Semiconductor Device Physics

Lecture 6

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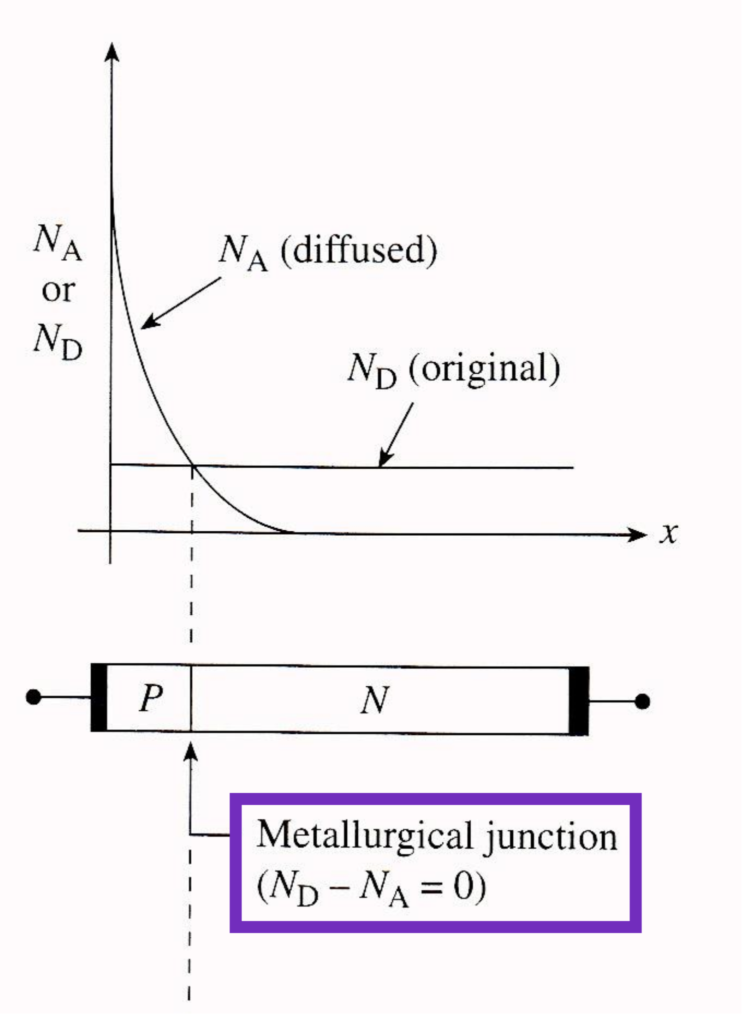
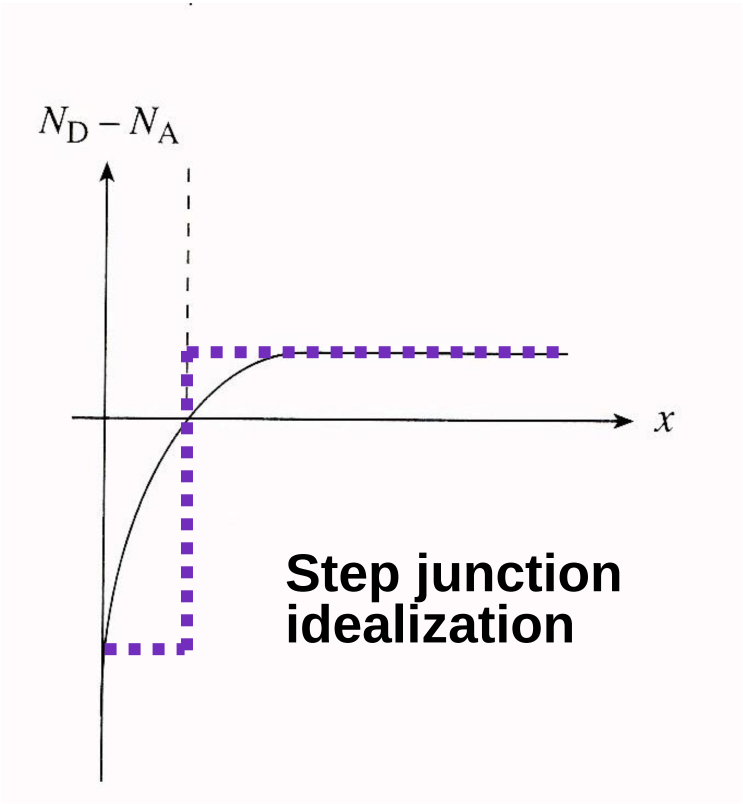
Chapter 5

pn Junction Electrostatics

Metallurgical Junction

- Doping profile





Poisson's Equation

- Poisson's equation is a well-known relationship in electricity and magnetism.
- It is now used because it often contains the starting point in obtaining quantitative solutions for the electrostatic variables.

