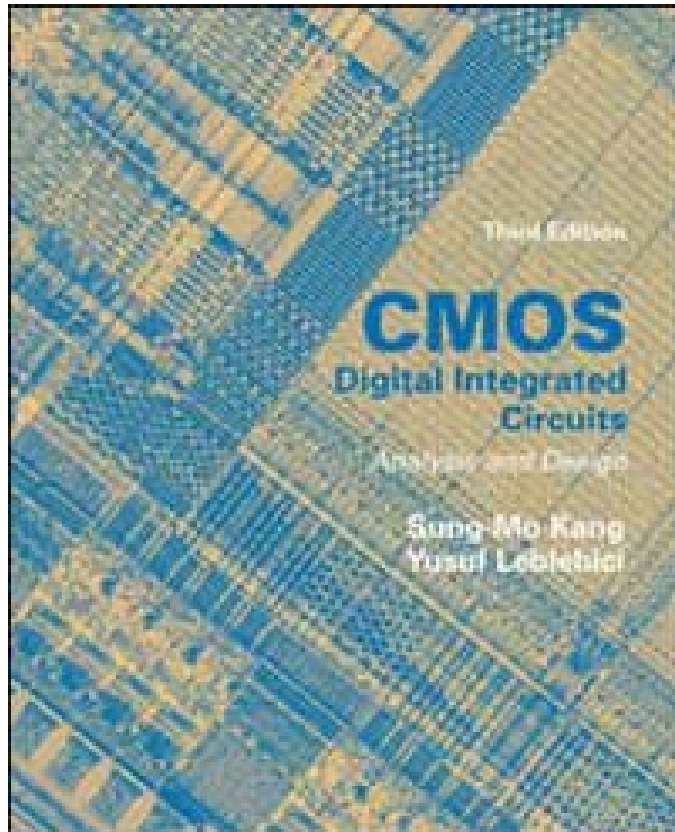


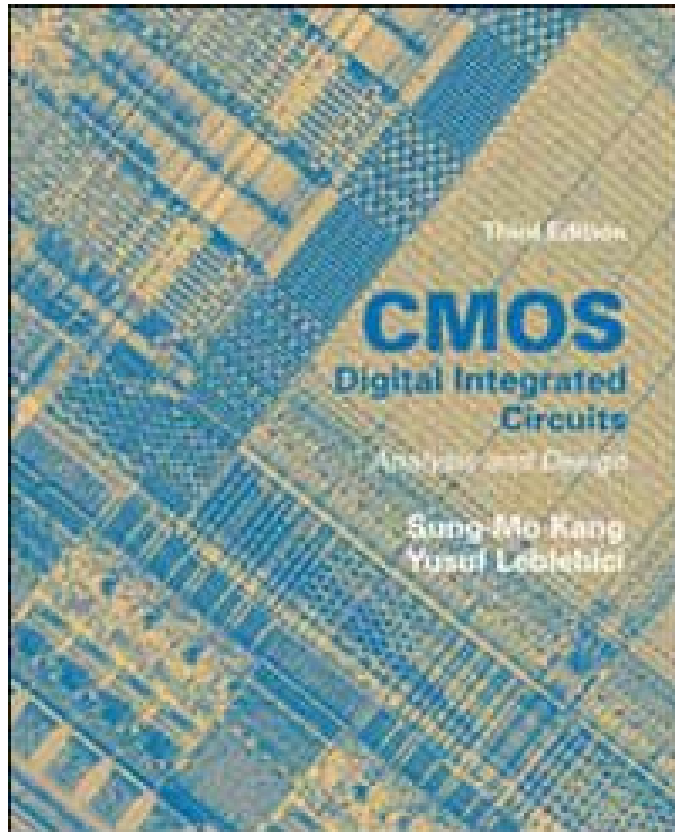
Digital IC Design and Architecture



MOS Transistor

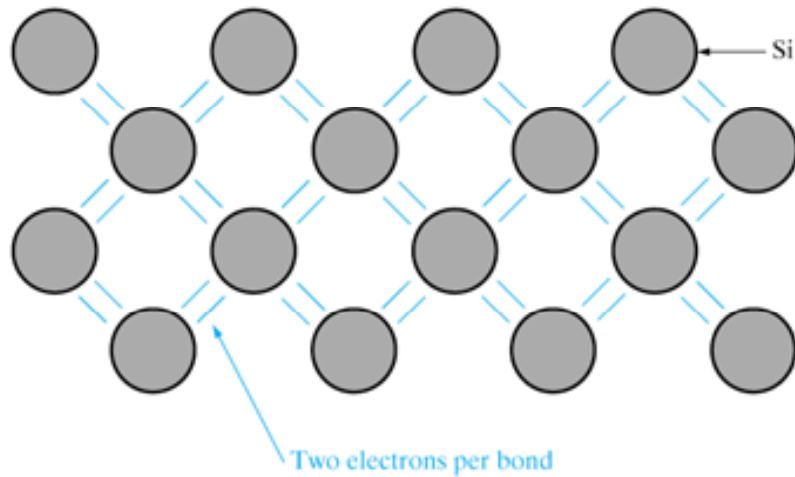
Goal of this chapter

- Present intuitive understanding of device operation
- Introduction of basic device equations of PN junction
- Introduction of basic device equations of FET
- Analysis of secondary and deep-sub-micron effects
- Review: ideal / non-ideal switch

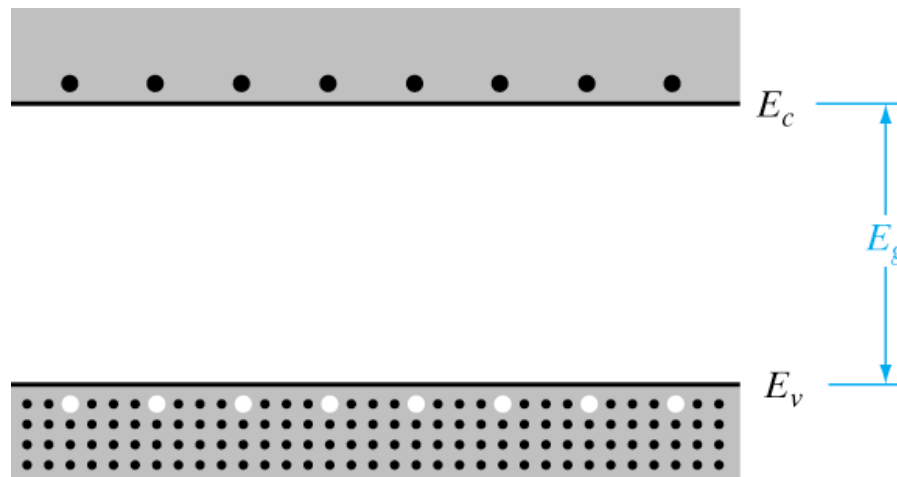


Properties of Si

Properties of Si

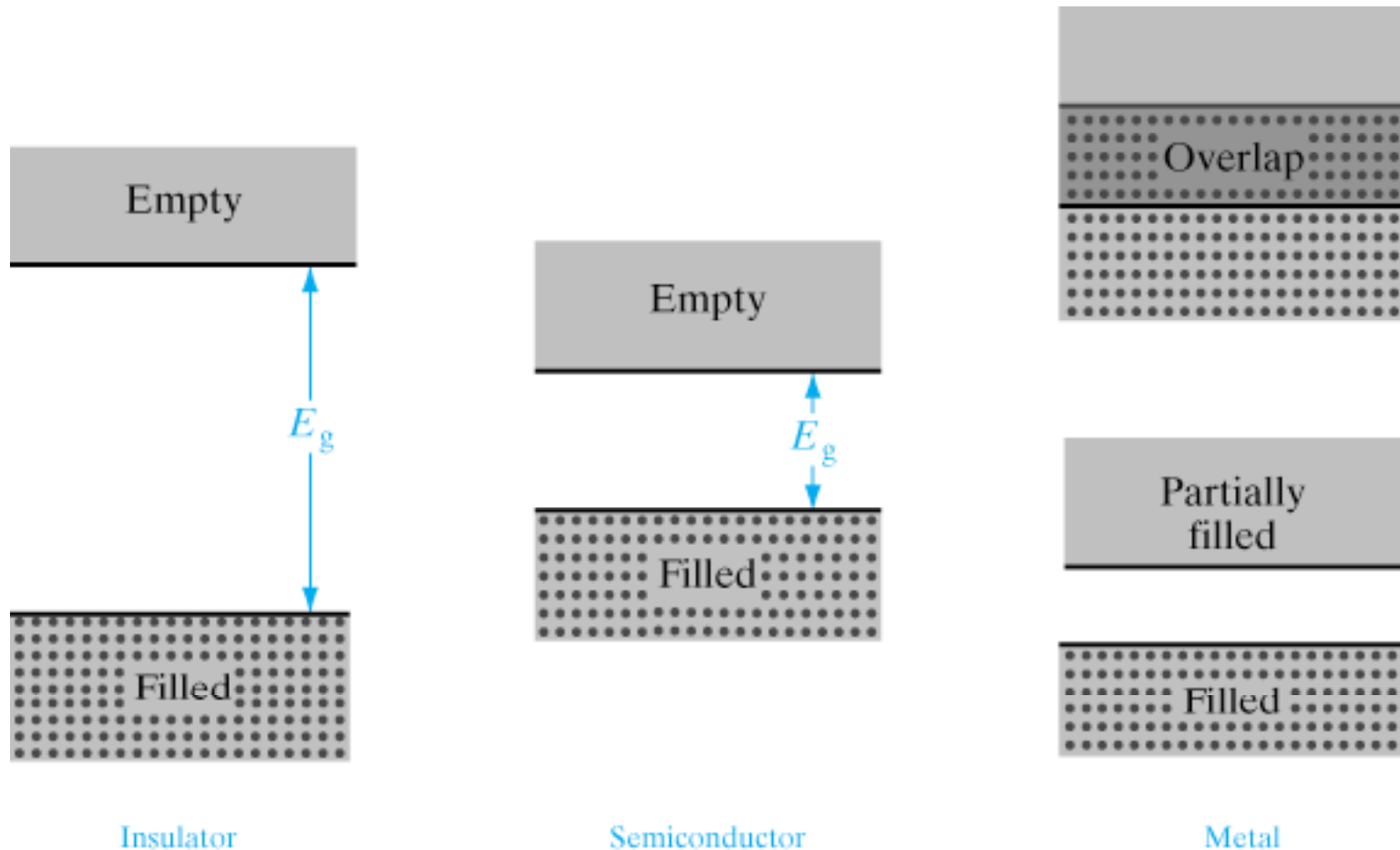


- covalent bonding in the Si crystal, viewed along a $\langle 100 \rangle$ direction



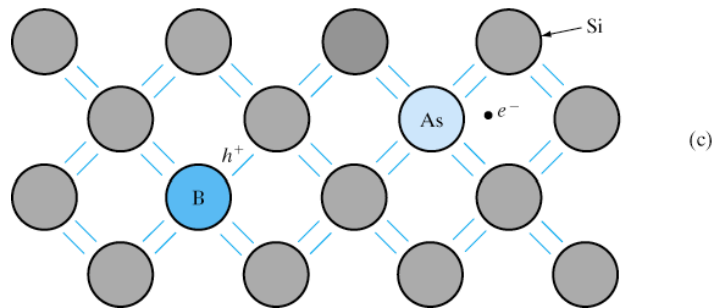
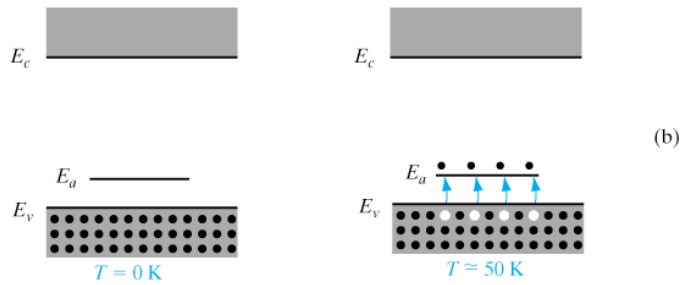
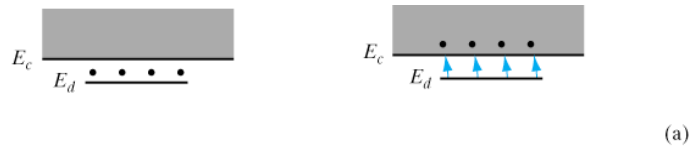
Energy band diagram of Si:
 $E_g = 1.1\text{eV}$

Typical band structures at 0 K



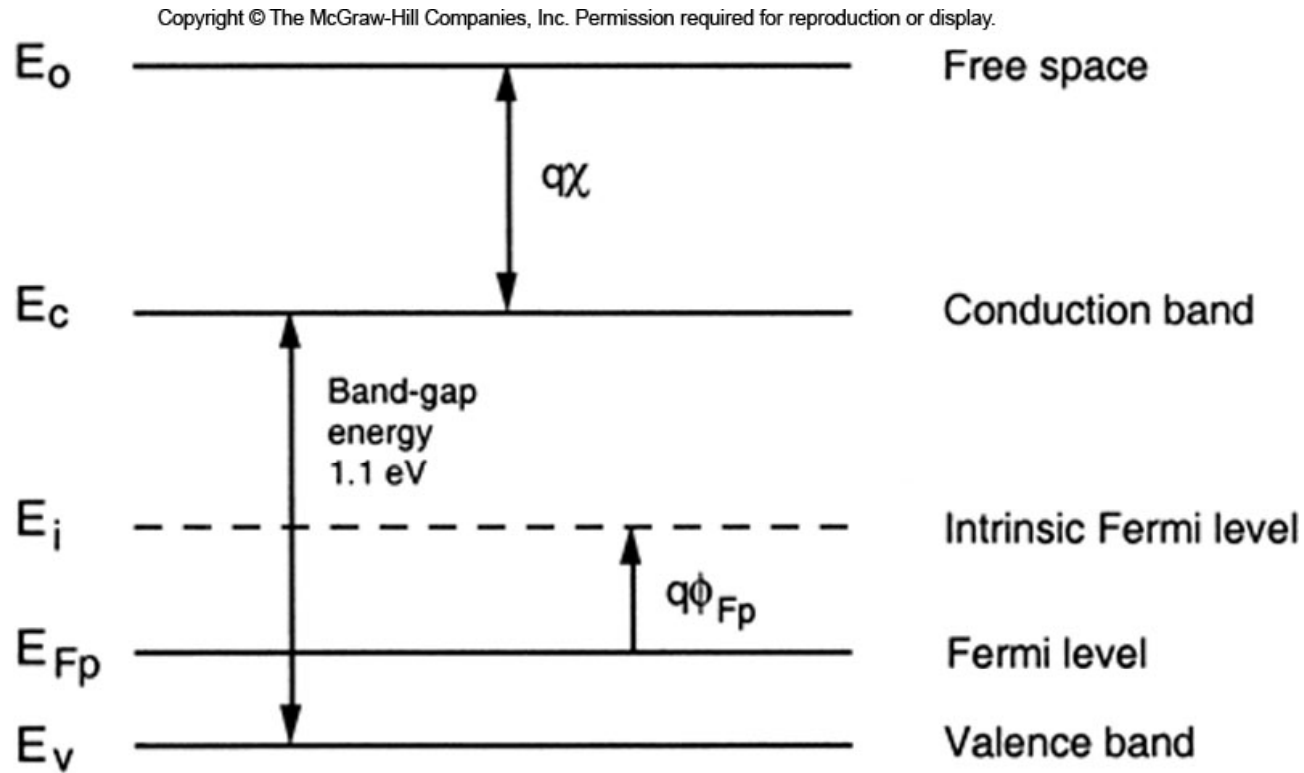
A. Insulators have large band gaps which prevents electrons to “jump” from valence to conduction band. B. Semiconductors have smaller band gaps such that electrons can be thermally excited to the conduction band C. In metals, large number of electrons exist in the valence band.

Doping of Silicon



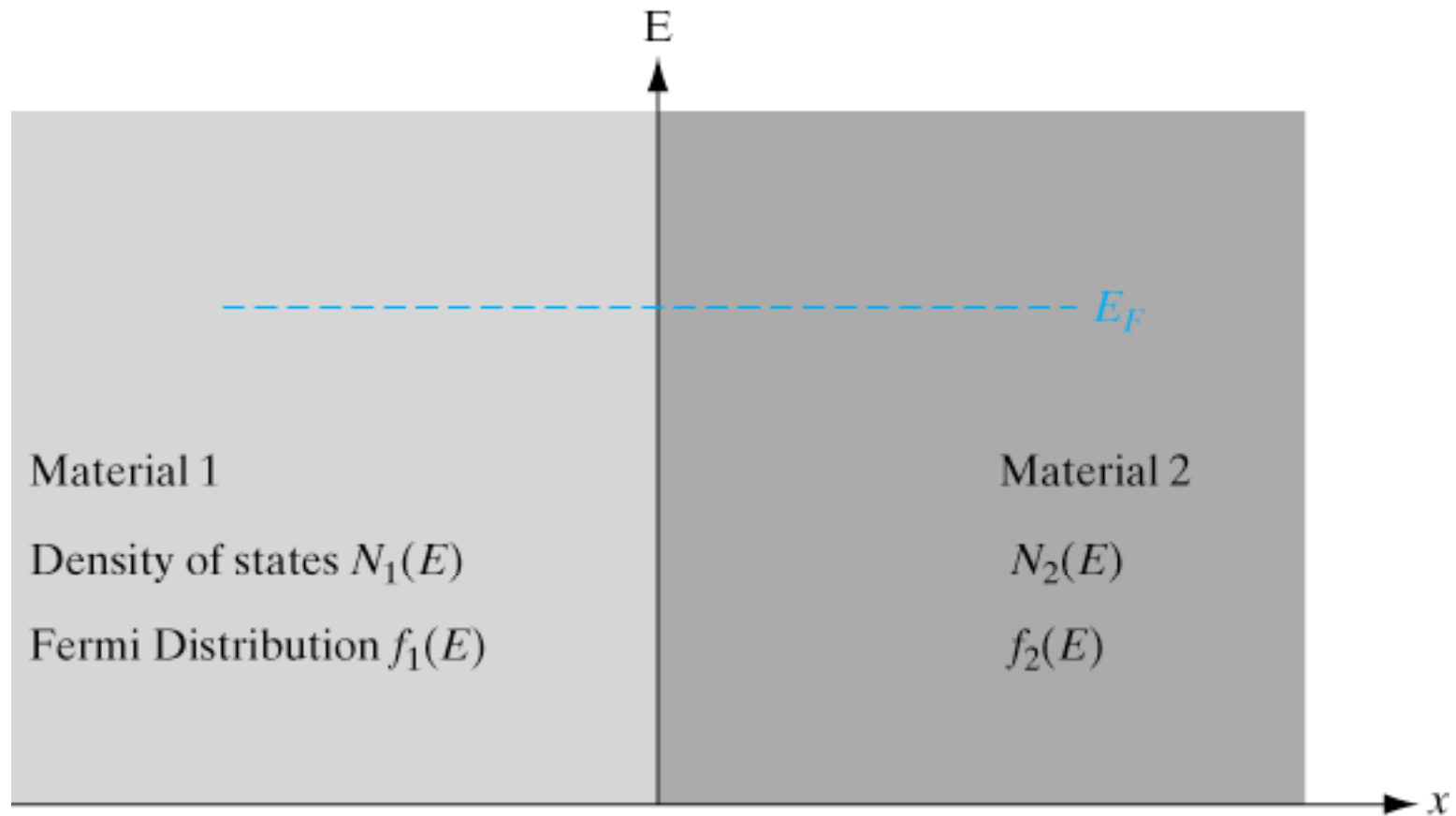
Energy band model and chemical bond model of dopants in semiconductors: (a) donation of electrons from donor level to conduction band; (b) acceptance of valence band electrons by an acceptor level, and the resulting creation of holes; (c) donor and acceptor atoms in the covalent bonding model of a Si crystal.

