Performance Analysis of Optical CDMA Communication Systems with PPM Signaling

Hossam M. H. Shalaby, *Member, IEEE*
Department of Electrical Engineering,
University of Alexandria, Alexandria 21544, Egypt.

Abstract—Direct-detection optical synchronous code-division multiple-access (CDMA) systems with M-ary pulse-position modulation (PPM) signaling are investigated. Optical orthogonal codes are used as the signature sequences of our system. A union upper bound on the bit error rate is derived taking into account the effect of the background noise, multiple-user interference, and receiver shot noise. The performance characteristics are then discussed for a variety of system parameters. Our results show that under average power and bit error rate constraints, there always exists a pulse position multiplicity that permit all the subscribers to communicate simultaneously.

I. Introduction

Optical code-division multiple-access (CDMA) systems have been given an increasing interest in the recent years [1–6]. This is due to the vast bandwidth offered by the optical links and the extra-high *optical* signal processing speed offered by the optical components. As a result, Optical CDMA can accommodate a larger number of simultaneous users than the radio-frequency techniques.

Most work done on optical CDMA has concentrated on the binary transmission of data, e.g., on-off keying (OOK). Very few suggested using M-ary transmission of data [4,5]. Lam and Hussain [4], for example, suggested an M-ary system in which each symbol is represented by one of M mutually orthogonal sequences (signatures). Thus a total of MN code sequences are required, where N is the number of users. In [5], Dale and Gagliardi suggested encoding the symbols using M-ary pulse-position modulation (PPM) format and then transmitting an aperiodic signature in place of the PPM pulse. In their analysis, Dale an Gagliardi assumed that the photodetector shot noise, dark current, and thermal noise can be modeled as Gaussian random processes. They showed that under fixed bit rate and chip time, there is no advantage in using PPM. On the other hand, they showed that PPM is superior to OOK if the average power rather than the chip time is the constraining factor.

Our main objective in this paper is to investigate the performance of direct-detection optical M-ary PPM-CDMA communication systems under the assumption of Poisson shot noise model for the receiver photodetector and synchronization between the users' symbols.

In PPM signaling format, each symbol is represented by a single laser pulse positioned in one of M (disjoint) possible time slots. The width of each slot is τ seconds. The entire symbol thus extends over a time frame of $T = M\tau$ seconds. This signaling format is attractive in optical communications because of its simple implementation and efficient use of the available source energy [8].

The model for an optical PPM-CDMA communication system is shown in Figs. 1 and 2. The transmitter in Fig. 1 is composed of N simultaneous users. Each user transmits M-ary continuous data symbols. The output symbol of the kth information source modulates the position of a laser pulse to form the PPM signal. This signal is then multiplied by a spreading sequence $a^k(t)$, which characterizes the kth user. The output waveform is finally transmitted over the optical channel. In asynchronous CDMA each optical signal is time delayed by Δ_k with respect to a reference instant $t = 0$. In synchronous CDMA, however, there are no time delays among the signals, $\Delta_k = 0$. An equivalent all-optical multiplier [2,4] can be just a tapped optical delay line [7]. At the receiving end, the received optical signal (composed of the sum of the N delayed users' optical signals in addition to the background noise) is multiplied by the same sequence $a^k(t)$ and then converted using the photodetector into an electric signal which is passed to the PPM decoder to obtain the data, Fig. 2. The PPM decoder is just a comparison between the photon counts over the M time slots: the number of the slot with the greatest count is declared to be the transmitted symbol. To make full use of the vast bandwidth available to the optical network, an equivalent all-optical receiver should be used, where the received optical signal is passed to a matched filter (a tapped optical delay line) and then photodetected. An integrator over chip intervals then follows the photodetector. Finally, the electric signal is sampled at M different instants $\{\tau, 2\tau, \ldots, M\tau\}$ to provide the photon counts over the M time slots.

The rest of the paper is organized as follows: Section II is devoted for the derivation of the bit error probabilities. In Section III we present some numerical results where we investigate the effect of some parameters (the background noise, the number of users, the pulse-position multiplicity, etc.) on the performance of the optical PPM-CDMA system. Finally, we give our conclusion in Section IV.

