Semiconductor Device Physics

Lecture 9

http://zitompul.wordpress.com

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Chapter 10

BJT Fundamentals

Bipolar Junction Transistors (BJTs)

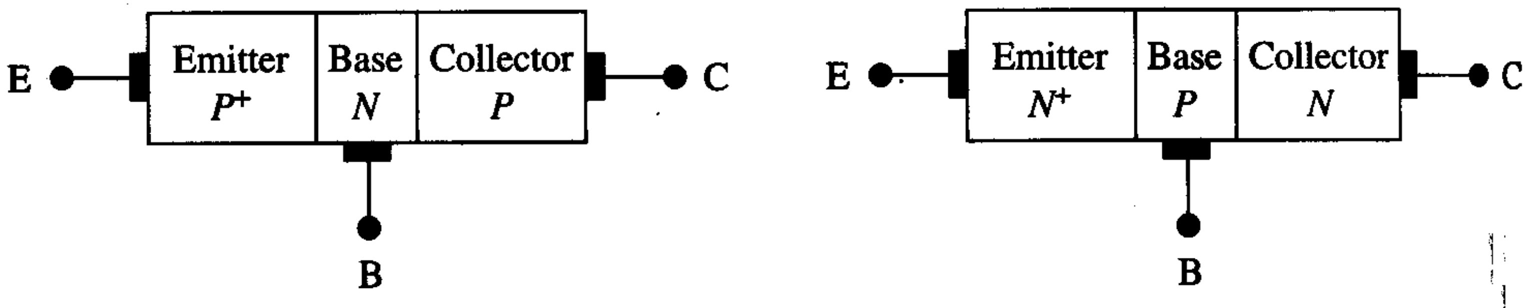
- Over the past decades, the higher layout density and low-power advantage of CMOS (**C**omplementary **M**etal–**O**xide–**S**emiconductor) has eroded away the BJT's dominance in integrated-circuit products.
  - Higher circuit density → better system performance

- BJTs are still preferred in **some** digital-circuit and analog-circuit applications because of their high speed and superior gain
  - Faster circuit speed (**+**)
  - Larger power dissipation (**–**)

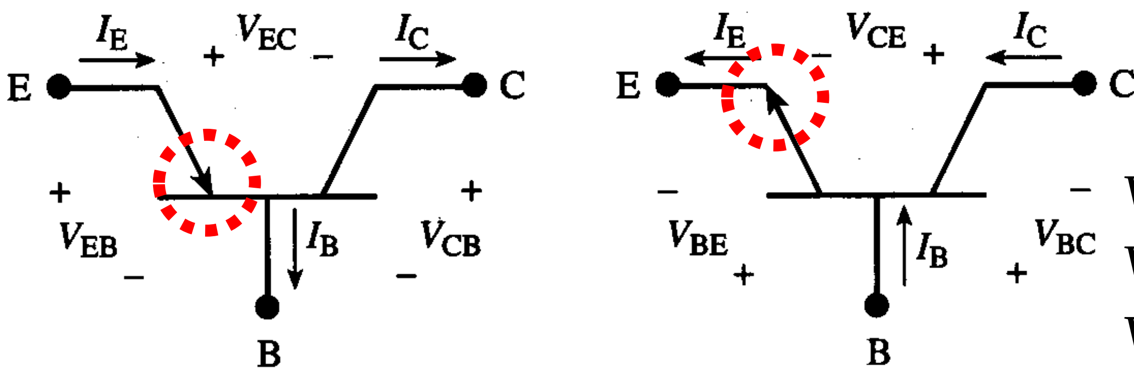
- ***Transistor:*** **current flowing between two terminals is controlled by a third terminal**

Introduction

- There are two types of BJT: *pnp* and *npn*.



$$\begin{aligned} V_{EB} &= V_E - V_B \\ V_{CB} &= V_C - V_B \\ V_{EC} &= V_E - V_C \\ &= V_{EB} - V_{CB} \end{aligned}$$

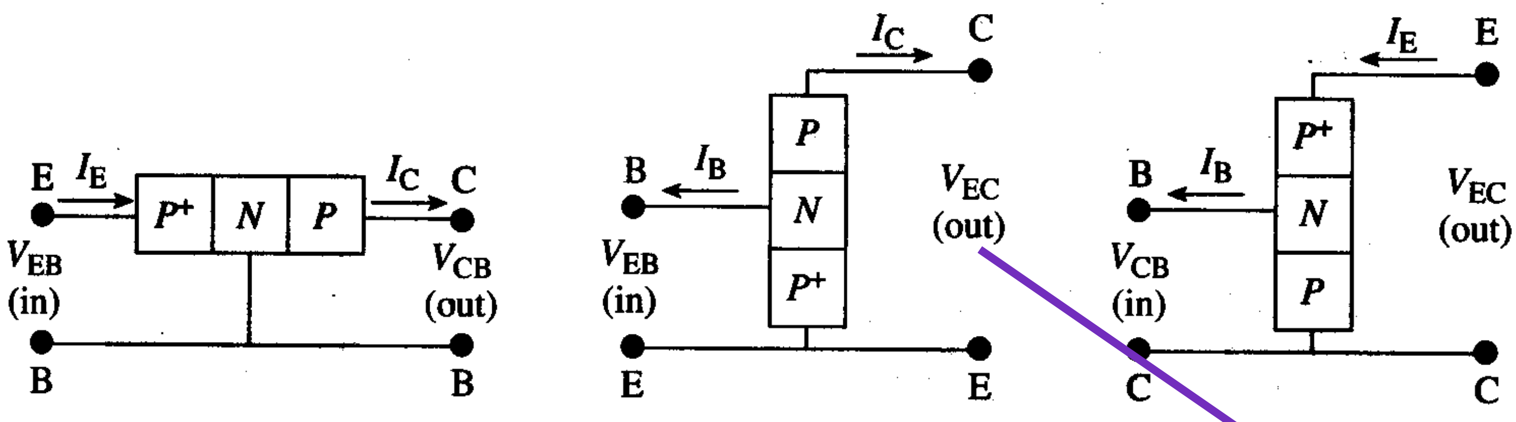


(a) *pnp* (b) *npn*

$$\begin{aligned} V_{BE} &= V_B - V_E \\ V_{BC} &= V_B - V_C \\ V_{CE} &= V_C - V_E \\ &= V_{CB} - V_{EB} \end{aligned}$$

- The convention used in the textbook does not follow IEEE convention, where currents flowing into a terminal is defined as positive.
- We will follow the normal convention: . . . . . .