Energy Band Diagram of p-type Silicon



$$\phi_p = \frac{kT}{q} \ln \frac{n_i}{N_A} \quad \phi_F = \frac{E_F - E_i}{q}$$

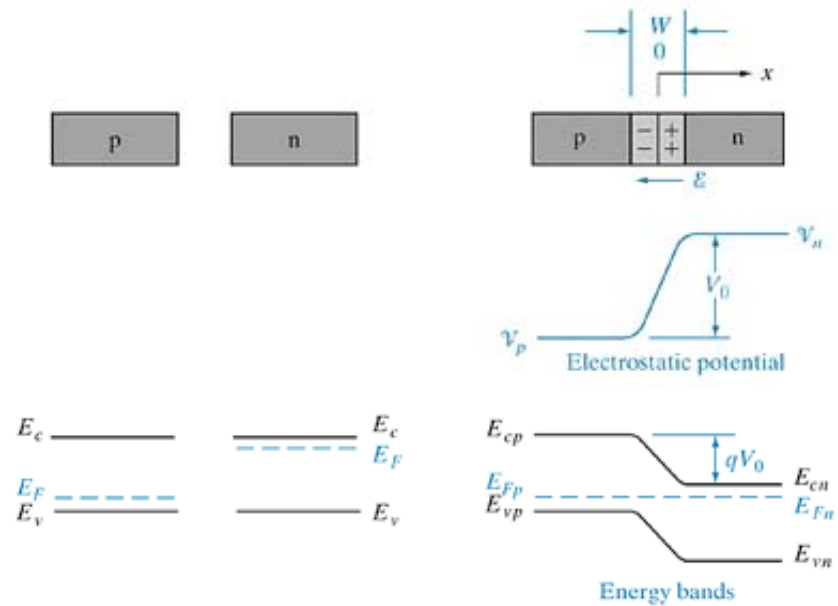
Junction between two materials



Two materials in intimate contact at equilibrium.

Note: *Since the net motion of electrons is zero, the equilibrium Fermi level must be constant throughout.*

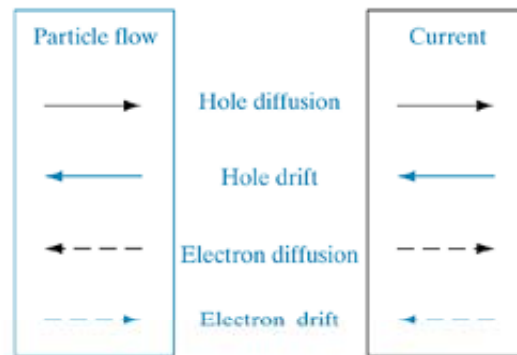
PN Junction: Energy Band



$$V_0 = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2}$$

(a)

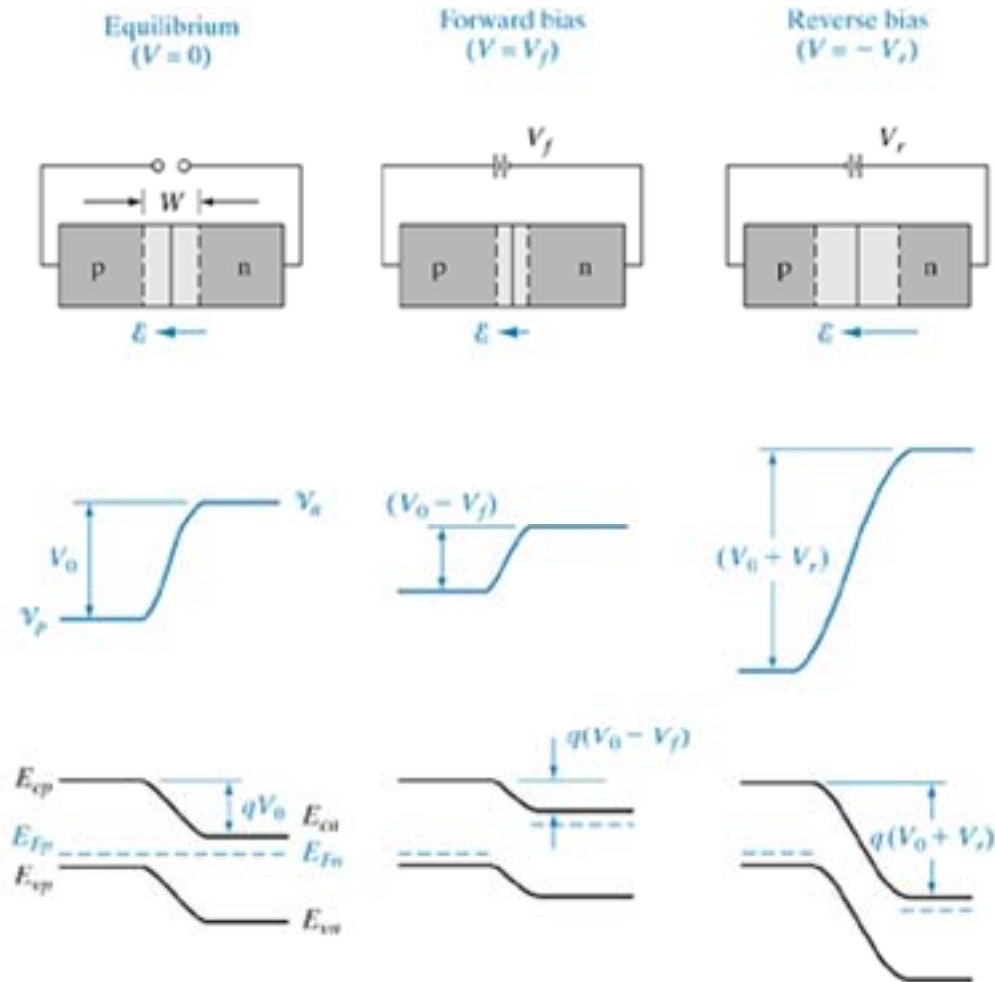
(b)

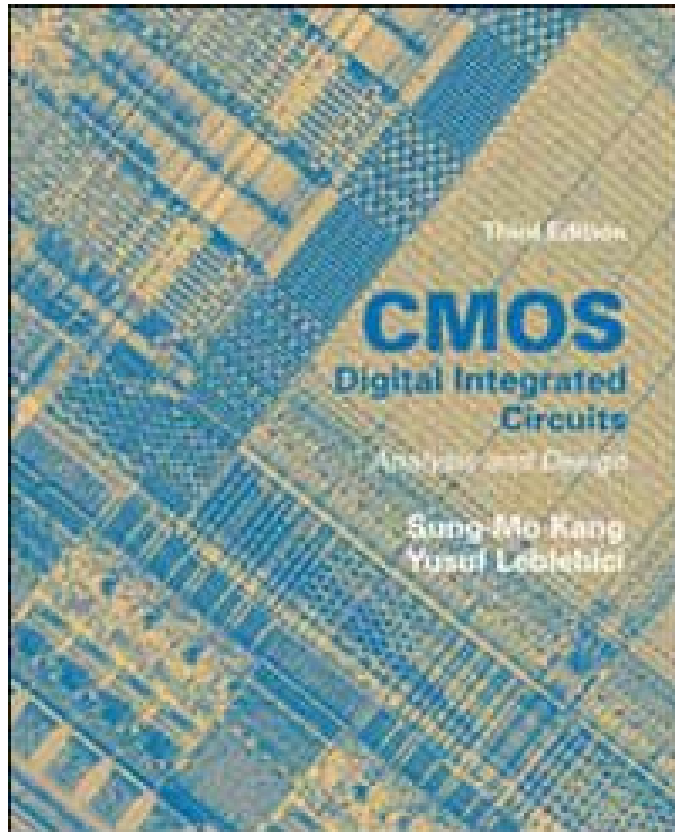


(c)

$$W = \left[\frac{2\epsilon V_0}{q} \left(\frac{N_a + N_d}{N_a N_d} \right) \right]^{1/2}$$

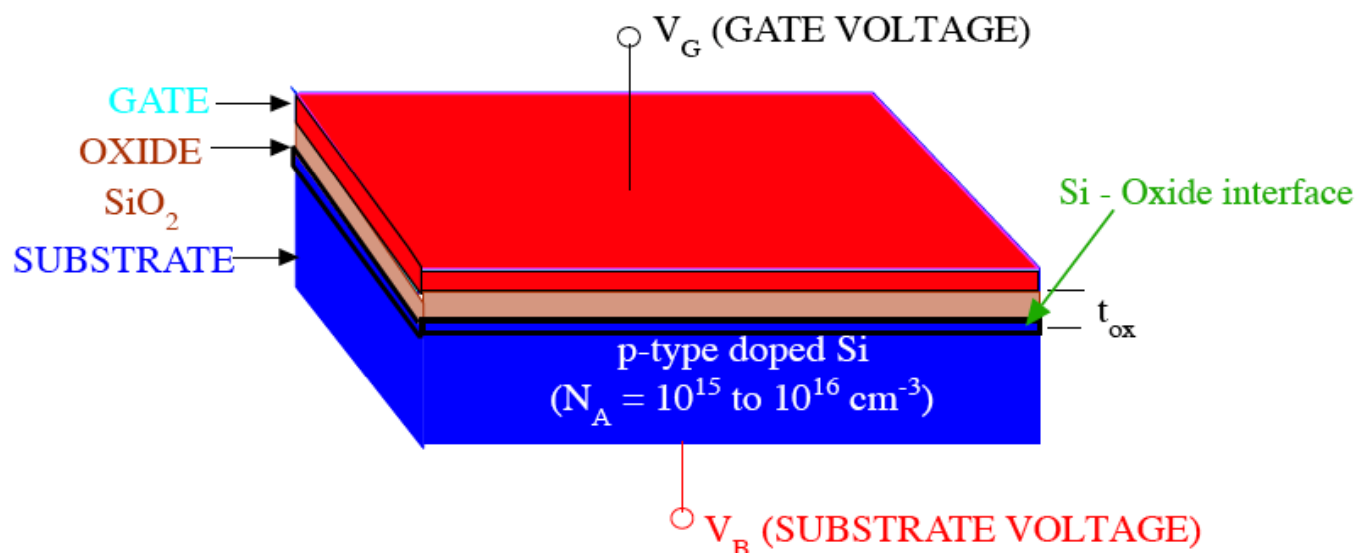
PN Junction Under External Bias





MOSFET Energy Band Diagram

Two terminal MOS Structure



EQUILIBRIUM: $n p = n_i^2$ ($n_i \approx 1.45 \times 10^{10} \text{ cm}^{-3}$)

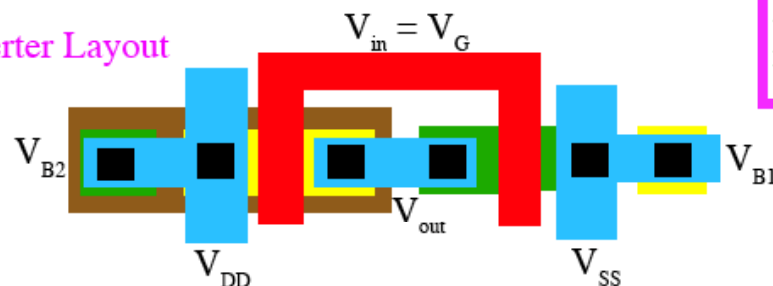
Let SUBSTRATE be uniformly doped @ N_A

$$n_{p0} \approx \frac{n_i^2}{N_A} \text{ and } p_{p0} = N_A$$

MASS ACTION LAW

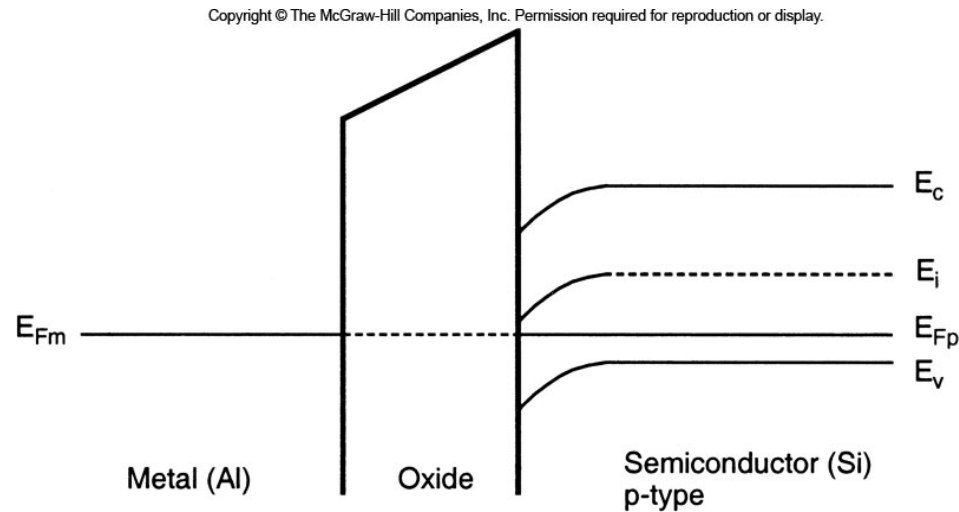
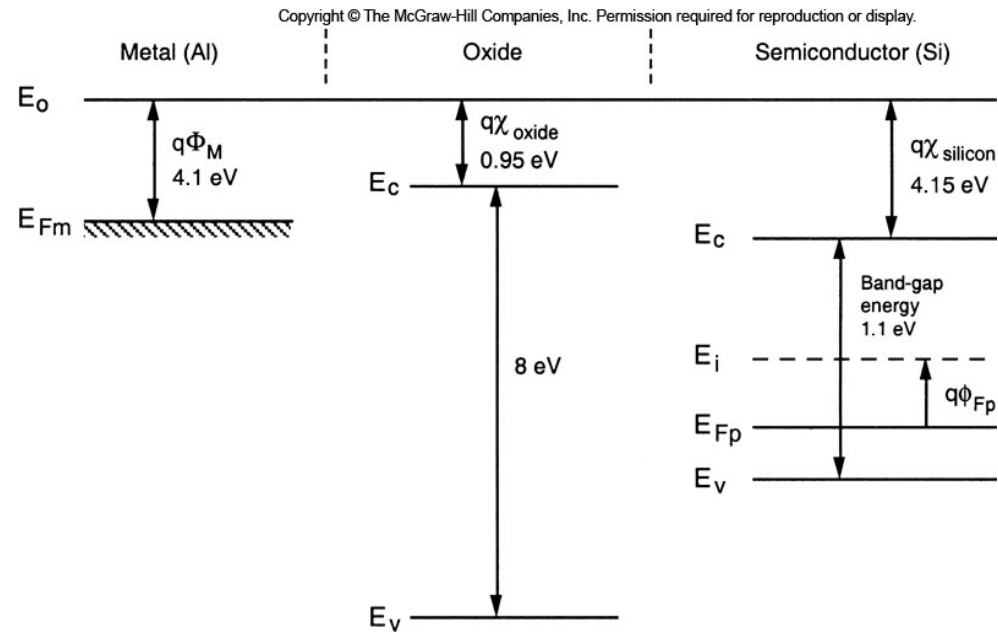
(BULK carrier concentrations)

