# Gradual MPPM: A Method to Improve Bandwidth Utilization Efficiency in Optical MPPM

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*Abstract*— **Gradual multi-pulse pulse position modulation (Gradual MPPM) is proposed as a new modulation technique in optical communications systems. The proposed modulation scheme achieves both higher transmission efficiency and lower levels of SER than ordinary MPPM scheme at the same average power level.**

## I. Introduction and System Model

All Multi-pulse position modulation (MPPM) was proposed in [1] to increase the bandwidth utilization efficiency of ordinary single pulse-position modulation (PPM). Instead of transmitting single optical pulse per frame, several pulses are allowed for transmission in order to increase number of symbols carried per frame [2]. Toward further increasing in number of transmitted symbols per frame while maintaining a reasonable small number of slots per frame, we proposed a new modulation scheme which called gradual multi-pulse position modulation (Gradual n-pulse M-PPM). In the ordinary multi-pulse pulse position modulation with frame size of M slots (n-pulse M-PPM), the transmitted frames contain $n$ optical pulses resulting in transmission of $\log_2\binom{M}{n}$ bits per frame. Clearly, in order to increase the number of transmitted bits per frame, the modulation constellations must be increased. Toward that, instead of transmitting $n$ optical pulses per frame, we allow the transmission of one or two up to $n$ optical pulses per frame As indicated in fig. 1. Thus, the number of transmitted bits per frame for gradual n-pulse M-PPM scheme is $\log_2 \sum_{i=1}^{n}\binom{M}{i}$ which is much larger than the case of ordinary scheme. Moreover, for the gradual scheme, the maximum value of $n$ could be increased to reach $M$ while in ordinary scheme this value is limited to $M/2$.

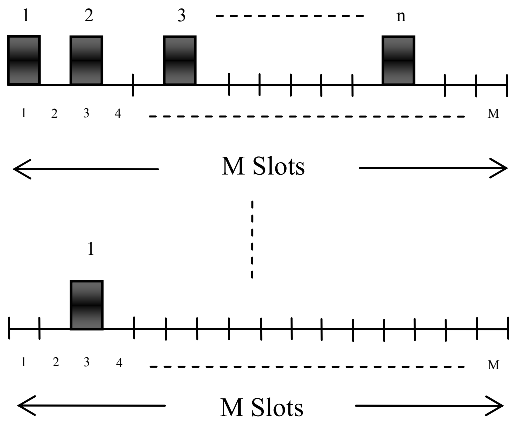


Fig. 1. Frame structure in gradual MPPM

The optimal decoding (maximum likelihood decoding) algorithm for the Gradual n-pulse M-PPM scheme in discrete memory-less channel is summarized in the following steps:

1) Sort the received photo-counts in a descending order.
2) Decode the first maximum photo-count as one.
3) For $i = 2$ to $n$
   If (next maximum $\geq Th$)
   Then: Decode it as one
   Else: Decode it and the remaining counts as zeros and end the algorithm.
4) Decode the remaining $M-n$ slots as zeros.

## II. Performance Evaluation

Toward the calculation of symbol error rate for gradual n-pulse M-PPM, we recall the calculation procedure of symbol error rate (SER) for the ordinary n-pulse M-PPM as stated in [3]. Here, we consider a discrete memory-less optical channel with Poisson distributions for both signal and background optical radiations with averages of $K_s$ and $K_b$ respectivly. The detected photon-counts associated with the slots of each frame are summarized in the received $M$-component count vector $\mathbf{k} = (k_1, k_2, k_3, \ldots, k_n, \ldots, k_M)$. Let $p_0(k_i)$ and $p_1(k_i)$ denote the count probabilities of slot $i \in \{1,2,\cdots,M\}$ in case of non-signal and signal slot, respectively and let $P_0(k_i)$ *and* $P_1(k_i)$ denote their cumulative distributions, respectively. Also, we define a threshold value $Th = K_s/\ln\left(1+\frac{K_s}{K_b}\right)$. In the calculations of probability to receive symbols correctly $p(c)$, we consider the following three cases:

**Case A**: In this case, we consider the transmitted symbols that contain only one signal slot and the remaining $M-1$ slots are non-signal slots. Let $k_{max} = max\,(k_2, k_3, \ldots, k_M)$ and let $l$ denotes the number of non-signal slots that have this count value. Here, the decoder chooses randomly one slot out of $l+1$ slots to be decoded as one and the remaining as zeros. The total probability to receive this symbol correctly is given by:

$$p(c)_A = \sum_{k_{max}=0}^{Th-1} \sum_{l=1}^{M-1} \sum_{m=0}^{1} I(l,m)\binom{M-1}{l} * p_0(k_{max})^l * P_0(k_{max}-1)^{M-1-l} p_1(k_{max})^m (1-P_1(k_{max}))^{1-m} \quad (1)$$

**Case B**: In this case, we compute the probability of correct transmission for symbols that contain number of signal slots $i$ greater than one and less than $n$ $(1 < i < n)$. Therefore, the probability to decode this symbol correctly is given by

$$p(c)_{Bi} = [1 - P_1(Th-1)]^i * [P_0(Th-1)]^{M-i} \quad (2)$$

**Case C**: In this case, we compute the probability of correct transmission for the symbols that contain $n$ signal slots. We obtain the probability of correct decoding for this symbol as:

$$p(c)_C = \sum_{k_{min}=Th}^{\infty} \sum_{l=0}^{M-n} \sum_{m=1}^{n} I(l,m) \binom{M-n}{l} p_0(k_{min})^l * P_0(k_{min}-1)^{M-n-l} \binom{n}{m} p_1(k_{min})^m (1-P_1(k_{min}))^{n-m} \quad (3)$$