II. Bit Error Rate of the Optical PPM-CDMA Systems

We assume that each user is assigned an optical code sequence (or signature sequence) of length L and weight ν. That is the kth user is assigned the code sequence $(a_0^k, \ldots, a_{L-1}^k)$, where $a_i^k \in \{0,1\}$ and the number of i's with $a_i^k = 1$ equals ν. The spreading signature waveform, $a^k(t)$, is assumed to be periodic of period τ (the slot width), hence it can be written as

$$a^k(t) = \sum_{i=-\infty}^{\infty} a_i^k P_{T_c}(t - iT_c) \ ,$$

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where $a_{i+L}^k = a_i^k$ for all integers i, $T_c = \tau/L$ is the chip time, and $P_{T_c}(\cdot)$ is a rectangular pulse of duration T_c:

$$P_\tau(t) \stackrel{\text{def}}{=} \begin{cases} 1 ; & \text{if } 0 < t < \tau, \\ 0 ; & \text{otherwise.} \end{cases}$$

The kth information source generates the data sequence $\{b_j^k\}_{j=-\infty}^{\infty}$, where $b_j^k \in \{0, \ldots, M-1\}$. This sequence modulates the position of a laser pulse so that the output of the optical PPM encoder can be written as

$$b^k(t) = \sum_{j=-\infty}^{\infty} \lambda_s P_\tau(t - b_j^k \tau - jT) ,$$

where λ_s is the signal photon rate (which is assumed to be constant for all the transmitters). This PPM signal is then multiplied by $a^k(t)$ to give the baseband signal of the kth user:

$$d^k(t) = b^k(t) a^k(t) .$$

We assume that the kth signal is associated with zero delay (synchronous transmitters). Hence the total signal waveform can be written as

$$s(t) = \sum_{k=1}^{N} d^k(t) .$$

The received waveform at the front end of each receiver is thus

$$r(t) = s(t) + n(t) ,$$

where $n(t)$ is the optical background noise. The input to the photodetector of user 1, Fig. 2, is thus given by

$$\begin{aligned} y(t) &= r(t) a^1(t) \\ &= [a^1(t)]^2 b^1(t) + a^1(t) n(t) + \sum_{k=2}^{N} a^1(t) a^k(t) b^k(t) . \end{aligned}$$

The first term in the last equation is the desired signal, the second term is due to the background noise, and the last term is due to the interference from other users. Since we are dealing with direct (noncoherent) detection, the optical signals and the optical noise are additive in intensity. The photon count over the ith slot of the kth user can be modeled as a conditional Poisson random variable Y_i^k [8]. Thus for the first user:

$$(\forall i \in \{0, \ldots, M-1\}) \qquad Y_i = Z_i + W_i + I_i ,$$

where we have assumed for simplicity that $Y_i^1 = Y_i$. Here W_i is a Poisson photon count due to the background noise. Z_i and I_i are conditionally independent Poisson photon counts given $\{b_j^k\}$ due to the desired signal and the multiple-user interference, respectively. To obtain the conditional mean of Y_i, we evaluate the conditional expectation of each of the three photon counts composing it. Denoting the background noise photon rate by λ_0, it is easy to see that

$$E[W_i] = \nu \lambda_0 T_c .$$

The Poisson random variable Z_i depends only on b_0^1. Hence its conditional mean value is given by

$$\begin{aligned} E[Z_i | b_0^1] &= \int_{i\tau}^{(i+1)\tau} [a^1(t)]^2 b^1(t) dt \\ &= \int_{i\tau}^{(i+1)\tau} \lambda_s [a^1(t)]^2 \sum_j P_\tau(t - b_j^1 \tau - jT) dt . \end{aligned}$$

Whence

$$E[Z_i | b_0^1] = \begin{cases} \nu \lambda_s T_c ; & \text{if } b_0^1 = i, \\ 0 ; & \text{otherwise.} \end{cases}$$

The conditional mean of the random variable I_i given data symbols $\{b_0^k\}_{k=2}^{N}$ can be written as

$$E[I_i | \{b_0^k\}_{=2}^{N}] = \lambda_s \sum_{k=2}^{N} \int_{i\tau}^{(i+1)\tau} a^1(t) a^k(t) P_\tau(t - b_0^k \tau - jT) dt .$$