N-well CMOS Inverter Layout



If n-type doped Si:
 $n_{n0} = N_D$ and $p_{n0} = n_i^2/N_D$

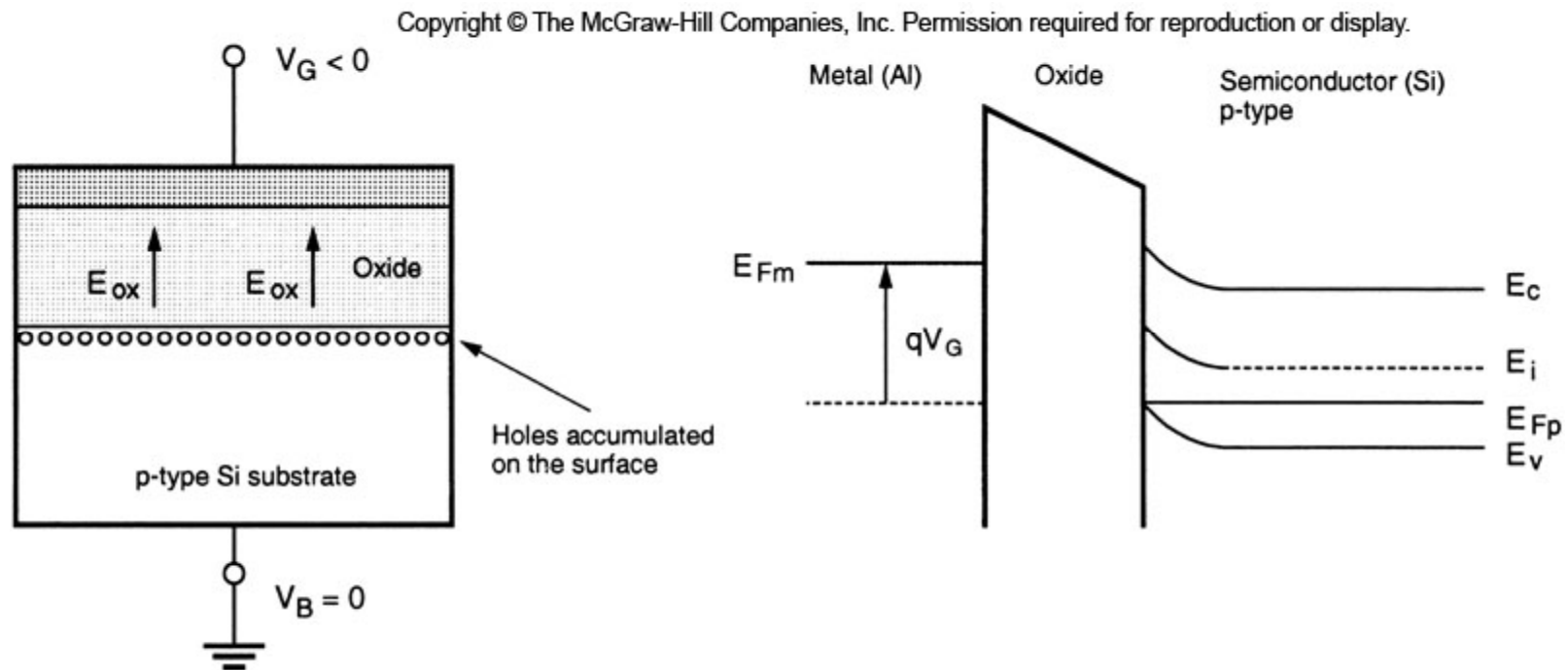
Energy Band Diagram of FET



Two-Terminal MOS Structure with External Bias

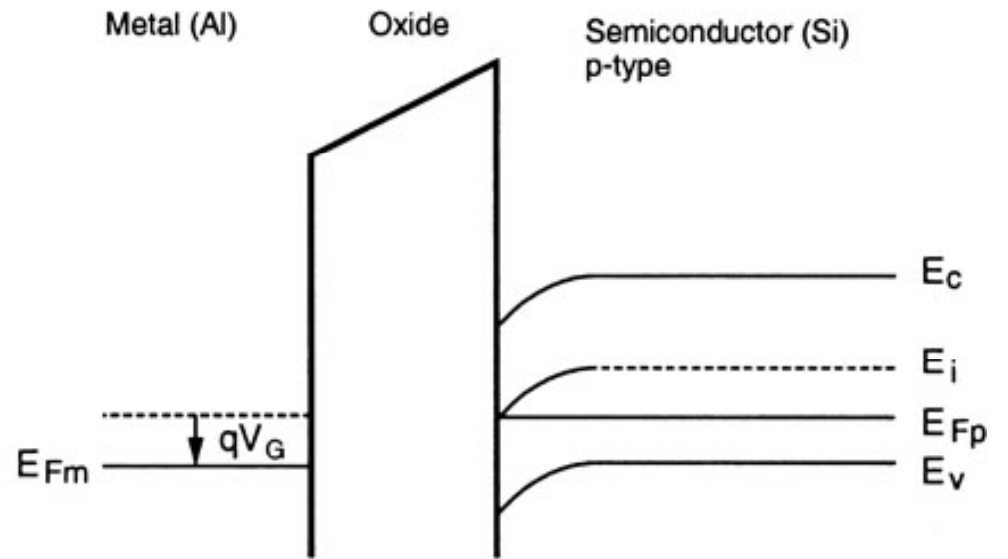
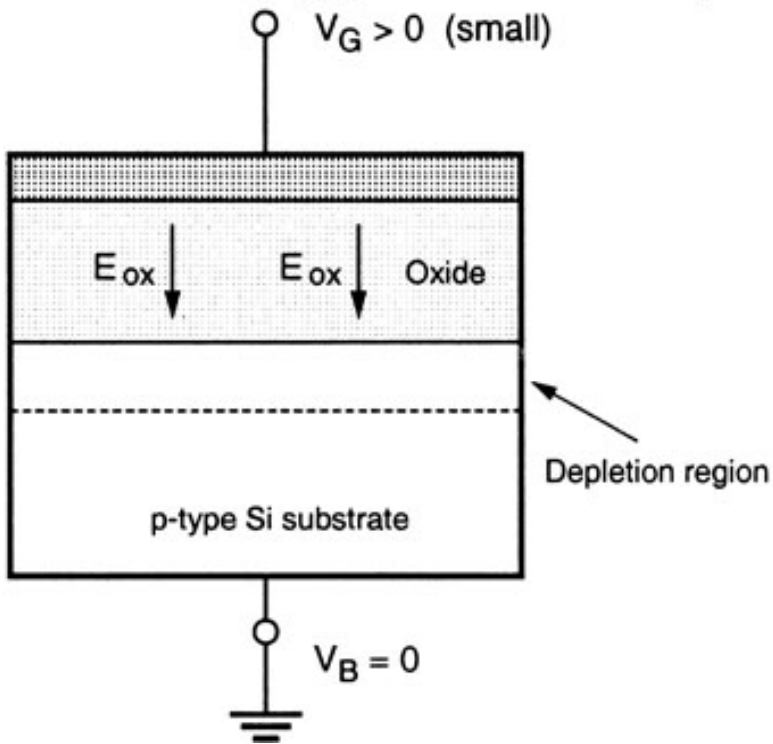
Three Regions of Operation:

1. Accumulation Region: $V_G < 0$
2. Depletion Region: $V_G > 0$, small
3. Inversion Region: $V_G > 0$, large



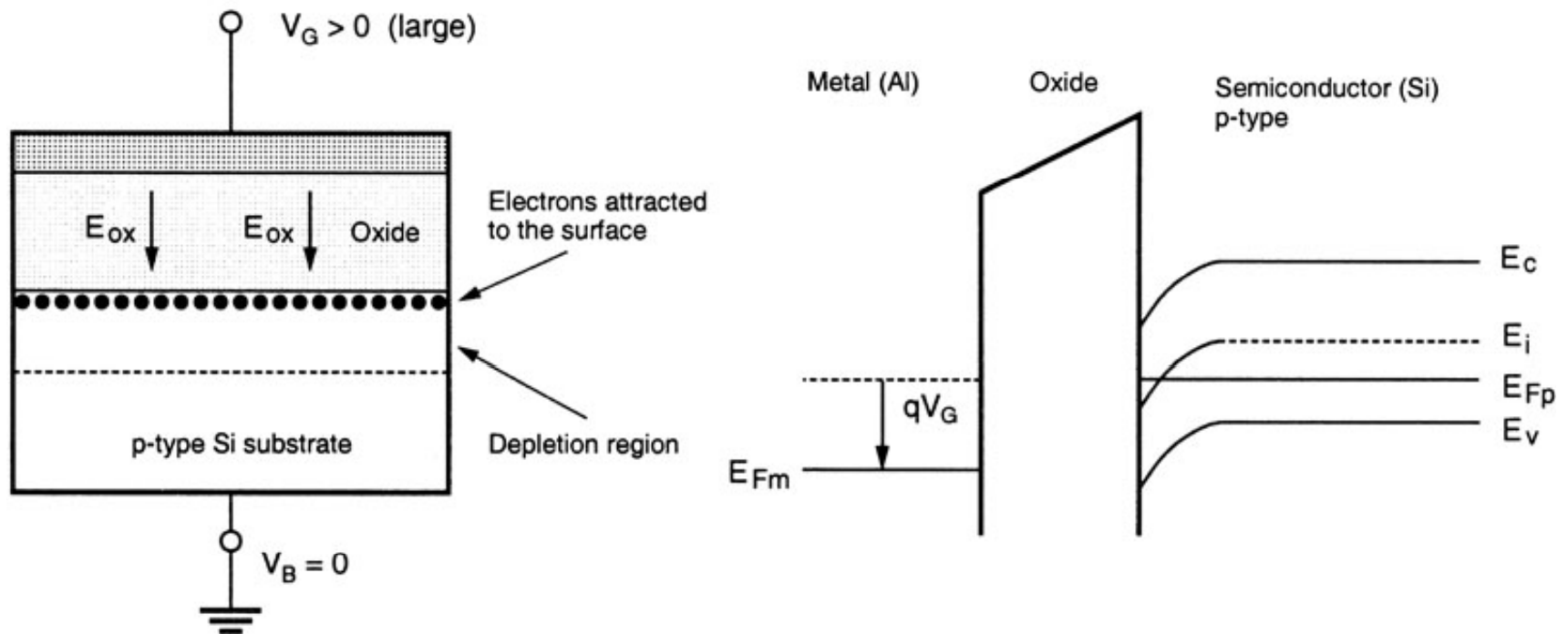
Two-Terminal MOS Structure Depletion Region

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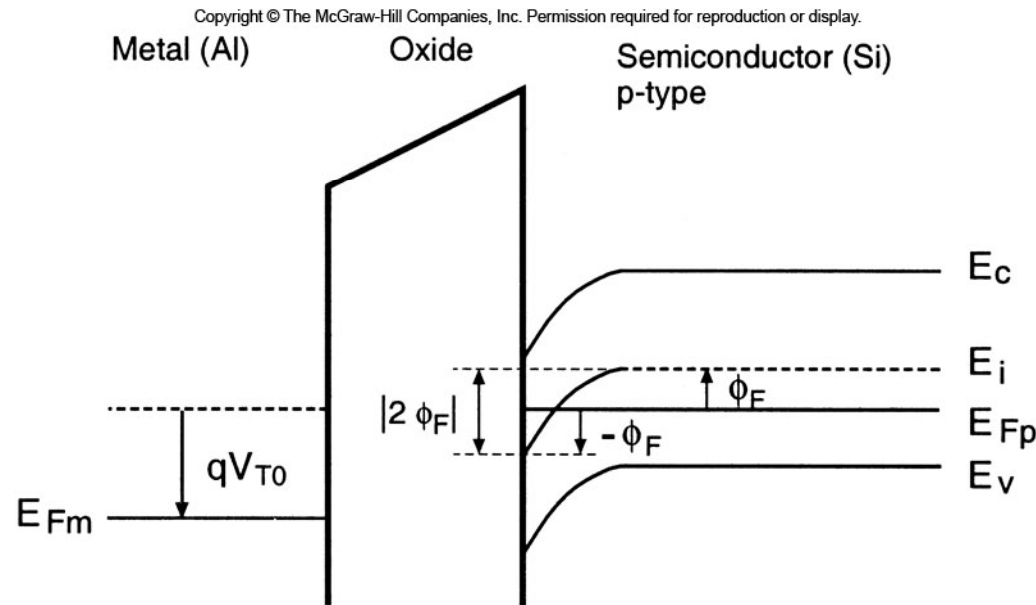


Two-Terminal MOS Structure Inversion Region

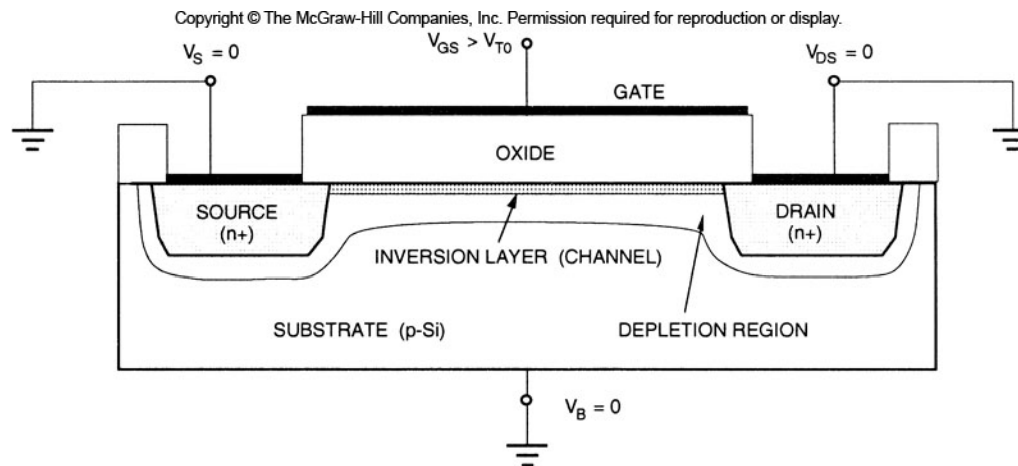
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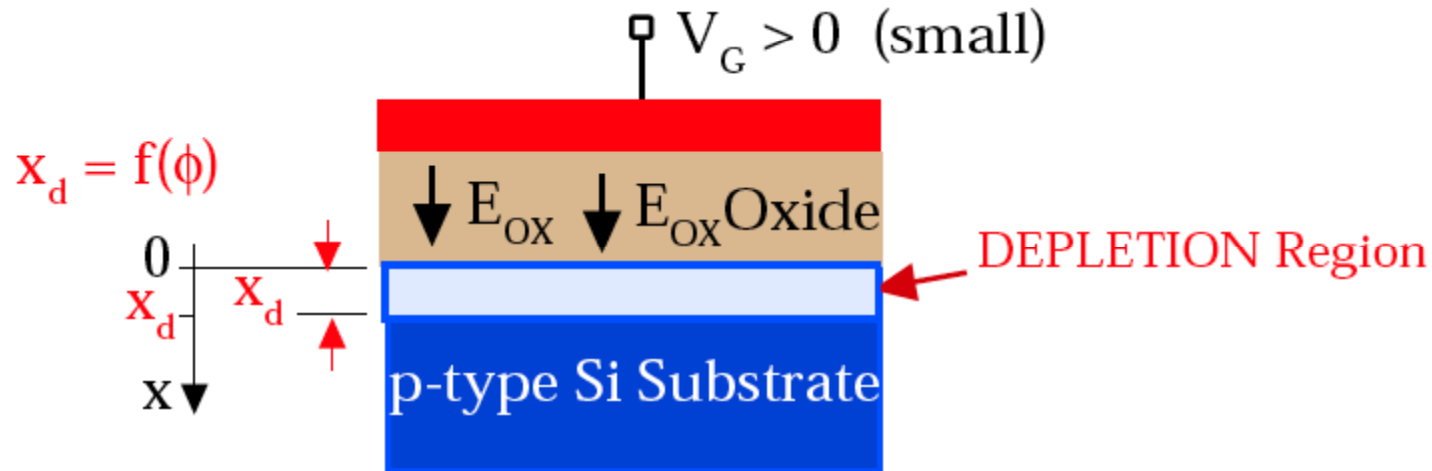
Two-Terminal MOS Structure Energy Band Diagram



Bending of the semiconductor bands at the onset of strong inversion: *the surface potential f_s is twice the value of f_F in the neutral p material.*



Two-Terminal MOS Structure Depletion Region



ϕ = voltage across the depletion region

$V_B = 0$ Mobile hole charge density in thin layer parallel to Si-Oxide interface

$$dQ = -qN_A dx$$

$(dV=dQ/C)$ $d\phi = \frac{-x}{\epsilon_{Si}} dQ = x \frac{qN_A}{\epsilon_{Si}} dx$ Change in surface potential to displace dQ by distance x into bulk (Poisson eq)

$$\int_{\phi_F}^{\phi_s} d\phi = \int_0^{x_d} x \frac{qN_A}{\epsilon_{Si}} dx \quad \longrightarrow \quad \phi_s - \phi_F = \frac{qN_A}{2\epsilon_{Si}} x_d^2$$

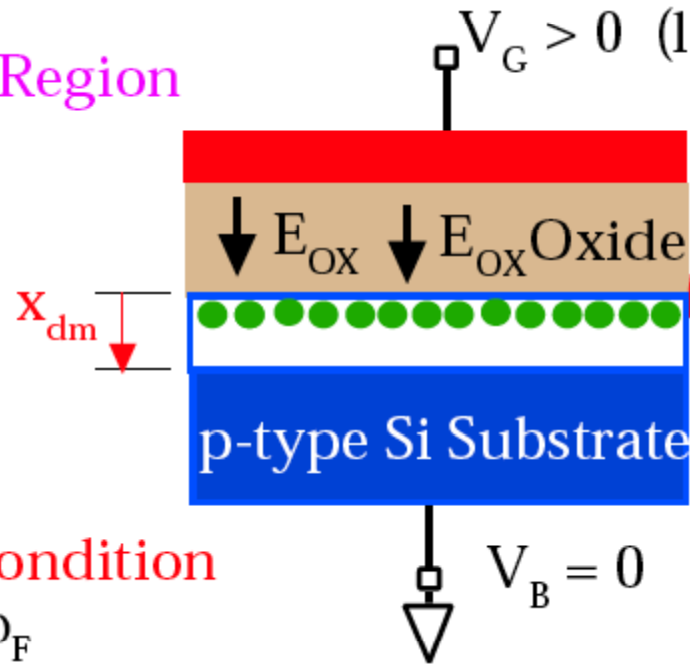
$$x_d = \sqrt{\frac{2\epsilon_{Si} |\phi_s - \phi_F|}{qN_A}}$$

Depletion Region Charge Density

$$Q = -qN_A x_d = -\sqrt{qN_A \epsilon_{Si} |\phi_s - \phi_F|}$$

Two-Terminal MOS Structure Inversion Region

Inversion Region



INVERSION Region
electrons
 attracted to Si-Oxide interface.
holes
 repelled deeper into Si bulk

Inversion Condition

$$\phi_s = -\phi_F$$

$$x_{dm} = x_d |_{\phi_s = -\phi_F} = \sqrt{\frac{2\epsilon_{Si}|-2\phi_F|}{qN_A}}$$

$$\phi_{Fp} = \frac{kT}{q} \ln \frac{n_i}{N_A} \quad (N_A \gg n_i)$$

$$Q_{B0} = Q |_{x_d = x_{dm}} = -qN_A x_{dm}$$

$$= -\sqrt{qN_A \epsilon_{Si}|-2\phi_F|}$$

Q_{B0} -> depletion charge density at surface inversion ($\phi = -\phi_F$)

$$\phi_{Fn} = \frac{kT}{q} \ln \frac{N_D}{n_i} \quad (N_D \gg n_i)$$

$$n_i = 1.45 \times 10^{10} \text{ cm}^{-3} \text{ @ room temp,}$$

$$k = 1.38 \times 10^{-23} \text{ J/}^\circ\text{K,}$$

$$q = 1.6 \times 10^{-19} \text{ C}$$

$$\Rightarrow kT/q = 26 \text{ mV @ room temp}$$

Threshold Voltage for MOS Transistors

n-channel enhancement

For $V_{SB} = 0$, the threshold voltage is denoted as V_{T0} or $V_{T0n,p}$ [$V_{T0} \rightarrow VT0$ in SPICE]

$$V_{T0} = \underbrace{\Phi_{GC}}_{V_{FB}} - 2\phi_F - \frac{Q_{ox}}{C_{ox}} - \frac{Q_{B0}}{C_{ox}} \quad \begin{array}{l} + \text{ for nMOS} \\ - \text{ for pMOS} \end{array}$$

$V_{FB} \rightarrow$ Flat Band Voltage [$V_{FB} = VFB$ in SPICE]

$[2\phi_F = PHI$ in SPICE]

$$Q_{B0} = -\sqrt{2qN_A\epsilon_{Si}|-2\phi_F|} \text{ C/cm}^2 \quad [N_A = NSUB \text{ in SPICE}]$$

$Q_{B0} \rightarrow$ depletion charge density at surface inversion ($\phi_s = -\phi_F$)

$$\Phi_{GC} = \phi_F(\text{substrate}) - \phi_M \quad \text{metal gate}$$

$$\Phi_{GC} = \phi_F(\text{substrate}) - \phi_F(\text{gate}) \quad \text{polysilicon gate}$$

$\Phi_{GC} = \phi_{F(\text{sub})} - \phi_{F(\text{gate})} \rightarrow$ work function between gate and channel

$$Q_{ox} = qN_{ox} \text{ C/cm}^2 \quad [Q_{ox} = qNSS \text{ in SPICE}]$$

$Q_{ox} \rightarrow$ charge density at gate Si-oxide interface due to impurities and lattice imperfections at the interface.

Threshold Voltage factors:

- > Gate conductor material;
- > Gate oxide material & thickness;
- > Substrate doping;
- > Impurities in Si-oxide interface;
- > Source-bulk voltage V_{SB} ;
- > Temperature.

