Analysis and Comparison of Fractional Frequency Reuse Schemes Based on Worst Case Signal to Interference Power Ratio in OFDMA Uplink

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***Abstract*—Fractional Frequency Reuse (FFR) is an efficient interference mitigation technique for modern cellular networks due to its low complexity, low coordination requirements and resource allocation flexibility. This work considers the use of FFR in the cellular uplink (UL) . We analyze the uplink worst case Signal to Interference power Ratio (SIR) for sectored FFR and compare its results with those of strict FFR and Soft FFR. A closed form expression is derived analytically for worst SIR and best inner radius. This analytical technique is utilized to configure a FFR solution for the UL of Orthogonal Frequency Division Multiple Access (OFDMA) cellular systems. The analysis is performed using two- tiers cellular network with uniform user density and for three different cases of FFR, sectored FFR, Soft Frequency Reuse (SFR), and strict FFR with Frequency Reuse Factor (FRF)= 3 and FRF= 4. Also the effect of power control exponent on those FFR schemes is studied. The inner radius configuration depends on equalizing the worst SIR for both inner and outer edges of the cell. Numerical results show that sectored FFR without power control yields the highest SIR followed by strict FFR with FRF= 4 . Power control exponent does not affect FFR with reuse three but strongly affects SFR and FRF= 4 as its increase reduces its SIR and inner radius. Sectored FFR without power control have the highest SE, hence sectored FFR highly balances the needs of interference reduction and resource efficiency but at the expense of increased system complexity.**

Keywords*—Worst case SIR, Inter cell interference, power control exponent, strict FFR, SFR, sectored FFR, and power control factor.

I. Introduction

Nowadays wireless cellular networks are evolving from voice-oriented to ubiquitous mobile-broadband data networks. So as to meet these growing requirements, 3GPP Long Term evaluation (LTE) [1] targets a spectrum reuse factor of one to achieve higher system capacity and Spectrum Efficiency (SE). However reuse one leads to strong Inter Cell Interference (ICI) by the users that use the same frequency in neighboring cells in Orthogonal Frequency Division Multiple Access (OFDMA) systems which are also known as Single Carrier (SC-) FDMA [2], [3] that utilized in LTE uplink (UL). The strongest interference yields from the users at the cell edges. Thus effective Inter-Cell Interference Coordination (ICIC) and spectrum utilization management is an important and challenging research area for wireless communication systems design.

Uplink performance analysis is highly recommended due to symmetric traffic applications like video-calls, social networking, and real time multimedia applications. Compared to downlink (DL) analysis, uplink analysis is more difficult as it faces three main constraints [4]. First, the interference in DL analysis comes from the fixed locations, while in UL analysis; interference is resulted by mobile devices spread across the network. Second, the UL analysis uses location dependent power control that makes the transmit power highly variable. Hence significance changes of interference statistics compared to the DL. The third, constraint is that maximum power and average transmit power are important for life time of battery powered mobile devices. Modeling uplink ICIC techniques is challenging due to modeling the varying interference generated by spread mobile devices with limited transmit power and highly variable transmit power due to power control. This paper focuses on one particular ICIC technique known as Fractional Frequency Reuse (FFR) [1], and [5–8]. Generally there are three main schemes of FFR : strict FFR, sectored FFR, and Soft Frequency Reuse(SFR) [5]. Strict FFR balances the link throughput across the coverage area where Cell Center Users (CCUs) use universal frequency reuse and Cell Edge Users (CEUs) use frequency reuse larger than one. FFR was first proposed for GSM networks [8].

The difficulty of modeling the cellular uplink arises from the correlation between mobile and base station locations; transmit power, and distribution of the resulting interference. One famous analytical approach is known as the wyner model [9] that is basically used for CDMA networks. This model assumes constant average ICI which is not exact assumption for OFDMA based networks [10]. A recent work for modeling the uplink based on stochastic geometry is presented in [11], where approximations for grid and Poisson Point Process (PPP) models are used to develop expressions for spectral efficiency (SE).