Let

$$I_i \stackrel{\text{def}}{=} \sum_{k=2}^{N} I_i^k , \tag{1}$$

where I_i is the interference in ith interval due to the kth user. This is a conditional Poisson random variable (given b_0^k) with expectation:

$$\begin{aligned} E[I_i^k | b_0^k] &= \lambda_s \int_{i\tau}^{(i+1)\tau} a^1(t) a^k(t) P_\tau(t - b_0^k \tau - jT) dt \\ &= \begin{cases} C_{1k} \lambda_s T_c ; & \text{if } b_0^k = i, \\ 0 ; & \text{otherwise} \end{cases} \\ &= C_{1k} \lambda_s T_c \delta_{b_0^k, i} , \end{aligned}$$

where $\delta_{b_0^k, i}$ is the Kronecker delta and C_{1k} denotes the cross-correlation between the first and kth code. In the remaining analysis we employ the optical orthogonal codes (OOC's) as our typical signature code sequences. OOC's with periodic cross-correlations and periodic auto-correlations bounded by one have been extensively studied in [6]. It has been shown that the maximum number of codes (subscribers) is at most

$$N \leq \frac{L-1}{\nu(\nu-1)} .$$

Thus we have

$$(\forall k, l \in \{1, \ldots, N\}, \ k \neq l) \qquad C_{kl} \leq 1 .$$

Assume, for simplicity, $C_{1k} = 1$ for every $k \in \{2, \ldots, N\}$ (worst case conditions). Hence

$$E[I_i^k | b_0^k] = \lambda_s T_c \delta_{b_0^k, i} .$$

Substituting in (1) yields

$$E[I_i | \{b_0^k\}_{k=2}^{N}] = \lambda_s T_c \sum_{k=2}^{N} \delta_{b_0^k, i} .$$

Define the following set of random variables

$$\kappa_i \stackrel{\text{def}}{=} \sum_{k=2}^{N} \delta_{b_0^k,i}, \qquad i \in \{0,\ldots,M-1\} .$$

We notice that each κ_i is a binomial random variable, i.e., for every $l \in \{0,\ldots,N-1\}$

$$\Pr\{\kappa_i = l\} = \binom{N-1}{l}\left(\frac{1}{M}\right)^l\left(1-\frac{1}{M}\right)^{N-1-l} .$$

Moreover, denote the vector $(\kappa_0,\ldots,\kappa_{M-1})$ by κ. It is easy to see that κ is a multinominal random vector with probability

$$\Pr\{\kappa = (l_0,\ldots,l_{M-1})\} = \frac{1}{M^{N-1}} \cdot \frac{(N-1)!}{l_0!l_1!\ldots l_{M-1}!} , \quad (2)$$

where $\sum_{j=0}^{M-1} l_j = N-1$. Assuming equally likely data symbols, the probability of a correct decision can be lower bounded by:

$$\begin{aligned} P_C \geq & \sum_{i=0}^{M-1} \Pr\{b_0^1 = i\} \times \\ & \qquad \Pr\{Y_i > Y_0,\ldots,Y_{i-1},Y_{i+1},\ldots,Y_{M-1}|b_0^1 = i\} \\ = & \Pr\{Y_0 > Y_1,\ldots,Y_{M-1}|b_0^1 = 0\} \\ = & \sum_{l_0=0}^{N-1}\sum_{l_1=0}^{N-1-l_0}\cdots\sum_{l_{M-1}=0}^{N-1-l_0-\cdots-l_{M-2}} \\ & \qquad \Pr\{Y_0 > Y_1,\ldots,Y_{M-1}|b_0^1 = 0, \kappa = l\}\Pr\{\kappa = l\}, \quad (3) \end{aligned}$$

where $l \stackrel{\text{def}}{=} (l_0,\ldots,l_{M-1})$ and

$$\begin{aligned} & \Pr\{Y_0 > Y_1,\ldots,Y_{M-1}|b_0^1 = 0, \kappa = l\} \\ & \quad = \sum_{k=0}^{\infty} e^{-(K_s+K_b+\lambda_s T_c l_0)} \frac{(K_s + K_b + \lambda_s T_c l_0)^k}{k!} \times \\ & \qquad \prod_{j=1}^{M-1}\left[\sum_{i=0}^{k-1} e^{-(K_b+\lambda_s T_c l_j)} \frac{(K_b + \lambda_s T_c l_j)^i}{i!}\right] , (4) \end{aligned}$$

where we have defined

$$\begin{aligned} K_s &\stackrel{\text{def}}{=} E[Z_0|b_0^1 = 0, \kappa = l] = E[Z_0|b_0^1 = 0] = \nu\lambda_s T_c , \\ K_b &\stackrel{\text{def}}{=} E[W_i|b_0^1 = 0, \kappa = l] = E[W_i] = \nu\lambda_0 T_c , \qquad (5) \end{aligned}$$

and we have used the identity

$$E[I_i|b_0^1 = 0, \kappa = l] = E[I_i|\kappa_i = l_i] = \lambda_s T_c l_i . \qquad (6)$$