Adjusting V_{T0} Using and an Added Channel Implant

$$V_{T0} = \Phi_{GC} - 2\phi_F - \frac{Q_{ox}}{C_{ox}} - \frac{Q_{B0}}{C_{ox}}$$

Intrinsic V_{T0} - no channel implant adjustment

$$V'_{T0} = V_{T0} + \Delta V_{T0} = \Phi_{GC} - 2\phi_F - \frac{Q_{ox}}{C_{ox}} - \frac{Q_{B0}}{C_{ox}} \pm \frac{qN_I}{C_{ox}}$$

Adjusted V'_{T0} - due to channel implant adjustment with carrier concentration N_I

$$\Delta V_{T0} = \pm \frac{qN_I}{C_{ox}}$$

$+\frac{qN_I}{C_{ox}}$ for p-type implant
 $-\frac{qN_I}{C_{ox}}$ for n-type implant

NOTE: When channel implant adjustment N_I is done as a step in the CMOS process, the SPICE parameter VT0 refers to the adjusted threshold voltage V'_{T0} .

Threshold Voltage for MOS Transistors

n-channel enhancement $V_{T0} = V_{FB} - 2\phi_F - \frac{Q_{B0}}{C_{ox}}$ for $V_{SB} = 0$

For $V_{SB} \neq 0$: the threshold voltage is denoted as V_T or $V_{Tn,p}$

$$Q_B = -\sqrt{2qN_A\epsilon_{Si}|-2\phi_F + V_{SB}|}, \quad Q_{B0} = -\sqrt{2qN_A\epsilon_{Si}|-2\phi_F|}$$

$$\begin{aligned} V_T &= V_{FB} - 2\phi_F - \frac{Q_{ox}}{C_{ox}} - \frac{Q_B}{C_{ox}} \\ &= \underbrace{V_{FB} - 2\phi_F - \frac{Q_{ox}}{C_{ox}} - \frac{Q_{B0}}{C_{ox}}}_{V_{T0}} - \frac{Q_B - Q_{B0}}{C_{ox}} \end{aligned}$$

where

$$\frac{Q_B - Q_{B0}}{C_{ox}} = \underbrace{\sqrt{\frac{2qN_A\epsilon_{Si}}{C_{ox}}}}_{\gamma} \left(\sqrt{|-2\phi_F + V_{SB}|} - \sqrt{|-2\phi_F|} \right)$$

(γ = Body-effect coefficient) [γ = GAMMA in SPICE]

$$V_T = V_{T0} + \gamma \left(\sqrt{|-2\phi_F + V_{SB}|} - \sqrt{|-2\phi_F|} \right)$$

Threshold Voltage for MOS Transistors

n-channel -> p-channel

****BE CAREFULL*** WITH SIGNS

- V_{FB} is negative in nMOS, positive in pMOS
- ϕ_F is negative in nMOS, positive in pMOS
- Q_{B0} , Q_B are negative in nMOS, positive in pMOS
- γ is positive in nMOS, negative in pMOS
- V_{SB} is positive in nMOS, negative in pMOS

NOTE: $\gamma \propto \frac{C_{BC}}{C_{GC}}$

EXAMPLE 3.2 Calculate the threshold voltage V_{T0n} at $V_{BS} = 0$, for a polysilicon gate n-channel MOS transistor with the following parameters:

substrate doping density $N_A = 10^{16} \text{ cm}^{-3}$,

gate oxide thickness $t_{ox} = 500 \text{ Angstroms}$,

flat band voltage $V_{FB} = -1.04 \text{ V}$,

oxide-sub interface charge $N_{ox} = 0 \text{ cm}^{-2} \Rightarrow Q_{ox} = 0$

dielectric permativities: $\epsilon_{ox} = 0.34 \times 10^{-12} \text{ Fcm}^{-1}$, $\epsilon_{Si} = 1.06 \times 10^{-12} \text{ Fcm}^{-1}$

$$\longrightarrow V_{T0n} = V_{FB} - 2\phi_{F(sub)} - \frac{Q_{ox}}{C_{ox}} - \frac{Q_{B0}}{C_{ox}} \longleftarrow$$

$\phi_{F(sub)}$:

$$\phi_{F(sub)} = \frac{kT}{q} \ln \frac{n_i}{N_A} = 0.026 \text{ V} \ln \left(\frac{1.45 \times 10^{10}}{10^{16}} \right) = -0.35 \text{ V}$$

C_{ox} :

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{0.34 \times 10^{-12} \text{ Fcm}^{-1}}{500 \times 10^{-8} \text{ cm}} = 6.8 \times 10^8 \text{ F/cm}^2$$

EXAMPLE 3-2 CONT.

$$V_{T0n} = V_{FB} - 2\phi_{F(\text{sub})} - \frac{Q_{B0}}{C_{ox}}$$

$$\epsilon_{ox} = 0.34 \times 10^{-12} \text{Fcm}^{-1},$$

$$\epsilon_{Si} = 1.06 \times 10^{-12} \text{Fcm}^{-1}$$

Q_{B0} :

$$Q_{B0} = -\sqrt{2qN_A\epsilon_{Si}|-2\phi_{F(\text{sub})}|}$$

$$= -\sqrt{2(1.6 \times 10^{19} \text{ C})(10^{16} \text{ cm}^{-3})(1.06 \times 10^{-12} \text{ Fcm}^{-1})|2 \times 0.35 \text{ V}|}$$

F = C/V

$$= -4.87 \times 10^{-8} \text{ C/cm}^2$$

$$\frac{Q_{B0}}{C_{ox}} = \frac{-4.87 \times 10^{-8} \text{ C/cm}^2}{6.8 \times 10^{-8} \text{ F/cm}^2} = -0.72 \text{ V}$$

$$V_{T0n} = -1.04 \text{ V} - 2(-0.35 \text{ V}) - (-0.72 \text{ V}) = 0.38 \text{ V}$$

EXAMPLE 3.3 Consider the n-channel MOS transistor in Exam₁ 18: 3.2:

- substrate doping density $N_A = 10^{16} \text{ cm}^{-3}$,
- gate oxide thickness $t_{\text{ox}} = 500 \text{ Angstroms}$,
- substrate Fermi potential $\Phi_{F(\text{sub})} = -0.35 \text{ V}$
- zero substrate bias threshold voltage $V_{T0n} = 0.38 \text{ V}$
- dielectric permittivities: $\epsilon_{\text{ox}} = 0.34 \times 10^{-12} \text{ Fcm}^{-1}$, $\epsilon_{\text{Si}} = 1.06 \times 10^{-12} \text{ Fcm}^{-1}$.

In digital circuit design, the condition $V_{\text{SB}} = 0$ can not always be guaranteed for all transistors. Plot the threshold voltage V_T as a function of V_{SB} .

$$\longrightarrow V_{Tn} = V_{T0n} + \gamma \left(\sqrt{|-2\phi_F + V_{\text{SB}}|} - \sqrt{|-2\phi_F|} \right) \longleftarrow$$

γ - Body-effect coefficient:

$$F = C/V$$

$$\begin{aligned} \gamma &= \frac{\sqrt{2qN_A \epsilon_{\text{Si}}}}{C_{\text{ox}}} = \frac{\sqrt{2(1.6 \times 10^{-19} \text{ C})(10^{16} \text{ cm}^{-3})(1.06 \times 10^{-12} \text{ Fcm}^{-1})}}{6.8 \times 10^8 \text{ F/cm}^2} \\ &= \frac{5.824 \times 10^{-8} \text{ C/V}^{-1/2} \text{ cm}^2}{6.8 \times 10^8 \text{ C/Vcm}^2} = 0.85 \text{ V}^{1/2} \end{aligned}$$

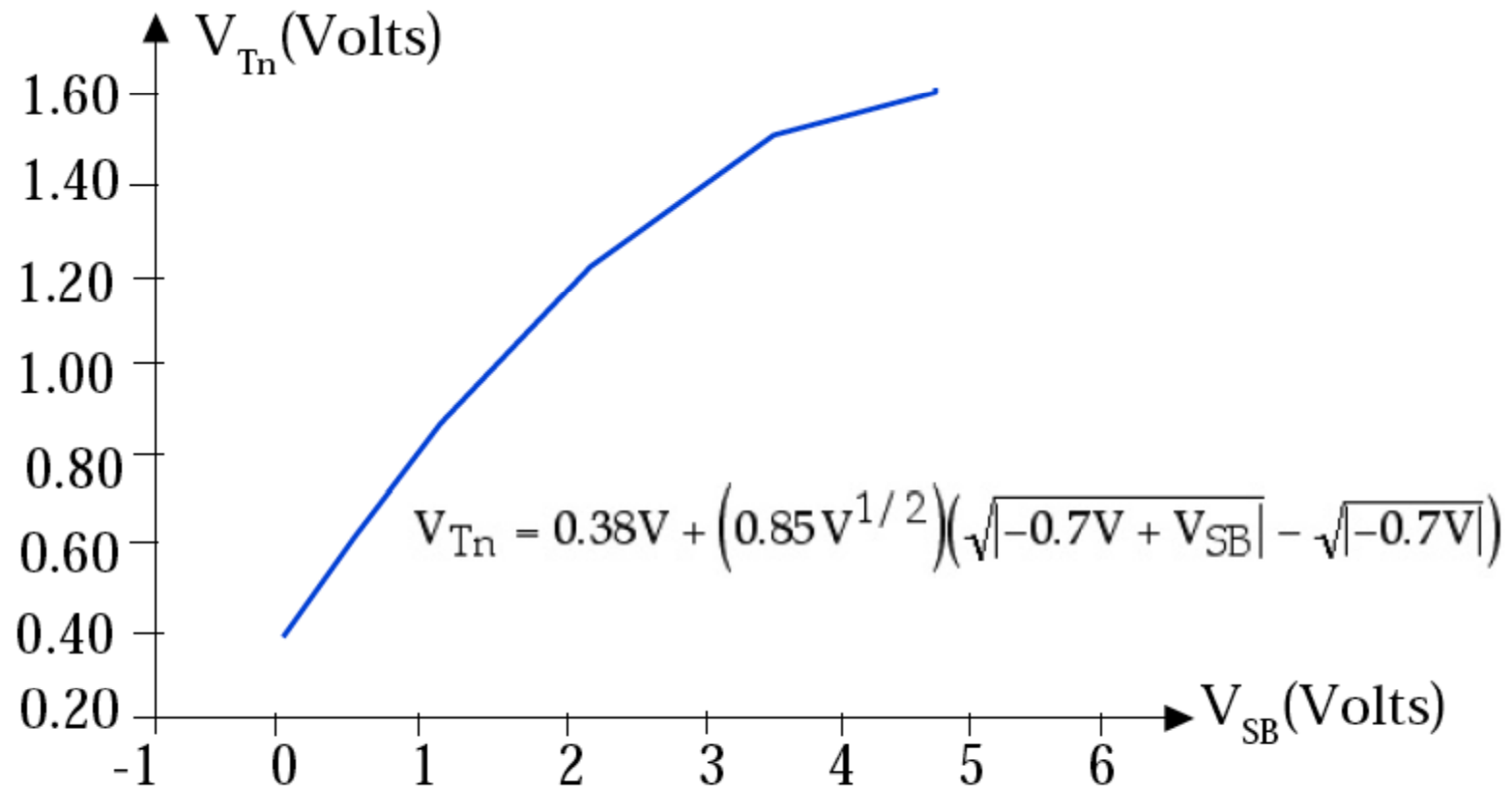
EXAMPLE 3-3 CONT.

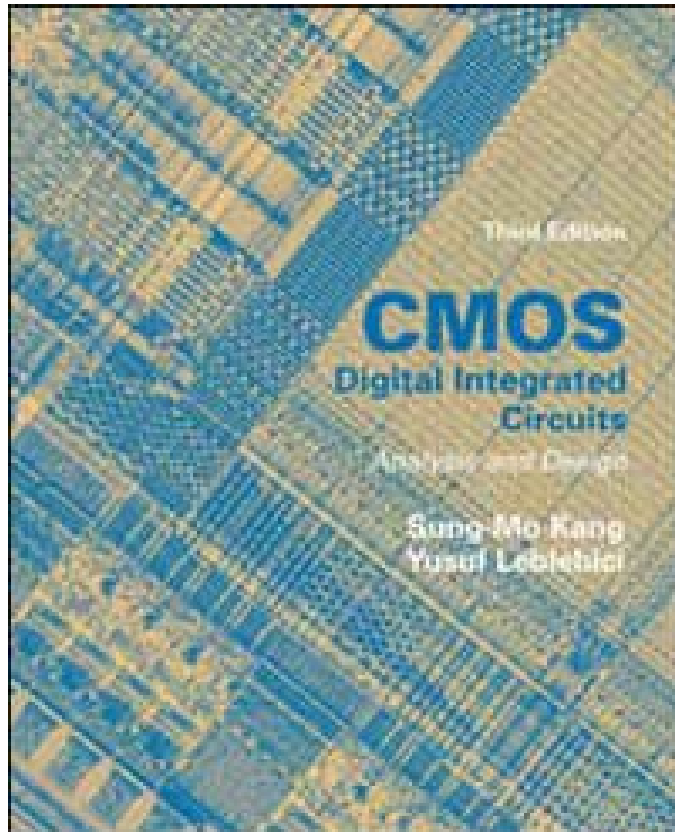
$$V_{Tn} = V_{T0n} + \gamma \left(\sqrt{|-2\phi_F + V_{SB}|} - \sqrt{|-2\phi_F|} \right)$$

where

$$V_{T0n} = 0.38V \quad (\text{from EX 3-2})$$

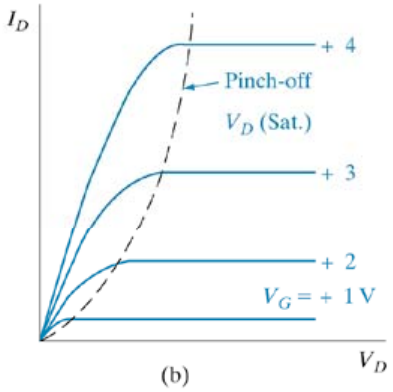
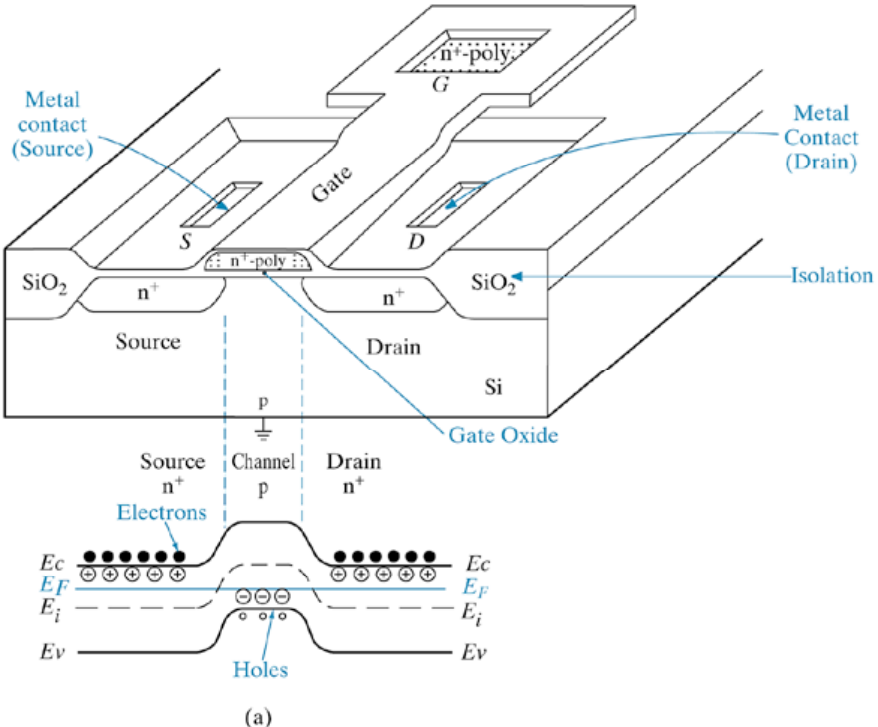
$$\gamma = 0.85 V^{1/2}$$