In addition to the spatial distribution of base stations (BSs) and mobiles, another modeling challenge arises from the use of open and closed loop fractional power control techniques [12]. In [13], an analytical model based on fluid model [14] was proposed. The model is based on approximations for hexagonal grid based base station (BS) locations to derive analytical expressions for SINR and SE of CEUs and CCUs. In [15], the authors consider a hexagonal network with two tiers of interference. They used markov chains to model the loads of the different base stations which allow iterative calculation of ICI. They discussed the tradeoff problem of static FFR between improvement in user outage at the cell edge, and sum-rate loss compared to universal reuse. In [16], the authors model the network as a hexagonal grid and calculate user's SINR as a function of their locations. Performance of SFR in the uplink have been studied using system level simulations based on hexagonal grid network with taking into consideration transmit power limitations. In [17], the authors tried to solve the uplink ICIC problem by using a decentralized adaptive soft frequency reuse scheme. They used physical resource block (PRB) reuse avoidance/minimization and cell edge bandwidth breathing to solve the problem. The authors in [18] investigated the performance of the OFDMA uplink with FFR schemes. They showed that SFR is well performing in the uplink as it does not depend on resource coordination. Network parameters like user distribution and the number of tiers of interfering mobiles are effective parameters on which uplink analysis depends [19]. In [20], the authors used system-level simulations of hexagonal base stations to derive traffic-adaptive SFR technique for the uplink. They showed the performance gains for interior and edge users' throughput by using simulations of hexagonal base stations grid aided by 3GPP standard body.

This paper presents an analytical design based on worst case SIR for the uplink of OFDMA cellular systems using FFR schemes, where a closed form solution for not only worst case SIR but also for best inner radius in OFDMA uplink cellular system is driven. By assuming the worst case SIR, unlike the existing studies in [12], [16], a system design that improves the lowest acceptable rate in cellular network is given. In [21], the authors have derived the uplink worst case SIR and best inner radius for SFR and strict FFR. In analysis of [21], the best inner radius was derived by equalizing the worst interior and edge SIRs in the two cases and their results indicated that FFR with FRF$= 4$ has the largest SIR value. In this paper we extend this work to sectored FFR case to clarify that sectored FFR is a good solution to the trade off problem between strict FFR and SFR in uplink too. Also, the SEs and outage probabilities of the three different FFR techniques are derived. Finally the effects of power control exponent on FRF$= 4$, sectored FFR, and SFR are discussed.

This paper also presents a new method to compute the inner region radius based on uplink worst SIR value in order to clarify inner radius that equalizes the worst edge SIR on both edges of exterior and interior regions. Paper organization is as follow: Section 2 details the system model and derives the worst SIR and best inner radius for sectored FFR. Section 3 repeats section 2 but for SFR. Section four presents the same analysis for strict FFR with reuse three and four. Numerical simulation results are provided in Sec. 5. Finally, Sec. 6 concludes the work.

II. System Model

Figure 1(a) illustrates the idea of strict FFR with reuse three, where R_{in} defines the interior region area illustrated by yellow circles. R_{ex}-R_{in} defines the exterior region illustrated white color between the hexagon and its inner yellow circle as in Fig. 1(a). The bandwidth is divided into two main parts. The first part is assigned for reuse one and the second part is divided equally into three parts for reuse three. For SFR the overall bandwidth is shared by all Base Stations (BSs) [5], [6]. The bandwidth is divided into major and minor sub bands. Major sub band is used for both center and edge users, while minor sub band is used only for center users. The transmission power for major sub-band is larger than minor sub band power. SFR with reuse three is shown in Fig. 1(b). Sectored FFR is similar to SFR except that interior users are served by omnidirectional antenna while exterior users are served by directional ones as shown in Fig. 1(c) [7]. Consider the uplink FFR-OFDMA cellular two-tiers network with 57 sectors that utilizes sectored FFR shown in Fig. 2. It is assumed that OFDMA system bandwidth is divided into four equal sub-bands which correspond to f1, f2, f3, and f4. Each BS uses tri-sectored directional antenna for exterior region and omnidirectional one for the interior region. The distance between any two adjacent BSs is $2d$, where $d = \frac{\sqrt{3}}{2}R$. Here, R is the cell radius. Interfering mobiles are given by the set K and the distance between an interfering mobile i and home BS is defined by D_i. The distance of an interfering mobile to its serving BS is denoted by R_i. It is assumed that the closest interfering mobile must be served by a different cell and cannot