Finally, the total probability of correct symbol transmission for gradual n-pulse M-PPM could be calculated as an average probability of the correct decoding over all the transmitted symbols. The SER is thus:

$$SER = 1 - p(c) = 1 - \frac{1}{\sum_{j=1}^{n}\binom{M}{j}} * \left[\binom{M}{1} * p(c)_A + \sum_{i=2}^{n-1}\left(\binom{M}{i} * p(c)_{Bi}\right) + \binom{M}{n} * p(c)_C\right] \quad (4)$$

## III. Numerical Results

In this section we compare the performance of the proposed Gradual n-pulse MPPM to the ordinary n-pulse M-PPM in terms of the average symbol error rate at the same average received power. To clarify the comparison, we use the same frame size $M$ and the same slot duration $T_s$ for both schemes resulting in the same frame rate. Furthermore, the background radiations (noise photons) are considered in the carried simulations by the mean of average number of the received background photons per slot $K_b$. The simulations are performed at two noise levels, which are $K_b = 1$ and $K_b = 5$. Also, the average power comparison could be replaced by the average number of received photons per frame $K_{av}$.

The results for the case $M = 8$ are shown in fig. 2, which carries the comparison between two specific schemes: 4-pluse 8-MPPM and gradual 3-pulse 8-MPPM. This selection achieves nearly the same bandwidth utilization with an advantage to the gradual one. The figure indicates the out-performance of the proposed schemes in achieving less SER at different values of average received photons per frame $K_{av}$. Specifically, at $K_{av} = 40$ photon, the gradual n-pulse M-PPM achieves 8dB reduction in SER at $K_b = 1$ and a reduction of 12 dB at $K_b = 5$. This large reduction in SER at higher $K_b$ levels makes the gradual scheme robust against back ground noise.

For the case of $M = 16$, as indicated in fig. 3, the comparison is carried between two modulation schemes which are 8-pluse 16-MPPM and gradual 6-pulse 16-MPPM. The figure emphasizes the superior performance of the proposed gradual scheme over the ordinary one.

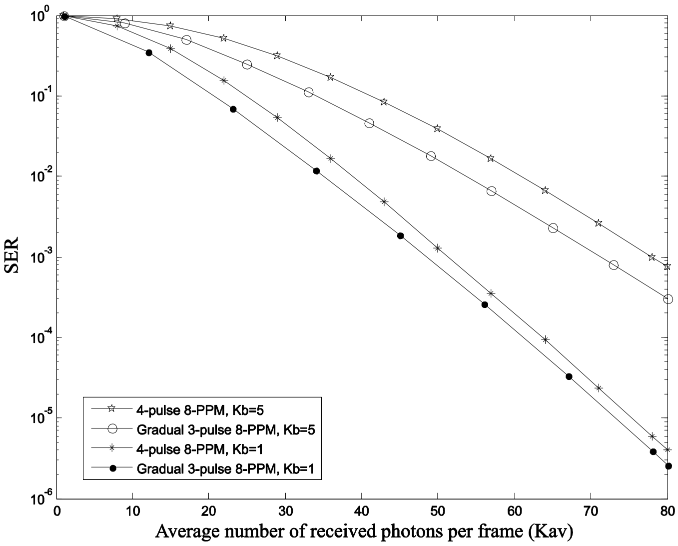


Fig. 2. Performance of Gradual 3-pulse 8-PPM

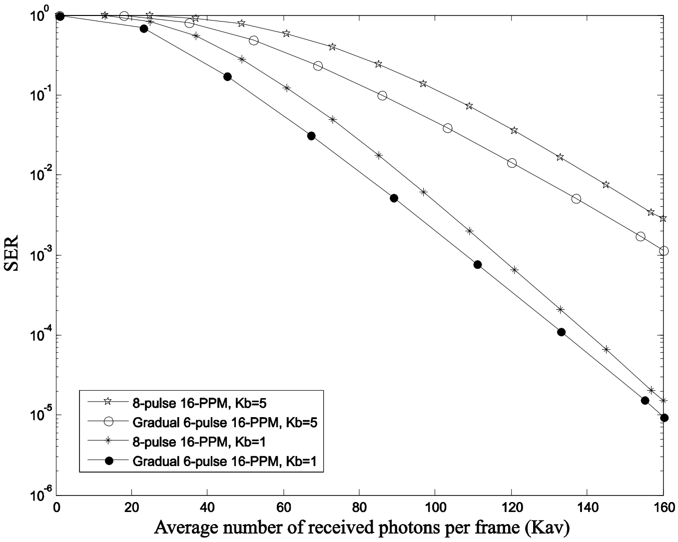


Fig. 3. Performance of Gradual 6-pulse 16-PPM

## V. Conclusion

A new modulation scheme, gradual n-pulse M-PPM, of the family of pulse position modulations is proposed for optical communications. The proposed scheme achieves much higher bandwidth utilization efficiency than that of the ordinary MPPM scheme. On a discrete memory-less channel, the maximum likelihood decoding criteria for the proposed scheme is derived resulting in a simple and fast decoding algorithm. The performance measure of the proposed scheme in terms of exact symbol error rate is obtained.

## References

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