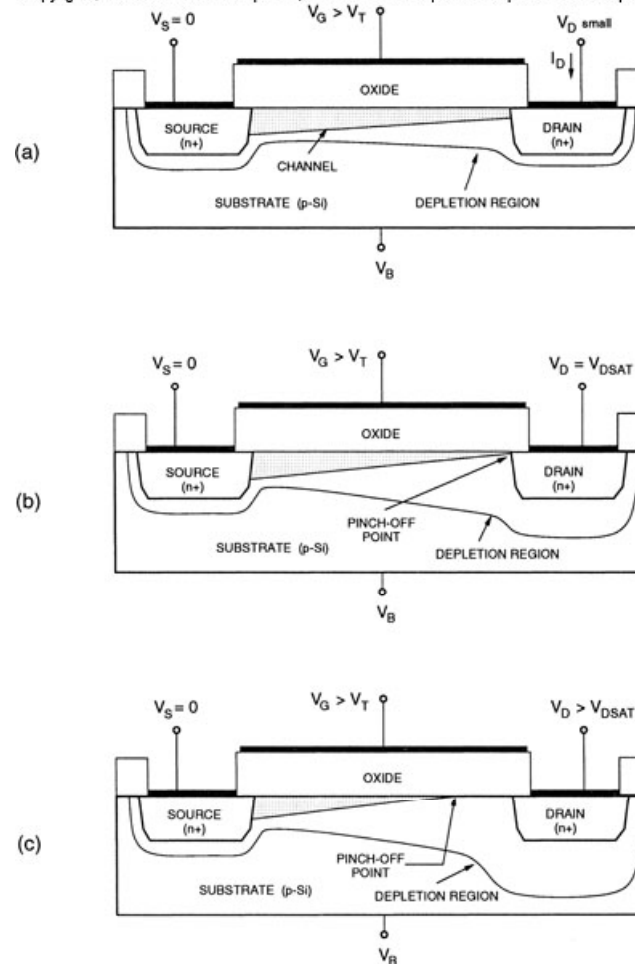
MOSFET I-V Characteristics

FET Band Diagram at Equilibrium



MOS Operation: A Qualitative View

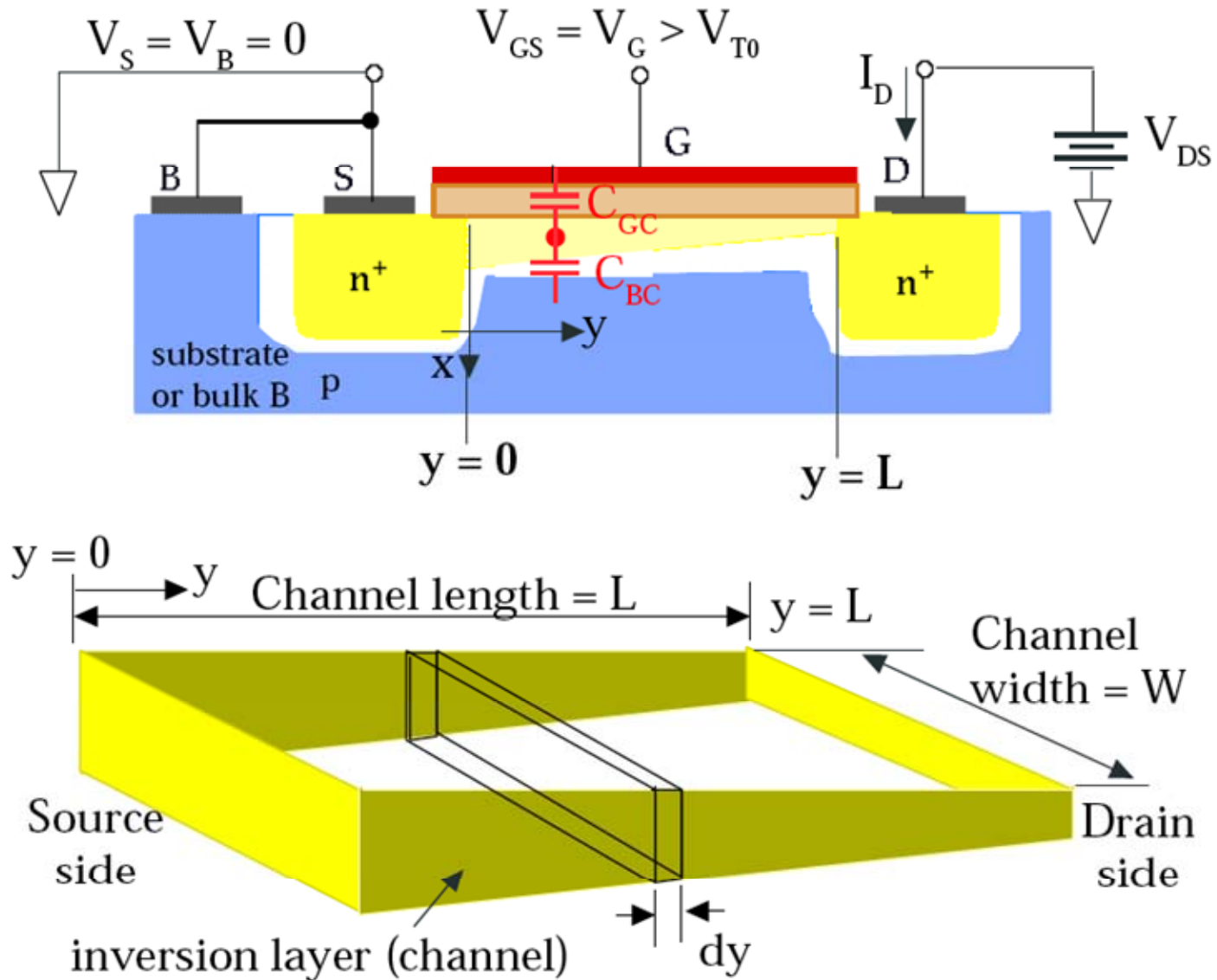
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n-channel MOSFET cross-sections under different operating conditions: (a) linear region for $V_G > V_T$ and $V_D < (V_G - V_T)$; (b) onset of saturation at pinch-off, $V_G > V_T$ and $V_D = (V_G - V_T)$; (c) strong saturation, $V_G > V_T$ and $V_D > (V_G - V_T)$.

Gradual Channel Approximation

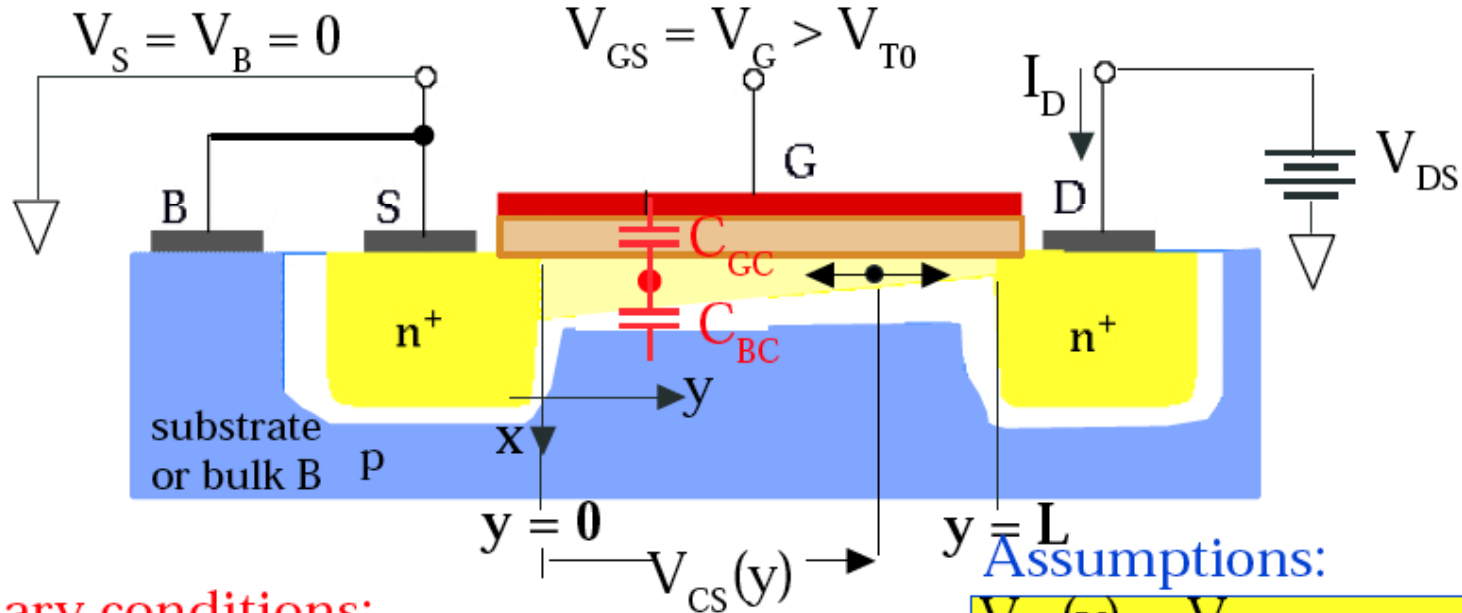
MOSFET CURRENT - VOLTAGE CHARACTERISTICS



Gradual Channel Approximation

MOSFET CURRENT - VOLTAGE CHARACTERISTICS

5



Boundary conditions:

$$\begin{aligned} V_{CS}(y=0) &= V_S = 0 \\ V_{CS}(y=L) &= V_{DS} \end{aligned}$$

Assumptions:

$$\begin{aligned} V_{T0}(y) &= V_{T0} \\ V_{GS} &> V_{T0} \\ V_{GD} &= V_{GS} - V_{DS} > V_{T0} \\ E_y &\gg E_x \end{aligned}$$

Mobile charge in channel:

$$Q_I(y) = -C_{ox} [V_{GS} - V_{CS}(y) - V_{T0}] \quad (\text{C/cm}^2)$$

$$\begin{aligned} \mu_n Q_I(y) &= \left(\frac{\text{cm}^2}{\text{V} \cdot \text{s}} \right) \left(\frac{\text{C}}{\text{cm}^2} \right) \\ &= \frac{\text{C/s}}{\text{V}} = \frac{I}{V} \end{aligned}$$

Incremental R for differential channel segment

$$dR = -\frac{dy}{W} \left(\frac{1}{\mu_n Q_I(y)} \right) \quad \begin{aligned} \mu_n &= \text{electron mobility} \\ &= \text{cm}^2/\text{Vsec} \\ &[\mu \rightarrow U0 \text{ in SPICE}] \end{aligned}$$

Gradual Channel Approximation

MOSFET CURRENT - VOLTAGE CHARACTERISTICS

Boundary conditions:

$$V_{CS}(y=0) = V_S = 0$$

$$V_{CS}(y=L) = V_{DS}$$

$$Q_I(y) = -C_{ox} [V_{GS} - V_{CS}(y) - V_{T0}]$$

$$dR = -\frac{dy}{W} \left(\frac{1}{\mu_n Q_I(y)} \right)$$

Voltage drop across

incremental segment dy

$$dV_{CS} = I_D dR = -\frac{I_D}{W \mu_n Q_I(y)} dy$$

Integrating along the channel $0 \leq y \leq L$ and $0 \leq V_{CS} \leq V_{DS}$:

$$\int_0^L I_D dy = -W \mu_n \int_0^{V_{DS}} Q_I(y) dV_{CS}$$

i.e.
$$\int_0^L I_D dy = W \mu_n C_{ox} \int_0^{V_{DS}} [V_{GS} - V_{CS} - V_{T0}] dV_{CS}$$

$$I_D y \Big|_{y=0}^{y=L} = W \mu_n C_{ox} \left[(V_{GS} - V_{T0}) V_{CS} - \frac{V_{CS}^2}{2} \right]_{V_{CS}=0}^{V_{CS}=V_{DS}}$$

$$= W \mu_n C_{ox} [(V_{GS} - V_{T0}) V_{DS} - \frac{V_{DS}^2}{2}]$$

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} [2(V_{GS} - V_{T0}) V_{DS} - V_{DS}^2]$$

I-V Equations of NFET

MOSFET CURRENT - VOLTAGE CHARACTERISTICS

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} [2(V_{GS} - V_{T0}) V_{DS} - V_{DS}^2]$$

$$= \frac{k'}{2} \frac{W}{L} [2(V_{GS} - V_{T0}) V_{DS} - V_{DS}^2]$$

$$= \frac{k}{2} [2(V_{GS} - V_{T0}) V_{DS} - V_{DS}^2]$$

$$k' = \mu_n C_{ox}$$

[k' -> KP in SPICE]

$$k = k' \frac{W}{L}$$

I-V Equations of NFET

MOSFET CURRENT - VOLTAGE CHARACTERISTICS

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} [2(V_{GS} - V_{T0}) V_{DS} - V_{DS}^2]$$

$$= \frac{k'}{2} \frac{W}{L} [2(V_{GS} - V_{T0}) V_{DS} - V_{DS}^2]$$

$$= \frac{k}{2} [2(V_{GS} - V_{T0}) V_{DS} - V_{DS}^2]$$

$$k' = \mu_n C_{ox}$$

[k' -> KP in SPICE]

$$k = k' \frac{W}{L}$$

NFET in saturation region

MOSFET CURRENT - VOLTAGE CHARACTERISTICS

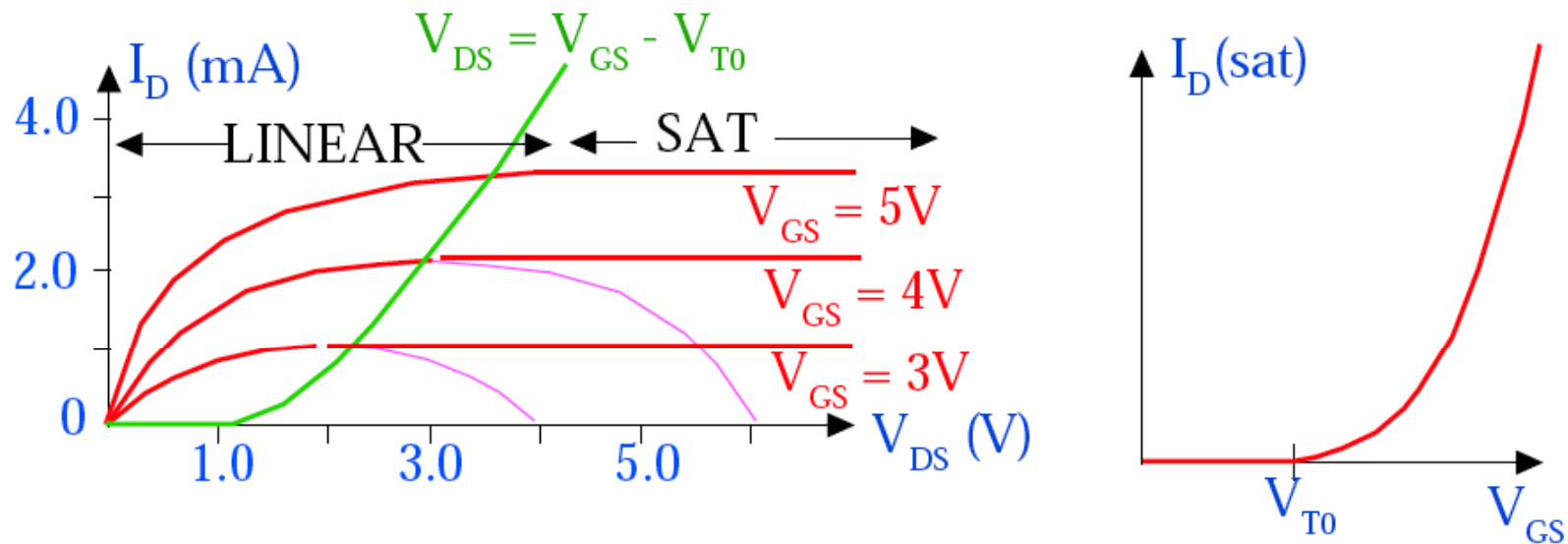
9

$$V_{DS} \geq V_{GS} - V_{T0} = V_{DSAT} \quad \text{SATURATION REGION}$$

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} [2(V_{GS} - V_{T0}) V_{DS} - V_{DS}^2] \quad @V_{DS} = V_{DSAT} = V_{GS} - V_{T0}$$

$$= \frac{\mu_n C_{ox}}{2} \frac{W}{L} [2(V_{GS} - V_{T0})(V_{GS} - V_{T0}) - (V_{GS} - V_{T0})^2]$$

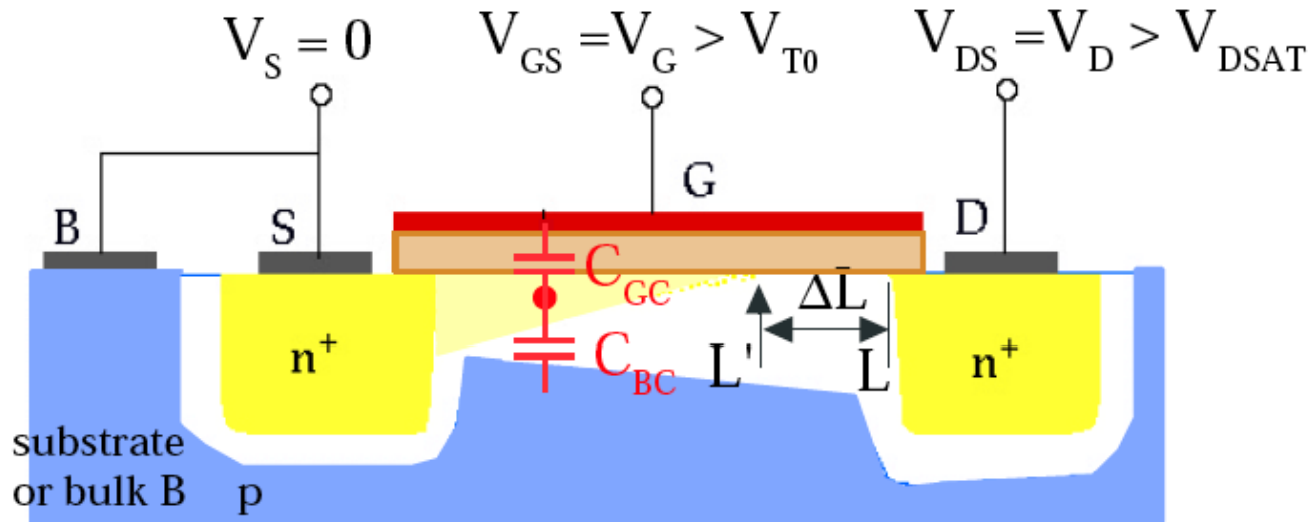
$$I_D(\text{sat}) = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_{T0})^2$$



NFET in saturation region: 2nd order effects

MOSFET CURRENT - VOLTAGE CHARACTERISTICS

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$$I_D(\text{sat}) = \frac{\mu_n C_{ox}}{2} \frac{W}{L'} (V_{GS} - V_{T0})^2 = \frac{\mu_n C_{ox}}{2} \frac{W}{L(1 - \frac{\Delta L}{L})} (V_{GS} - V_{T0})^2$$

where $\Delta L \propto \sqrt{V_{DS} - V_{DSAT}}$

empirical relation: $\frac{1}{1 - \frac{\Delta L}{L}} = 1 + \lambda V_{DS}$ [$\lambda \rightarrow$ LAMBDA in SPICE]

λ = channel length modulation coefficient (V^{-1})

NFET in saturation region: complete

MOSFET CURRENT - VOLTAGE CHARACTERISTICS

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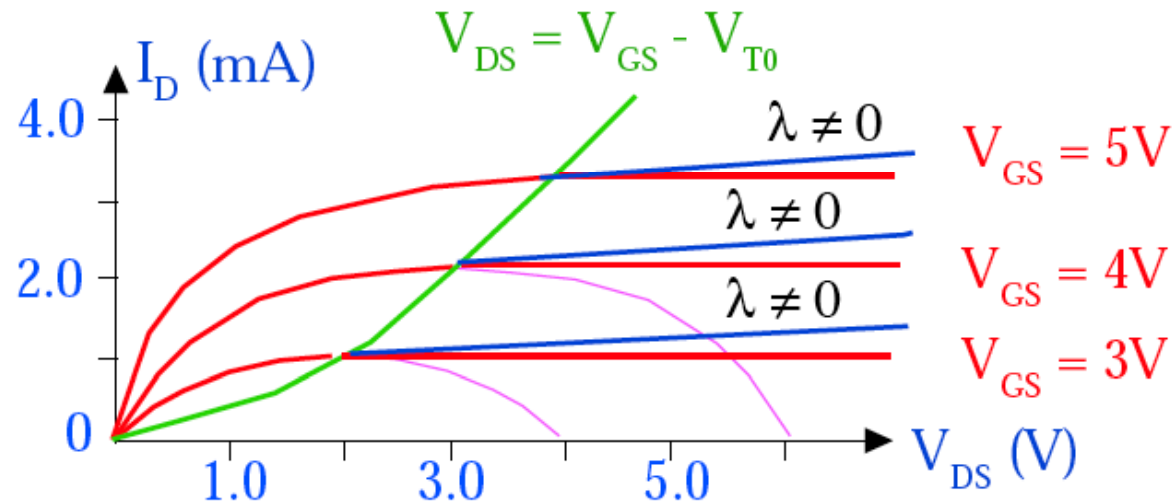
$$I_D(\text{sat}) = \frac{\mu_n C_{\text{ox}}}{2} \frac{W}{L'} (V_{\text{GS}} - V_{\text{T0}})^2 = \frac{\mu_n C_{\text{ox}}}{2} \frac{W}{L(1 - \frac{\Delta L}{L})} (V_{\text{GS}} - V_{\text{T0}})^2$$

$$\frac{1}{1 - \frac{\Delta L}{L}} = 1 + \lambda V_{\text{DS}} \quad \text{assume } \lambda V_{\text{DS}} \ll 1$$

$$I_D(\text{sat}) = \frac{\mu_n C_{\text{ox}}}{2} \frac{W}{L} (V_{\text{GS}} - V_{\text{T0}})^2 (1 + \lambda V_{\text{DS}})$$

$$I_D(\text{lin}) = \frac{\mu_n C_{\text{ox}}}{2} \frac{W}{L} [2(V_{\text{GS}} - V_{\text{T0}})V_{\text{DS}} - V_{\text{DS}}^2] (1 + \lambda V_{\text{DS}})$$