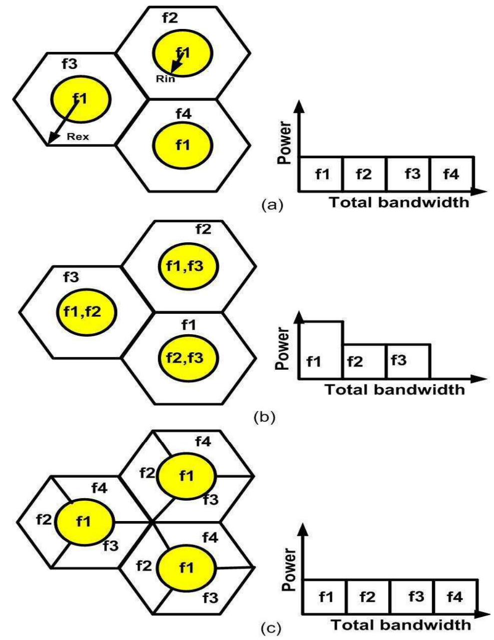


Fig. 1: FFR concept. (a) Strict FFR. (b) SFR. (c)Sectored FFR.

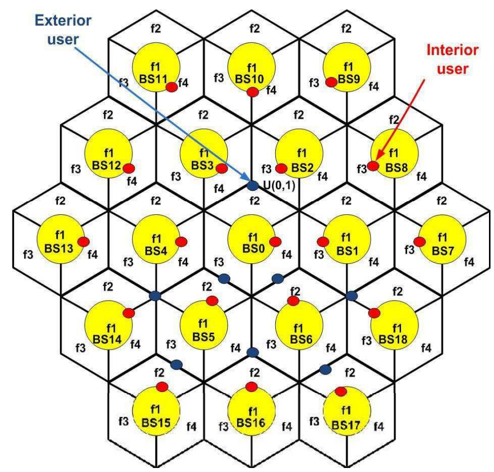


Fig. 2: UL two tiers network with sectored FFR

be closer to the home cell than the transmitting mobile which means $D_i > R_i$. For system simplicity, channel variations due to path loss only is considered[1], i.e., we use the exponent path loss model [23] given by $G = G_0 r^{-\alpha}$, where α is the path loss exponent. The parameter r is the distance between the BS and its serving mobile User Equipment (UE). The constant G_0 is given by $G_0 = (\frac{c}{4\pi f})^2$, where f is the center frequency and c is the speed of light. Each sub band exposes to exponentially distributed fast fading power g with unit mean and Gaussian noise with power σ^2.

The transmit powers used by mobiles rely on fractional path loss inversion of the form $Pr^{\alpha\mu}$, where $\mu \in [0, 1]$ is the power control exponent and P is the fixed received power for all mobiles. One specific formula of P_{UL} that is used by 3GPP

[1]Shadowing between BS and its serving mobile user is neglected because it can easily overcome with utilizing power control [22]

standards like LTE is given as [24]:

$$P_{UL} = \mu(SNR_0 + \sigma^2) + P_{max}(1 - \mu) \tag{1}$$

where P_{max} is the maximum UL transmit power, σ^2 is the noise power, and SNR_0 is the target threshold in dB. The uplink power control strategy of the mobiles is as follow: when $\mu = 0$ all mobiles transmit with constant power $P_{UL} = P_{max}$.This occurs when the mobile is far away from its serving BS. While selecting $\mu = 1$ leads to desired signal received power at the base station of gP_{UL} regardless of the location of the user.

Considering small-scale Rayleigh fading between the mobiles and the base station under consideration, and a constant baseline mobile transmit power of μ^{-1}. Thus the received power of the desired signal at the serving base station is given by $gr^{\alpha(\mu-1)}$. Next,we assume that all the mobiles utilize distance-proportional fractional power control of the form $R_i^{\alpha\mu}$. Thus, as a user moves closer to the desired base station, the transmit power required to maintain the same received signal power decreases, which is an important consideration for battery-powered mobile devices. The associated SIR at a base station located at origin is given by

$$SIR = \frac{gPr^{\alpha(\mu-1)}}{I_K} \tag{2}$$

where I_k for a given set of interfering mobiles K is given by

$$I_K = \sum_{i \in K} g_i P(R_i^{\alpha})^{\mu} D_i^{-\alpha} \tag{3}$$

If a user lies uniformly in at a distance r from BS_0 (see Fig. 2), upon its location this user can be specified as either inner or exterior user. To compute the worst case SIR for inner user, we consider 18 interfering inner mobile users, located in other cells. The location of interfering users is selected carefully to be the closest locations to the home base station for strong interference. The signal to Interference power ratio (SIR) of that user is given by:

$$SIR_{in} = \frac{gPr^{\alpha(\mu-1)}}{gPr^{\alpha\mu}[6(2d-r)^{-\alpha} + 6(4d-r)^{-\alpha} + 6(3R-r)^{-\alpha}}$$
$$= \frac{r^{-\alpha}}{6[(\sqrt{3}R-r)^{-\alpha} + (2\sqrt{3}R-r)^{-\alpha} + (3R-r)^{-\alpha}]} \tag{4}$$

