Semiconductor Device Physics

Lecture 2

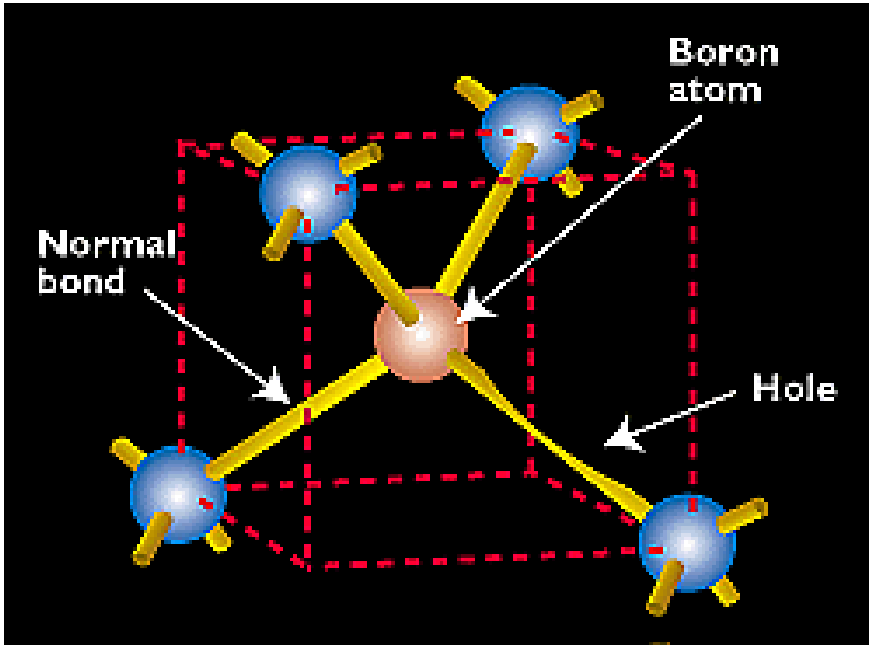
http://zitompul.wordpress.com

2013

Manipulation of Carrier Numbers – Doping

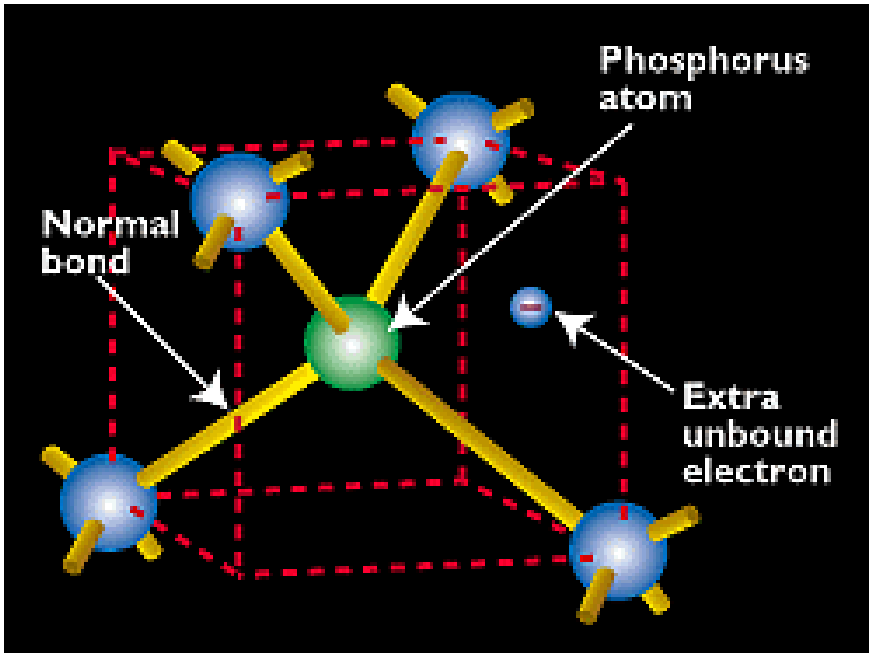
- By substituting a Si atom with a special impurity atom (elements from **Group III** or **Group V**), a hole or conduction electron can be created.

Acceptors: B, Ga, In, Al



Boron, Gallium Indium, Aluminum

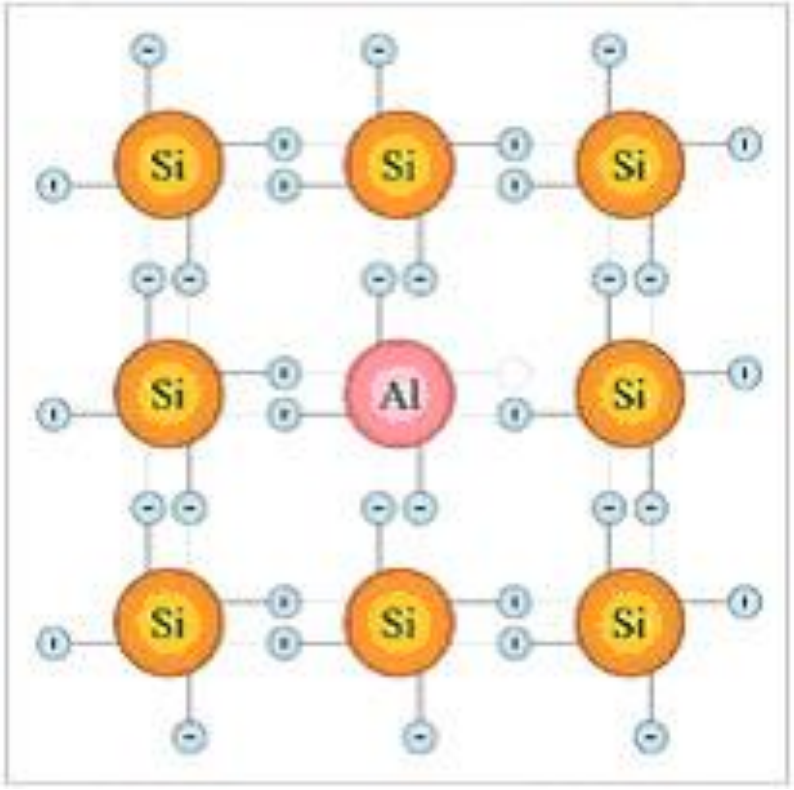
Donors: P, As, Sb



Phosphorus, Arsenic, Antimony

Doping Silicon with Acceptors

- **Example:** Aluminum atom is doped into the Si crystal.