Here K_s and K_b denote the average photon counts per symbol due to the signal and noise, respectively. Using the relation $P_E = 1 - P_C$, we can obtain an upper bound on the word error probability. Finally, the equivalent bit error probability P_b can be found from the fact that $P_b = \frac{M/2}{M-1}P_E$.

III. Numerical Results

It is too expensive to perform numerical calculations using the expressions derived in the previous section especially for large values of M. We thus employ a union bound on the error rate to simplify the calculations. Using (3) we can write

$$\begin{aligned} P_E &\leq 1 - \Pr\{Y_0 > Y_1,\ldots,Y_{M-1}|b_0^1 = 0\} \\ &\leq \sum_{i=1}^{M-1} \Pr\{Y_i \geq Y_0|b_0^1 = 0\} \\ &= (M-1)\Pr\{Y_1 \geq Y_0|b_0^1 = 0\} \\ &= (M-1)\sum_{l_0=0}^{N-1}\sum_{l_1=0}^{N-1-l_0} \Pr\{Y_1 \geq Y_0|b_0^1 = 0, \kappa_0 = l_0, \kappa_1 = l_1\} \\ &\qquad \times \Pr\{\kappa_0 = l_0, \kappa_1 = l_1\} , \end{aligned}$$

where

$$\begin{aligned} & \Pr\{\kappa_0 = l_0, \kappa_1 = l_1\} \\ & \quad = \binom{N-1}{l_0}\left(\frac{1}{M}\right)^{l_0}\left(1-\frac{1}{M}\right)^{N-1-l_0} \times \\ & \qquad \binom{N-1-l_0}{l_1}\left(\frac{1}{M-1}\right)^{l_1}\left(1-\frac{1}{M-1}\right)^{N-1-l_0-l_1} \\ & \quad = \frac{(N-1)!}{l_0!l_1!(N-1-l_0-l_1)!} \cdot \frac{(M-2)^{N-1-l_0-l_1}}{M^{N-1}} , \end{aligned}$$

$$\begin{aligned} & \Pr\{Y_1 \geq Y_0|b_0^1 = 0, \kappa_0 = l_0, \kappa_1 = l_1\} \\ & \quad = \sum_{k=0}^{\infty} e^{-(K_b+\lambda_s T_c l_1)} \frac{(K_b + \lambda_s T_c l_1)^k}{k!} \times \\ & \qquad \left[\sum_{i=0}^{k} e^{-(K_s+K_b+\lambda_s T_c l_0)} \frac{(K_s + K_b + \lambda_s T_c l_0)^i}{i!}\right] , \end{aligned}$$

and K_s, K_b are as given in (5). Finally the bit error rate is bounded as $P_b \leq \frac{M/2}{M-1}P_E$.

In our numerical evaluations we assume that the bit rate (throughput) is fixed at R_0. Normalizing the slot-width with each different value of M is thus mandatory. Hence

$$\tau = \frac{\log M}{MR_0} . \qquad (7)$$

K_b in (5) can now be written as

$$K_b = \left(\frac{\lambda_0}{R_0}\right)\frac{\nu \log M}{ML} ,$$

where the ratio λ_0/R_0 denotes the average background noise photons per nat time. We consider two different types of energy constraints on the laser source. The first type is average energy per pulse constraint, which is equivalent to $K_s/\nu =$ constant. The second type is average power constraint. This is equivalent to $K_s/T =$ constant or, using (7), $K_s/\log M =$ constant, i.e., fixed energy per information nat.