For exterior user who lies at one of the corner edges of the home cell at distance equals the cell radius R, we consider seven nearest interfering mobile users that use the same frequency and lie at the closest distance to BS_0 as shown in Fig. 2 (blue dots). Thus, a general SIR expression for exterior user uplink SIR is as follow:

$$SIR_{sectored} = \frac{gpR^{\alpha(\mu-1)}}{gp[R^{\alpha\mu}(6R)^{-\alpha} + (2R+\sqrt{3}R)^{\alpha\mu}(1.5\sqrt{3}R)^{-\alpha}]}$$
$$= \frac{1}{6^{-\alpha} + (2+\sqrt{3})^{\alpha\mu}(1.5\sqrt{3})^{-\alpha}} \tag{5}$$

From Eqs. (4), and (5) the worst SIR depends not only on path loss exponent α but also on the power control exponent μ. In order to find the best sectored FFR inner radius, our technique calculates the worst SIR for inner and exterior users. The worst SIR for exterior user that utilizes sectored FFR is previously calculated in Eq (5). For reuse one case the worst SIR is given in terms of inner cell radius r is provided in (4). By equalizing the worst SIR in the two cases, the result is an equation of one unknown which is the inner radius r as shown in Eqs. (6) and (7) for sectored FFR case.

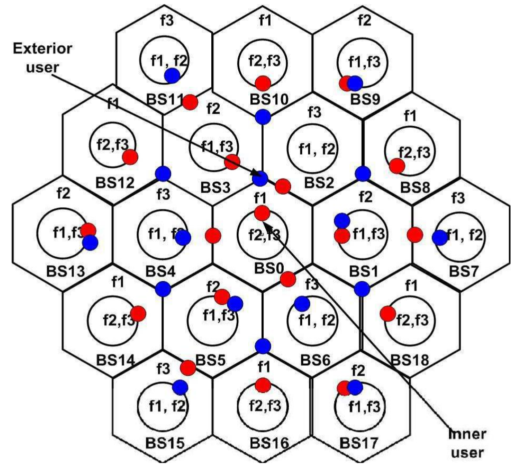


Fig. 3: UL two tiers network with SFR.

$$\text{SIR}_{in} = \text{SIR}_{sectored} \tag{6}$$

For the best inner radius equation will be

$$\frac{r^{-\alpha}}{6[(\sqrt{3}R-r)^{-\alpha} + (2\sqrt{3}R-r)^{-\alpha} + (3R-r)^{-\alpha}]} = \frac{1}{6^{-\alpha} + (2+\sqrt{3})^{\alpha\mu}(1.5\sqrt{3})^{-\alpha}} \tag{7}$$

High SE is obtained by maintaining high SIR in the system. SE for user i is shown in Eq.(8), where Ω is the inverse of the reuse factor.

$$SE_i = \Omega log_2(1 + SIR_i) \tag{8}$$

A user's QoS can be measured by his outage probability O which defined as the probability that a user's SIR falls below certain threshold value SIR_{th}, and is given by: [[12], [17]]

$$O_i = \text{prob}(SIR_i \preceq SIR_{th}) = 1 - \Pi_{k \in Z} \frac{1}{1 + SIR_{th}(\frac{r_k^{-\alpha}}{r^{-\alpha}})} \tag{9}$$