Circuit Configurations

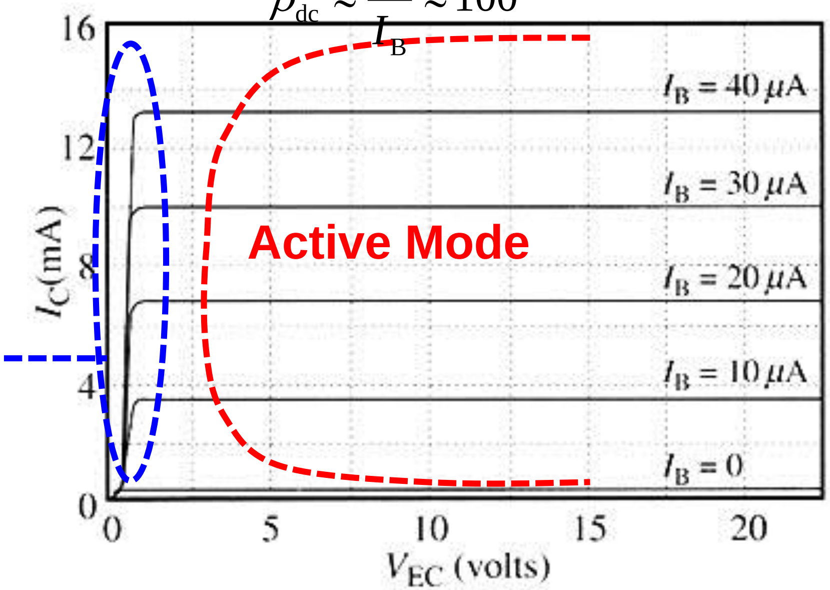


(a) Common base (b) Common emitter (c) Common collector

Most popular configuration

$$\beta_{dc} \approx \frac{I_C}{I_B} \approx 100$$

Common-Emitter
***I–V* Characteristics**

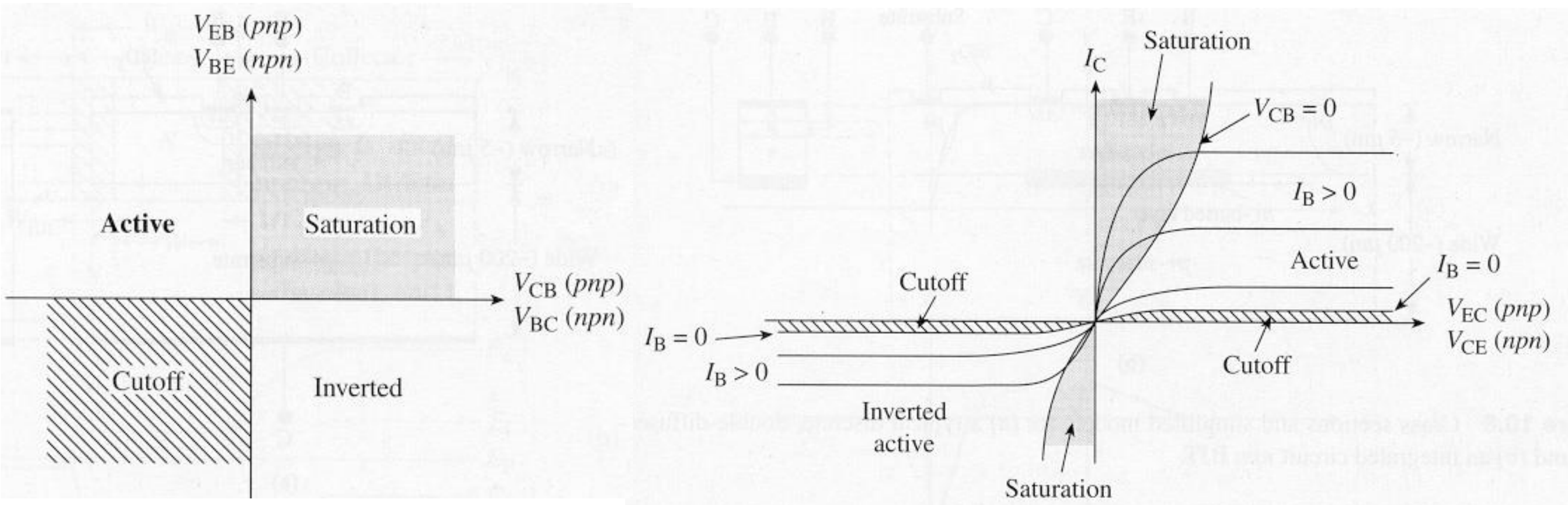


Saturation Mode
$I_C < \beta I_B$

In active mode, β_{dc} is the common emitter dc current gain

Modes of Operation

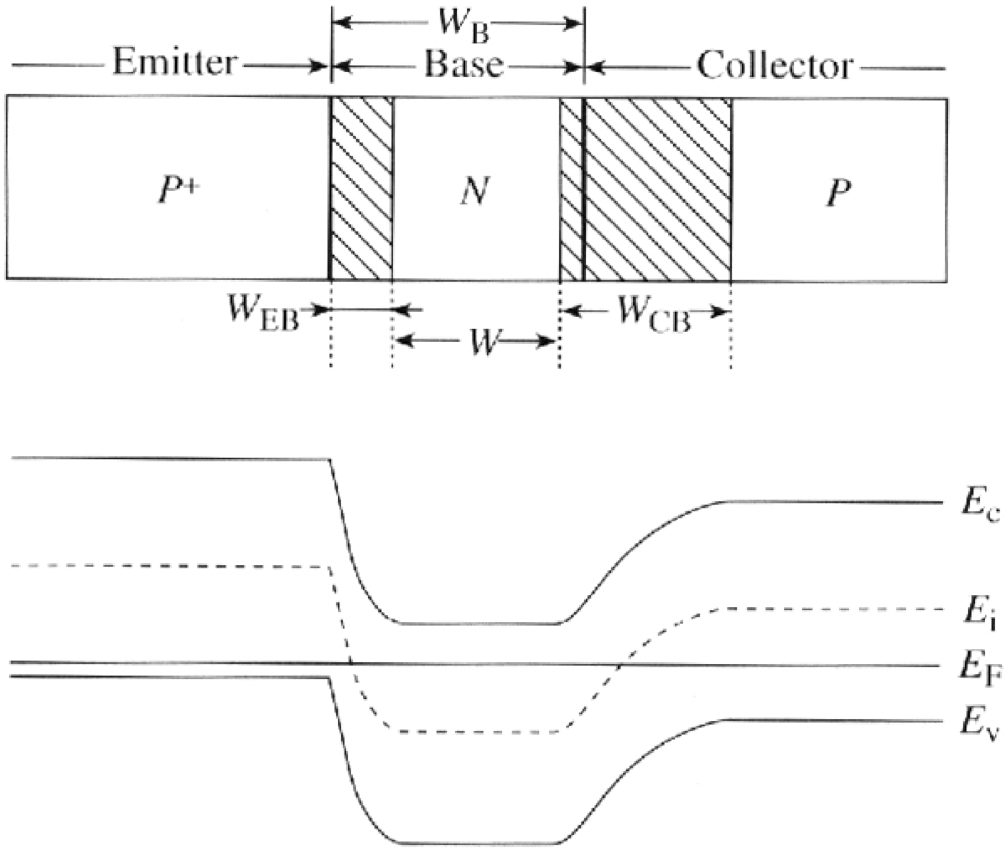
Common-Emitter Output Characteristics



Mode	E-B Junction	C-B Junction
Saturation	forward bias	forward bias
Active/Forward	forward bias	reverse bias