$$\nabla \cdot \mathcal{E} = \frac{\rho}{K_S \varepsilon_0}$$

$$\nabla \cdot \mathbf{D} = \rho_v$$

$$\mathbf{D} = \varepsilon \mathbf{E}$$

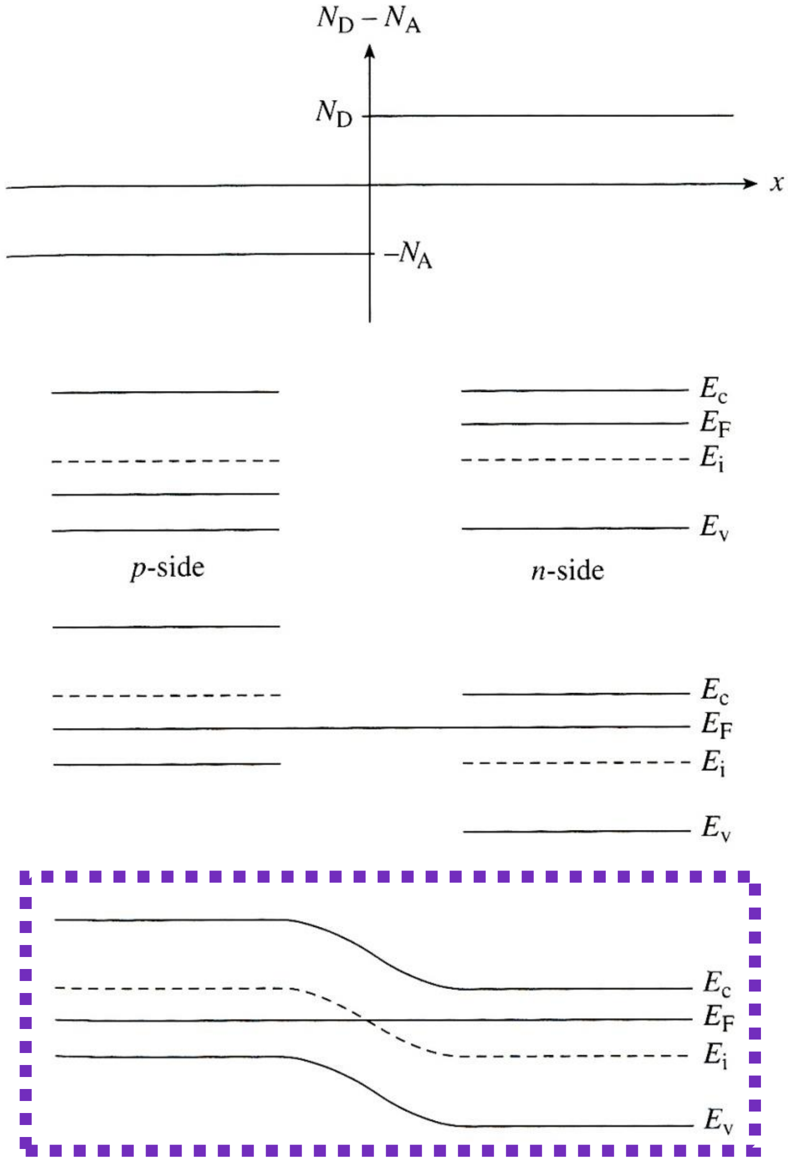
$$\varepsilon = K_S \varepsilon_0$$

- In one-dimensional problems, Poisson's equation simplifies to:

$$\frac{\partial \mathcal{E}}{\partial x} = \frac{\rho}{K_S \varepsilon_0}$$

Equilibrium Energy Band Diagram

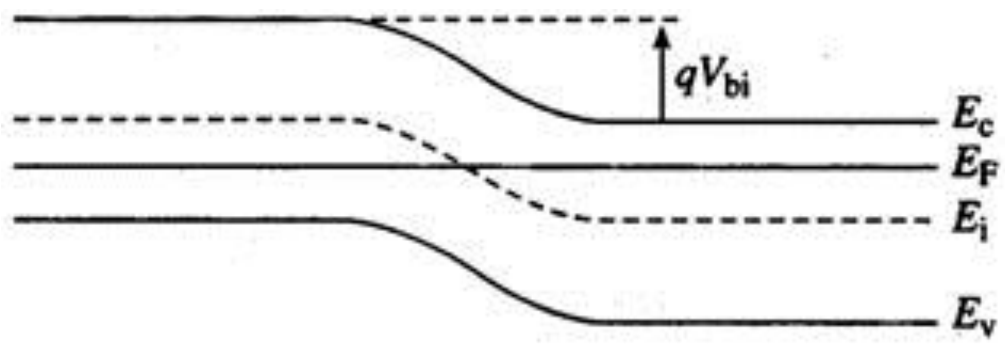
- *pn*-Junction diode



Qualitative Electrostatics

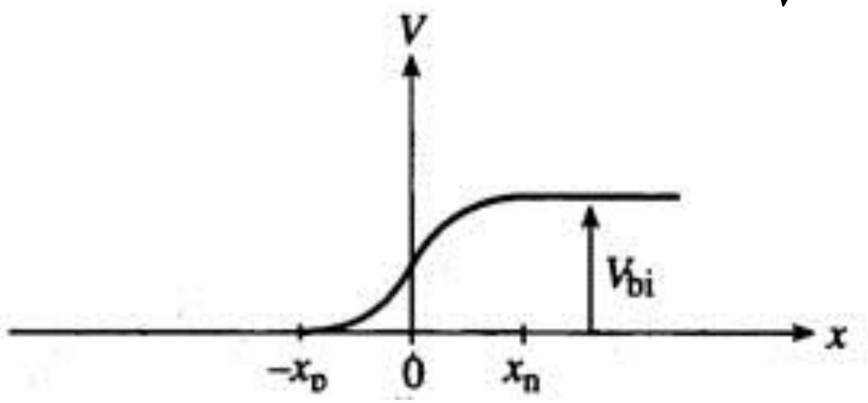
- Equilibrium condition

Band diagram



$$V = -\frac{1}{q}(E_c - E_{ref})$$

Electrostatic potential

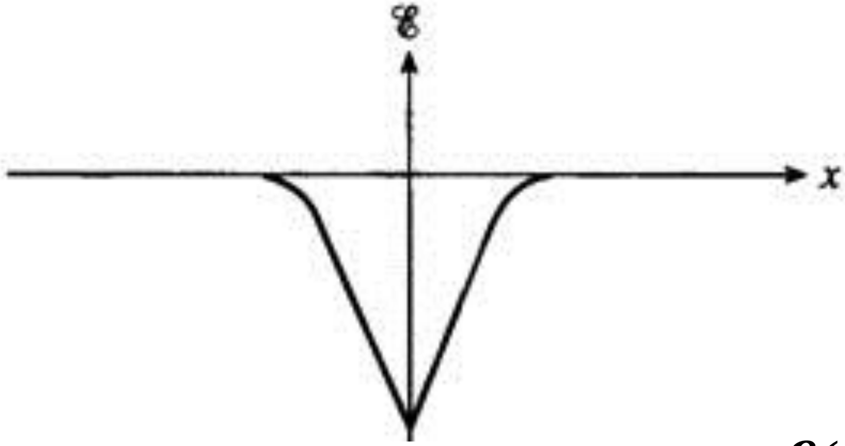


$$V(x) = -\int \mathcal{E} dx$$

Qualitative Electrostatics

- Equilibrium condition

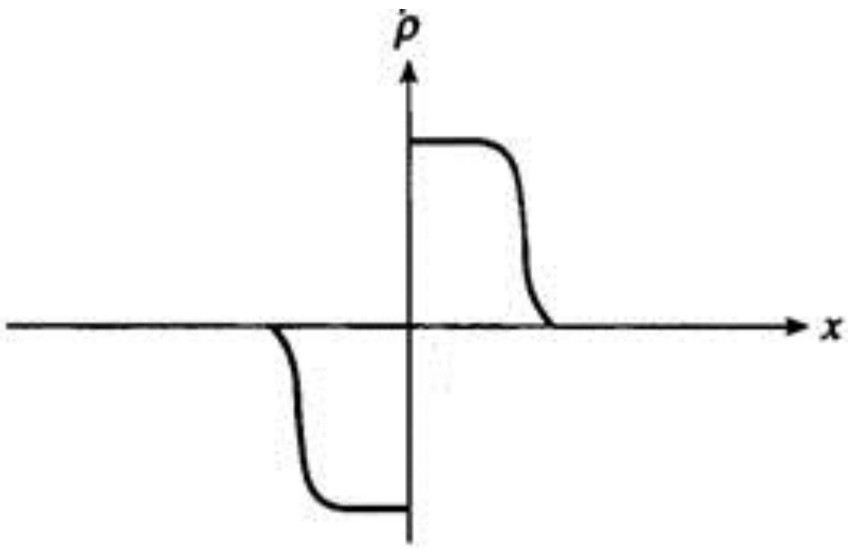
Electric field



$$\mathcal{E} = -\frac{dV}{dx}$$

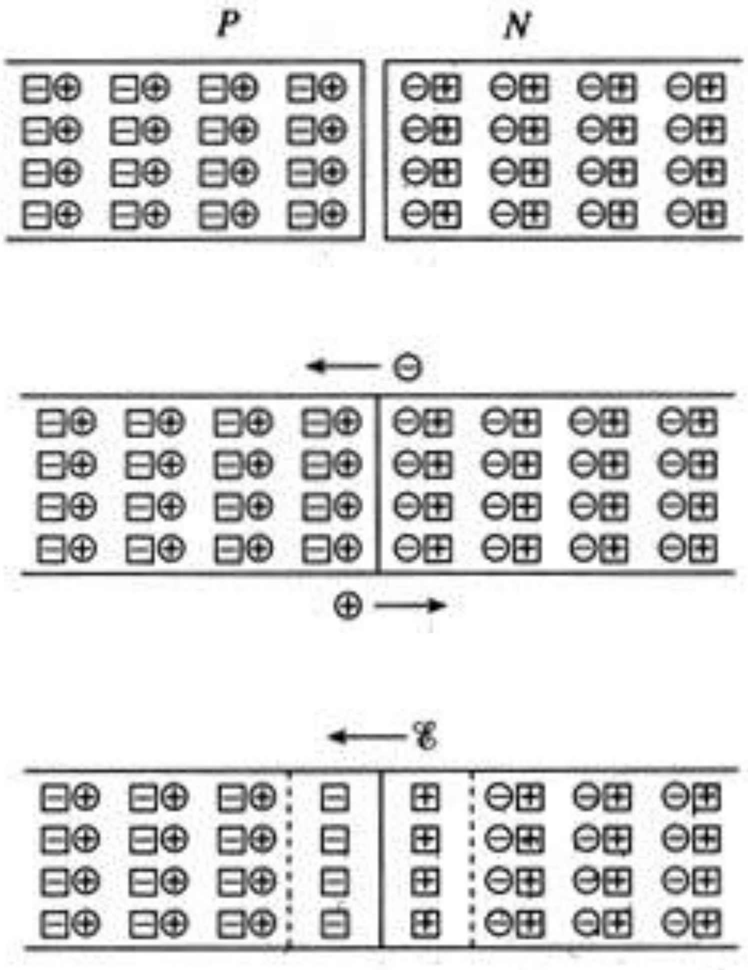
$$\mathcal{E}(x) = \int \frac{\rho}{K_S \varepsilon_0} \partial x$$

