Semiconductor Device Physics

Lecture 5

http://zitompul.wordpress.com

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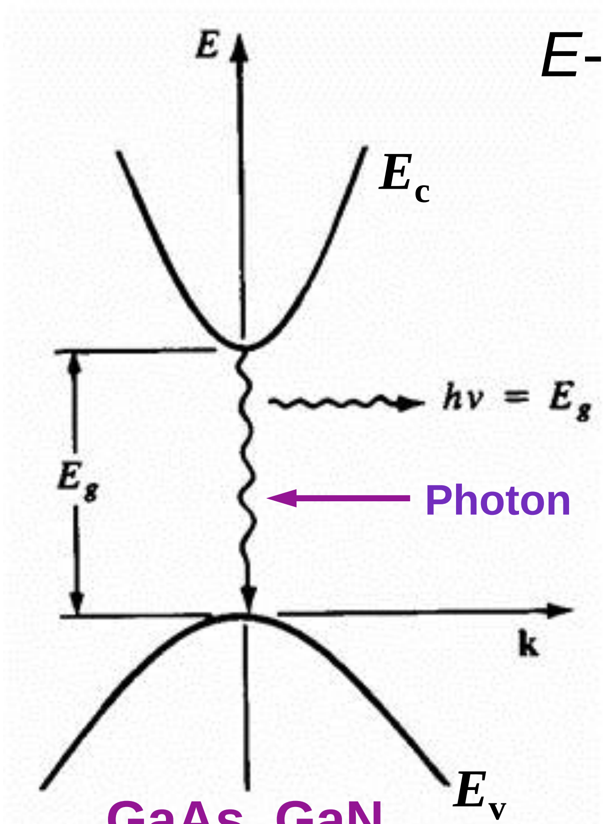
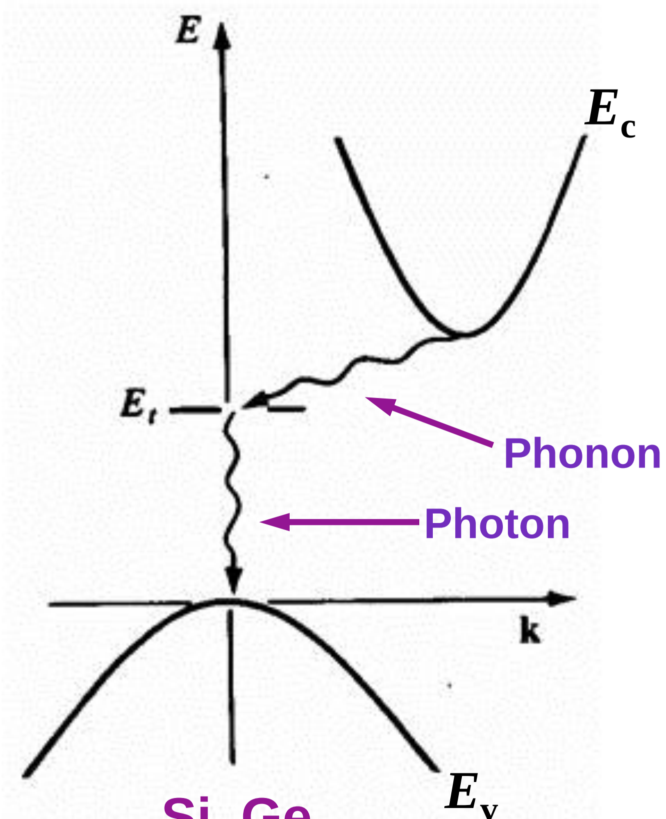
Direct and Indirect Semiconductors

E-**k** Diagrams



GaAs, GaN
(direct semiconductors)



Si, Ge
(indirect semiconductors)

- **Little change in momentum is required for recombination**
- **Momentum is conserved by photon (light) emission**

- **Large change in momentum is required for recombination**
- **Momentum is conserved by mainly phonon (vibration) emission + photon emission**

Excess Carrier Concentrations

Values under arbitrary conditions

Deviation from equilibrium values

Equilibrium values

$$\Delta n \equiv n - n_0$$

$$\Delta p \equiv p - p_0$$

- Positive deviation $\Delta n, \Delta p > 0$ corresponds to a carrier excess, while negative deviation $\Delta n, \Delta p < 0$ corresponds to a carrier deficit.
- Charge neutrality condition:

$$\Delta n = \Delta p$$

"Low-Level Injection"

- Often, the disturbance from equilibrium is small, such that the majority carrier concentration is not affected significantly.
- However, the minority carrier concentration can be significantly affected.