Inverted	reverse bias	forward bias
Cutoff	reverse bias	reverse bias

BJT Electrostatics

- Under equilibrium and normal operating conditions, the BJT may be viewed electrostatically as two independent *pn* junctions.



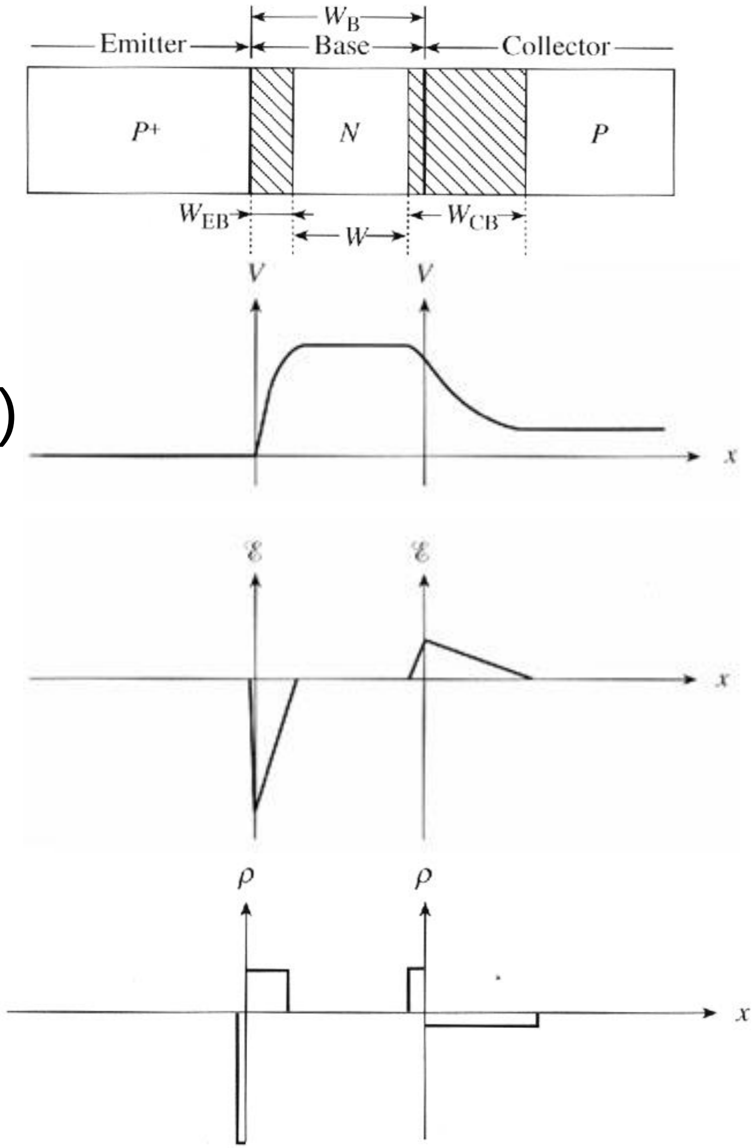
$$N_{\mathrm{AE}} >> N_{\mathrm{DB}} > N_{\mathrm{AC}}$$

$$W_{\mathrm{CB}} > W_{\mathrm{EB}}$$

$$W = W_{\mathrm{B}} - x_{\mathrm{nEB}} - x_{\mathrm{nCB}}$$

***W* : quasineutral base width**

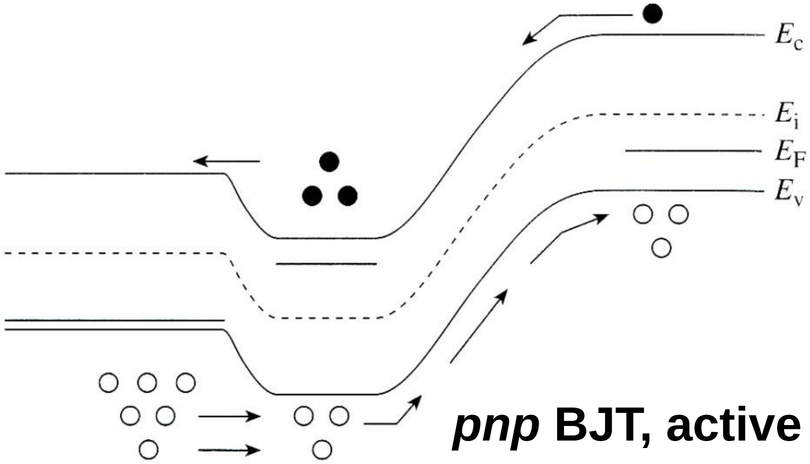
BJT Electrostatics



- Electrostatic potential, $V(x)$
- Electric field, $\mathcal{E}(x)$
- Charge density, $\rho(x)$