Charge density

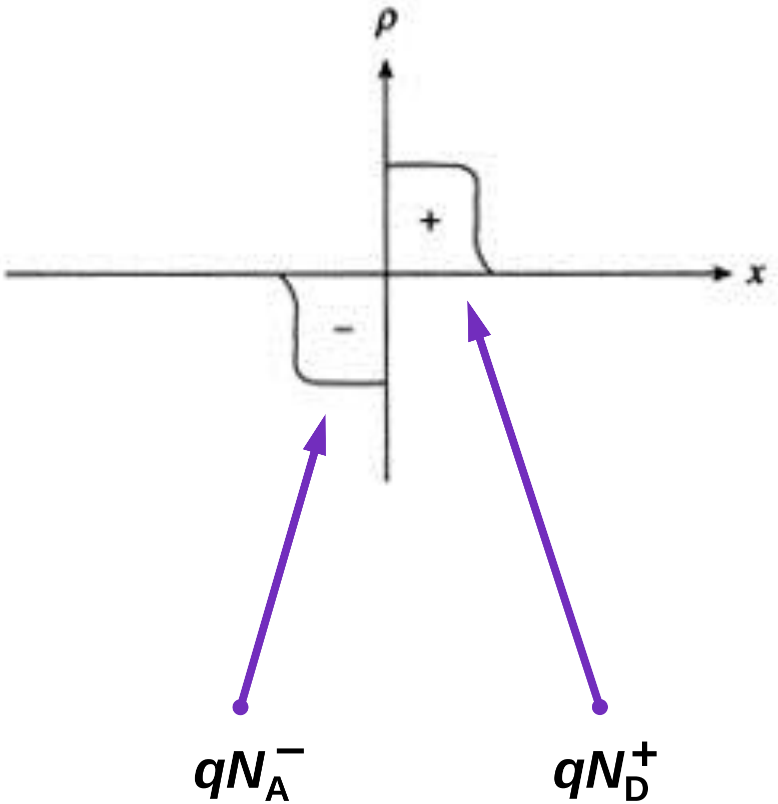


$$\frac{\partial \mathcal{E}}{\partial x} = \frac{\rho}{K_S \varepsilon_0}$$

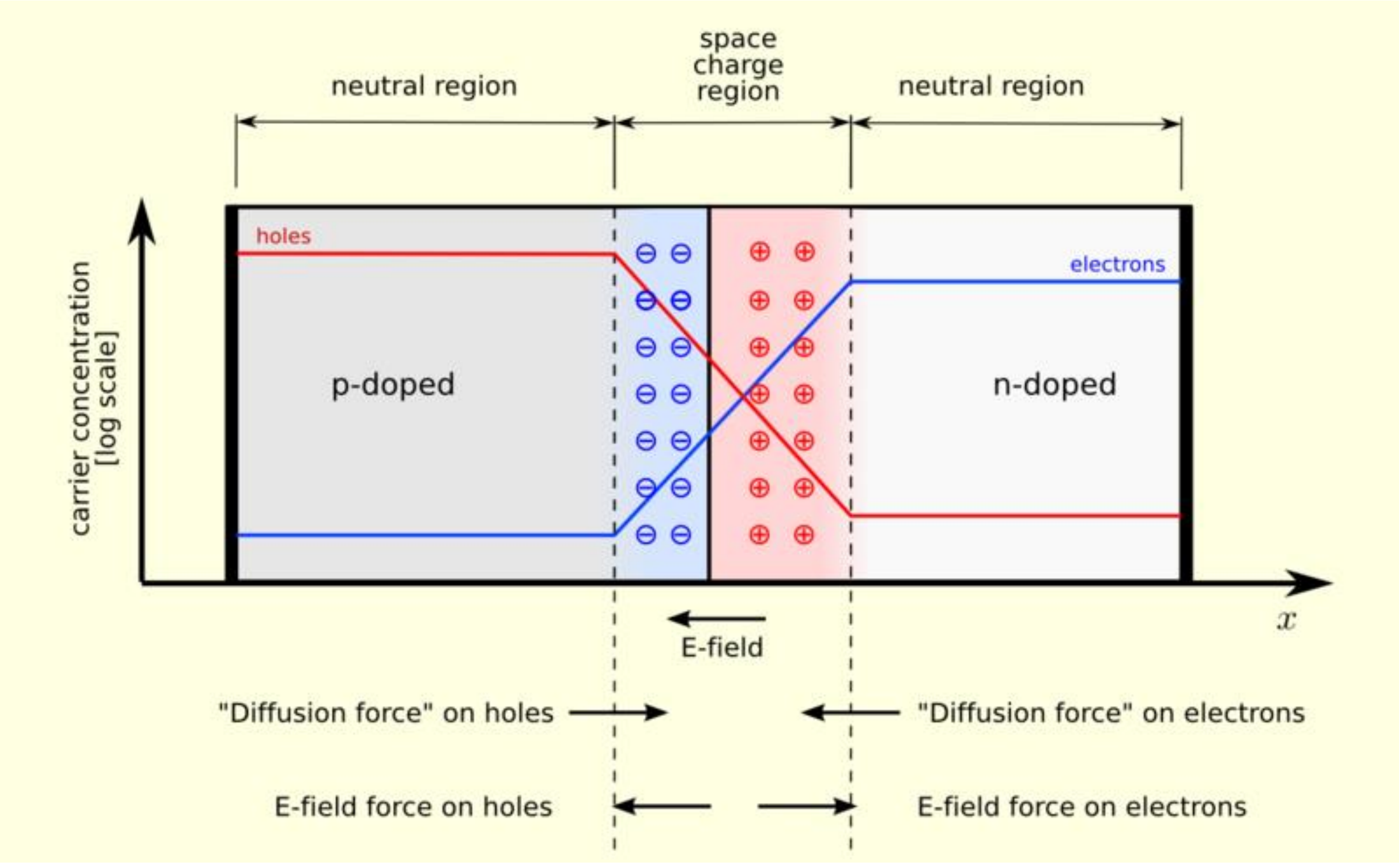
Formation of *pn* Junction and Charge Distribution



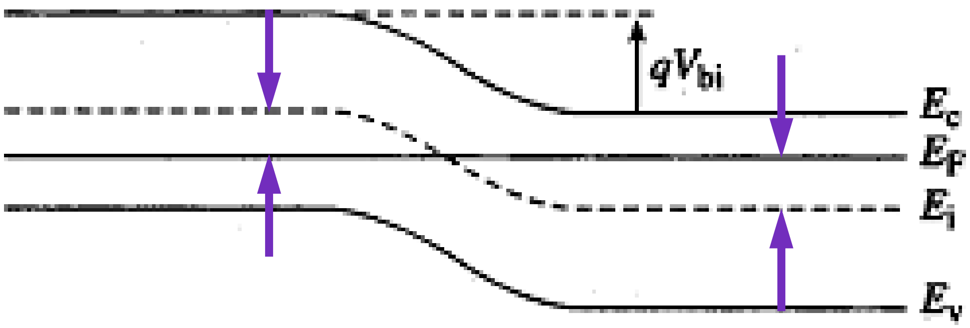
$$\rho = q(p - n + N_D^+ - N_A^-)$$



Formation of *pn* Junction and Charge Distribution



Built-In Potential V_{bi}



- V_{bi} **for several materials:**
  - **Ge ≤ 0.66 V**
  - **Si ≤ 1.12 V**
  - **GeAs ≤ 1.42 V**