LEVEL 1
Model



NFET equations

MOSFET CURRENT - VOLTAGE CHARACTERISTICS

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SUBSTRATE BIAS EFFECT

$$V_T = V_{T0} + \gamma \left(\sqrt{|2\phi_F - V_{SB}|} - \sqrt{|2\phi_F|} \right)$$

$$I_D(\text{lin}) = \frac{\mu_n C_{ox} W}{2 L} \left[2(V_{GS} - V_T(V_{SB}))V_{DS} - V_{DS}^2 \right] (1 + \lambda V_{DS})$$

$$I_D(\text{sat}) = \frac{\mu_n C_{ox} W}{2 L} (V_{GS} - V_T(V_{SB}))^2 (1 + \lambda V_{DS})$$

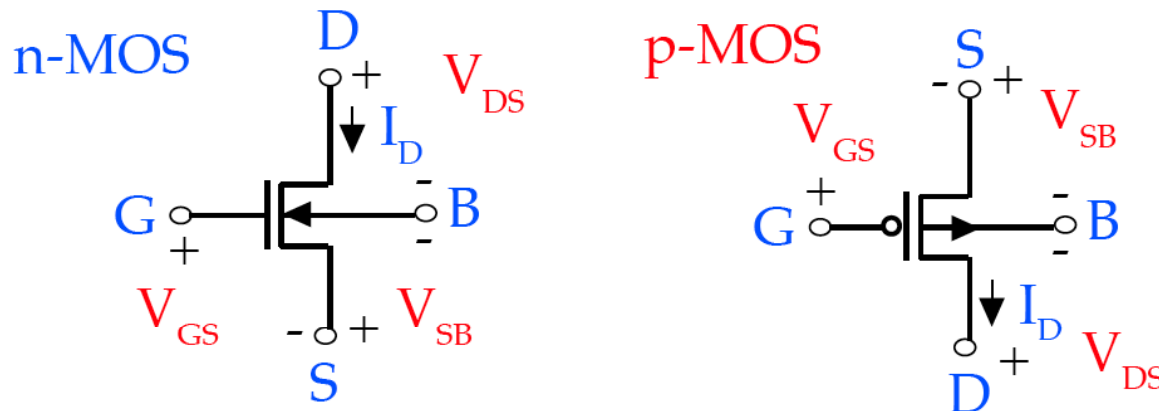
LEVEL 1
Model

$$I_D = f(V_{GS}, V_{DS}, V_{SB})$$

PMOS and NMOS equations

MOSFET CURRENT - VOLTAGE CHARACTERISTICS

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n-MOS $I_D = 0$ for $V_{GS} \leq V_T$

$$I_D(\text{lin}) = \frac{\mu_n C_{ox} W}{2 L} \left[2(V_{GS} - V_T(V_{SB}))V_{DS} - V_{DS}^2 \right] (1 + \lambda V_{DS}) \quad V_{GS} > V_T, V_{DS} < V_{GS} - V_T$$

$$I_D(\text{sat}) = \frac{\mu_n C_{ox} W}{2 L} (V_{GS} - V_T(V_{SB}))^2 (1 + \lambda V_{DS}) \quad V_{GS} > V_T, V_{DS} \geq V_{GS} - V_T$$

p-MOS $I_D = 0$ for $V_{GS} \geq V_T$

$$I_D(\text{lin}) = \frac{\mu_p C_{ox} W}{2 L} \left[2(V_{GS} - V_T(V_{SB}))V_{DS} - V_{DS}^2 \right] (1 + \lambda V_{DS}) \quad V_{GS} < V_T, V_{DS} > V_{GS} - V_T$$

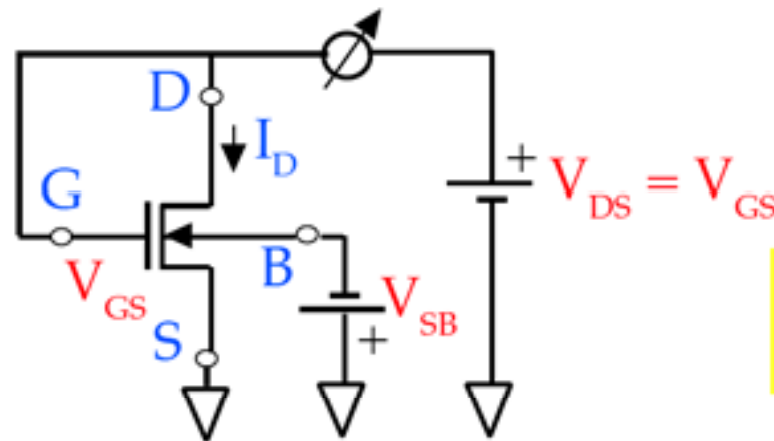
$$I_D(\text{sat}) = \frac{\mu_p C_{ox} W}{2 L} (V_{GS} - V_T(V_{SB}))^2 (1 + \lambda V_{DS}) \quad V_{GS} < V_T, V_{DS} \leq V_{GS} - V_T$$

MOSFET CURRENT - VOLTAGE CHARACTERISTICS

MEASUREMENT OF PARAMETERS (V_{T0} , γ , λ , k_n , k_p)

$$k_n = \mu_n C_{ox} \frac{W}{L}$$

$$k_p = \mu_p C_{ox} \frac{W}{L}$$



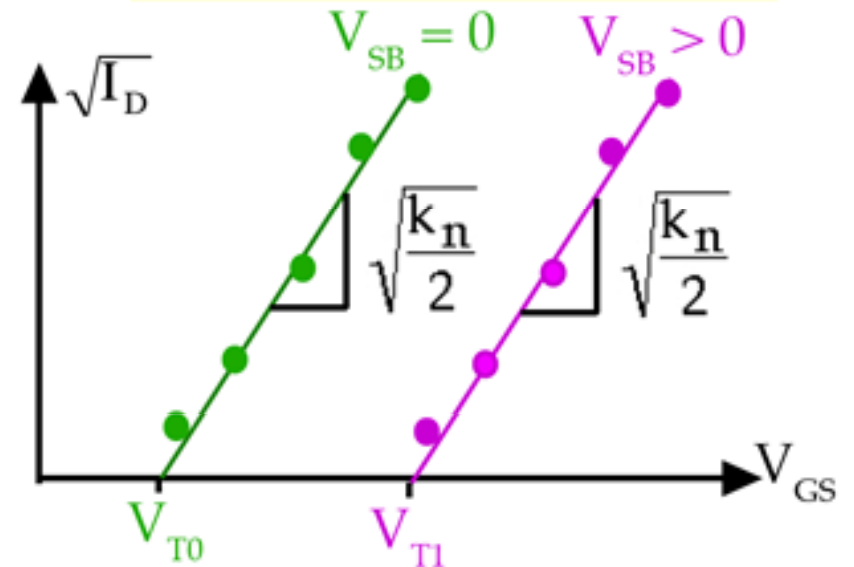
$$I_D(\text{sat}) = \frac{k_n}{2} (V_{GS} - V_{T0})^2$$

$$\sqrt{I_D(\text{sat})} = \sqrt{\frac{k_n}{2}} (V_{GS} - V_{T0})$$

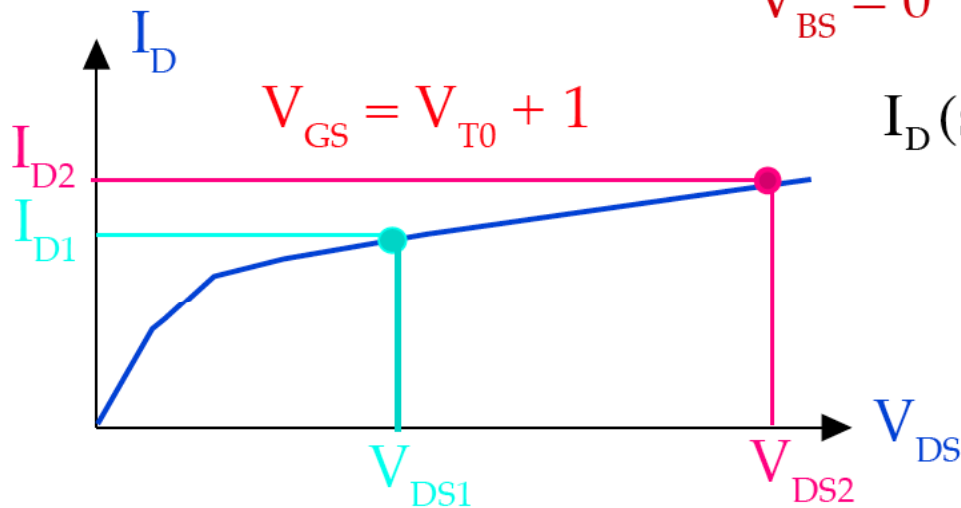
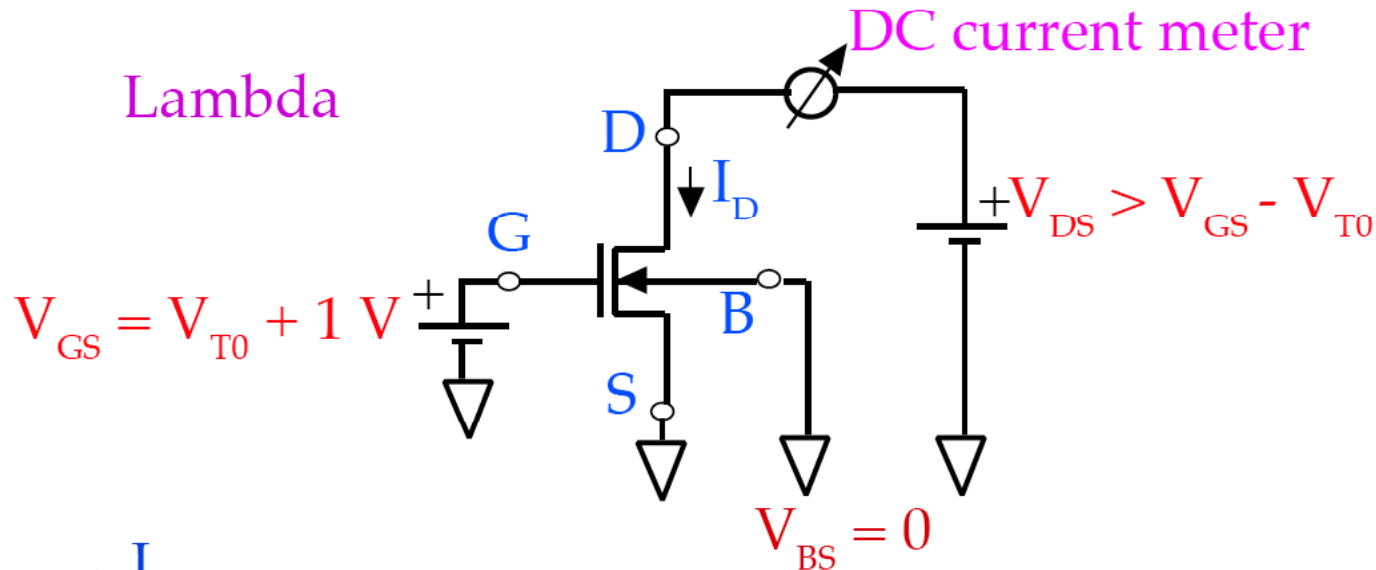
Gamma

$$V_T = V_{T0} + \gamma (\sqrt{|-2\phi_F + V_{SB}|} - \sqrt{-2\phi_F})$$

$$\gamma = \frac{V_T(V_{SB}) - V_{T0}}{\sqrt{|-2\phi_F + V_{SB}|} - \sqrt{-2\phi_F}}$$



MOSFET CURRENT - VOLTAGE CHARACTERISTICS



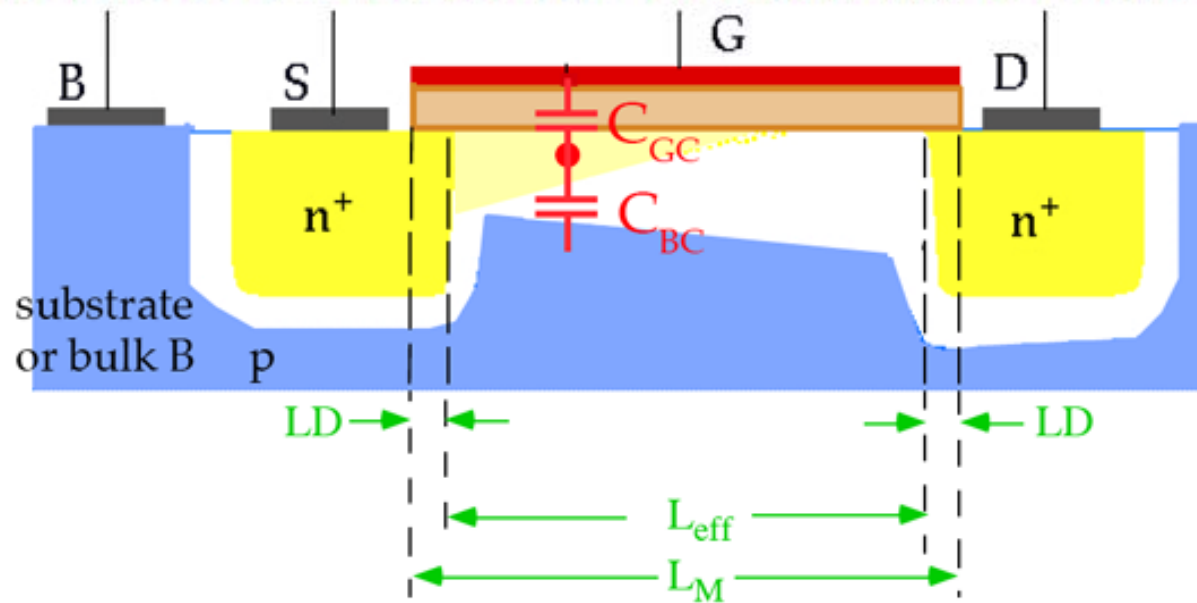
$$I_D(\text{sat}) = k_n (V_{GS} - V_{T0})^2 (1 + \lambda V_{DS})$$

$$V_{GS} = V_{T0} + 1\text{ V}$$

$$\frac{I_{D2}}{I_{D1}} = \frac{1 + \lambda V_{DS2}}{1 + \lambda V_{DS1}}$$

$$\lambda = \frac{I_{D1} - I_{D2}}{V_{DS1}I_{D2} - V_{DS2}I_{D1}}$$

EFFECTIVE CHANNEL LENGTH AND WIDTH



SPICE Parameters

$$L_{eff} = L_M - 2LD - DL$$

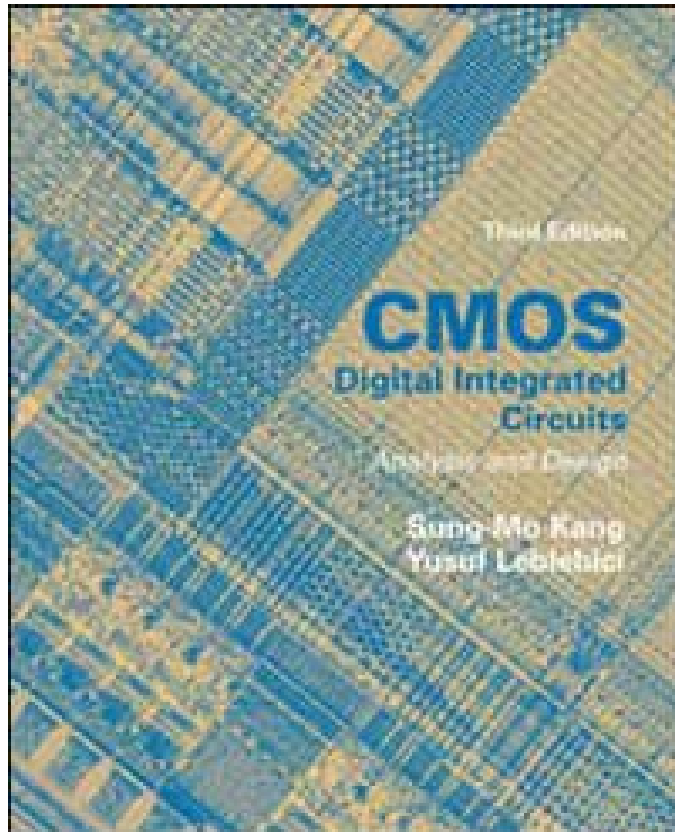
LD -> under diffusion

DL -> error in photolith and etch

$$W_{eff} = W_M - DW$$

SPICE Parameters

DW -> error in photolith and etch



MOSFET Scaling

MOSFET - SCALING

SCALING -> refers to ordered reduction in dimensions of the MOSFET and other VLSI features

- Reduce Size of VLSI chips.
- Change operational characteristics of MOSFETs and parasitics.
- Physical limits restrict degree of scaling that can be achieved.

SCALING FACTOR = $\alpha > 1$ --> S

First-order "constant field" MOS scaling theory:

The electric field E is kept constant, and the scaled device is obtained by applying a dimensionless scale-factor α to reduce dimensions by $(1/\alpha)$ and maintain E unchanged:

- All dimensions, including those vertical to the surface $(1/\alpha)$
- device voltages $(1/\alpha)$
- the concentration densities (α) .

$$(1/\alpha)/(1/\alpha) = 1$$

$$\alpha(1/\alpha) = 1$$

$$E_{\text{ox}} = V_{\text{GS}} / t_{\text{ox}}$$

\Leftrightarrow

$$E = \frac{q}{\epsilon} N_A x$$

MOSFET - SCALING

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Alternative Scaling Rules:

Constant Voltage Scaling, i.e. V_{DD} is kept constant, while the process dimensions are scaled by $(1/\alpha)$.