BJT Design

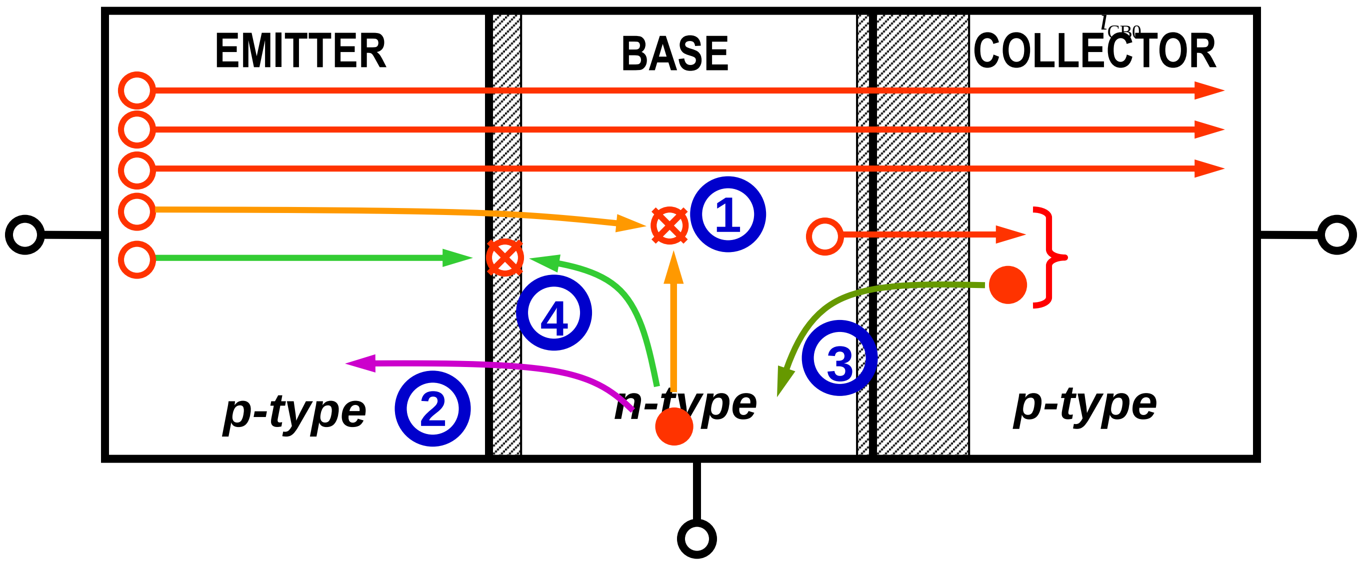
- Important features of a good transistor:
  - Injected minority carriers do not recombine in the neutral base region ➔ short base, $W \ll L_p$ for *pnp* transistor
  - Emitter current is comprised almost entirely of carriers injected into the base rather than carriers injected into the emitter ➔ the emitter must be doped heavier than the base



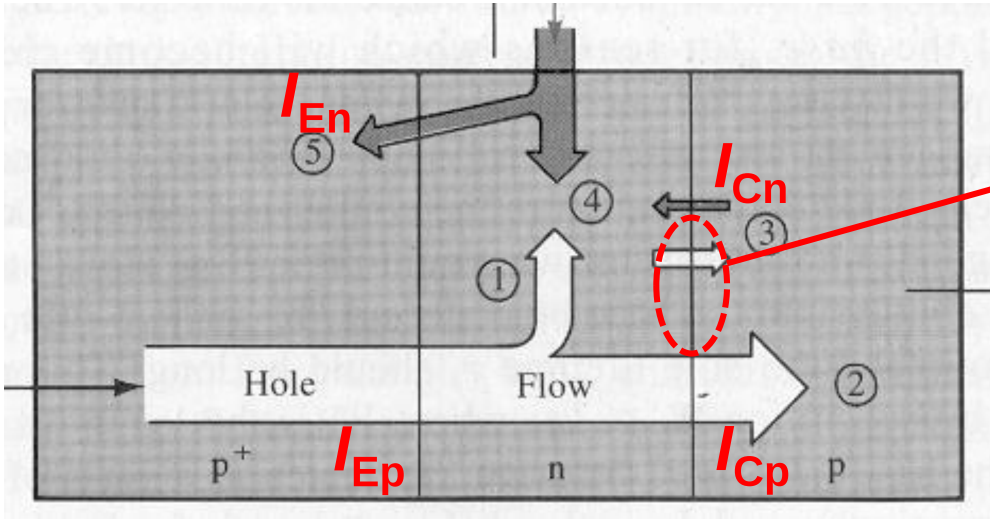
pnp **BJT, active mode**

Base Current (Active Bias)

- The base current consists of majority carriers (electrons) supplied for:
    1. Recombination of injected minority carriers in the base
    2. Injection of carriers into the emitter
    3. Reverse saturation current in collector junction
    4. Recombination in the base-emitter depletion region



BJT Performance Parameters (*pnp*)



- Emitter Efficiency

$$\gamma = \frac{I_{\mathrm{Ep}}}{I_{\mathrm{E}}} = \frac{I_{\mathrm{Ep}}}{I_{\mathrm{Ep}} + I_{\mathrm{En}}}$$

  - Decrease ⑤ relative to ① and ② to increase efficiency

- Base Transport Factor

$$\alpha_{\mathrm{T}} = \frac{I_{\mathrm{Cp}}}{I_{\mathrm{Ep}}}$$

  - Decrease ① relative to ② to increase transport factor

Common base dc current gain: $\alpha_{\mathrm{dc}} = \gamma \alpha_{\mathrm{T}}$

Collector Current (Active Bias)

- The collector current is composed of:
  - Holes injected from emitter, which do not recombine in the base ②
  - Reverse saturation current of collector junction ③