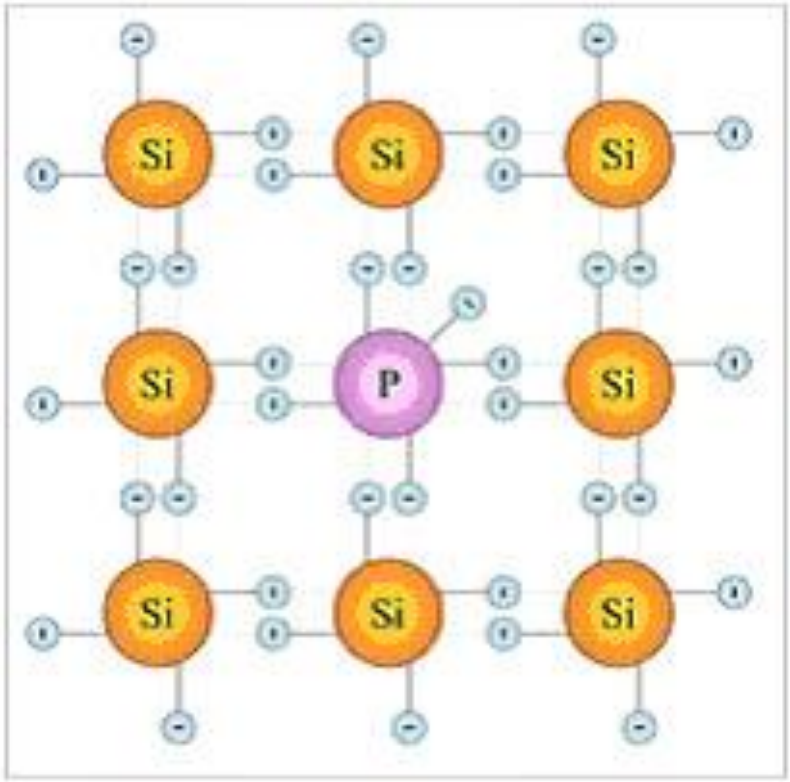


Al^- is immobile

- The **Al** atom accepts an electron from a neighboring **Si** atom, resulting in a missing bonding electron, or “hole”.
- The hole is free to roam around the **Si** lattice, and as a moving positive charge, the hole carries current.

Doping Silicon with Donors

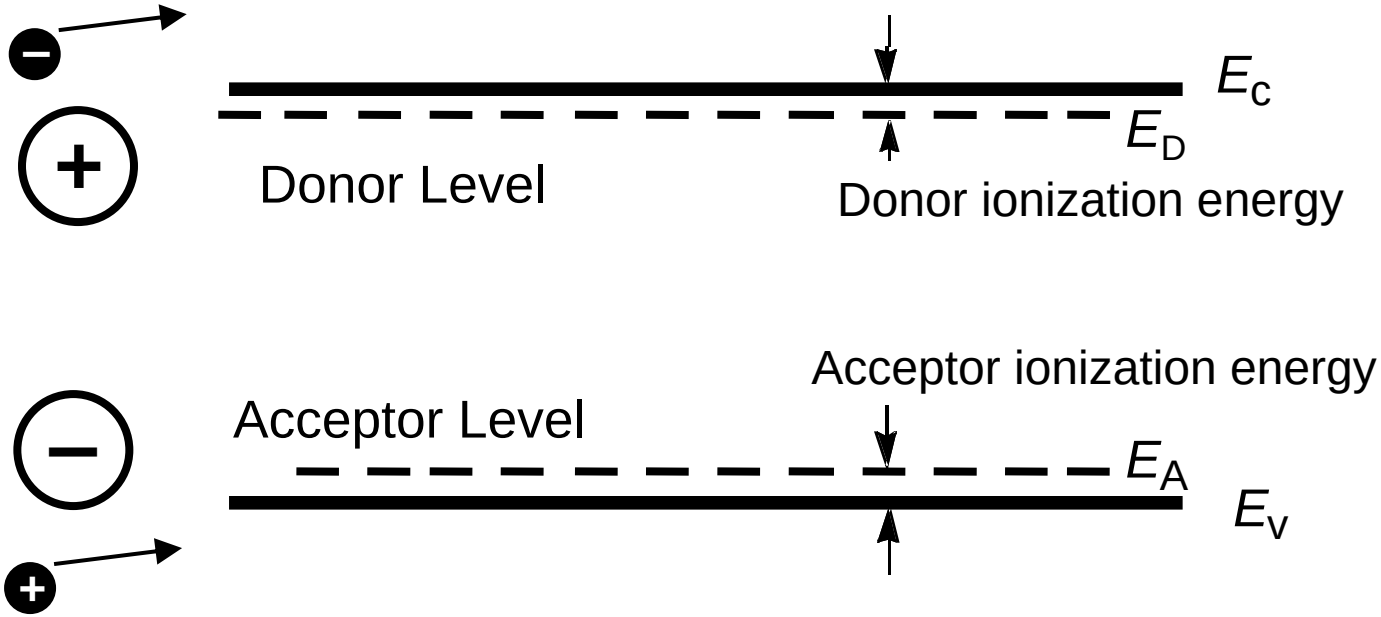
- **Example:** Phosphorus atom is doped into the Si crystal.



P^+ is immobile

- The loosely bounded fifth valence electron of the **P** atom can "break free" easily and becomes a mobile conducting electron.
- This electron contributes in current conduction.

Donor / Acceptor Levels (Band Model)

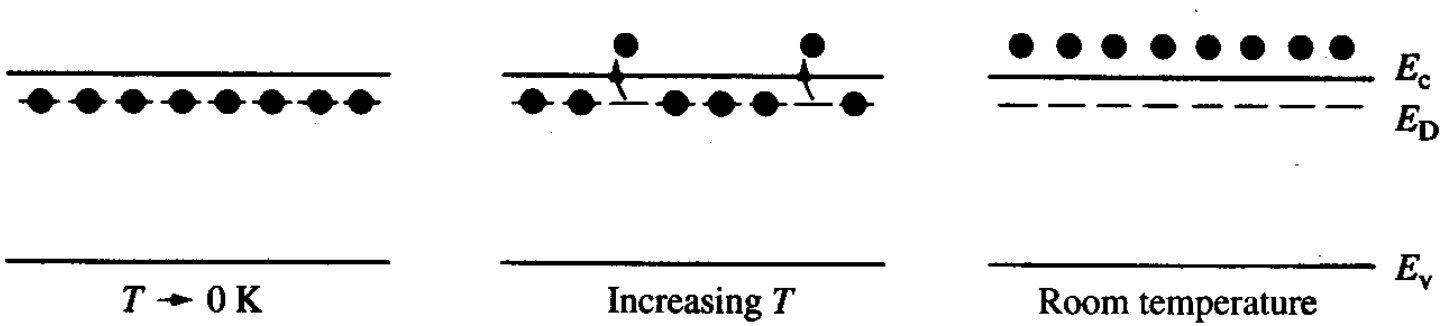


Ionization energy of selected donors and acceptors in Silicon (E_G = 1.12 eV)

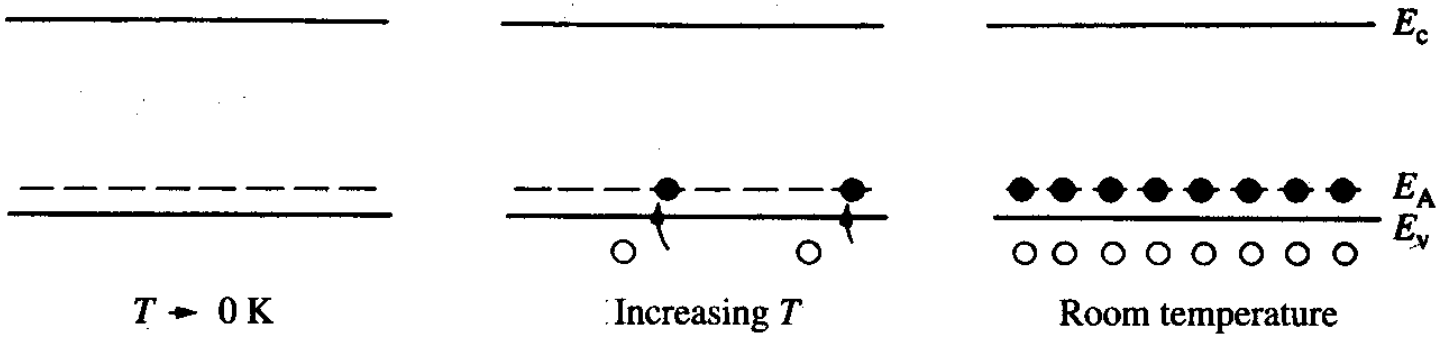
	Donors			Acceptors		
Ionization energy of dopant	Sb	P	As	B	Al	In
$E_C - E_D$ *or* $E_A - E_V$ (meV)	39	45	54	45	67	160