$$qV_{bi} = (E_F - E_i)_{n-side} + (E_i - E_F)_{p-side}$$

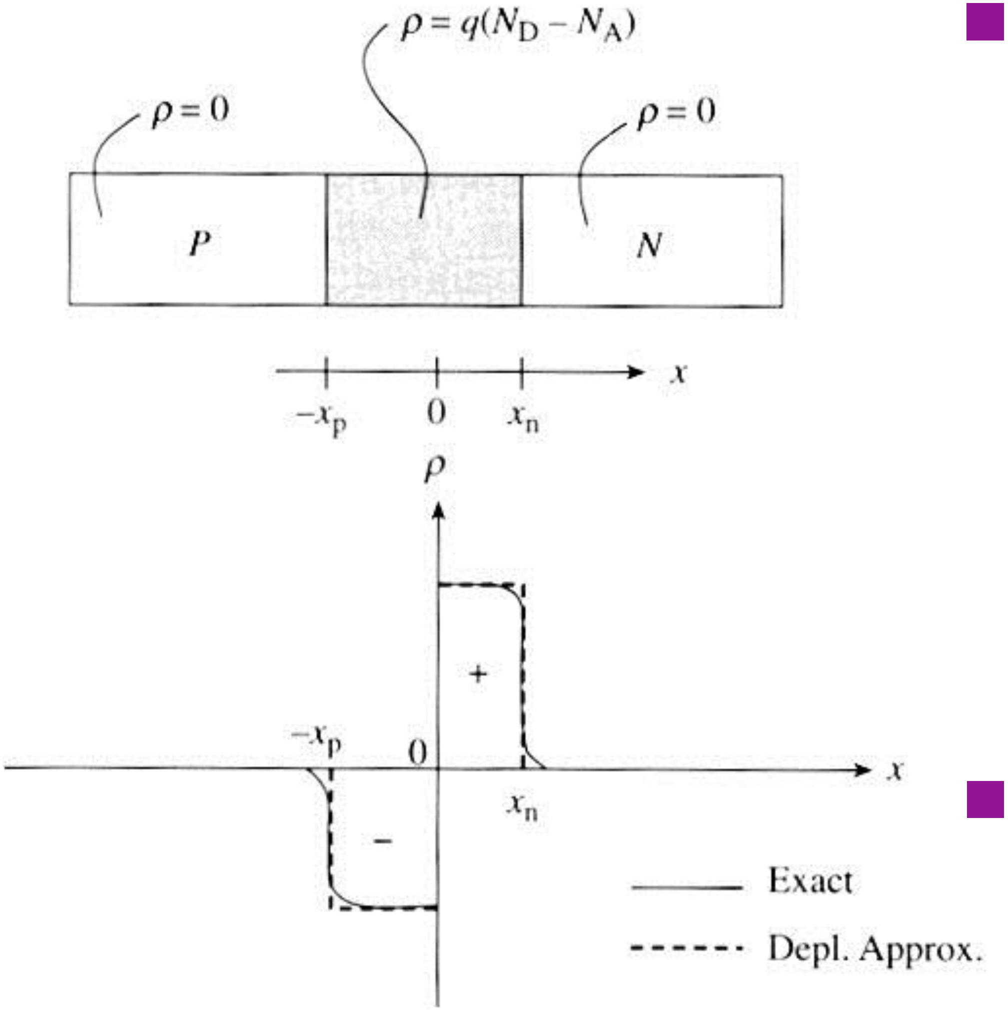
- For **non-degenerately doped** material,

$$(E_F - E_i)_{n\text{-side}} = kT \ln\left(\frac{n}{n_i}\right) = kT \ln\left(\frac{N_D}{n_i}\right)$$

$$(E_i - E_F)_{p\text{-side}} = kT \ln\left(\frac{p}{n_i}\right) = kT \ln\left(\frac{N_A}{n_i}\right)$$

$$qV_{bi} = kT \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

The Depletion Approximation



- On the ***p*-side**, $\rho = -qN_A$

$$\frac{d\mathcal{E}}{dx} = -\frac{qN_A}{\varepsilon_S}$$

$$\mathcal{E}(x) = -\frac{qN_A}{\varepsilon_S} x + c_1$$

$$\mathcal{E}(x) = -\frac{qN_A}{\varepsilon_S}(x + x_p)$$

with boundary $\mathcal{E}(-x_p) = 0$

- On the ***n*-side**, $\rho = qN_D$

$$\mathcal{E}(x) = -\frac{qN_D}{\varepsilon_S}(x_n - x)$$

with boundary $\mathcal{E}(x_n) = 0$

Step Junction with $V_A=0$

- Solution for ρ

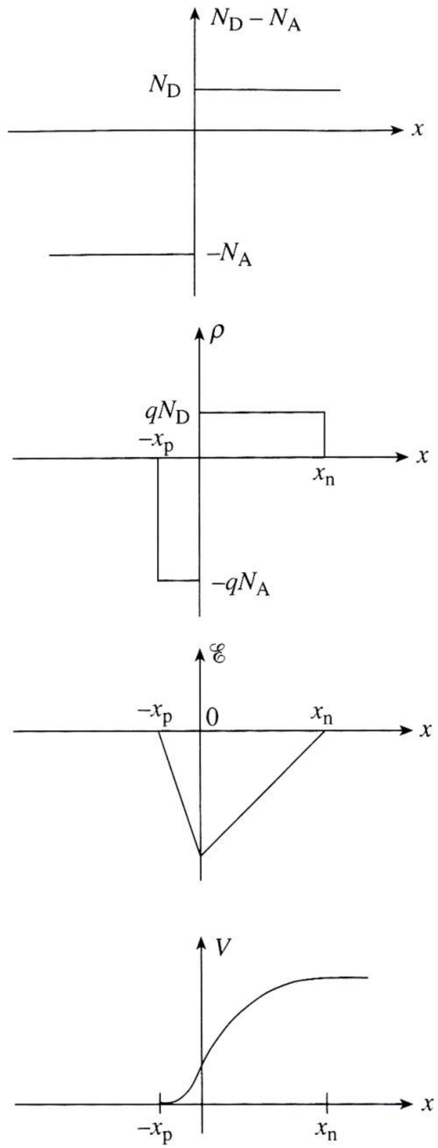
$$\rho = \begin{cases} -qN_A, & -x_p \le x \le 0 \\ qN_D, & 0 \le x \le x_n \\ 0, & \text{otherwise} \end{cases}$$