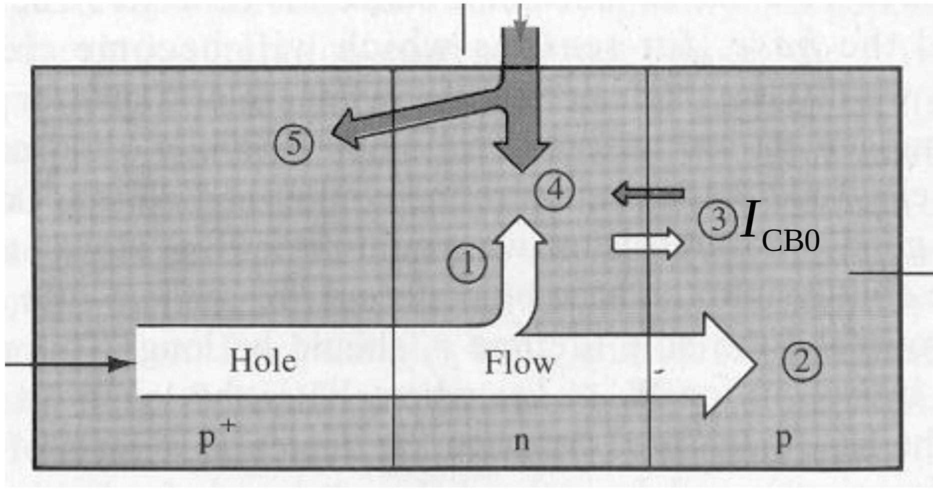
$$I_C = \alpha_{dc} I_E + I_{CB0}$$

I_{CB0} :collector current when $I_E = 0$

$$I_C = \alpha_{dc}(I_C + I_B) + I_{CB0}$$

$$I_C = \frac{\alpha_{dc}}{1-\alpha_{dc}} I_B + \frac{I_{CB0}}{1-\alpha_{dc}}$$

$$I_C = \beta_{dc} I_B + I_{CE0}$$



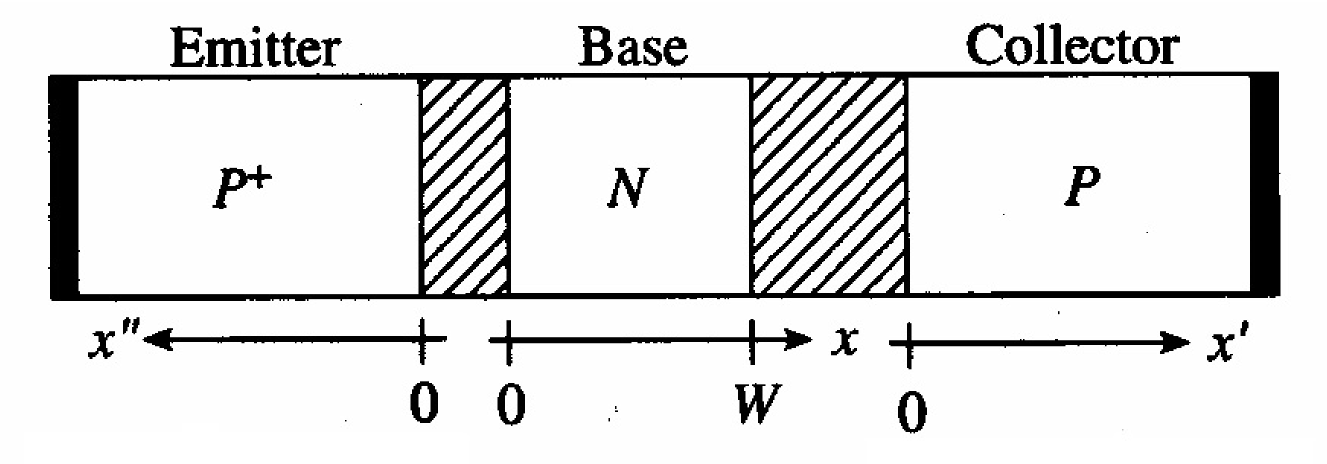
Common emitter dc current gain:

$$\beta_{dc} = \frac{\alpha_{dc}}{1-\alpha_{dc}} \approx \frac{I_C}{I_B}$$

Chapter 11

BJT Static Characteristics

Notation (*pnp* BJT)



$$N_E = N_{AE} \qquad N_B = N_{DB} \qquad N_C = N_{AC}$$

Minority carrier constants:

$$D_E = D_N \qquad D_B = D_P \qquad D_C = D_N$$

$$\tau_E = \tau_n \qquad \tau_B = \tau_p \qquad \tau_C = \tau_n$$

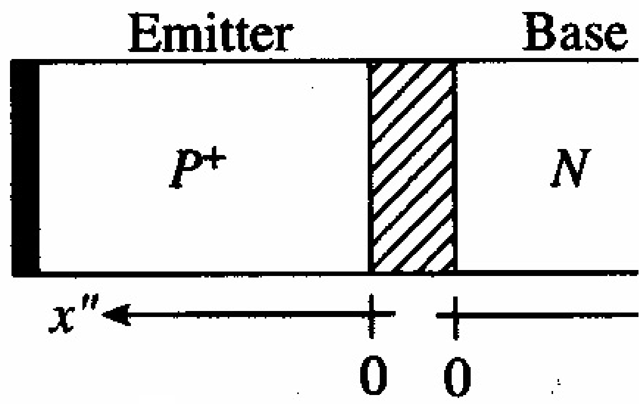
$$L_E = L_N \qquad L_B = L_P \qquad L_C = L_N$$

$$n_{E0} = n_{p0} = n_i^2 / N_E \qquad p_{B0} = p_{n0} = n_i^2 / N_B \qquad n_{C0} = n_{p0} = n_i^2 / N_C$$

Emitter Region

- Diffusion equation:

$$0 = D_E \frac{d^2 \Delta n_E}{dx''^2} - \frac{\Delta n_E}{\tau_E}$$



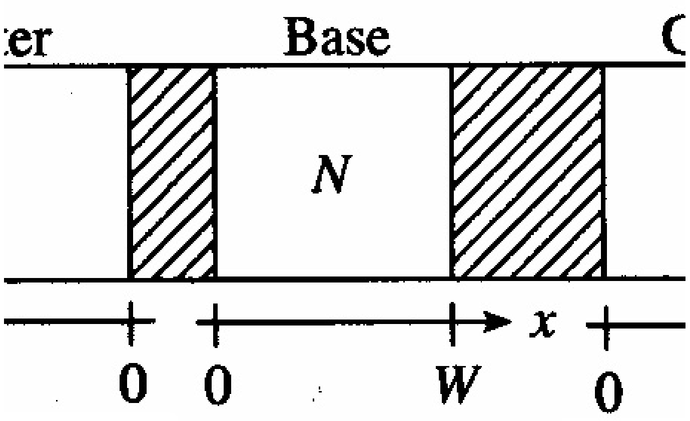
- Boundary conditions:

$$\Delta n_E (x'' \to \infty) = 0$$
$$\Delta n_E (x'' = 0) = n_{E0} (e^{qV_{EB}/kT} - 1)$$

Base Region

- Diffusion equation:

$$0 = D_{\mathrm{B}} \frac{d^2 \Delta p_{\mathrm{B}}}{dx^2} - \frac{\Delta p_{\mathrm{B}}}{\tau_{\mathrm{B}}}$$