Dopant Ionization (Band Model)

Donor atoms



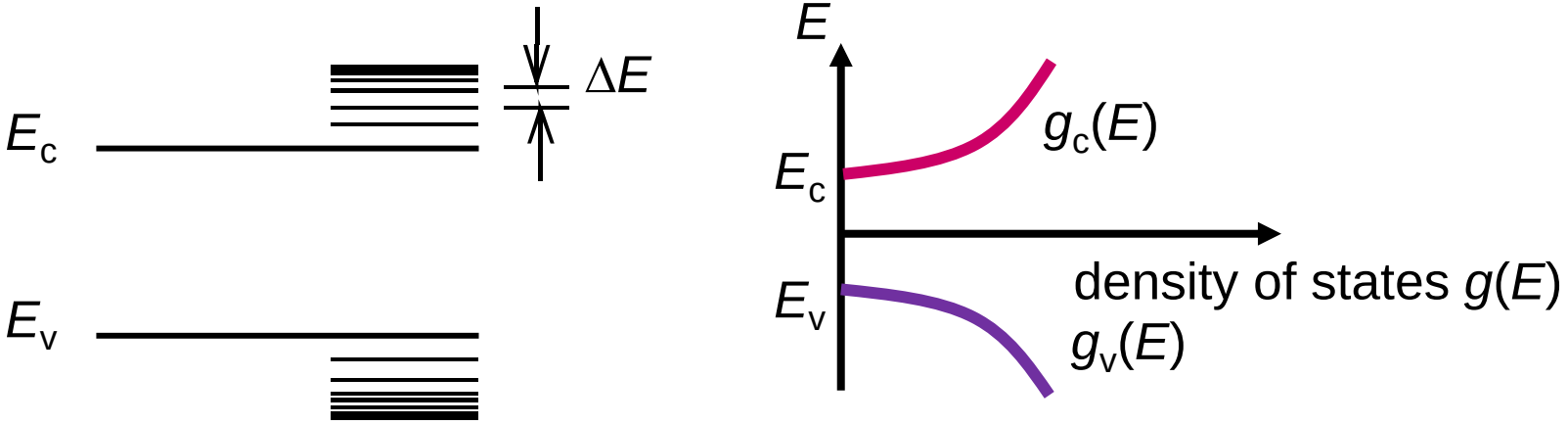
Acceptor atoms



Carrier-Related Terminology

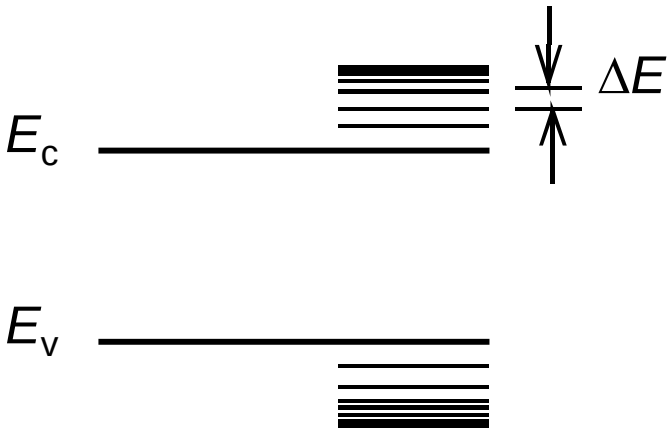
- **Donor**: impurity atom that increases *n* (conducting electron).
  Acceptor: impurity atom that increases *p* (hole).
- ***n*-type material**: contains more electrons than holes.
  ***p*-type material**: contains more holes than electrons.
- **Majority carrier**: the most abundant carrier.
  Minority carrier: the least abundant carrier.
- **Intrinsic semiconductor**: undoped semiconductor $n = p = n_i$.
  Extrinsic semiconductor: doped semiconductor.

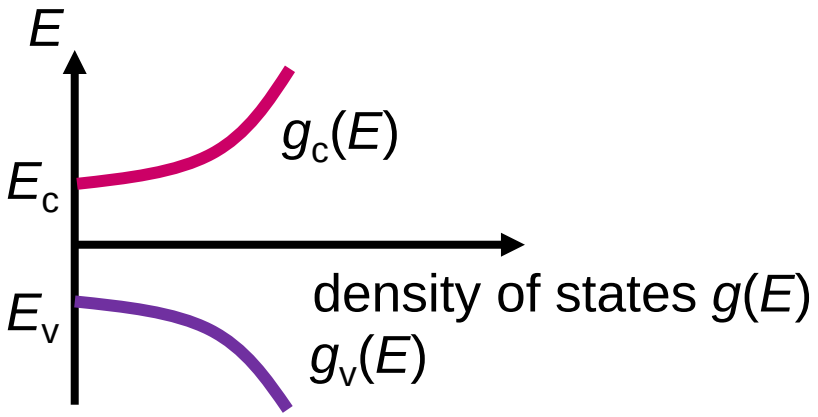
Density of States



- $g(E)$ is the number of states per cm^3 per eV.
- $g(E)dE$ is the number of states per cm^3 in the energy range between E and $E+dE$).

Density of States





- Near the band edges:

m_n^* : effective mass of electron

For Silicon at 300 K,

$m_n^* = 1.18 m_o$

$m_p^* = 0.81 m_o$

$m_o = 9.1 \times 10^{-31}$ kg

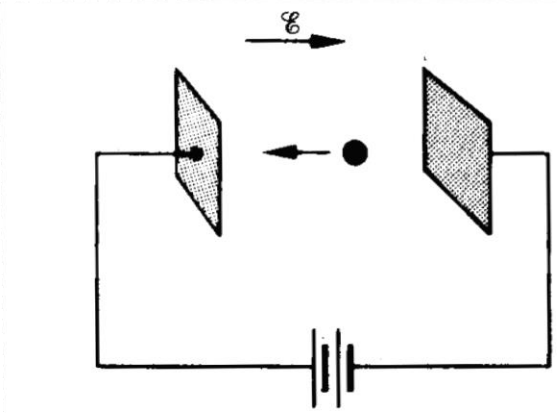
$$g_c(E) = \frac{m_n^* \sqrt{2m_n^*(E - E_c)}}{\pi^2 \hbar^3} \qquad E \geq E_c$$

$$g_v(E) = \frac{m_p^* \sqrt{2m_p^*(E_v - E)}}{\pi^2 \hbar^3} \qquad E \leq E_v$$

m_o: electron rest mass

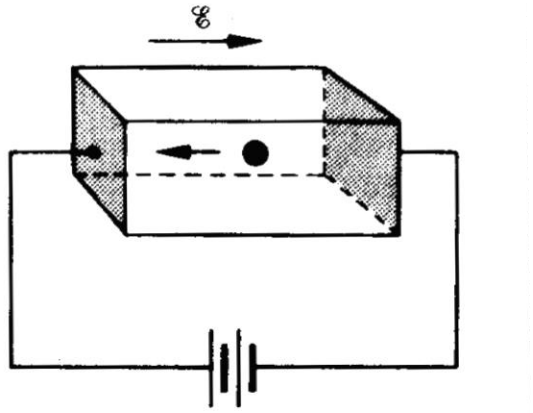
Electrons as Moving Particles

- **In free space**



$$\vec{F} = -q\vec{E} = m_0\vec{a}$$

- **In semiconductor**



$$\vec{F} = -q\vec{E} = m_n^*\vec{a}$$

m_o: electron rest mass

m_n*: effective mass of electron

Effective masses at 300 K

	Si	Ge	GaAs
m_n^*/m_0	1.18	0.55	0.066
m_p^*/m_0	0.81	0.36	0.52

Fermi Function

- The probability that an available state at an energy *E* will be occupied by an electron is specified by the following probability distribution function:

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

k **: Boltzmann constant**
T **: temperature in Kelvin**

- E_F is called the **Fermi energy** or the **Fermi level**.

$$\text{If } E >> E_F, \quad f(E) \to 0$$
$$\text{If } E << E_F, \quad f(E) \to 1$$
$$\text{If } E = E_F, \quad f(E) = 1/2$$

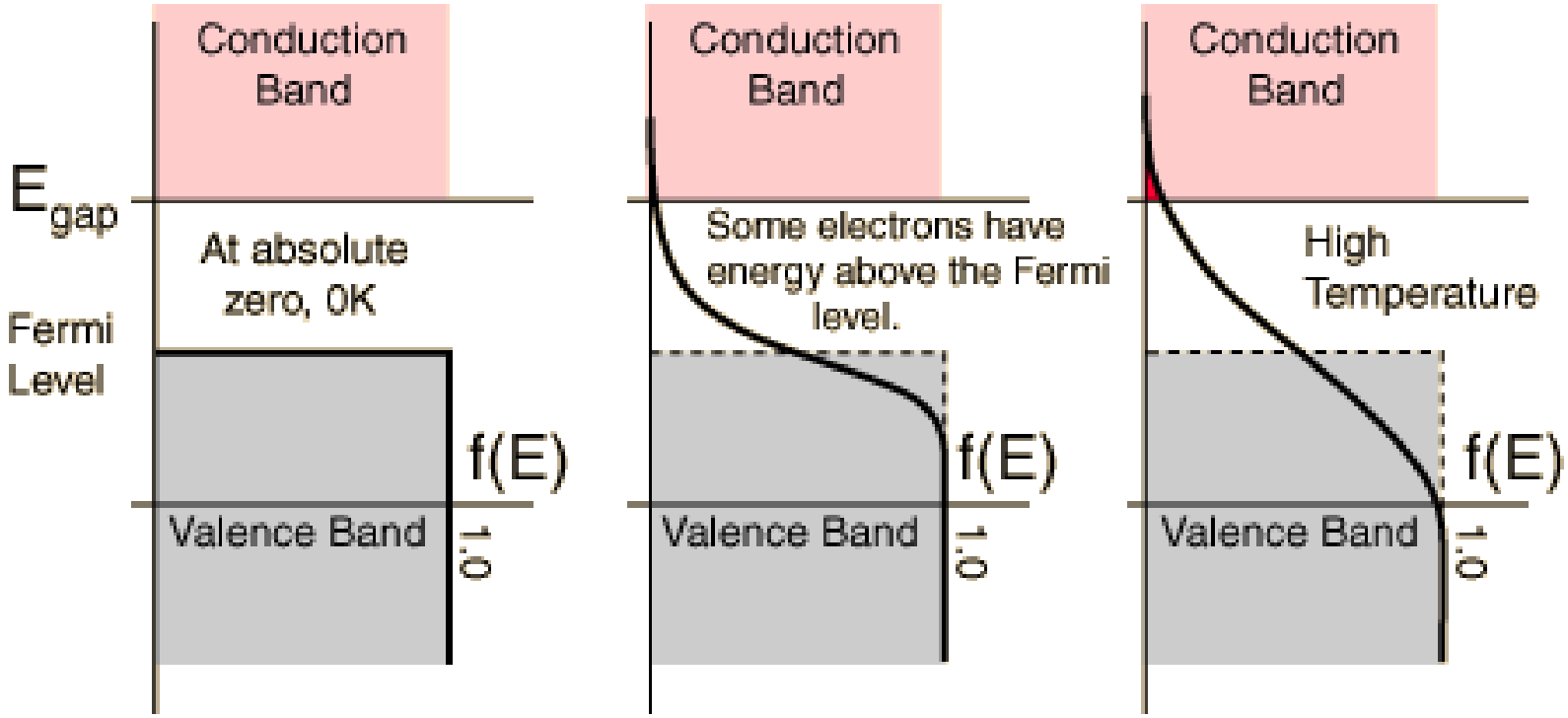
Boltzmann Approximation of Fermi Function

- The Fermi Function that describes the probability that a state at energy *E* is filled with an electron, under equilibrium conditions, is already given as:

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

- Fermi Function can be approximated, using Boltzmann Approximation, as:
  - $f(E) \cong e^{-(E - E_F)/kT}$ if $E - E_F > 3kT$
  - $1 - f(E) \cong e^{(E - E_F)/kT}$ if $E_F - E > 3kT$

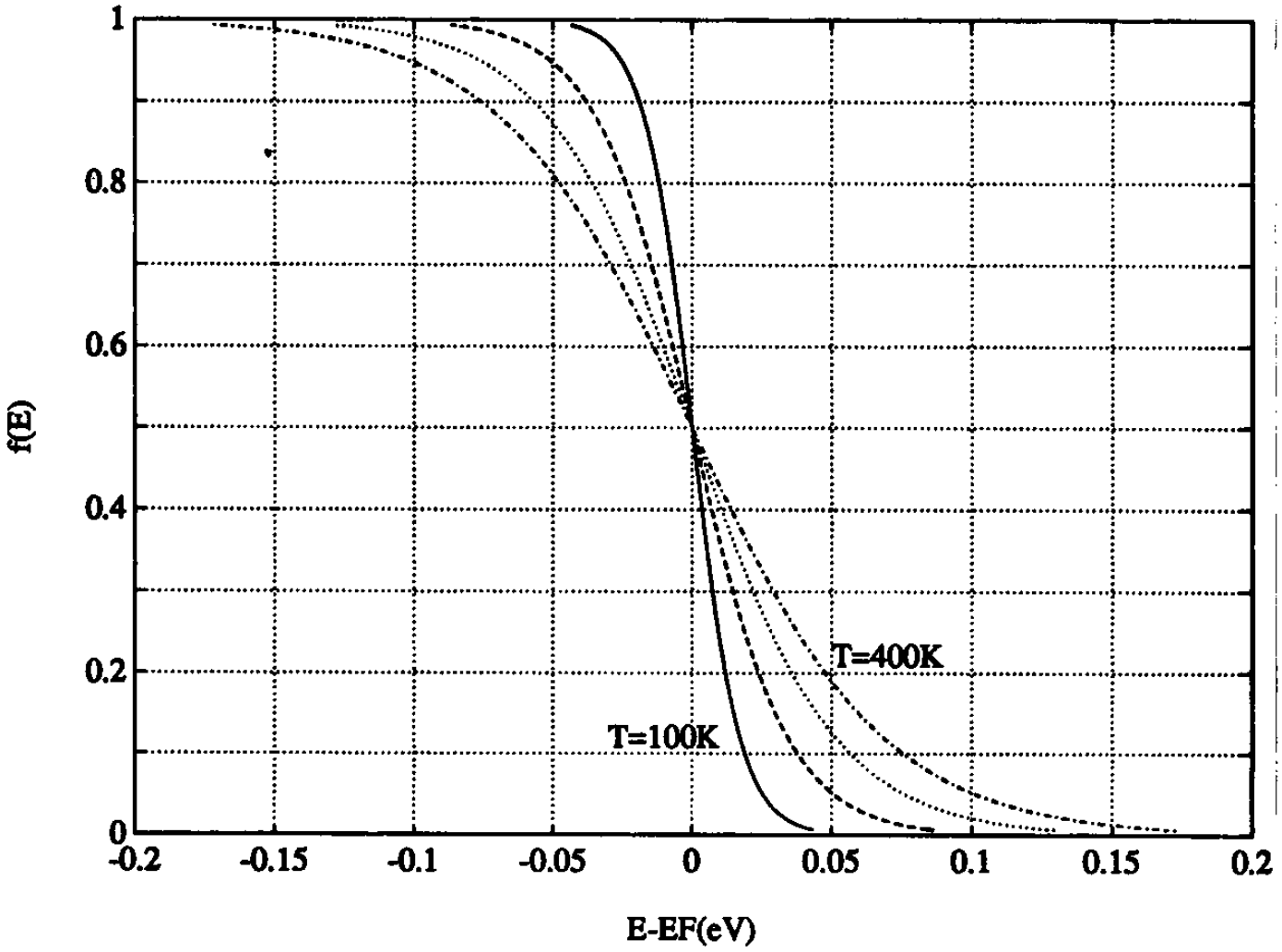
Effect of Temperature on *f*(*E*)