III. SOFT FREQUENCY REUSE (SFR)

Figure 3 shows two tiers network that uses the concept of soft frequency reuse. In this case, there are two different power levels according to the mobile user location if it is CEU or CCU. β is the power ratio of $\frac{P_0}{P_i}$, where P_0 is the transmitted power of exterior mobile user to its serving BS and P_i is the transmitted power of interior user to its serving BS. ICI comes from both interior and exterior uplinks. As shown in Fig. 1(b) the power control factor $\beta \geq 1$ is applied. Interior user transmit power will equal $Pr^{\alpha\mu}$. A practical range of β in uplink is between 2 and 100 [24], [25]. SIR equation for SFR will be different from sectored FFR case because of the power level difference. For an interior user served by home base station BS_0 and far away from it by distance r. The SIR equation in case of SFR will be as following:

$$SIR_{SFRinner} = \frac{gPr^{\alpha(\mu-1)}}{P\sum_{z \in Z_{inner}} g_z r_z^{\alpha\mu} D_z^{-\alpha} + \beta P \sum_{z \in Z_{edge}} g_z R_z^{\alpha\mu} D_z^{-\alpha}} \tag{10}$$

where, Z_{inner} is the set of all interfering mobile stations transmitting to its serving base stations on the same sub band as user y. Z_{edge} is the same of Z_{inner} but for edge mobile users that use the same frequency of the interior user. In this case final SIR expression will equal

$$SIR_{SFRinner} = \frac{r^{\alpha(\mu-1)}}{3\beta(\frac{\sqrt{3}}{2}R^{\alpha})^{\mu}A + 3(r^{\alpha})^{\mu}B} \quad (11)$$

where

$$A = (\frac{\sqrt{3}}{2}R)^{-\alpha} + (3\frac{\sqrt{3}}{2}R)^{-\alpha}$$

$$B = (\sqrt{3}R - r)^{-\alpha} + (2\sqrt{3}R - r)^{-\alpha} + 2(3R - r)^{-\alpha}$$

If we define X as the ratio of inner and outer radius of the cell, i.e., $x = \frac{r}{R}$, then Eq.(11) will equal

$$SIR_{SFRinner} = \frac{x^{\alpha(\mu-1)}}{3\beta(\frac{\sqrt{3}}{2})^{\alpha\mu}A_x + 3x^{\alpha\mu}B_x} \quad (12)$$

where

$$A_x = (0.5\sqrt{3})^{-\alpha} + (1.5\sqrt{3})^{-\alpha}$$

$$B_x = (\sqrt{3} - x)^{-\alpha} + (2\sqrt{3} - x)^{-\alpha} + 2(3 - x)^{-\alpha}$$

For exterior mobile user faraway from BS_0 by distance equals cell radius R, its worst SIR will equal

$$SIR_{SFRouter} = \frac{g\beta PR^{\alpha(\mu-1)}}{P\sum_{z\in Z_{inner}} g_z(r_z^{\alpha})^{\mu}D_z^{-\alpha} + \beta P\sum_{z\in Z_{edge}} g_z(R_z^{\alpha})^{\mu}D_z^{-\alpha}}$$
$$= \frac{\beta R^{\alpha(\mu-1)}}{\beta(R^{\alpha})^{\mu}6(2R)^{-\alpha} + 6(r^{\alpha})^{\mu}[(\sqrt{3}R - r)^{-\alpha} + (2\sqrt{3}R - r)^{-\alpha}]}$$
$$= \frac{\beta}{6x^{\alpha\mu}[(\sqrt{3} - x)^{-\alpha} + (2\sqrt{3} - x)^{-\alpha}] + 6\beta 2^{-\alpha}} \quad (13)$$

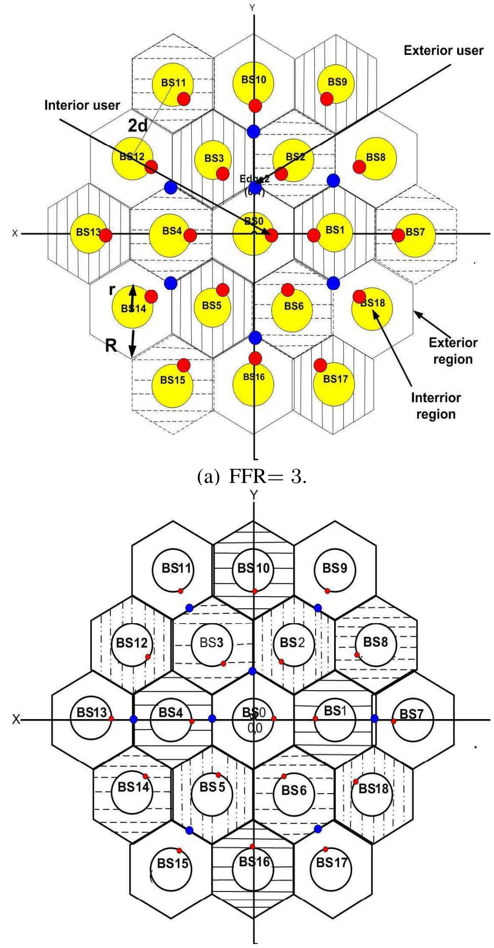
The best inner radius r is calculated by equalizing Eq. (12), and last step of Eq. (13). This yields an equation for one unknown which is given as X. Hence, we can easily calculate inner radius r for a given cell radius R.