- Solution for $\mathcal{E}$

$$\mathcal{E}(x) = \begin{cases} -\dfrac{qN_A}{\varepsilon_S}(x_p + x), & -x_p \le x \le 0 \\ -\dfrac{qN_D}{\varepsilon_S}(x_n - x), & 0 \le x \le x_n \end{cases}$$

- Solution for V

$$V(x) = \begin{cases} \dfrac{qN_A}{2\varepsilon_S}(x_p + x)^2, & -x_p \le x \le 0 \\ V_{bi} - \dfrac{qN_D}{2\varepsilon_S}(x_n - x)^2, & 0 \le x \le x_n \end{cases}$$



Relation between $\rho(x)$, $\mathcal{E}(x)$, and $V(x)$

1. Find the profile of the built-in potential V_{bi}
2. Use the depletion approximation $\rightarrow \rho(x)$
   - With depletion-layer widths x_p, x_n unknown
3. Integrate $\rho(x)$ to find $\mathcal{E}(x)$
   - Boundary conditions $\mathcal{E}(-x_p) = 0$, $\mathcal{E}(x_n)=0$
4. Integrate $\mathcal{E}(x)$ to obtain $V(x)$
   - Boundary conditions $V(-x_p) = 0$, $V(x_n) = V_{bi}$
5. For $\mathcal{E}(x)$ to be continuous at $x = 0$, $N_A x_p = N_D x_n$
   - Solve for x_p, x_n

Bonus Question

- Answer the following questions in a report.
- Can the built-in potential of a pn junction in equilibrium be measured directly. Justify your answer.
- What happens if V_A is increased beyond the built in potential V_{bi} in the forward biased region.
  - Keep all bonus reports and submit them upon requesting that.
  - Only non-cheated reports will be rewarded while students who will submit copied reports will be punished.

Step Junction with $V_A=0$

- At $x = 0$, expressions for *p*-side and *n*-side for the solutions of $\mathcal{E}$ and V must be equal:

$$N_A x_p = N_D x_n$$

$$\frac{qN_A}{2\varepsilon_S}(x_p)^2 = V_{bi} - \frac{qN_D}{2\varepsilon_S}(x_n)^2$$

Depletion Layer Width

- Eliminating x_p,

$$x_n = \sqrt{\frac{2\varepsilon_S}{q}\frac{N_A}{N_D(N_A+N_D)}V_{bi}}$$

- Eliminating x_n,

$$x_p = \sqrt{\frac{2\varepsilon_S}{q}\frac{N_D}{N_A(N_A+N_D)}V_{bi}} = \frac{N_D}{N_A}x_n$$

- Summing

$$x_n + x_p = W \equiv \sqrt{\frac{2\varepsilon_S}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)V_{bi}}$$

Exact solution, try to derive

One-Sided Junctions

- If $N_A >> N_D$ as in a p^+n junction,

$$W \approx x_n \approx \sqrt{\frac{2\varepsilon_S}{q}\frac{V_{bi}}{N_D}} \ , \ x_p = x_n \frac{N_D}{N_A} \approx 0$$

- If $N_D >> N_A$ as in a n^+p junction,

$$W \approx x_p \approx \sqrt{\frac{2\varepsilon_S}{q}\frac{V_{bi}}{N_A}} \ , x_n = x_p \frac{N_A}{N_D} \approx 0$$

- Simplifying,

$$W \approx \sqrt{\frac{2\varepsilon_S}{q}\frac{V_{bi}}{N}}$$

where N denotes the lighter dopant density

Example: Depletion Layer Width

- A p^+n junction has $N_A = 10^{20}$ cm^{-3} and $N_D = 10^{17}$cm^{-3}, at 300 K.

a) What is V_{bi}?

$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right) = 25.86 \text{ mV} \times \ln\left(\frac{10^{17} \cdot 10^{20}}{(10^{10})^2}\right) = \underline{\underline{1.012 \text{ V}}}$$

b) What is W?

$$W \approx \sqrt{\frac{2\varepsilon_S V_{bi}}{qN_D}} = \left(\frac{2 \times 11.9 \times 8.854 \times 10^{-14} \times 1.012}{1.602 \times 10^{-19} \times 10^{17}}\right)^{1/2} = \underline{\underline{0.115\ \mu\text{m}}}$$

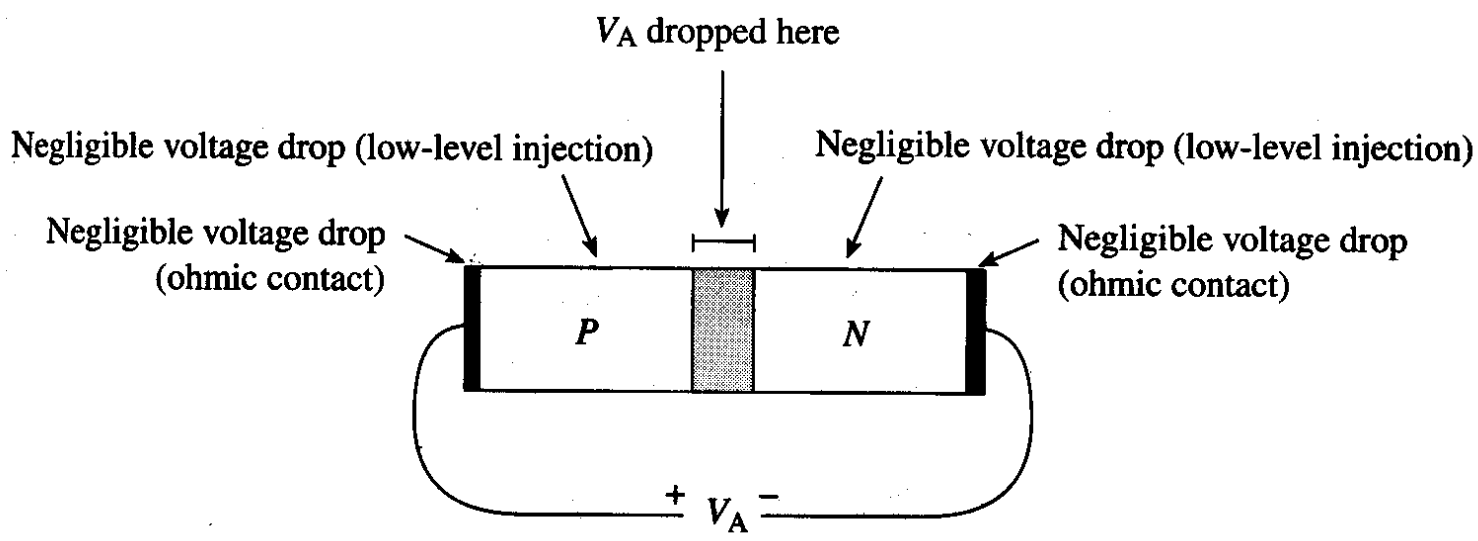
c) What is x_n?