No electrons can be above the valence band at 0K, since none have energy above the Fermi level and there are no available energy states in the band gap.

At high temperatures, some electrons can reach the conduction band and contribute to electric current.

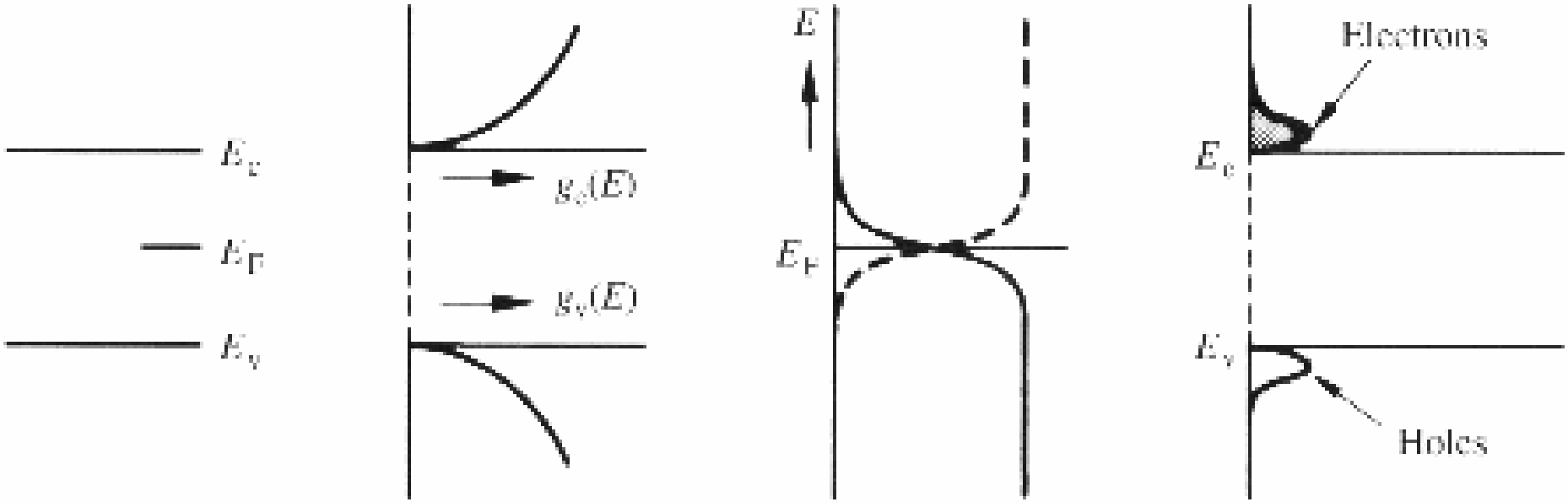
Effect of Temperature on *f*(*E*)



Equilibrium Distribution of Carriers

- $n(E)$ is obtained by multiplying $g_c(E)$ and $f(E)$,
  $p(E)$ is obtained by multiplying $g_v(E)$ and $1–f(E)$.

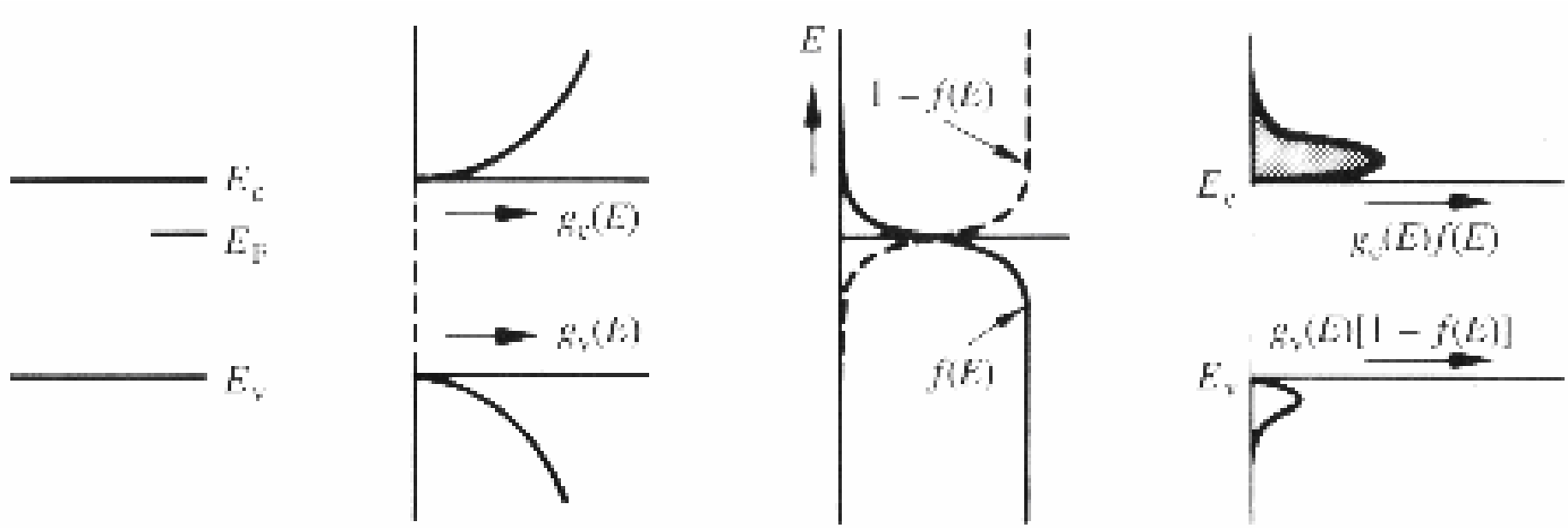
- **Intrinsic semiconductor material**



Energy band diagram | **Density of states** | **Probability of occupancy** | **Carrier distribution**

