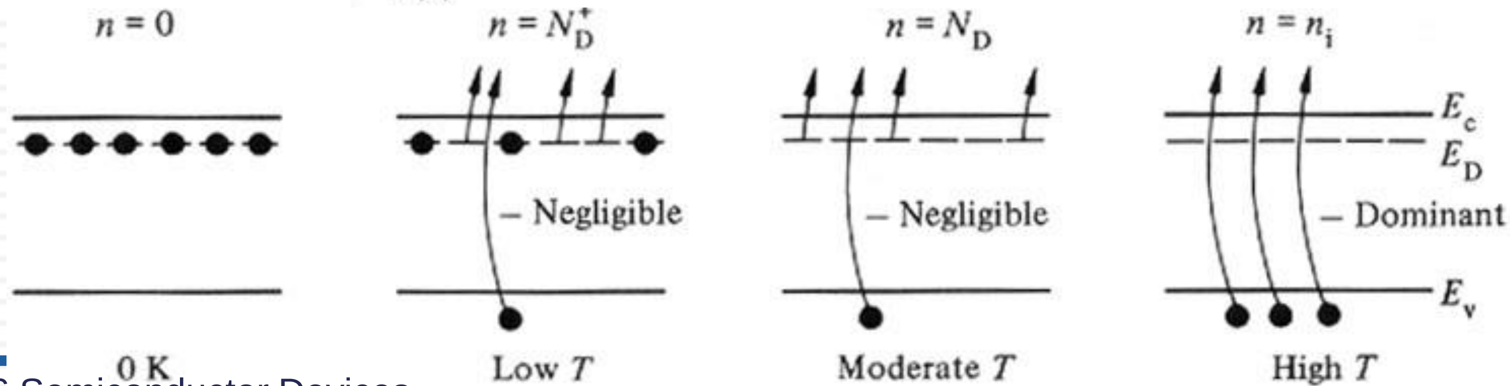
Carrier Concentration *vs.* Temperature

Phosphorus-doped Si



$N_D = 10^{15}$ cm^{-3}

- n : number of majority carrier
- N_D : number of donor electron
- n_i : number of intrinsic conductive electron



1.11 Chapter Summary

Energy band diagram. Acceptor. Donor. m_n, m_p.
Fermi function. E_f .

$$n = N_c e^{-(E_c - E_f)/kT}$$

$$p = N_v e^{-(E_f - E_v)/kT}$$

$$n = N_d - N_a$$

$$p = N_a - N_d$$

$$np = n_i^{\,2}$$