A. Fixed Energy per Pulse

In this part we study the effect of some system parameters on the performance (bit error rate) of optical PPM-CDMA systems under a constraint on the transmitted energy per pulse. This constraint is equivalent to fixed photon count per symbol (K_s). The product $\lambda_s T_c$ in (6) is thus equal to $\frac{K_s}{\nu}$, which denotes the average signal photon count per pulse. The bit error probability with $N = 10$, $\nu = 5$, $L = 500$, and $\lambda_0/R_0 = 100$ has been evaluated and plotted in Fig. 3 for different values of K_s and M. We have found that the effect of the background noise on the performance is negligible, which indicates that the degradation in performance under same signal energy is mainly due to the multiple-user interference. Since the spreading code length (L) affects only the received background noise power, it has also a negligible effect on the performance. On the other hand, the effect of the pulse position multiplicity (M) and the code weight (ν) on the performance is remarkable. It is seen from Fig. 3 that the performance gets better as M increases. Moreover, this improvement is associated with a save in energy because the transmitted photons per nat ($K_s/\log M$) decreases as M increases. Fig. 4 demonstrates the effect of the code weight. It is seen from this figure that the performance gets worse when decreasing ν. Indeed, lowering ν will decrease the average signal energy with respect to the interference energy which leads to a more frequent wrong decisions and hence worse bit error rate.

B. Average Power Constraint

In this part we examine the performance of the above system under a constraint on the average power (or energy per information nat). Let μ denote the number of the transmitted photons per nat, K_s in (5) is now equal to $K_s = \mu \log M$. Fig. 5 shows the bit error rate versus the average photons per nat for a fixed number of users, weight, and chip size, and different values of M. As expected, when M increases, the system performance improves significantly. As an example, for a system with 10 users, weight/chip size of 5/500, background noise count of 100 photons/nat time, and signal energy of 50 photons/nat, the bit error rate equals 6.99×10^{-2} if $M = 2$, 6.7×10^{-4} if $M = 16$, and 3.21×10^{-8} if $M = 256$. The improve in this case is better than in case A because here the energy per pulse ($\mu \log M/\nu$) increases with M, however in case A the same energy per pulse (which is fixed) is used to transmit more information as M increases.

The effect of the maximum number of simultaneous users is explored in Fig. 6 under average power constraint. Since we employ OOC's with auto- and cross-correlations that are bounded by one, the maximum number of subscribers is at most $N \leq \frac{L-1}{\nu(\nu-1)}$. As an example, consider a system with energy constraint of 30 photons/nat employing OOC's with $L = 500$ and $\nu = 5$. This system can accommodate at most 24 subscribers. Let the bit error rate be required to be less than 10^{-5}. From Fig. 6 (with $\lambda_0/R_0 = 100$) we see that systems with small values of M can not accommodate much simultaneous users. For example the number of users is at most 4, 4, or 5, when M equals 2, 4, or 16, respectively. If, on the other hand, $M = 64$, at most 10 users can communicate simultaneously and achieve the above error rate constraint. When $M = 256$, however, all the 24 subscribers can communicate reliably. The above example demonstrates that energy and bit error rate constraints may limit the maximum number of simultaneous users. However, increasing the value of the pulse position multiplicity may permit all the subscribers to communicate simultaneously.

IV. Concluding Remarks

Direct-detection optical synchronous CDMA systems with PPM signaling has been studied in details. We considered optical orthogonal codes, with cross-correlations bounded by one, as the signature code sequences in our system. The Poisson shot noise model has been assumed for the receiver photodetector. The background noise and multiple-user interference have been accounted for in estimating the bit error rate. In our numerical evaluation we derived a union upper bound on the probability of error to simplify the calculations. We have evaluated the performance under the restriction of fixed throughput rate. Our results demonstrate that under average power and bit error rate constraints, a pulse position multiplicity $M^* > 0$ always exists so that if $M \geq M^*$, all the subscribers can communicate simultaneously.

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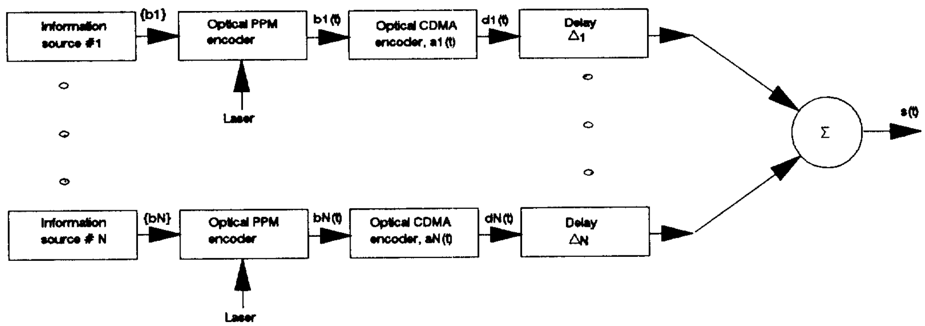

Fig. 1. Optical CDMA system with PPM signaling.