Equilibrium Distribution of Carriers

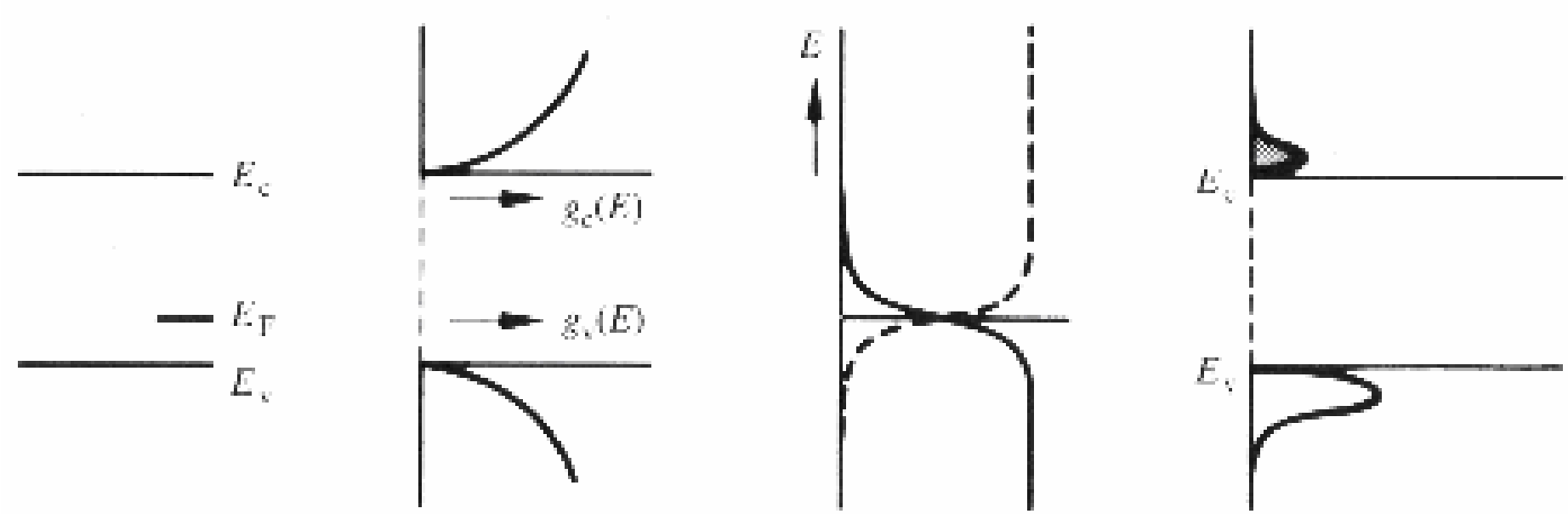
- ***n*-type semiconductor material**



Energy band diagram | **Density of States** | **Probability of occupancy** | **Carrier distribution**

Equilibrium Distribution of Carriers

- ***p*-type semiconductor material**



Energy band diagram | **Density of States** | **Probability of occupancy** | **Carrier distribution**

Important Constants

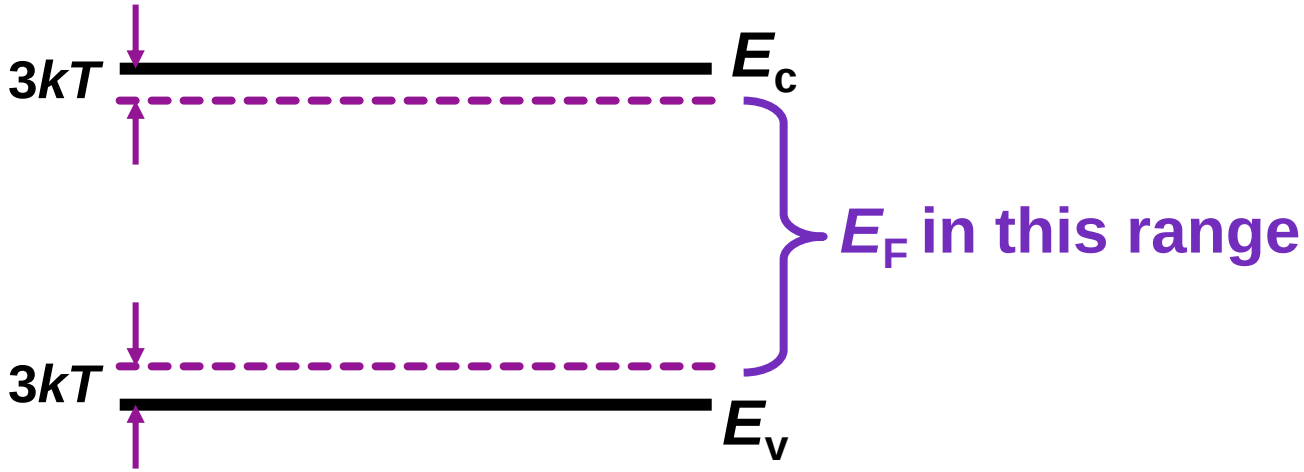
- Electronic charge, $q = 1.6\times10^{-19}$ C
- Permittivity of free space, $\varepsilon_o = 8.854\times10^{-12}$ F/m
- Boltzmann constant, $k = 8.62\times10^{-5}$ eV/K
- Planck constant, $h = 4.14\times10^{-15}$ eV·s
- Free electron mass, $m_o = 9.1\times10^{-31}$ kg
- Thermal energy, $kT = 0.02586$ eV (at 300 K)
- Thermal voltage, $kT/q = 0.02586$ V (at 300 K)

$$\hbar = \frac{h}{2\pi}$$

Nondegenerately Doped Semiconductor

- The expressions for *n* and *p* will now be derived in the range where the Boltzmann approximation can be applied:

$$E_v + 3kT \le E_F \le E_c - 3kT$$



- The semiconductor is said to be **nondegenerately doped (lightly doped)** in this case.

Degenerately Doped Semiconductor

- If a semiconductor is very heavily doped, the Boltzmann approximation is not valid.
- For Si at $T = 300$ K,
  $E_c - E_F > 3kT$ if $N_D > 1.6 \times 10^{18}$ cm^{-3}
  $E_F - E_v > 3kT$ if $N_A > 9.1 \times 10^{17}$ cm^{-3}
- The semiconductor is said to be **degenerately doped (heavily doped)** in this case.

- **N_D = total number of donor atoms/cm^3**
- **N_A = total number of acceptor atoms/cm^3**

Equilibrium Carrier Concentrations

- Integrating $n(E)$ over all the energies in the conduction band to obtain n (conduction electron concentration):

$$n = \int_{E_c}^{E_{top}} g_c(E) f(E) dE$$

- By using the Boltzmann approximation, and extending the integration limit to ∞,

$$n = N_C e^{(E_F - E_c)/kT} \quad \text{where} \quad N_C = 2\left[\frac{m_n^* kT}{2\pi\hbar^2}\right]^{3/2}$$

  - **N_C = "effective" density of conduction band states**
  - **For Si at 300 K, $N_C = 3.22 \times 10^{19}$ cm^{-3}**

Equilibrium Carrier Concentrations

- Integrating $p(E)$ over all the energies in the conduction band to obtain p (hole concentration):

$$p = \int_{E_{\text{bottom}}}^{E_V} g_v(E)\left[1 - f(E)\right]dE$$

- By using the Boltzmann approximation, and extending the integration limit to ∞,

$$p = N_V e^{(E_v - E_F)/kT} \quad \text{where} \quad N_V = 2\left[\frac{m_p^* kT}{2\pi\hbar^2}\right]^{3/2}$$