$$x_n \approx W \approx \underline{\underline{0.115\ \mu\text{m}}}$$

d) What is x_p?

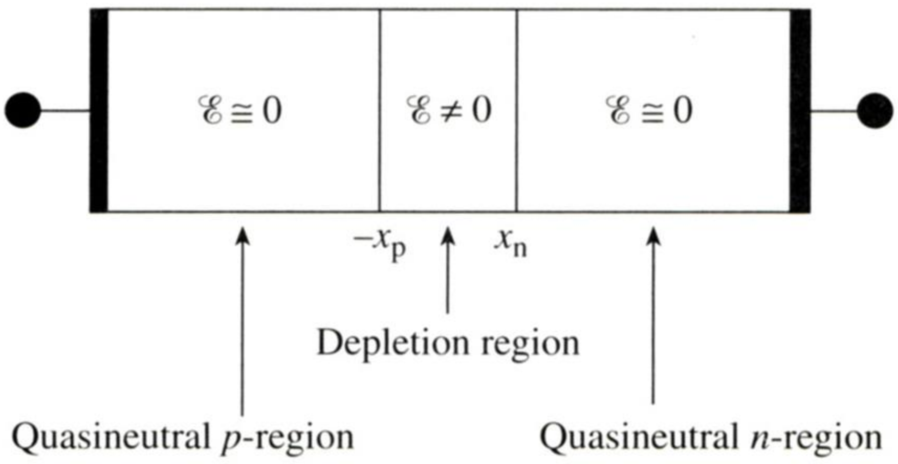
$$x_p = x_n \frac{N_D}{N_A} = 0.115 \mu\text{m} \times 10^{-3} = \underline{\underline{1.15\ \text{Å}}}$$

Step Junction with $V_A \neq 0$



- **To ensure low-level injection conditions, reasonable current levels must be maintained → V_A should be small**

Step Junction with $V_A \neq 0$



- In the quasineutral, regions extending from the contacts to the edges of the depletion region, minority carrier diffusion equations can be applied since $\mathcal{E} \approx 0$.
- In the depletion region, the continuity equations are applied.

Step Junction with $V_A \neq 0$

- Built-in potential V_{bi} (non-degenerate doping):

$$V_{bi} = \frac{kT}{q}\ln\left(\frac{N_A}{n_i}\right) + \frac{kT}{q}\ln\left(\frac{N_D}{n_i}\right) = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

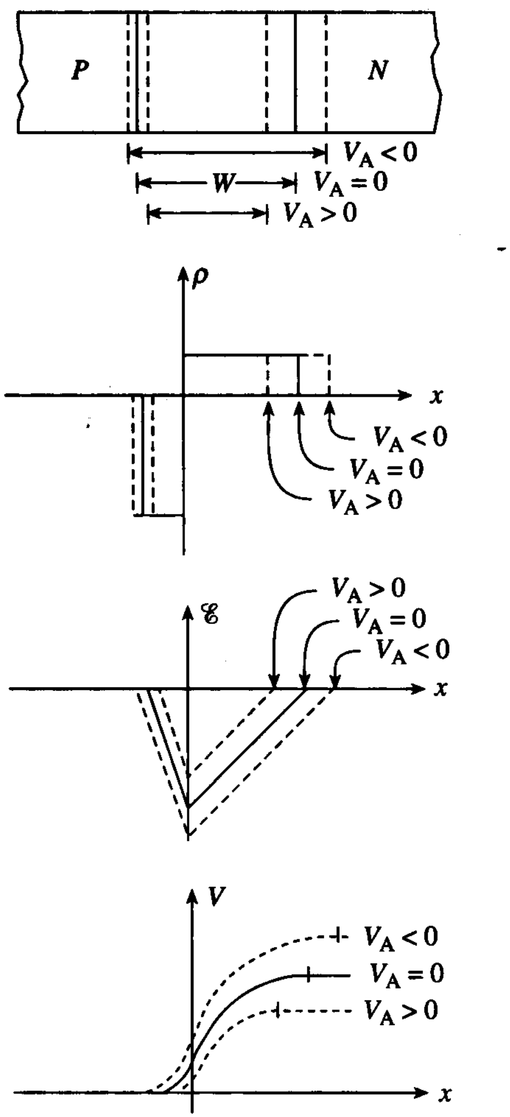
- Depletion width W :