  - For an *n*-type material $\Delta p << n_0, \quad n \approx n_0$ $\quad \boxed{\Delta p >> p_0}$
  - For a *p*-type material $\Delta n << p_0, \quad p \approx p_0$ $\quad \boxed{\Delta n >> n_0}$

- This condition is called "low-level injection condition".
- The *workhorse* of the diffusion in low-level injection condition is the minority carrier (which number increases significantly) while the majority carrier is practically undisturbed.

Photoconductor

- **Photoconductivity** is an optical and electrical phenomenon in which a material becomes more electrically conductive due to the absorption of electro-magnetic radiation such as visible light, ultraviolet light, infrared light, or gamma radiation.
- When light is absorbed by a material like semiconductor, the number of free electrons and holes changes and raises the electrical conductivity of the semiconductor.
- To cause excitation, the light that strikes the semiconductor must have enough energy to raise electrons across the band gap.

Example: Photoconductor

- Consider a sample of Si at 300 K doped with 10^{16} cm^{-3} Boron, with recombination lifetime 1 μs. It is exposed continuously to light, such that electron-hole pairs are generated throughout the sample at the rate of 10^{20} per cm^3 per second, *i.e.* the **generation rate G_L** = 10^{20}/cm^3/s.

a) What are p_0 and n_0?

$$p_0 = 10^{16}\,\mathrm{cm}^{-3}$$

$$n_0 = \frac{n_i^2}{p_0} = \frac{\left(10^{10}\right)^2}{10^{16}} = 10^4\,\mathrm{cm}^{-3}$$

b) What are Δn and Δp?

$$\Delta p = \Delta n = G_L \cdot \tau = 10^{20} \times 10^{-6} = 10^{14}\ \mathrm{cm}^{-3}$$

- **Hint: In steady-state (equilibrium), generation rate equals recombination rate**

Example: Photoconductor

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c) What are *p* and *n*?

$$p = p_0 + \Delta p = 10^{16} + 10^{14} \approx 10^{16}\,\text{cm}^{-3}$$

$$n = n_0 + \Delta n = 10^{4} + 10^{14} \approx 10^{14}\,\text{cm}^{-3}$$

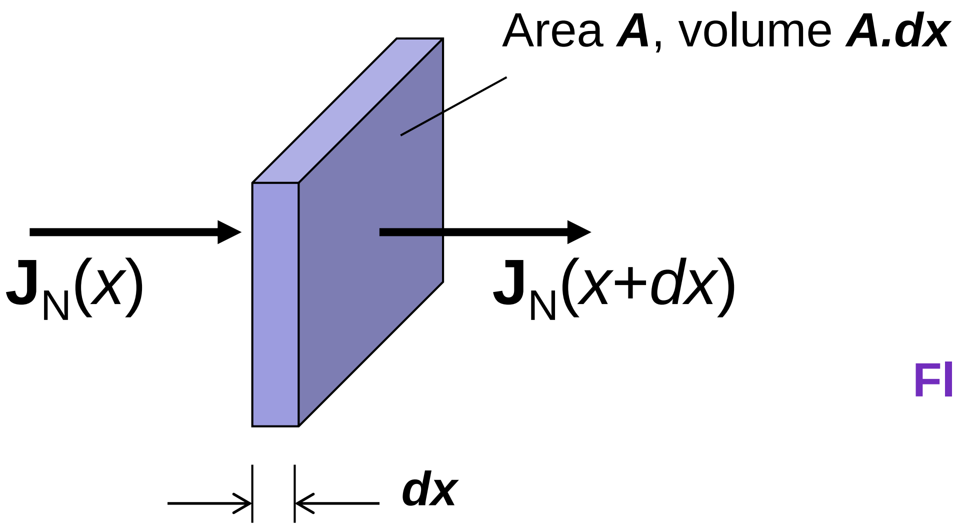
d) What are *np* product?