IV. Strict FFR with FRF= 3 and 4

Figure 4 (a) and (b) shows the two tiers, strict FFR, OFDMA network with FRF= 3 and 4, respectively. If we repeat the same UL analysis of the previous two sections on that case for an exterior user, its worst case SIR at FRF= 3, and 4 will respectively change to:

$$SIR_{FFR3} = \frac{gPR^{\alpha(\mu-1)}}{gP(R^{\alpha})^{\mu}6(2R)^{-\alpha}} = \frac{2^{\alpha-1}}{3} \quad (14)$$

$$SIR_{FFR4} = \frac{gPR^{\alpha(\mu-1)}}{gP[6(R\sqrt{3}/2))^{\alpha\mu}((3R\sqrt{3})/2)^{-\alpha}]}$$
$$= \frac{1}{6(\sqrt{3}/2)^{\alpha\mu}(1.5\sqrt{3})^{-\alpha}} \quad (15)$$



(a) FFR= 3.

(b) FFR= 4.

Fig. 4: UL two tiers network with FRF=3 and 4.

For optimum inner radius in FRF= 3 will be as follow:

$$\frac{r^{-\alpha}}{(\sqrt{3}R - r)^{-\alpha} + (2\sqrt{3}R - r)^{-\alpha} + (3R - r)^{-\alpha}} = 2^{\alpha} \quad (16)$$

In case of strict FFR with FRF= 4 the optimum inner radius equation is given as:

$$\frac{r^{-\alpha}}{6[(\sqrt{3}R - r)^{-\alpha} + (2\sqrt{3}R - r)^{-\alpha} + (3R - r)^{-\alpha}]}$$
$$= \frac{[1.5\sqrt{3}]^{\alpha}}{6[\frac{\sqrt{3}}{2}]^{\alpha\mu}} \quad (17)$$

V. Numerical Results

Figure 5 shows the relation between SIR and α for different possible values of α using Eqs. (4), (5), (13) and (14) of reuse one, sectored FFR, strict FFR with FRF = 3, and FRF = 4, respectively. Practically, the SIR at different locations is greater than worst SIR at the cell corner. It is noted that as α increases, the worst SIR also increase. This is because as α increases (attenuation increases) both the received signal and interfering signals powers decrease. However the decrease in the interfering signal is more than that of received signal resulting in an increase of SIR. It is clear that the power control exponent μ has large effect on sectored FFR and strict

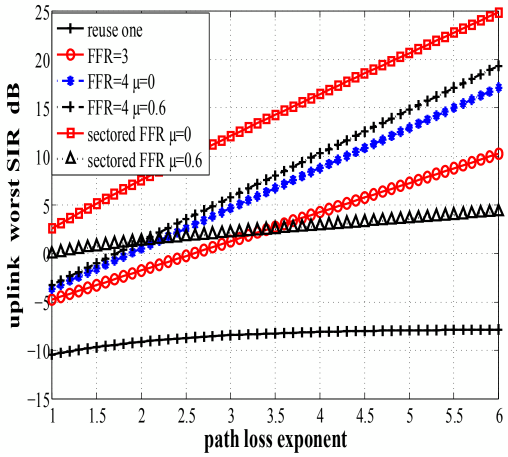


Fig. 5: Worst SIR vs path loss exponent for Reuse one, FFR3, and sectored FFR, FFR4 with and without power control.