  - **N_V = "effective" density of valence band states**
  - **For Si at 300 K, $N_V = 1.83 \times 10^{19}$ cm^{-3}**

Intrinsic Carrier Concentration

- Relationship between E_F and n, p :

$$n = N_C e^{(E_F - E_c)/kT}$$

$$p = N_V e^{(E_v - E_F)/kT}$$

- For intrinsic semiconductors, where $n = p = n_i$,

$$np = n_i^2$$

$$n_i = \sqrt{N_C N_V}\, e^{-E_G/2kT}$$

- **E_G : band gap energy**

Intrinsic Carrier Concentration

$$np = (N_C e^{(E_F - E_c)/kT}) \cdot (N_V e^{(E_v - E_F)/kT})$$

$$= N_C N_V e^{(E_v - E_c)/kT}$$

$$= N_C N_V e^{-E_G/kT}$$

$$n_i = \sqrt{N_C N_V} e^{-E_G/2kT}$$

Alternative Expressions: $n(n_i, E_i)$ and $p(n_i, E_i)$

- In an intrinsic semiconductor, $n = p = n_i$ and $E_F = E_i$, where E_i denotes the intrinsic Fermi level.

$$n = N_C e^{(E_F - E_c)/kT}$$

$$n_i = N_C e^{(E_i - E_c)/kT}$$

$$\Rightarrow N_C = n_i e^{-(E_i - E_c)/kT}$$

$$n = n_i e^{-(E_i - E_c)/kT} \cdot e^{(E_F - E_c)/kT}$$

$$n = n_i e^{(E_F - E_i)/kT}$$

$$E_F = E_i + kT \ln\left(\frac{n}{n_i}\right)$$

$$p = N_V e^{(E_v - E_F)/kT}$$

$$p_i = N_V e^{(E_v - E_i)/kT}$$

$$\Rightarrow N_V = n_i e^{-(E_v - E_i)/kT}$$

$$p = n_i e^{-(E_v - E_i)/kT} \cdot e^{(E_v - E_F)/kT}$$

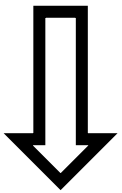
$$p = n_i e^{(E_i - E_F)/kT}$$

$$E_F = E_i - kT \ln\left(\frac{p}{n_i}\right)$$

Intrinsic Fermi Level, E_i

- To find E_F for an intrinsic semiconductor, we use the fact that $n = p$.

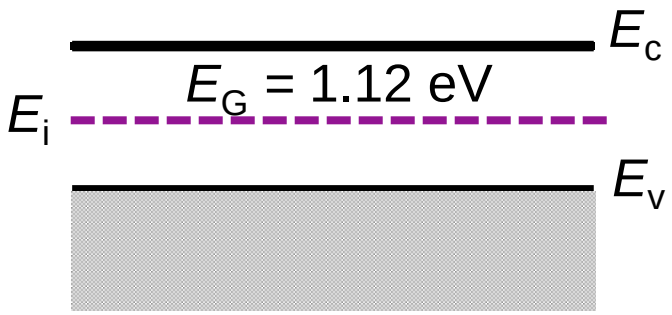
$$N_C e^{(E_i - E_c)/kT} = N_V e^{(E_v - E_i)/kT}$$



$$E_i = \frac{E_c + E_v}{2} + \frac{kT}{2} \ln\left(\frac{N_V}{N_C}\right)$$

$$E_i = \frac{E_c + E_v}{2} + \frac{3kT}{4} \ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$E_i \cong \frac{E_c + E_v}{2}$$



Si

- **E_i lies (almost) in the middle between E_c and E_v**

Example: Energy-Band Diagram

- For Silicon at 300 K, where is E_F if $n = 10^{17}$ cm^{-3} ?

Silicon at 300 K, $n_i = 10^{10}$ cm^{-3}

$$E_F = E_i + kT \ln\left(\frac{n}{n_i}\right)$$

$$= 0.56 + 8.62 \cdot 10^{-5} \cdot 300 \cdot \ln\left(\frac{10^{17}}{10^{10}}\right) \text{ eV}$$

$$= 0.56 + 0.417 \text{ eV}$$

$$= \underline{\underline{0.977 \text{ eV}}}$$

Charge Neutrality and Carrier Concentration

- N_D^+: concentration of ionized donor (cm^{-3})
- N_A^-: concentration of ionized acceptor (cm^{-3})?

- Charge neutrality condition: $p - n + N_D^+ - N_A^- = 0$

- Setting $N_D^+ = N_D$ and $N_A^- = N_A$, as at room temperature almost all of donor and acceptor sites are ionized,

$$p - n + N_D - N_A = 0, \qquad p = \frac{n_i^2}{n}$$

$$\frac{n_i^2}{n} - n + N_D - N_A = 0$$

$$n^2 - n(N_D - N_A) - n_i^2 = 0$$

- E_i quadratic equation in n

- Assumptions: **nondegeneracy** (*np* product relationship applies) and **total ionization**.

Charge-Carrier Concentrations

- The solution of the previous quadratic equation for *n* is:

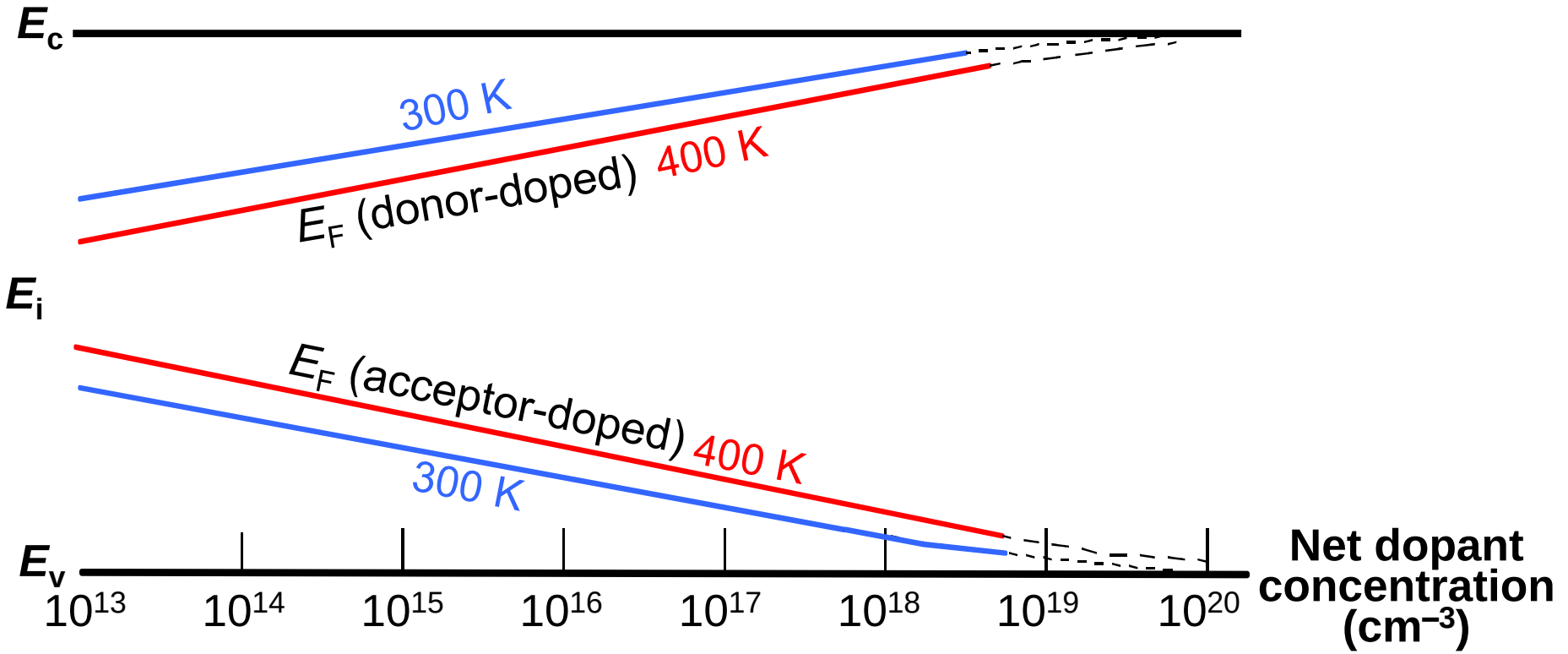
$$n = \frac{N_D - N_A}{2} + \left[\left(\frac{N_D - N_A}{2} \right)^2 + n_i^2 \right]^{1/2}$$

