Chip-Level Receivers in Optical Overlapping PPM-CDMA

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Abstract—**A chip-level detector for optical overlapping pulse-position modulation code-division multiple-access (OPPM-CDMA) communication systems is proposed. The bit error rate of the proposed system is derived and compared to some traditional receivers under the constraints of fixed data rate and laser pulsewidth. These traditional receivers include OOK-, PPM-, and OPPM-CDMA correlators. The throughput limitations of all these receivers are also presented and compared. Our results reveal that a significant improvement in the performance is gained when using the proposed scheme. A throughput limitation of about 7.5 times greater than that of OOK-CDMA correlator is reported on the average.**

I. Introduction

One of the most serious drawbacks in the area of direct detection optical CDMA [1]–[4] when compared to optical TDMA is the throughput limitation of the former. This restricts full utilization of the vast bandwidth offered by the optical channel. To come across this stipulation we have proposed utilizing overlapping pulse-position modulation (OPPM) scheme rather than on-off keying (OOK) in the transmitter of the optical CDMA system [5]. We have reported that when using OPPM-CDMA, the throughput can be increased by a factor 5 above that when using OOK-CDMA. In [6] we have suggested a new detector (called chip-level detector) that can be used in place of the correlator at the receiver side. It has been shown that:

i) The performance (in terms of bit error rate) of the chip-level receiver is better than that of the correlation receiver and is *asymptotically* optimal.
ii) The complexity of the chip-level receiver is independent of the number of users, and therefore the system is much more practical than the optimum receiver.

In this paper we propose applying both the aforementioned techniques in optical CDMA systems. Namely at the transmitter side we adopt OPPM-CDMA scheme, whereas at the receiver end we use a chip-level detector.

In our analysis we employ optical orthogonal codes (OOCs) as the signature code sequences [1]. In order to ensure minimal interference among the users we choose OOCs with periodic cross-correlations and out-of-phase periodic auto-correlations that are bounded by 1 only.

The remaining of this paper is organized as follows. Section II is devoted for the description of the optical OPPM-CDMA system. The proposed chip-level receiver for OPPM-CDMA is presented in Section III. The theoretical analysis for this receiver is given in Section IV. In Section V we demonstrate some of our numerical calculations, where we compare the performance of the proposed system with other optical direct-detection CDMA systems. Our conclusion in given in Section VI.

II. Optical OPPM-CDMA system Transmitter

The OPPM-CDMA communication system transmitter is composed of N simultaneous sources of information (users). Each user produces continuous data symbols. We assume that the symbols can takes values in the set $\{0, 1, \ldots, M-1\}$. The data of each user modulates a laser source using OPPM scheme. Each modulated signal is then permitted to be spread over a spreading interval of width τ. An overlap with depth $(1-\frac{1}{\gamma})\tau, \gamma \in \{1, 2\cdots, M\}$, is allowed between any two adjacent spreading intervals. Here γ is called the index of overlap. The spreading signature code sequence is assumed to have a length L and a weight w. If wrapped signals are allowed, then the OPPM time frame T is given by

$$T = \frac{M}{\gamma}\tau = \frac{M}{\gamma}LT_c\ ,$$

where T_c is the pulsewidth (chip time.) For the spreading sequence to fit properly within the spreading interval, $\frac{L}{\gamma}$ must be an integer. An example of the transmitted signal formats of a single user is shown in Fig. 1.

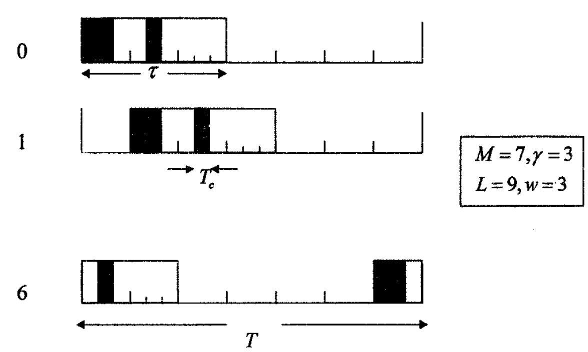

Fig. 1. An example of the transmitted signal formats of a single user in an OPPM-CDMA system with M=7, γ=3, L=9 and w=3. A signature code of 110010000 is assumed.