- All dimensions, including those vertical to the surface $(1/\alpha)$
- device voltages (1)
- the concentration densities (α^2) to preserve charge-field relations.

$$1/(1/\alpha) = \alpha$$

$$\alpha^2(1/\alpha) = \alpha$$

$$E_{ox} = V_{GS} / t_{ox}$$

\Leftrightarrow

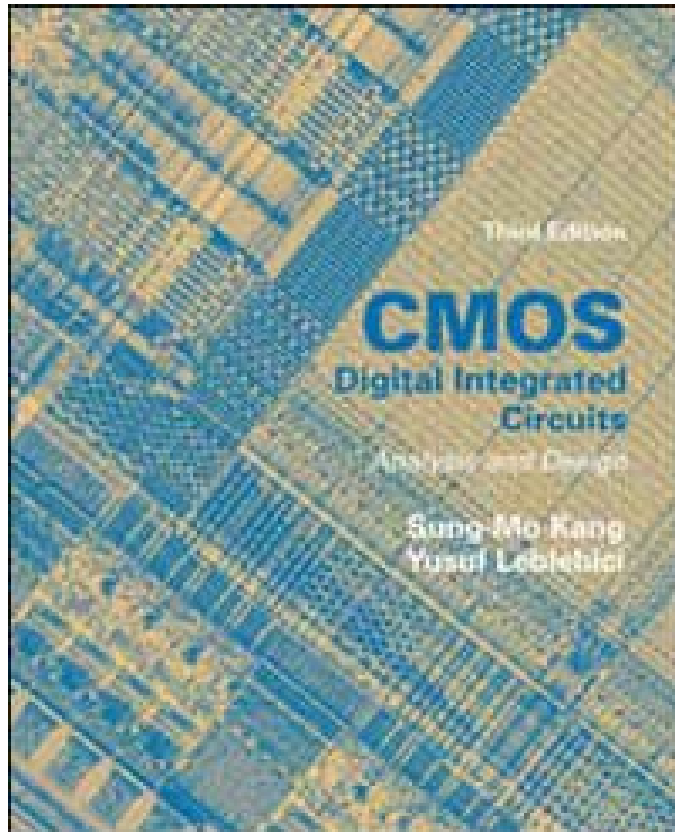
$$E = \frac{q}{\epsilon} N_A x$$

Lateral Scaling: only the gate length is scaled $L = 1/\alpha$ (gate-shrink).

Scaling Effects

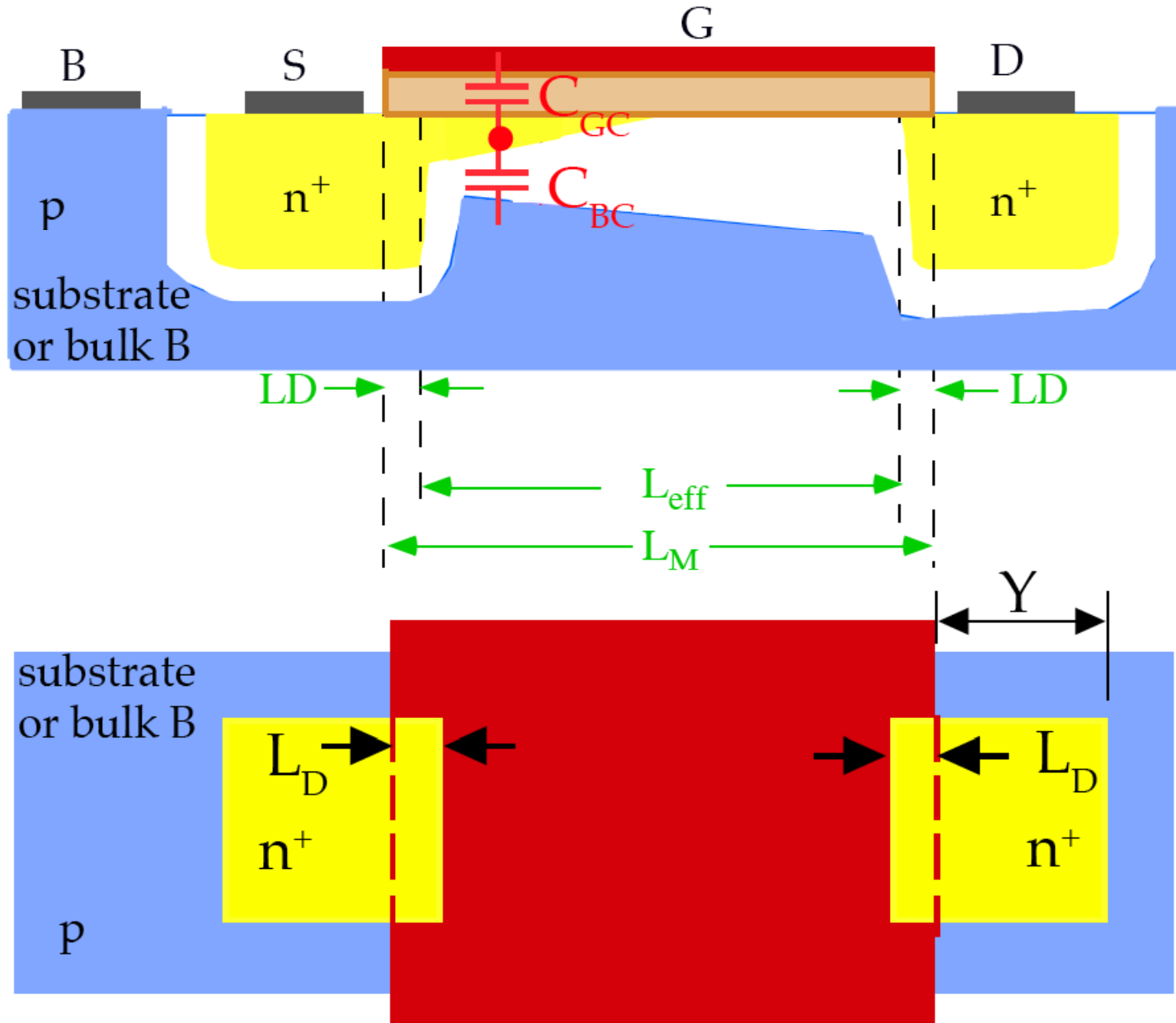
Influence of Scaling on MOS Device Performance

PARAMETER	SCALING MODEL		
	Constant Field	Constant Voltage	Lateral
Length (L)	$1/\alpha$	$1/\alpha$	$1/\alpha$
Width (W)	$1/\alpha$	$1/\alpha$	1
Supply Voltage (V)	$1/\alpha$	1	1
Gate Oxide thickness (t_{ox})	$1/\alpha$	$1/\alpha$	1
Junction depth (X_j)	$1/\alpha$	$1/\alpha$	1
Substrate Doping (N_A)	α	α^2	1
<hr/>			
Current (I) - $(W/L) (1/t_{ox})V^2$	$1/\alpha$	α	α
Power Dissipation (P) - IV	$1/\alpha^2$	α	α
Power Density (P/Area)	1	** (α^3) **	α^2
Electric Field Across Gate Oxide - V/t_{ox}	1	α	1
Load Capacitance (C) - $WL (1/t_{ox})$	$1/\alpha$	$1/\alpha$	$1/\alpha$
Gate Delay (T) - VC/I	$1/\alpha$	$1/\alpha^2$	$1/\alpha^2$

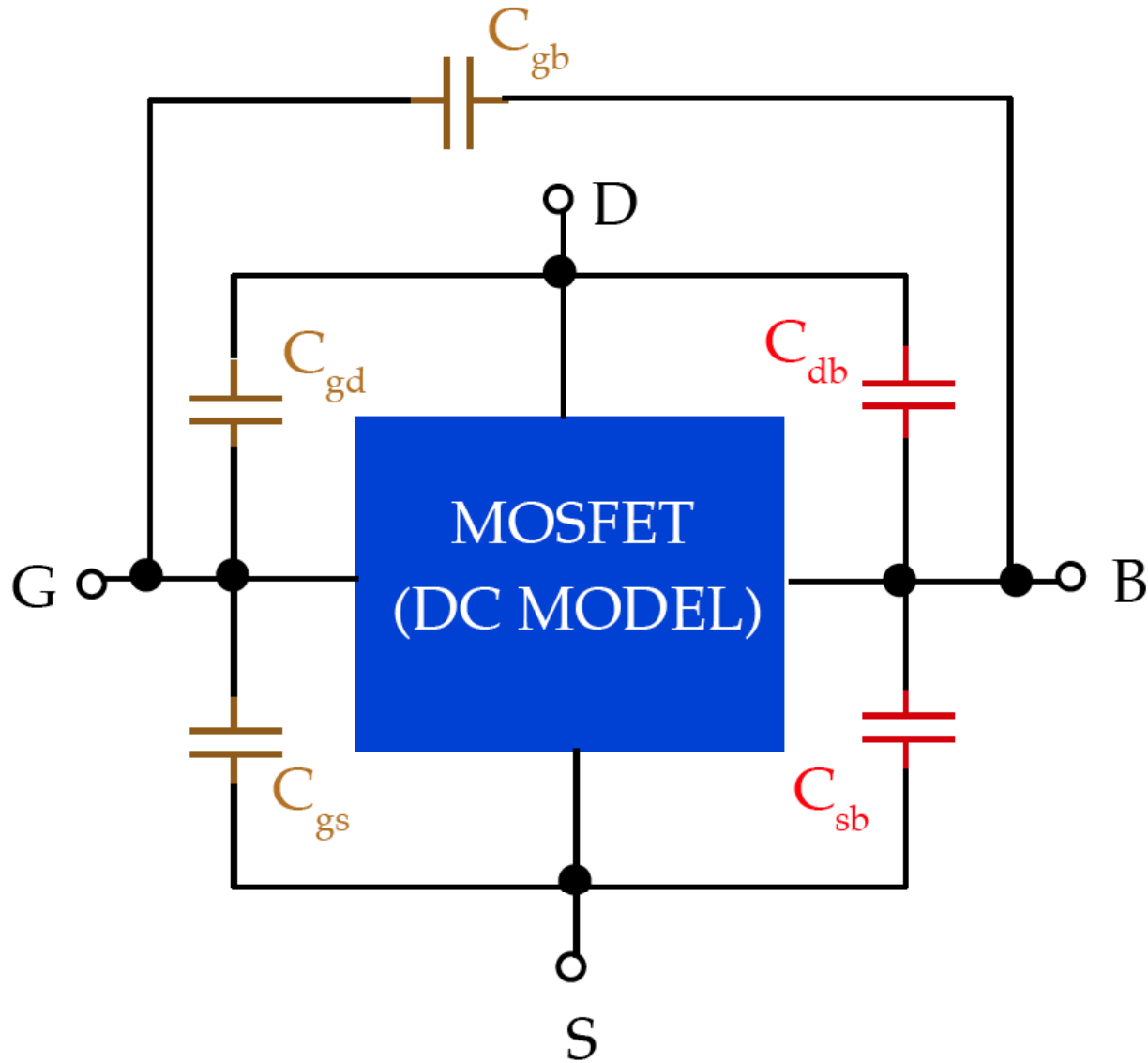


MOSFET Capacitance Models

MOSFET CAPACITANCES



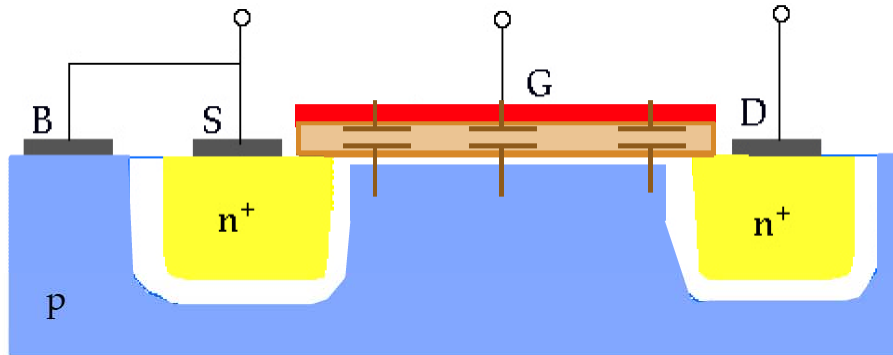
MOSFET CAPACITANCES



C_{gd} , C_{gs} , C_{gb} -> Oxide Capacitances

C_{db} , C_{sb} -> Junction Capacitances

b. Gate - Channel C_{gb} , C_{gs} and C_{gd}
MOSFET - Cut-off Region

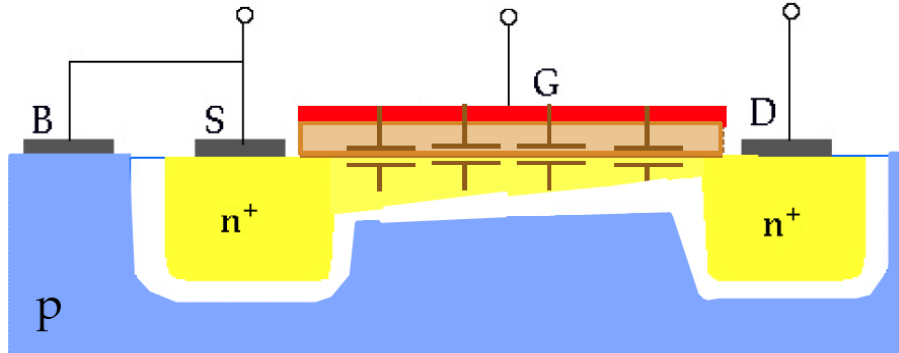


$$C_{gb} = C_{ox} W L_{eff}$$

$$C_{gs} = C_{gd} = 0$$

(no conducting channel in cut-off)

MOSFET - Linear Region

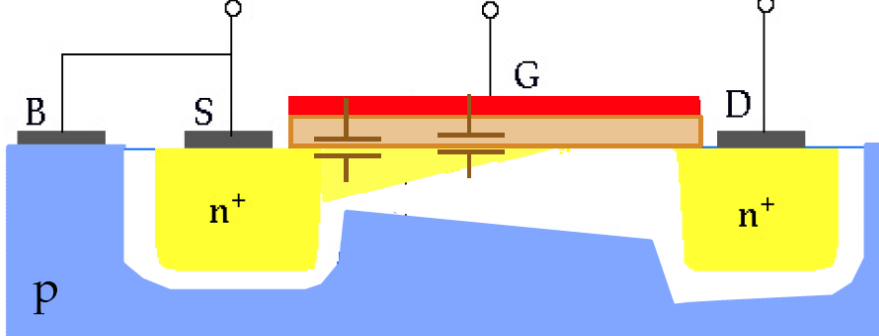


$$C_{gb} = 0$$

$$C_{gs} = (1/2) C_{ox} W L_{eff}$$

$$C_{gd} = (1/2) C_{ox} W L_{eff}$$

MOSFET - Saturation Region



$$C_{gb} = 0$$

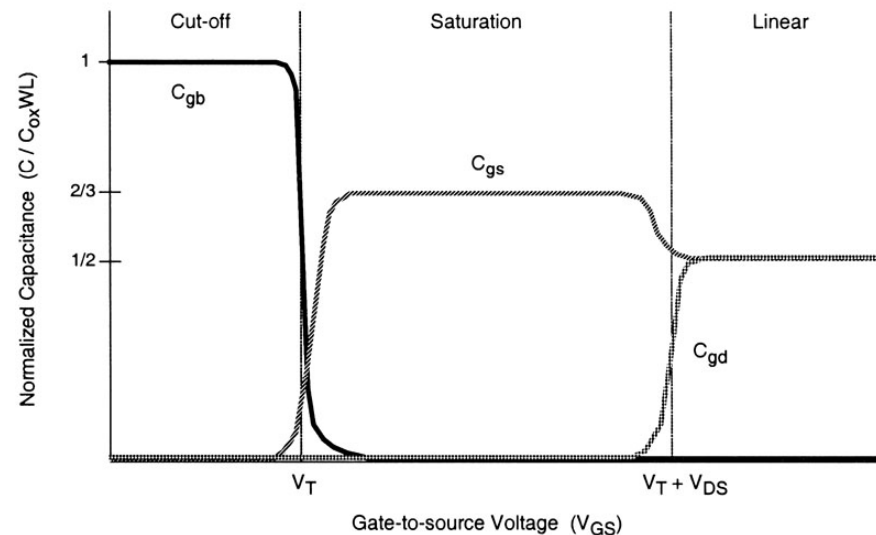
$$C_{gs} = (2/3) C_{ox} W L_{eff}$$

$$C_{gd} = 0$$

Capacitance Summary

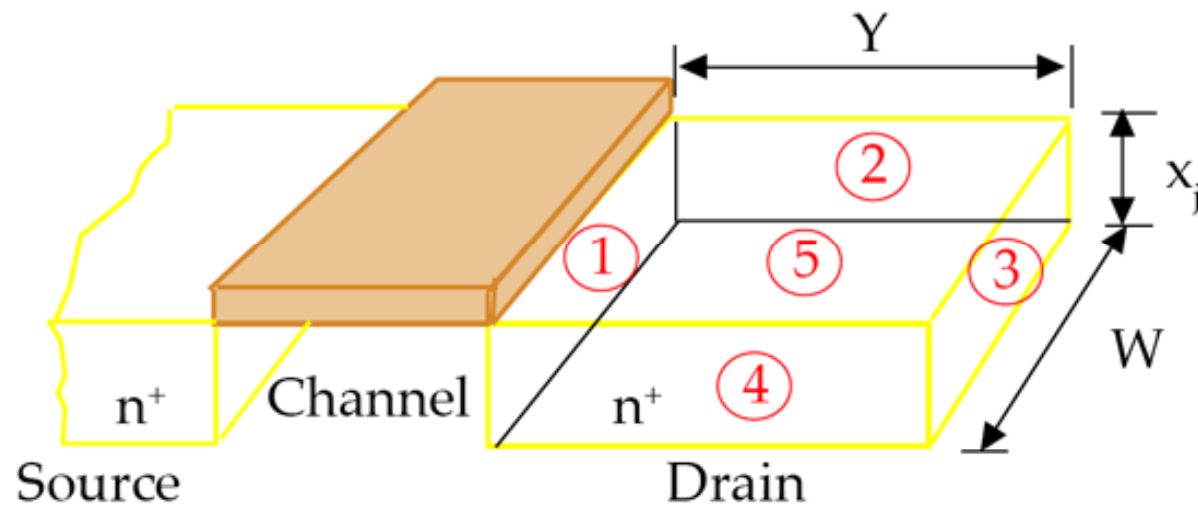
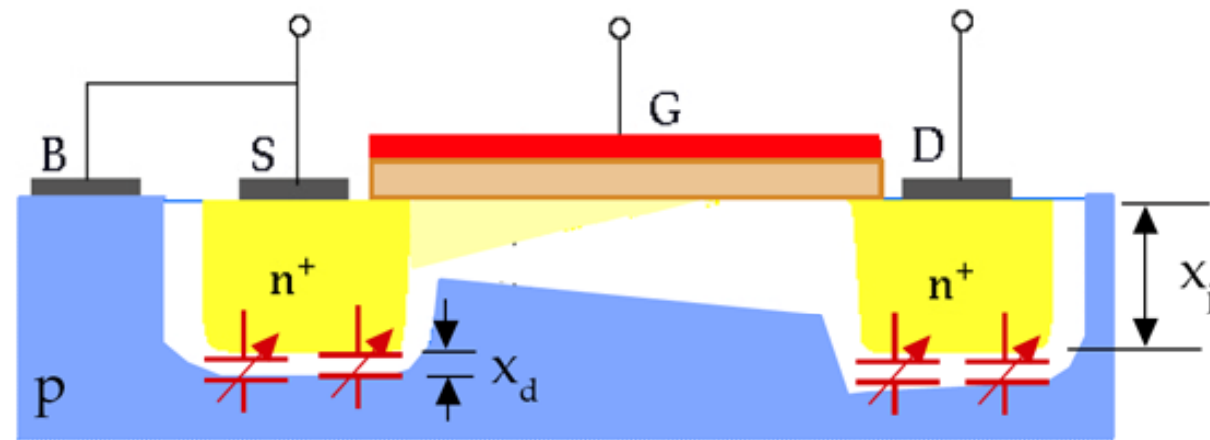
Capacitance	Cut-off	Linear	Saturation
$C_{gb}(\text{total})$	$C_{ox} WL_{eff} + C_{GB0}$	$0 + C_{GB0}$	$0 + C_{GB0}$
$C_{gd}(\text{total})$	$0 + C_{GD0}$	$0.5C_{ox} WL_{eff} + C_{GD0}$	$0 + C_{GD0}$
$C_{gs}(\text{total})$	$0 + C_{GS0}$	$0.5C_{ox} WL_{eff} + C_{GS0}$	$(2/3)C_{ox} WL_{eff} + C_{GS0}$