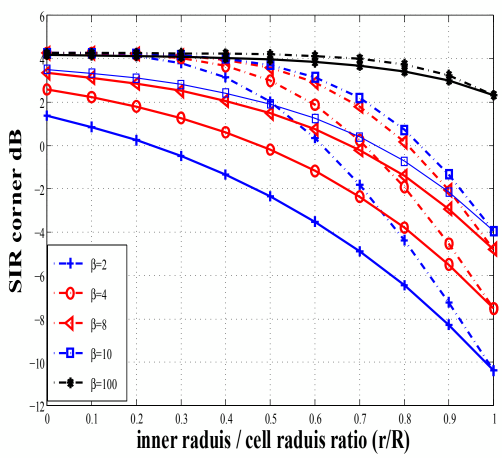


Fig. 6: Worst SIR vs path loss exponent for SFR at $\beta = 2, 4, 8, 10$, and 200. $\alpha = 4$, no power control $\mu = 0$(lines), with power control $\mu = 0.6$ (dotted lines).

FFR with FRF$= 4$ schemes according to their corresponding equations. For the same value of α, FFR with reuse four has the largest SIR value. The effect of power control in uplink is shown in the figure, in the case of FRF$= 4$ with $\mu = 0.6$ have the larger value than $\mu = 0$, while in sectored FFR it has a reverse effect due to the use of directional antenna. It is obvious from the curve that sectored FFR without power control has the highest values of SIR because of utilizing directional antenna that provides equal power at its coverage area, hence no need for power control. Also reuse one has lowest values of SIR due to large interference at the edges. For $\alpha = 3.5$, the worst SIR $= -7, 4, 6, 9, 15, and$ 3 dB for reuse one, FRF $= 3$, FRF $= 4$ with no power control, FRF $= 4$ with power control $\mu = 0.6$, sectored FFR without power control, and sectored FFR with power control, respectively.

Figure 6 previews worst SIR of SFR against the ratio of inner radius to cell radius at different values of power control factor β which are 2, 4, 8, 10, and 100. The figure is drawn using equation (13) for $\alpha = 4$. The relation is drawn twice for no power control exponent ($\mu = 0$) represented by solid lines, and $\mu = 0.6$ represented by dotted lines. The effect of power control on mobile is obvious from the figure as the SIR

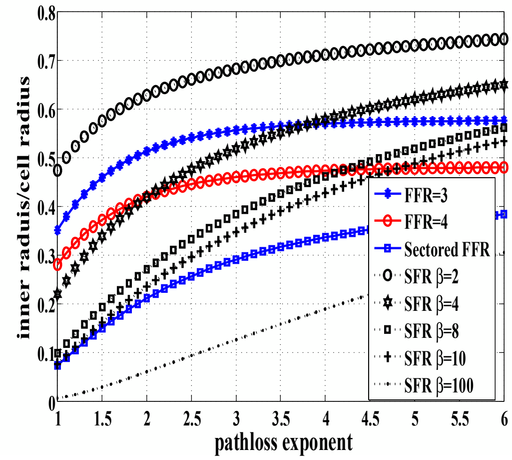


Fig. 7: Inner radius to cell radius ratio versus path loss exponent at no power control ($\mu = 0$).

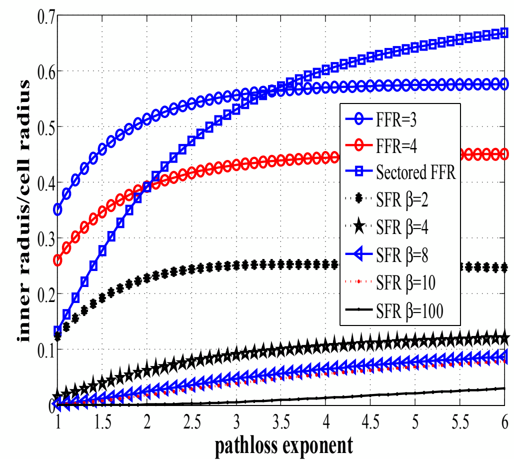


Fig. 8: Inner radius to cell radius ratio versus path loss exponent at power control ($\mu = 0.6$).