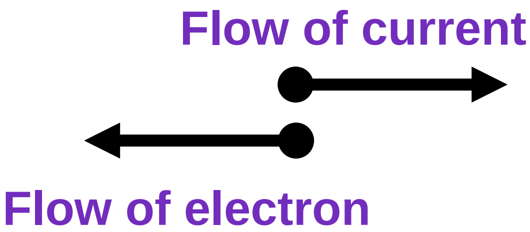
$$np \approx 10^{16} \cdot 10^{14} = 10^{30}\,\text{cm}^{-3} >> n_i^2$$

- **Note: The *np* product can be very different from n_i^2 in case of perturbed / agitated semiconductor**

Continuity Equation

- Consider carrier-flux into/out of an infinitesimal volume:





$$A dx \cdot \left(\frac{\partial n}{\partial t} \right) = \frac{1}{q} \left[\mathbf{J}_N(x+dx) - \mathbf{J}_N(x) \right] A$$

Continuity Equation

$$\mathbf{J}_N(x+dx) = \mathbf{J}_N(x) + \frac{\partial \mathbf{J}_N(x)}{\partial x}dx$$

- **Taylor's Series Expansion**

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial \mathbf{J}_N(x)}{\partial x}$$

The Continuity Equations

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial \mathbf{J}_N(x)}{\partial x} + \left.\frac{\partial n}{\partial t}\right|_{\substack{\text{thermal}\\ \text{R-G}}} + \left.\frac{\partial n}{\partial t}\right|_{\substack{\text{other}\\ \text{processes}}}$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial \mathbf{J}_P(x)}{\partial x} + \left.\frac{\partial p}{\partial t}\right|_{\substack{\text{thermal}\\ \text{R-G}}} + \left.\frac{\partial p}{\partial t}\right|_{\substack{\text{other}\\ \text{processes}}}$$

Minority Carrier Diffusion Equation

- The minority carrier diffusion equations are derived from the general continuity equations, and are applicable only for minority carriers.
- Simplifying assumptions:
  - The electric field is small, such that:

$$\mathbf{J}_{\mathrm{N}} = q\mu_{\mathrm{n}} n\mathcal{E} + qD_{\mathrm{N}}\frac{\partial n}{\partial x} \approx qD_{\mathrm{N}}\frac{\partial n}{\partial x}$$

• **For *p*-type material**

$$\mathbf{J}_{\mathrm{P}} = q\mu_{\mathrm{p}} p\mathcal{E} - qD_{\mathrm{P}}\frac{\partial p}{\partial x} \approx -qD_{\mathrm{P}}\frac{\partial p}{\partial x}$$

• **For *n*-type material**

  - Equilibrium minority carrier concentration n_0 and p_0 are independent of x (uniform doping).
  - Low-level injection conditions prevail.

Minority Carrier Diffusion Equation

- Starting with the continuity equation for electrons:

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial \mathbf{J}_{\mathrm{N}}(x)}{\partial x} - \frac{\Delta n}{\tau_{\mathrm{n}}} + G_{\mathrm{L}}$$

$$\left.\frac{\partial n}{\partial t}\right|_{\substack{\text{thermal}\\ \text{R-G}}} = -\frac{\Delta n}{\tau_{\mathrm{n}}}$$

$$\frac{\partial (n_0 + \Delta n)}{\partial t} = \frac{1}{q}\frac{\partial}{\partial x}\left[q D_{\mathrm{N}} \frac{\partial (n_0 + \Delta n)}{\partial x} \right] - \frac{\Delta n}{\tau_{\mathrm{n}}} + G_{\mathrm{L}}$$

$$\left.\frac{\partial n}{\partial t}\right|_{\substack{\text{other}\\ \text{processes}}} = G_{\mathrm{L}}$$

- The **minority carrier lifetime** τ is the average time for excess minority carriers to “survive” in a sea of majority carriers.

- Therefore

$$\frac{\partial \Delta n}{\partial t} = D_{\mathrm{N}} \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_{\mathrm{n}}} + G_{\mathrm{L}}$$

- Similarly

$$\frac{\partial \Delta p}{\partial t} = D_{\mathrm{P}} \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau_{\mathrm{p}}} + G_{\mathrm{L}}$$

Carrier Concentration Notation

- The subscript "*n*" or "*p*" is now used to explicitly denote *n*-type or *p*-type material.
  - p_n is the hole concentration in *n*-type material
  - n_p is the electron concentration in *p*-type material

- Thus, the minority carrier diffusion equations are:

$$\frac{\partial \Delta n_\mathrm{p}}{\partial t} = D_\mathrm{N} \frac{\partial^2 \Delta n_\mathrm{p}}{\partial x^2} - \frac{\Delta n_\mathrm{p}}{\tau_\mathrm{n}} + G_\mathrm{L}$$

$$\frac{\partial \Delta p_\mathrm{n}}{\partial t} = D_\mathrm{P} \frac{\partial^2 \Delta p_\mathrm{n}}{\partial x^2} - \frac{\Delta p_\mathrm{n}}{\tau_\mathrm{p}} + G_\mathrm{L}$$