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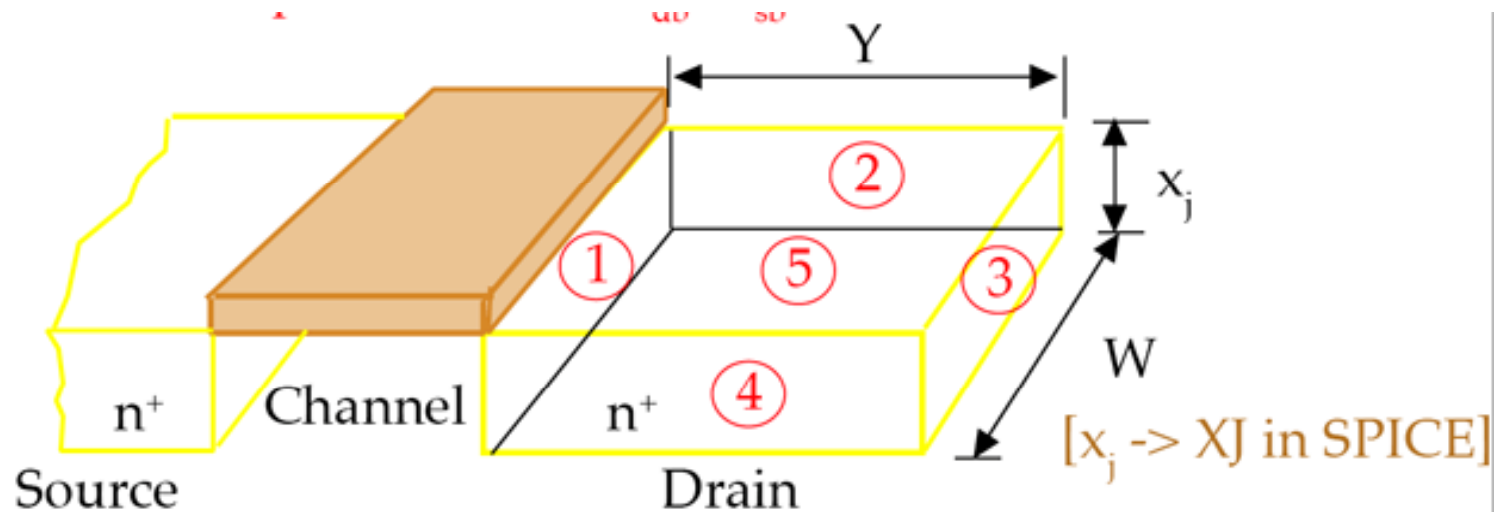


Drain/Source Capacitance

JUNCTION Capacitances $\rightarrow C_{db}, C_{sb}$



Drain/Source Capacitance



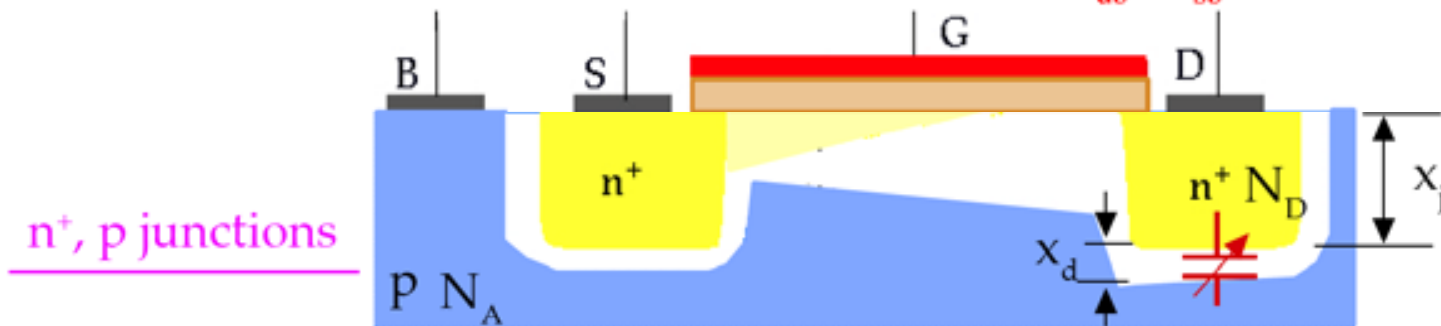
Junction	Area	Type
①	$W x_j$	n^+/p
②	$Y x_j$	n^+/p^+
③	$W x_j$	n^+/p^+
④	$Y x_j$	n^+/p^+
⑤	WY	n^+/p

p - Substrate $\rightarrow N_A$
 p^+ - Channel-stop $\rightarrow 10N_A$

Depletion Region Capacitance

Depletion Region Capacitances $\rightarrow C_{db}, C_{sb}$

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$$x_d = \sqrt{\frac{2\epsilon_{Si}}{q} \left(\frac{1}{N_A} + \frac{1}{N_D} \right) (\phi_0 + V)}$$

$V = \text{Ext Bias} \rightarrow V_{SB}, V_{DB}$

$$\phi_0 = \frac{kT}{q} \ln \left(\frac{N_A N_D}{n_i^2} \right)$$

built-in junction potential

$Q_j = \text{depletion-region charge}$ $A = \text{junction area}$

$$C_j(V) = \left| \frac{dQ_j}{dV} \right| = \frac{AC_{j0}}{\left(1 + \frac{V}{\phi_0}\right)^m} = \frac{(AS, D) \cdot CJ}{\left(1 + \frac{V}{PB}\right)^m} \quad (F)$$

$m = \text{grading coefficient}$
 $m = 1/2$ for abrupt junction

[AS, AD \rightarrow Source, Drain Areas in SPICE]
 [CJ $\rightarrow C_{j0}$ in SPICE]
 [PB $\rightarrow \phi_0$ in SPICE]
 [MJ $\rightarrow m$ in SPICE]

where

$$CJ = C_{j0} = \sqrt{\frac{q\epsilon_{Si}}{2} \left(\frac{N_A N_D}{N_A + N_D} \right) \frac{1}{\phi_0}} \quad (F/cm^2)$$

Depletion Region Capacitance

n⁺, p Junctions

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$$C_j(V) = \left| \frac{dQ_j}{dV} \right| = \frac{AC_{j0}}{\left(1 + \frac{V}{\phi_0}\right)^m} = \frac{(AS, D) \cdot CJ}{\left(1 + \frac{V}{PB}\right)^{MJ}} \quad (F) \quad \begin{array}{l} [AS, AD \rightarrow \text{Source, Drain} \\ \text{Areas in SPICE}] \\ [CJ \rightarrow C_{j0} \text{ in SPICE}] \end{array}$$

$$CJ = C_{j0} = \sqrt{\frac{q \epsilon_{Si}}{2} \left(\frac{N_A N_D}{N_A + N_D} \right) \frac{1}{\phi_0}} \quad (F/cm^2) \quad \begin{array}{l} [PB \rightarrow \phi_0 \text{ in SPICE}] \\ [MJ \rightarrow m \text{ in SPICE}] \end{array}$$

$$C_j(V) = A C_{j0} \text{ when } V = 0$$

EQUIVALENT LINEAR LARGE SIGNAL CAPACITANCE

$$C_j(V) \approx AC_{j0} \cdot \frac{\phi_0}{(V_2 - V_1)(1-m)} \left[\left(1 + \frac{V_2}{\phi_0}\right)^{1-m} - \left(1 + \frac{V_1}{\phi_0}\right)^{1-m} \right]$$

$0 < K_{eq} < 1 \rightarrow$ Voltage Equivalence Factor

where $V_1 \leq V \leq V_2$

$V =$ Ext Bias $\rightarrow V_{SB}, V_{DB}$

$$C_j(V) = AC_{j0} K_{eq} = (AS, D) \cdot CJ \cdot K_{eq}$$

Depletion Region Capacitance

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n⁺, p⁺ junctions (Sidewalls)

$$C_{j0sw} = \sqrt{\frac{\epsilon_{Si} q}{2} \left(\frac{N_A(sw) N_D}{N_A(sw) + N_D} \right) \frac{1}{\phi_{0sw}}} \quad (\text{F/cm}^2)$$

[PS, PD -> Source, Drain Perimeters in SPICE]

[CJSW -> C_{jsw} in SPICE]

[PBSW -> ϕ_{0sw} in SPICE]

[MJSW -> m(sw) in SPICE]

[XJ -> x_j in SPICE]

Since all sidewalls have depth = x_j:

$$C_{jsw} = C_{j0sw} x_j \quad (\text{F/cm})$$

EQUIVALENT LARGE SIGNAL CAPACTIANCE

$$C_{jsw}(V) \approx PC_{jsw} K_{eq}(sw) \quad P = \text{sidewall perimeter}$$

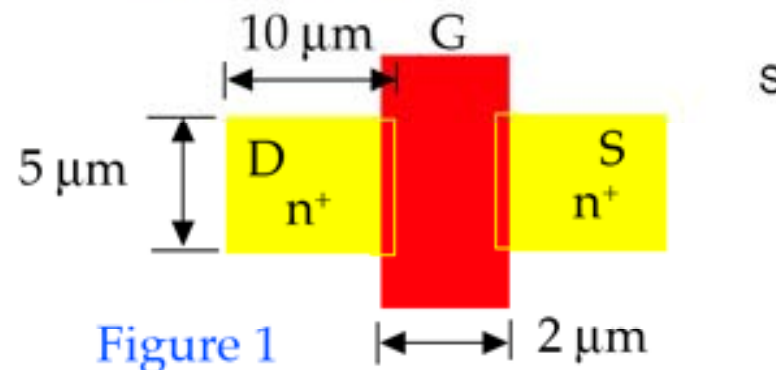
$$K_{eq}(sw) = \frac{\phi_{0sw}}{(V_2 - V_1)(1 - m(sw))} \left[\left(1 + \frac{V_2}{\phi_{0sw}}\right)^{1 - m(sw)} - \left(1 + \frac{V_1}{\phi_{0sw}}\right)^{1 - m(sw)} \right]$$

m(sw) = 1/2 for an abrupt junction

EXAMPLE 4 :

Determine the **total junction capacitance at the drain**, i.e. C_{db} , for the n-channel enhancement MOSFET in Fig. 1. The process parameters are

$$\begin{aligned} CJ &= 1.35 \times 10^{-8} \text{ F/cm}^2 \\ CJSW &= 5.83 \times 10^{-12} \text{ F/cm} \\ PB &= 0.896 \text{ V} \\ PBSW &= 0.975 \text{ V} \\ XJ &= 1 \times 10^{-4} \text{ cm} \\ MJ &= MJSW = \frac{1}{2} \end{aligned}$$



Source, Drain are surrounded by p^+ channel-stop. The substrate is biased at 0V. Assume the drain voltage range is 0.5 V to 5.0 V.

$$C_j(V) = AC_{j0} K_{eq} = AD \cdot CJ \cdot K_{eq}$$

$$K_{eq} = \frac{PB}{(V_2 - V_1)(1 - MJ)} \left[\left(1 + \frac{V_2}{PB}\right)^{1 - MJ} - \left(1 + \frac{V_1}{PB}\right)^{1 - MJ} \right]$$

$$= \frac{2 \cdot 0.896 V}{(5 V - 0.5 V)} \left[\left(1 + \frac{5 V}{0.896 V}\right)^{1/2} - \left(1 + \frac{0.5 V}{0.896 V}\right)^{1/2} \right] = 0.52$$

$$CJ = 1.35 \times 10^{-8} \text{ F/cm}^2$$

$$CJSW = 5.83 \times 10^{-12} \text{ F/cm}$$

$$PB = 0.896 \text{ V}$$

$$PBSW = 0.975 \text{ V}$$

$$XJ = 1 \times 10^{-4} \text{ cm}$$

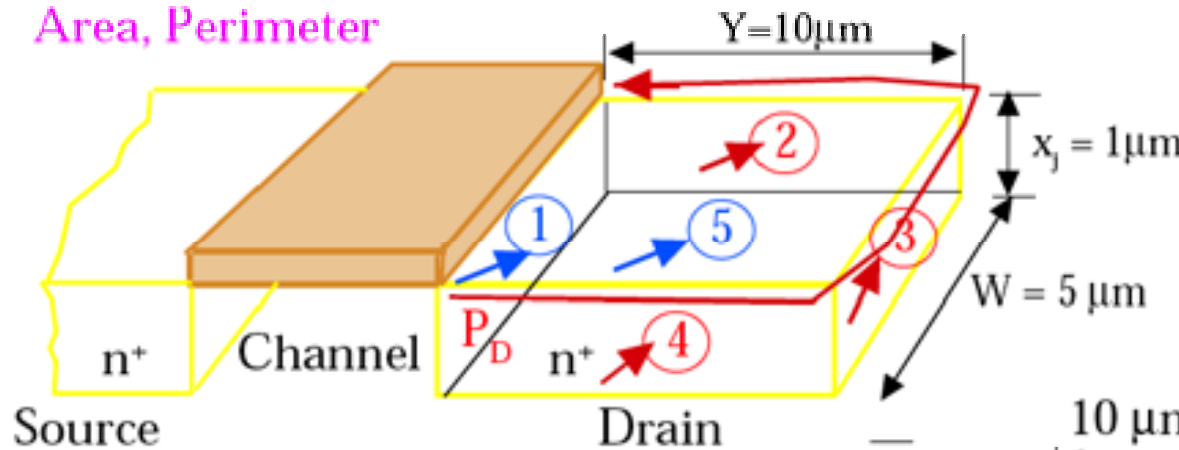
$$MJ = MJSW = 1/2$$

$$C_{jsw}(V) = PC_{jsw} K_{eq}(sw) = PD \cdot CJSW \cdot K_{eq}(sw)$$

$$K_{eq}(sw) = \frac{PBSW}{(V_2 - V_1)(1 - MJSW)} \left[\left(1 + \frac{V_2}{PBSW}\right)^{1 - MJSW} - \left(1 + \frac{V_1}{PBSW}\right)^{1 - MJSW} \right]$$

$$= \frac{2 \cdot 0.975 V}{(5 V - 0.5 V)} \left[\left(1 + \frac{5 V}{0.975 V}\right)^{1/2} - \left(1 + \frac{0.5 V}{0.975 V}\right)^{1/2} \right] = 0.53 \approx K_{eq}$$

Area, Perimeter



- $CJ = 1.35 \times 10^{-8} \text{ F/cm}^2$
- $CJSW = 5.83 \times 10^{-12} \text{ F/cm}$
- $PB = 0.896 \text{ V}$
- $PBSW = 0.975 \text{ V}$
- $XJ = 1 \times 10^{-4} \text{ cm}$
- $MJ = MJSW = 1/2$

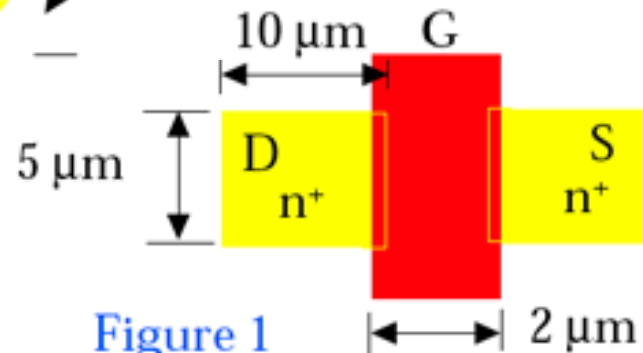


Figure 1

A_D : n^+/p junctions:

$$A_D = (5 \times 1) \mu\text{m}^2 + (10 \times 5) \mu\text{m}^2 = 55 \mu\text{m}^2$$

P_D : n^+/p^+ junctions:

$$P_D = 2Y + W = 20 \mu\text{m} + 5 \mu\text{m} = 25 \mu\text{m}$$

$$C_j(V) = AC_{j0}K_{eq} = AD \cdot CJ \cdot K_{eq} = (55 \times 10^{-8} \text{ cm}^2) \cdot (1.35 \times 10^{-8} \text{ F/cm}^2) \cdot 0.52 = 3.86 \text{ fF}$$

$$C_{jsw}(V) = PC_{jsw}K_{eq}(sw) = PD \cdot CJSW \cdot K_{eq}(sw) = (2.5 \times 10^{-3} \text{ cm}) \cdot (5.83 \times 10^{-12} \text{ F/cm}) \cdot 0.53 = 7.72 \text{ fF}$$

$$C_{db} = AD \cdot CJ \cdot K_{eq} + PD \cdot CJSW \cdot K_{eq}(sw) = 11.58 \text{ fF}$$

Important 2nd Order Effects

Short Channel Effects - $L_{\text{eff}} \rightarrow x_j$

Narrow Channel Effects - $W \rightarrow x_{\text{dm}}$

Subthreshold Current - $V_{\text{GS}} < V_{\text{T0}}$

Short Channel Effect - $L_{eff} \rightarrow X_j$ (source, drain diffusion depth)

$$V_{T0} \text{ (short channel)} = V_{T0} \text{ (long channel)} - \Delta V_{T0}$$

$$\Delta V_{T0} \approx 8.15 \times 10^{-20} \eta \frac{v_{DS}}{L_{eff}^3 C_{ox}} \quad (\text{Lvl 3})$$

[SPICE Parameter: ETA $\rightarrow \eta$ = imperical parameter]

Narrow Channel Effect - $W \rightarrow X_{dm}$ (depletion region depth)

$$V_{T0} \text{ (narrow channel)} = V_{T0} \text{ (long channel)} + \Delta V_{T0}$$

$$\Delta V_{T0} \approx \frac{\delta (\pi \epsilon_{Si} |2\Phi_F|)}{4W C_{ox}} \quad (\text{Lvl 2 \& 3})$$

[SPICE Parameter: DELTA $\rightarrow \delta$ = imperical parameter]

Subthreshold Current - $V_{GS} < V_{T0}$

$$I_D \text{ (weak inversion)} = I_{on} e^{(V_{GS} - V_{on})/(q/nkT)} \quad (\text{Spice Model})$$

$I_{on} = I_D$ in strong inversion and $V_{GS} = V_{on}$ is the boundary weak and strong inversion