increases in case of $\mu = 0.6$ than no power exponent case. The best SIR occurs for $\beta = 100$ and $\mu = 0.6$. While the lowest SIR value occurs when $\beta = 2$ and $\mu = 0$. SIR is proportional to both values of β and μ. For larger values of $\frac{r}{R}$, SIR degrades quickly due to the increased ICI resulted from neighboring cells. Figures 7 and 8 present the relation between the ratio of inner radius to cell radius versus path loss exponent for three main types of FFR with and without power control exponent respectively. Figure 7 shows the no power control case where $\mu = 0$. Sectored FFR inner radius is small because of no power control. It is clear that strict FFR with FRF $= 3$ curve does not change in the two figures due to the negligible effect of μ. On the contrary for SFR case, power control exponent reduces the overall ratio of inner to cell radius for the same path loss exponent as shown in the two figures. It is clear that power control exponent greatly affects sectored FFR inner radius. Figure 9 previews the SE of edge user with path loss exponent in case of sectored FFR, Strict FFR with FRF$= 3$ and 4, and reuse one. The curves are drawn by substituting SIR values into Eq.(5) to get their SEs. SE is calculated using 10^{-5} bit error rate. Sectored FFR without power control has the best SE due to the efficient use of not only powers but also frequencies in the cell. The second best SE is sectored FFR

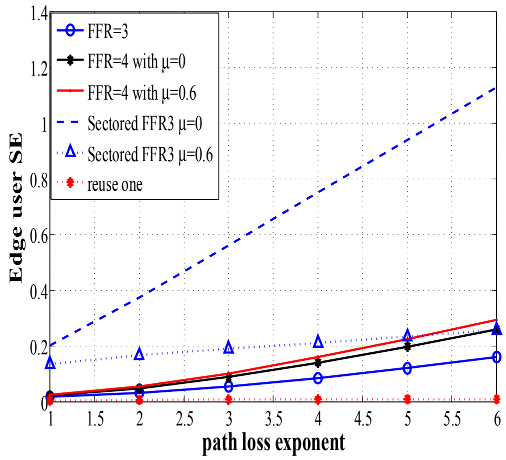


Fig. 9: Edge user SE vs path loss exponent for Reuse one, FFR3, and sectored FFR,FFR4 with and without power control.

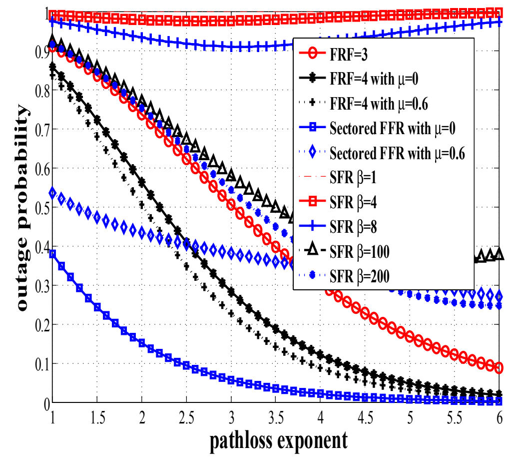


Fig. 10: Outage probability VS path loss exponent for FRF= 3, FRF= 4 without power control $\mu = 0$, FRF= 4 with power control $\mu = 0.6$, sectored FFR with and without power control, and SFR at $\beta = 1, 4, 8, 100, and\ 200$

with power control, and the lowest curve is frequency reuse one because it suffers from the largest interference. Figure 10 shows the outage probability (1-coverage probability) using Eq.(9) at $SIR_{th} = 0$ dB. It is clear from the curve that reuse one owns the largest outage probability while sectored FFR without power control has the lowest one. Strict FFR with FRF= 4 with power control has lower outage than of wihout power control and FRF= 3, respectively.

VI. CONCLUSIONS

FFR is one of the most important ICI mitigation techniques in OFDMA based Cellular systems. In this paper, we presented a new analytical method for analyzing not only worst case SIR but also for best inner radius in the uplink of cellular networks utilizing FFR schemes. These expressions are based on the distribution of the mobile users in two tiers network so as to compute the worst SIR. The problem of calculating the universal reuse inner distance is solved by equalizing worst SIR of both inner and outer regions. The approach was applied to three different FFR schemes sectored FFR, SFR, and Strict FFR with FRF= 3 and = 4, respectively. It was found that the worst SIR depends basically on the path loss exponent, and partially on the power control exponent in uplink case. The effect of power control exponent was also studies on the three main FFR schemes as it was found that it affects SFR and FFR with reuse four greatly and has no effect on strict FFR with reuse three. The outage probabilities and SEs of the different FFR schemes were obtained. Sectored FFR is a good solution for the trade off between strict FFR and SFR but at the expense of increased system complexity. Combined log normal shadowing and Rayleigh fading analysis will be considered as a future work.

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