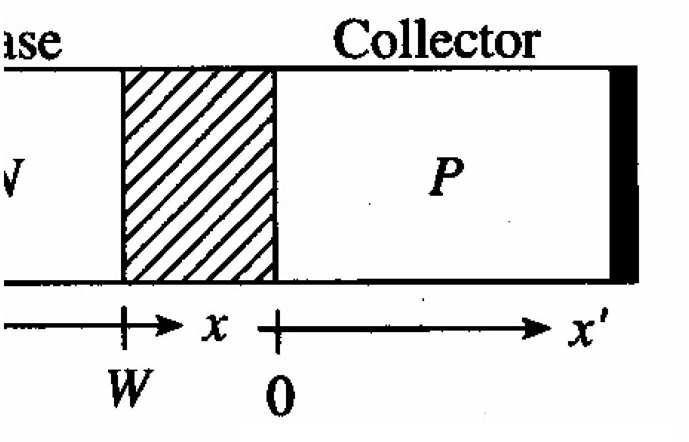
- Boundary conditions:

$$\Delta p_{\mathrm{B}}(0) = p_{\mathrm{B0}}(e^{qV_{\mathrm{EB}}/kT} - 1)$$
$$\Delta p_{\mathrm{B}}(W) = p_{\mathrm{B0}}(e^{qV_{\mathrm{CB}}/kT} - 1)$$

Collector Region

- Diffusion equation:

$$0 = D_{\mathrm{C}} \frac{d^2 \Delta n_{\mathrm{C}}}{dx'^2} - \frac{\Delta n_{\mathrm{C}}}{\tau_{\mathrm{C}}}$$



- Boundary conditions:

$$\Delta n_{\mathrm{C}}(x' \to \infty) = 0$$

$$\Delta n_{\mathrm{C}}(x' = 0) = n_{\mathrm{C0}}(e^{qV_{\mathrm{CB}}/kT} - 1)$$

Ideal Transistor Analysis

- Solve the minority-carrier diffusion equation in each quasi-neutral region to obtain excess minority-carrier profiles
  - Each region has different set of boundary conditions

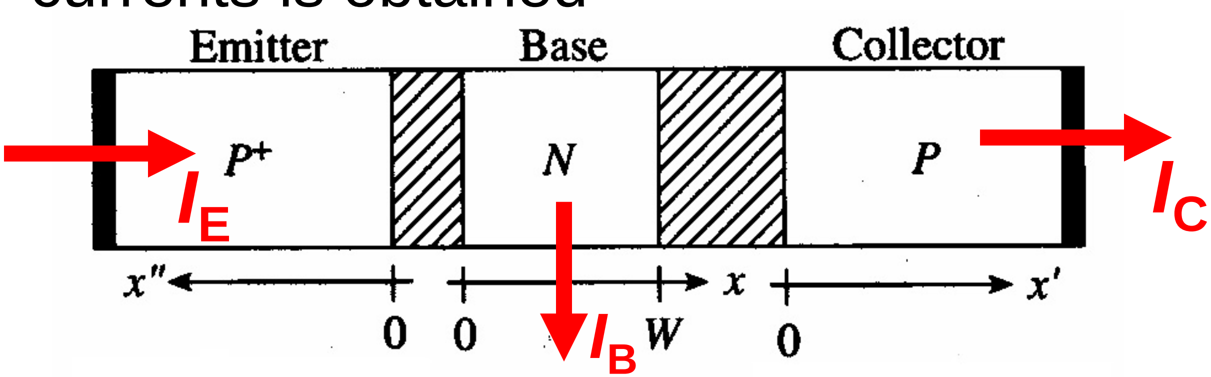
$\Delta n_E(x''),$
$\Delta p_B(x),$
$\Delta n_C(x')$

- Evaluate minority-carrier diffusion currents at edges of depletion regions

$$I_{En} = -qAD_E \left.\frac{d\Delta n_E}{dx''}\right|_{x''=0} \qquad I_{Ep} = -qAD_B \left.\frac{d\Delta p_B}{dx}\right|_{x=0}$$

$$I_{Cn} = qAD_C \left.\frac{d\Delta n_C}{dx'}\right|_{x'=0} \qquad I_{Cp} = -qAD_B \left.\frac{d\Delta p_B}{dx}\right|_{x=W}$$

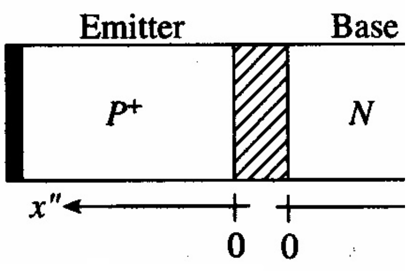
- Add hole and electron components together ➔ terminal currents is obtained



$$I_E = I_{Ep} + I_{En}$$
$$I_C = I_{Cp} + I_{Cn}$$
$$I_B = I_E - I_C$$

Emitter Region Solution

- Diffusion equation: $$0 = D_E \frac{d^2 \Delta n_E}{dx''^2} - \frac{\Delta n_E}{\tau_E}$$



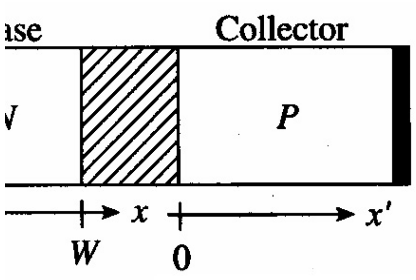
- General solution: $$\Delta n_E(x'') = A_1 e^{-x''/L_E} + A_2 e^{x''/L_E}$$

- Boundary conditions: $$\Delta n_E(x'' \to \infty) = 0$$ $$\Delta n_E(x'' = 0) = n_{E0}(e^{qV_{EB}/kT} - 1)$$

- Solution $$\Delta n_E(x'') = n_{E0}(e^{qV_{EB}/kT} - 1)e^{-x''/L_E}$$

$$I_{En} = -qAD_E \left.\frac{d\Delta n_E}{dx''}\right|_{x''=0} = qA\frac{D_E}{L_E} n_{E0}(e^{qV_{EB}/kT} - 1)$$

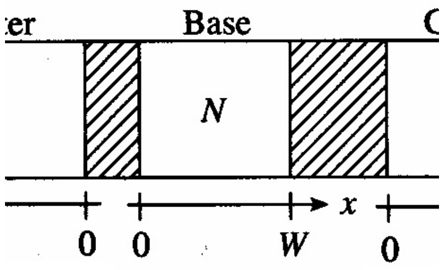
Collector Region Solution



- Diffusion equation: $$0 = D_C \frac{d^2 \Delta n_C}{dx'^2} - \frac{\Delta n_C}{\tau_C}$$
- General solution: $$\Delta n_C(x') = A_1 e^{-x'/L_C} + A_2 e^{x'/L_C}$$
- Boundary conditions: $$\Delta n_C(x' \to \infty) = 0$$ $$\Delta n_C(x' = 0) = n_{C0}(e^{qV_{CB}/kT} - 1)$$
- Solution $$\Delta n_C(x') = n_{C0}(e^{qV_{CB}/kT} - 1)e^{-x'/L_C}$$

$$I_{Cn} = qAD_C \left.\frac{d\Delta n_C}{dx'}\right|_{x'=0} = -qA\frac{D_C}{L_C} n_{C0}(e^{qV_{CB}/kT} - 1)$$