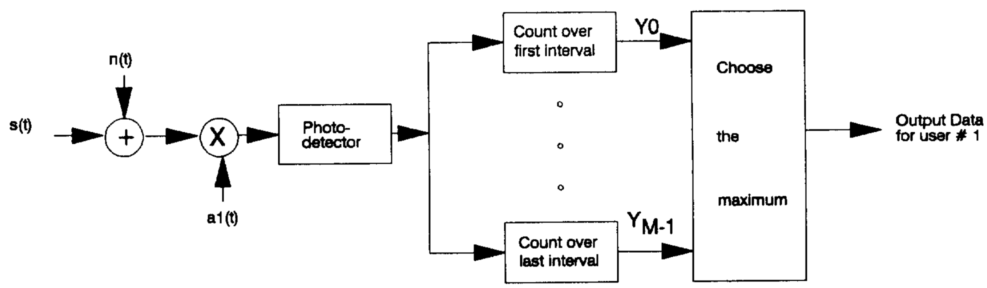

Fig. 2. Direct-detection optical PPM-CDMA system model.

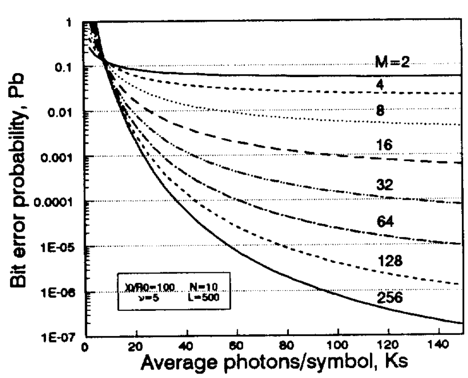

Fig. 3. Bit error probability versus Ks, M.

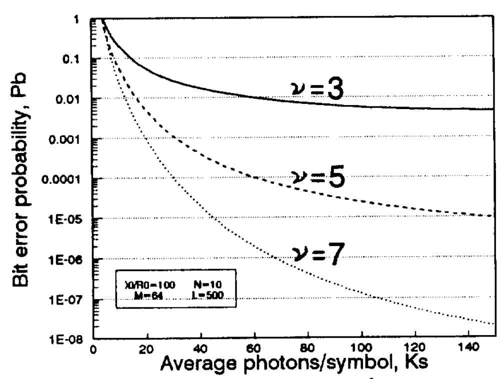

Fig. 4. Bit error probability versus Ks, ν.

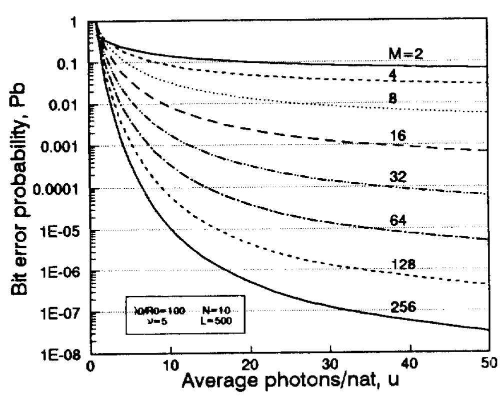

Fig. 5. Bit error probability versus u.

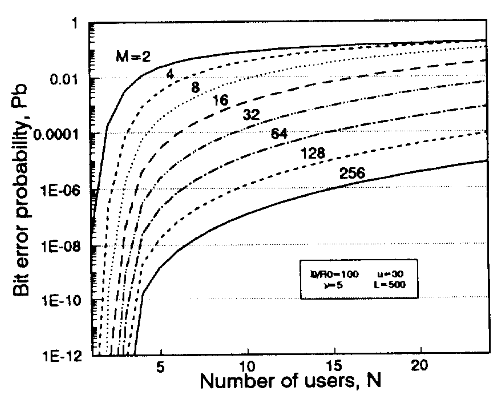

Fig. 6. Bit error probability versus the number of users.