- New quadratic equation can be constructed and the solution for *p* is:

$$p = \frac{N_A - N_D}{2} + \left[\left(\frac{N_A - N_D}{2} \right)^2 + n_i^2 \right]^{1/2}$$

- **Carrier concentrations depend on *net* dopant concentration $N_D - N_A$ or $N_A - N_D$**

Dependence of E_F on Temperature

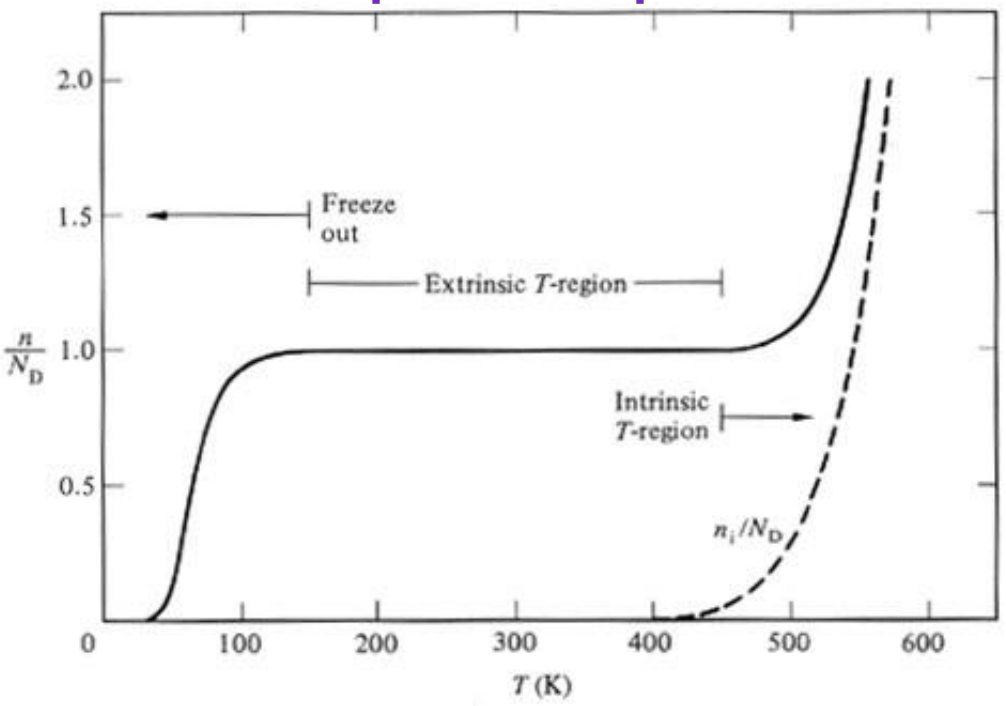


$$E_F = E_i + kT \ln\left(\frac{n}{n_i}\right), \quad \text{donor-doped}$$

$$E_F = E_i - kT \ln\left(\frac{p}{n_i}\right), \quad \text{acceptor-doped}$$

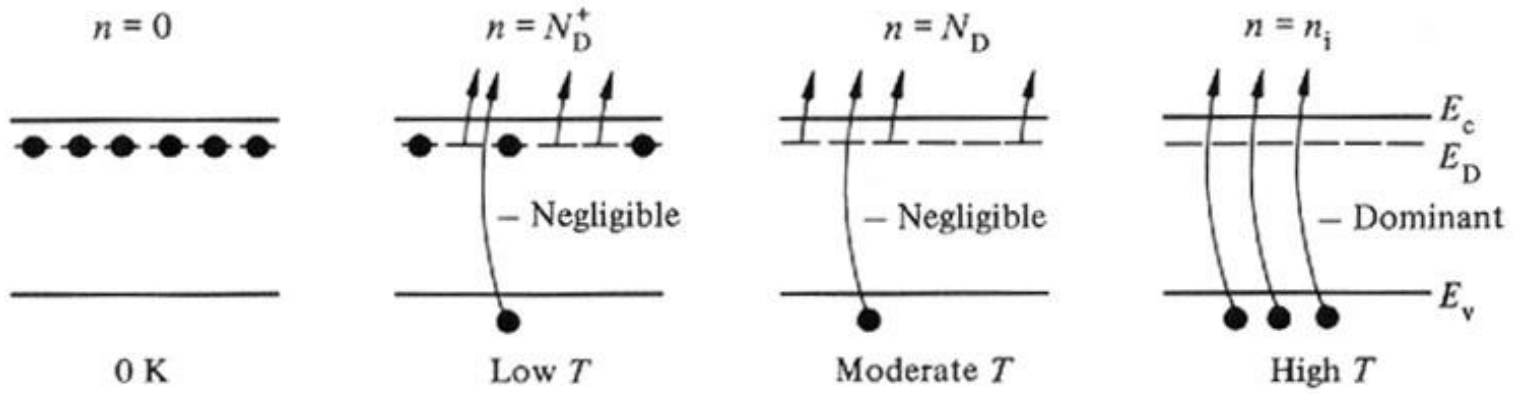
Carrier Concentration *vs.* Temperature

Phosphorus-doped Si



$N_D = 10^{15}\ cm^{-3}$

- n : number of majority carrier
- N_D : number of donor electron
- n_i : number of intrinsic conductive electron



Homework 1

- 1. (Problem 2.6 in Pierret) (4.2)
  - a) Under equilibrium condition at $T > 0\ K$, what is the probability of an electron state being occupied if it is located at the Fermi level?
  - b) If E_F is positioned at E_c, determine (numerical answer required) the probability of finding electrons in states at $E_c + kT$.
  - c) The probability a state is filled at $E_c + kT$ is equal to the probability a state is empty at $E_c + kT$. Where is the Fermi level located?
- 2. (2.36)
  - a) Determine for what energy above E_F (in terms of kT) the Boltzmann approximation is within 1 percent of the exact Fermi probability function .
  - b) Give the value of the exact probability function at this energy.

Homework 1

- 3.a. (4.2)
  Calculate the equilibrium hole concentration in silicon at T = 400 K if the Fermi energy level is 0.27 eV above the valence band energy.
- 3.b. (E4.3)
  Find the intrinsic carrier concentration in silicon at:
  (i) T = 200 K and (ii) T = 400 K.
- 3.c. (4.13)
  Silicon at T = 300 K contains an acceptor impurity concentration of $N_A = 10^{16}$ cm^{-3}. Determine the concentration of donor impurity atoms that must be added so that the silicon is n-type and the Fermi energy level is 0.20 eV below the conduction band edge.