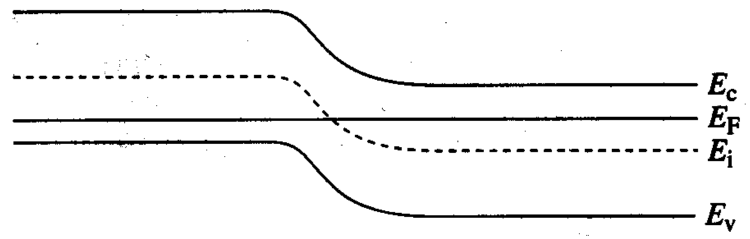
$$W \equiv x_p + x_n = \sqrt{\frac{2\varepsilon_S}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_{bi} - V_A)}$$

$$x_p = \sqrt{\frac{2\varepsilon_S}{q}\frac{N_D}{N_A(N_A + N_D)}(V_{bi} - V_A)}, \quad x_n = \sqrt{\frac{2\varepsilon_S}{q}\frac{N_A}{N_D(N_A + N_D)}(V_{bi} - V_A)}$$

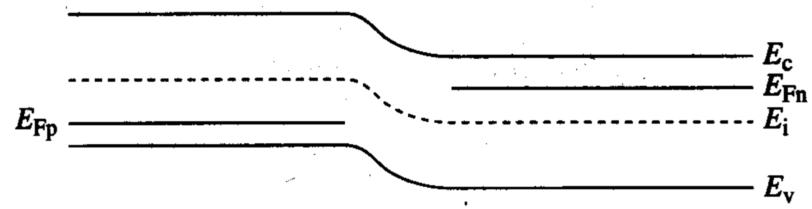
$$x_p = \frac{N_D}{N_A + N_D}W, \quad x_n = \frac{N_A}{N_A + N_D}W$$

Effect of Bias on Electrostatics

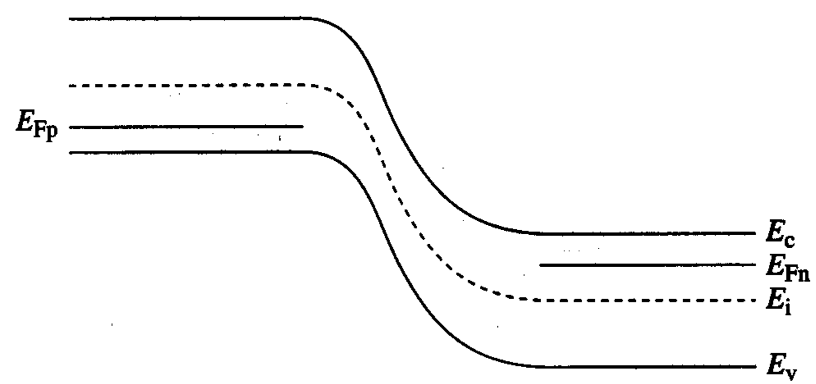




(a) Equilibrium ($V_A = 0$)



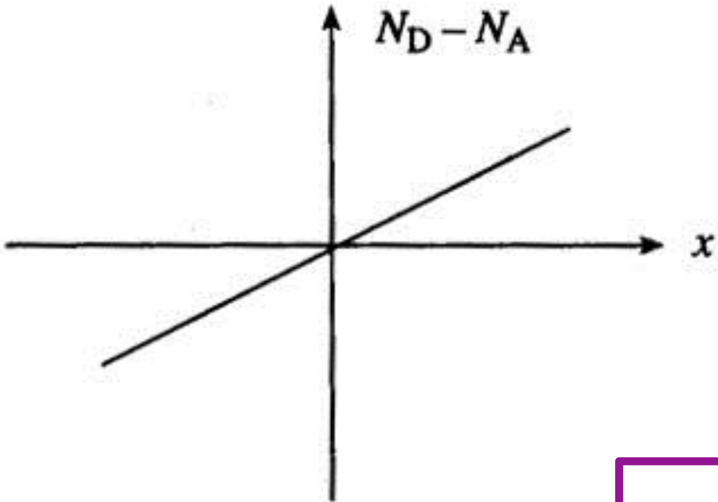
(b) Forward bias ($V_A > 0$)

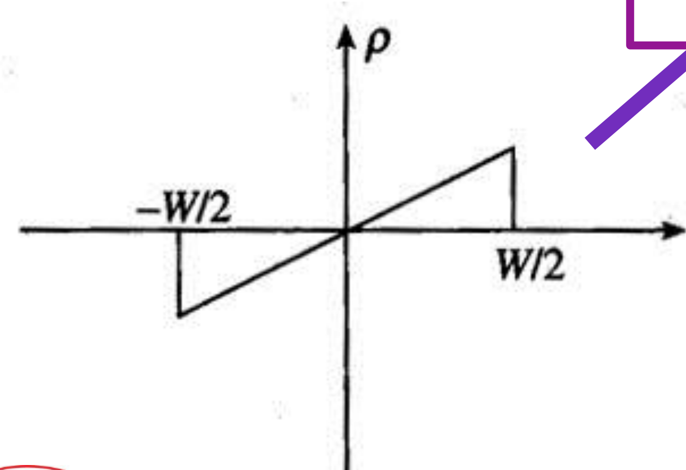


(c) Reverse bias ($V_A < 0$)

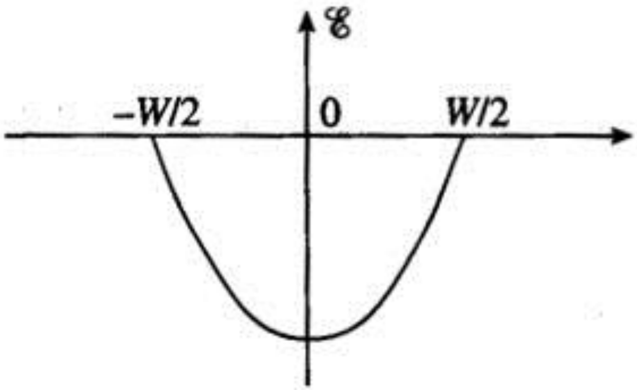
- **If voltage drop ↓, then depletion width ↓**
- **If voltage drop ↑, then depletion width ↑**

Linearly-Graded Junction





$$\mathcal{E} = \frac{1}{\varepsilon_S} \int \rho dx$$



$$V = -\int \mathcal{E} dx$$