Base Region Solution



- Diffusion equation: $$0 = D_{\mathrm{B}} \frac{d^2 \Delta n_{\mathrm{B}}}{dx^2} - \frac{\Delta p_{\mathrm{B}}}{\tau_{\mathrm{B}}}$$

- General solution: $$\Delta p_{\mathrm{B}}(x) = A_1 e^{-x/L_{\mathrm{B}}} + A_2 e^{x/L_{\mathrm{B}}}$$

- Boundary conditions: $$\Delta p_{\mathrm{B}}(0) = p_{\mathrm{B0}}(e^{qV_{\mathrm{EB}}/kT} - 1)$$
$$\Delta p_{\mathrm{B}}(W) = p_{\mathrm{B0}}(e^{qV_{\mathrm{CB}}/kT} - 1)$$

- Solution

$$\Delta p_{\mathrm{B}}(x) = p_{\mathrm{B0}}(e^{qV_{\mathrm{EB}}/kT} - 1)\left(\frac{e^{(W-x)/L_{\mathrm{B}}} - e^{-(W-x)/L_{\mathrm{B}}}}{e^{W/L_{\mathrm{B}}} - e^{-W/L_{\mathrm{B}}}}\right) + p_{\mathrm{B0}}(e^{qV_{\mathrm{CB}}/kT} - 1)\left(\frac{e^{x/L_{\mathrm{B}}} - e^{-x/L_{\mathrm{B}}}}{e^{W/L_{\mathrm{B}}} - e^{-W/L_{\mathrm{B}}}}\right)$$

Base Region Solution

- Since $\sinh(\xi) = \dfrac{e^{\xi} - e^{-\xi}}{2}$
- We can write $\Delta p_B(x) = p_{B0}(e^{qV_{EB}/kT} - 1)\left(\dfrac{e^{(W-x)/L_B} - e^{-(W-x)/L_B}}{e^{W/L_B} - e^{-W/L_B}}\right)$

$$+ p_{B0}(e^{qV_{CB}/kT} - 1)\left(\frac{e^{x/L_B} - e^{-x/L_B}}{e^{W/L_B} - e^{-W/L_B}}\right)$$

as

$$\Delta p_B(x) = p_{B0}(e^{qV_{EB}/kT} - 1)\frac{\sinh\left[(W-x)/L_B\right]}{\sinh(W/L_B)} + p_{B0}(e^{qV_{CB}/kT} - 1)\frac{\sinh(x/L_B)}{\sinh(W/L_B)}$$

Base Region Solution

- Since $$\frac{d}{d\xi}\sinh(\xi)=\frac{d}{d\xi}\left(\frac{e^{\xi}-e^{-\xi}}{2}\right)=\frac{e^{\xi}+e^{-\xi}}{2}=\cosh(\xi)$$

$$I_{\mathrm{Ep}}=-qAD_{\mathrm{B}}\left.\frac{d\Delta p_{\mathrm{B}}}{dx}\right|_{x=0}$$
$$=qA\frac{D_{\mathrm{B}}}{L_{\mathrm{B}}}p_{\mathrm{B0}}\left[\frac{\cosh(W/L_{\mathrm{B}})}{\sinh(W/L_{\mathrm{B}})}(e^{qV_{\mathrm{EB}}/kT}-1)-\frac{1}{\sinh(W/L_{\mathrm{B}})}(e^{qV_{\mathrm{CB}}/kT}-1)\right]$$

$$I_{\mathrm{Cp}}=-qAD_{\mathrm{B}}\left.\frac{d\Delta p_{\mathrm{B}}}{dx}\right|_{x=W}$$
$$=qA\frac{D_{\mathrm{B}}}{L_{\mathrm{B}}}p_{\mathrm{B0}}\left[\frac{1}{\sinh(W/L_{\mathrm{B}})}(e^{qV_{\mathrm{EB}}/kT}-1)-\frac{\cosh(W/L_{\mathrm{B}})}{\sinh(W/L_{\mathrm{B}})}(e^{qV_{\mathrm{CB}}/kT}-1)\right]$$

Terminal Currents

- Since $I_{\mathrm{E}} = I_{\mathrm{En}} + I_{\mathrm{Ep}},\ I_{\mathrm{C}} = I_{\mathrm{Cn}} + I_{\mathrm{Cp}}$
- Then $\Rightarrow I_{\mathrm{E}} = qA\left[\left(\frac{D_{\mathrm{E}}}{L_{\mathrm{E}}} n_{\mathrm{E0}} + \frac{D_{\mathrm{B}}}{L_{\mathrm{B}}} p_{\mathrm{B0}} \frac{\cosh(W/L_{\mathrm{B}})}{\sinh(W/L_{\mathrm{B}})}\right)(e^{qV_{\mathrm{EB}}/kT} - 1) - \left(\frac{D_{\mathrm{B}}}{L_{\mathrm{B}}} p_{\mathrm{B0}} \frac{1}{\sinh(W/L_{\mathrm{B}})}\right)(e^{qV_{\mathrm{CB}}/kT} - 1)\right]$

$$\Rightarrow I_{\mathrm{C}} = qA\left[\left(\frac{D_{\mathrm{B}}}{L_{\mathrm{B}}} p_{\mathrm{B0}} \frac{1}{\sinh(W/L_{\mathrm{B}})}\right)(e^{qV_{\mathrm{EB}}/kT} - 1) - \left(\frac{D_{\mathrm{C}}}{L_{\mathrm{C}}} n_{\mathrm{C0}} + \frac{D_{\mathrm{B}}}{L_{\mathrm{B}}} p_{\mathrm{B0}} \frac{\cosh(W/L_{\mathrm{B}})}{\sinh(W/L_{\mathrm{B}})}\right)(e^{qV_{\mathrm{CB}}/kT} - 1)\right]$$

$$\Rightarrow I_{\mathrm{B}} = I_{\mathrm{E}} - I_{\mathrm{C}}$$

Simplified Relationships

- To achieve high current gain, a typical BJT will be constructed so that $W \ll L_B$.
- Using the limit value $\lim_{\xi \to 0} \sinh(\xi) = \xi$