- **Partial Differential Equation (PDE)!**
- **The so called "Heat Conduction Equation"**

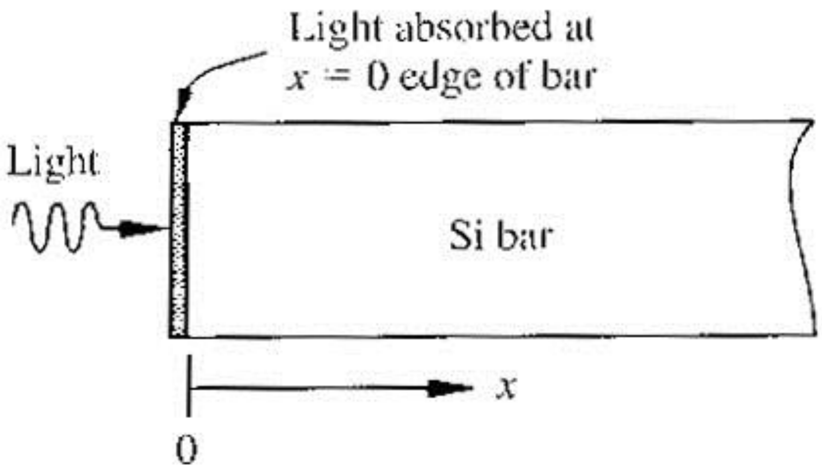
Simplifications (Special Cases)

- Steady state: $$\frac{\partial \Delta n_{\mathrm{p}}}{\partial t} = 0, \quad \frac{\partial \Delta p_{\mathrm{n}}}{\partial t} = 0$$
- No diffusion current: $$D_{\mathrm{N}} \frac{\partial^2 \Delta n_{\mathrm{p}}}{\partial x^2} = 0, \quad D_{\mathrm{P}} \frac{\partial^2 \Delta p_{\mathrm{n}}}{\partial x^2} = 0$$
- No thermal R–G: $$\frac{\Delta n_{\mathrm{p}}}{\tau_{\mathrm{n}}} = 0, \quad \frac{\Delta p_{\mathrm{n}}}{\tau_{\mathrm{p}}} = 0$$
- No other processes: $$G_{\mathrm{L}} = 0$$

  - **Solutions for these common special-case diffusion equation are provided in the textbook**

Minority Carrier Diffusion Length

- Consider the special case:
  - Constant minority-carrier (hole) injection at $x = 0$
  - Steady state, no light absorption for $x > 0$



$$0 = D_P \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p}$$

$$\Delta p_n(0) = \Delta p_{n0}$$

$$G_L = 0 \text{ for } x > 0$$

$$\Downarrow$$

$$\frac{\partial^2 \Delta p_n}{\partial x^2} = \frac{\Delta p_n}{D_P \tau_p} = \frac{\Delta p_n}{L_P{}^2}$$

- The **hole diffusion length** L_P is defined to be: $L_P = \sqrt{D_P \tau_p}$

Similarly, $L_N = \sqrt{D_N \tau_n}$

Minority Carrier Diffusion Length

- The general solution to the equation $\dfrac{\partial^2 \Delta p_n}{\partial x^2} = \dfrac{\Delta p_n}{{L_P}^2}$ is:

$$\Delta p_n(x) = Ae^{-x/L_P} + Be^{x/L_P}$$

- *A* and *B* are constants determined by boundary conditions:

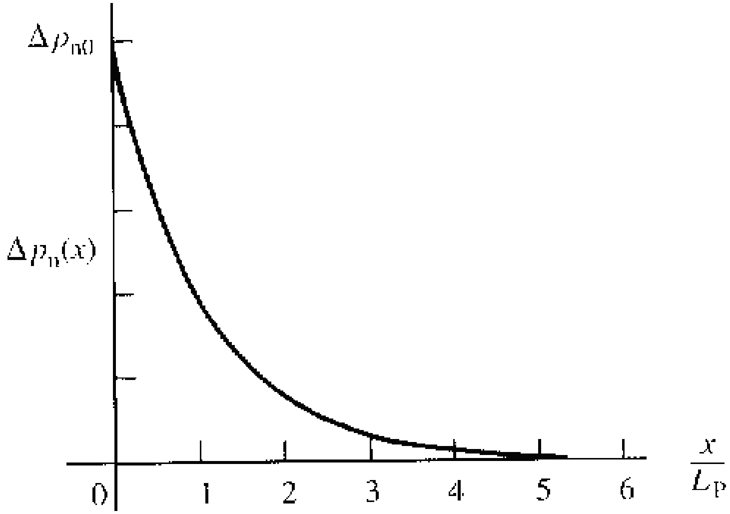
$$\Delta p_n(\infty) = 0 \quad \Rightarrow \quad B = 0$$

$$\Delta p_n(0) = \Delta p_{n0} \quad \Rightarrow \quad A = \Delta p_{n0}$$

- Therefore, the solution is:

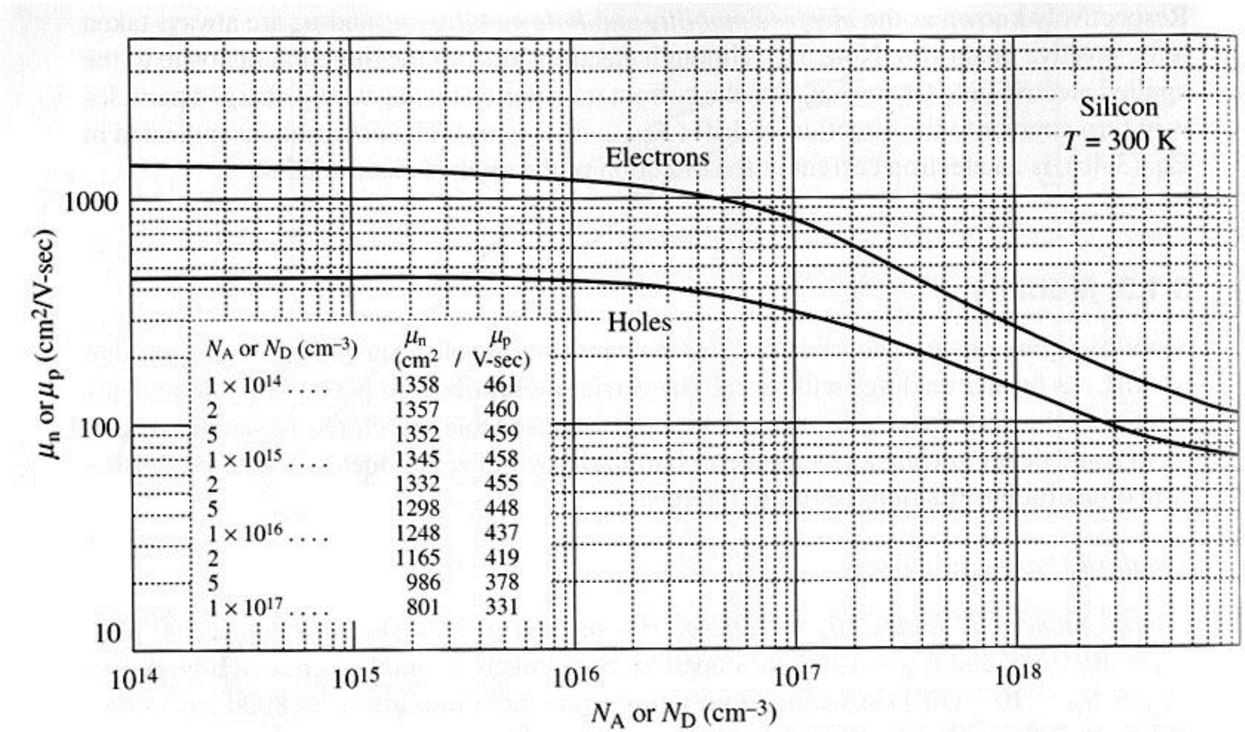
$$\Delta p_n(x) = \Delta p_{n0} e^{-x/L_P}$$

  - **Physically, L_P and L_N represent the average distance that a minority carrier can diffuse before it recombines with a majority carrier.**



Example: Minority Carrier Diffusion Length

- Given $N_D = 10^{16}$ cm^{-3}, $\tau_p = 10^{-6}$ s. Calculate L_P.