$$\lim_{\xi \to 0} \cosh(\xi) = 1 + \frac{\xi^2}{2}$$

- We will have

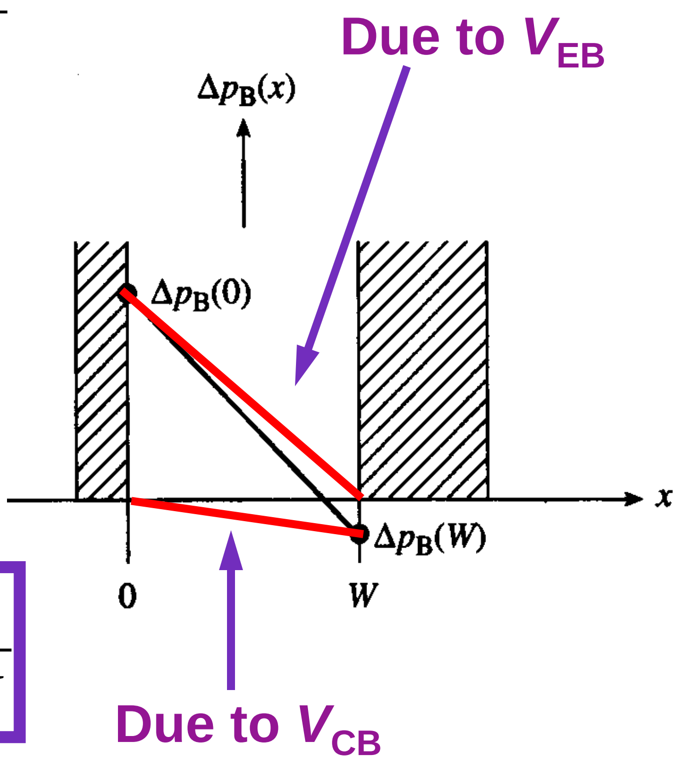
$$\Delta p_B(x) \approx p_{B0}(e^{qV_{EB}/kT} - 1)\left(1 - \frac{x}{W}\right)$$

$$+ p_{B0}(e^{qV_{CB}/kT} - 1)\left(\frac{x}{W}\right)$$

$\Delta p_B(0)$

$\Delta p_B(W)$

$$\Delta p_B(x) = \Delta p_B(0) + \left[\Delta p_B(W) - \Delta p_B(0)\right]\frac{x}{W}$$



Performance Parameters

- For specific condition of
  - "Active Mode": emitter junction is forward biased and collector junction is reverse biased
  - $W << L_B$, $n_{E0}/p_{B0} = N_B/N_E$

$$\gamma = \frac{1}{1 + \frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E}} \qquad \alpha_T = \frac{1}{1 + \frac{1}{2}\left(\frac{W}{L_B}\right)^2}$$

$$\alpha_{dc} = \frac{1}{1 + \frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2}, \quad \beta_{dc} = \frac{1}{\frac{D_E}{D_B}\frac{N_B}{N_E}\frac{W}{L_E} + \frac{1}{2}\left(\frac{W}{L_B}\right)^2}$$

Ebers-Moll BJT Equations

I_{F0}

$$\Rightarrow I_E = qA\left[\left(\frac{D_E}{L_E}n_{E0} + \frac{D_B}{L_B}p_{B0}\frac{\cosh(W/L_B)}{\sinh(W/L_B)}\right)\left(e^{qV_{EB}/kT}-1\right) - \left(\frac{D_B}{L_B}p_{B0}\frac{1}{\sinh(W/L_B)}\right)\left(e^{qV_{CB}/kT}-1\right)\right]$$

$\alpha_F I_{F0} = \alpha_R I_{R0}$

$$\Rightarrow I_C = qA\left[\left(\frac{D_B}{L_B}p_{B0}\frac{1}{\sinh(W/L_B)}\right)\left(e^{qV_{EB}/kT}-1\right) - \left(\frac{D_C}{L_C}n_{C0} + \frac{D_B}{L_B}p_{B0}\frac{\cosh(W/L_B)}{\sinh(W/L_B)}\right)\left(e^{qV_{CB}/kT}-1\right)\right]$$

I_{R0}

I_{F0}, I_{R0}: Emitter and Collector Diode Saturation Currents

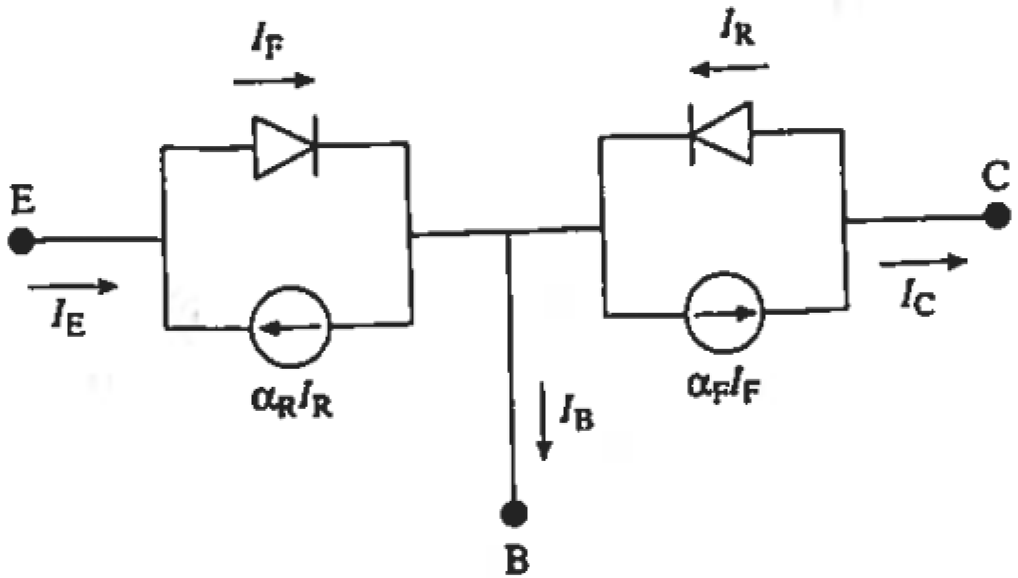
α_F and α_R: Forward and Reverse Gains

- Rewriting I_E and I_C equations yields:

$$I_E = I_{F0}(e^{qV_{EB}/kT} - 1) - \alpha_R I_{R0}(e^{qV_{CB}/kT} - 1)$$

$$I_C = \alpha_F I_{F0}(e^{qV_{EB}/kT} - 1) - I_{R0}(e^{qV_{CB}/kT} - 1)$$

- Those equations can be represented by the Ebers-Moll BJT model shown below:



$$I_F = I_{F0}(e^{qV_{EB}/kT} - 1)$$

$$I_R = I_{R0}(e^{qV_{CB}/kT} - 1)$$