- From the plot,

$$\mu_p = 437 \ \text{cm}^2/\text{V}\cdot\text{s}$$

$$D_P = \frac{kT}{q}\mu_p$$

$$= 25.86 \ \text{mV} \cdot 437 \ \text{cm}^2/\text{V}\cdot\text{s}$$

$$= 11.3 \text{cm}^2/\text{s}$$

$$L_P = \sqrt{D_P \tau_p}$$

$$= \sqrt{11.3 \text{cm}^2/\text{s} \cdot 10^{-6} \ \text{s}}$$

$$= 3.361 \times 10^{-3} \ \text{cm}$$

$$= 33.61 \mu\text{m}$$

Quasi-Fermi Levels

- Whenever $\Delta n = \Delta p \neq 0$ then $np \neq n_i^2$ and we are at non-equilibrium conditions.
- In this situation, now we would like to preserve and use the relations:

$$n = n_i e^{(E_F - E_i)/kT}, \quad p = n_i e^{(E_i - E_F)/kT}$$

- On the other hand, both equations imply $np = n_i^2$, which does not apply anymore.
- The solution is to introduce to quasi-Fermi levels F_N and F_P such that:

$$n = n_i e^{(F_N - E_i)/kT} \qquad p = n_i e^{(E_i - F_P)/kT}$$

$$F_N = E_i + kT \ln\left(\frac{n}{n_i}\right) \qquad F_P = E_i - kT \ln\left(\frac{p}{n_i}\right)$$

- **The quasi-Fermi levels is useful to describe the carrier concentrations under non-equilibrium conditions**

Example: Quasi-Fermi Levels

- Consider a Si sample at 300 K with $N_D = 10^{17}\ \text{cm}^{-3}$ and $\Delta n = \Delta p = 10^{14}\ \text{cm}^{-3}$.

a) What are p and n?

- **The sample is an *n*-type**

$$n_0 = N_D = 10^{17}\ \text{cm}^{-3},\ \ p_0 = \frac{n_i^2}{n_0} = 10^3\ \text{cm}^{-3}$$

$$n = n_0 + \Delta n = 10^{17} + 10^{14} \approx 10^{17}\ \text{cm}^{-3}$$

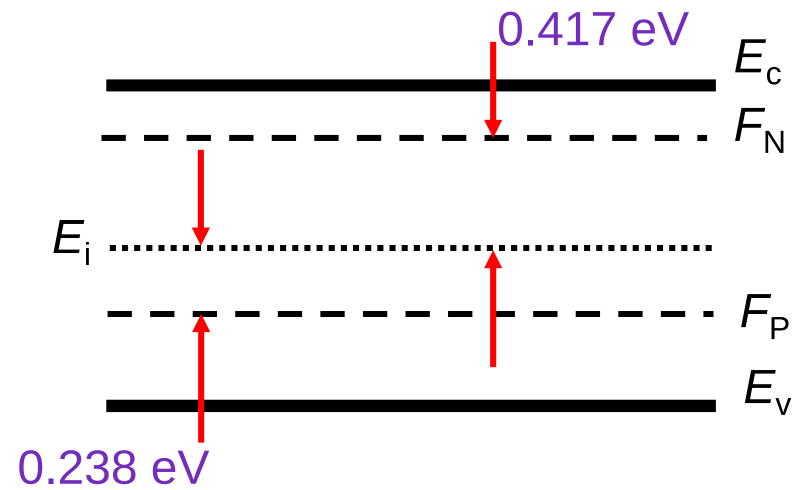
$$p = p_0 + \Delta p = 10^3 + 10^{14} \approx 10^{14}\ \text{cm}^{-3}$$

b) What is the np product?

$$np \approx 10^{17} \cdot 10^{14} = 10^{31}\ \text{cm}^{-3}$$

Example: Quasi-Fermi Levels

- Consider a Si sample at 300 K with $N_D = 10^{17}$ cm^{-3} and $\Delta n = \Delta p = 10^{14}$ cm^{-3}.



c) Find F_N and F_P?

$$F_N = E_i + kT \ln(n/n_i)$$

$$F_N - E_i = 8.62\times10^{-5}\cdot300\cdot\ln\left(10^{17}/10^{10}\right)$$

$$= \underline{\underline{0.417\text{ eV}}}$$

$$F_P = E_i - kT \ln(p/n_i)$$

$$E_i - F_P = 8.62\times10^{-5}\cdot300\cdot\ln\left(10^{14}/10^{10}\right)$$

$$= \underline{\underline{0.238\text{ eV}}}$$

$$np = n_i e^{(F_N - E_i)/kT}\cdot n_i e^{(E_i - F_P)/kT}$$

$$= 10^{10} e^{\frac{0.417}{0.02586}}\cdot 10^{10} e^{\frac{0.238}{0.02586}}$$

$$= 1.000257\times10^{31}$$

$$\approx 10^{31}\text{cm}^{-3}$$