III. Optical OPPM-CDMA Chip-Level Receiver

At the receiving end, the received waveform is composed of the sum of N delayed and attenuated signals from each user in addition to the photodetector's dark current noise. The block diagram of this receiver is shown in Fig. 2. The photodetected received signal is integrated over each chip and then sampled at the end of each mark chip and passed to the decision subsystem to decide on the data. To make full use of the vast bandwidth available at the optical network, an equivalent all-optical receiver is shown in Fig. 3, where the received optical signal is *optically* sampled at the correct mark chips. Each sampled signal is then photodetected and integrated over the entire time frame T and is further *electronically* sampled by the end of the time frame. Finally it is passed to the decision subsystem.

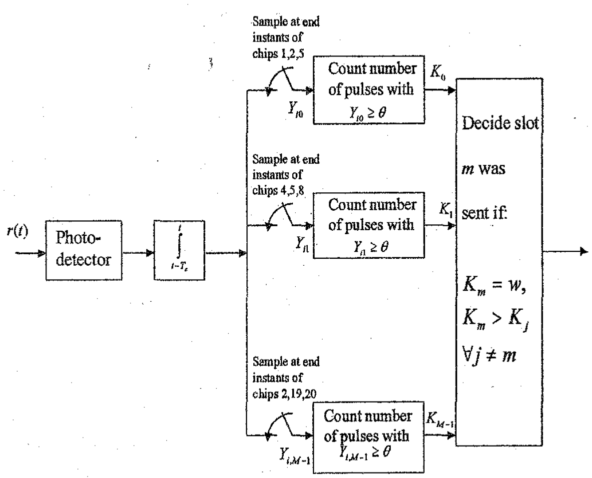

Fig. 2. An optical direct-detection OPPM-CDMA chip-level receiver.

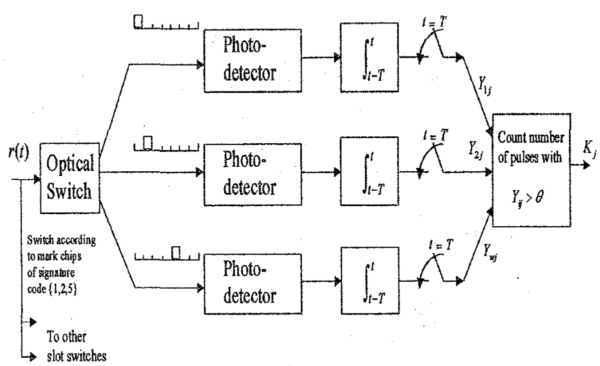

Fig. 3. An all-optical version of the OPPM-CDMA chip-level receiver.

IV. Theoretical Analysis

A. Bit Error Probability

Let $Y_{ij},\ i \in \mathcal{X} = \{1,2,\dots,w\},\ j \in \mathcal{M} = \{0,1,\dots,M-1\}$, be the photon count collected from chip number i of the mark positions of slot number j. Y_{ij} is a compound Poisson random variable. Let $\kappa_{ij},\ i \in \mathcal{X},\ j \in \mathcal{M}$ be the number of pulses that cause interference to chip i of slot j. Further let the vector $(\kappa_{1j}, \kappa_{2j}, \dots, \kappa_{ij})^T$ be denoted by κ_j^i. Assuming frame-level synchronization among the transmitters, it is easy to check that κ_j^i is a multinomial random vector with parameters $\frac{\gamma w}{ML}$ and $N-1$:

$$\Pr\{\kappa_j^i = l_j^i\} = \frac{(N-1)!}{l_{1j}!l_{2j}!\dots l_{ij}!s_{ij}!} \times \left(\frac{\gamma w}{ML}\right)^{N-1-s_{ij}} \left(1 - i\frac{\gamma w}{ML}\right)^{s_{ij}},$$

where

$$l_j^i = (l_{1j}, l_{2j}, \dots, l_{ij})^T,$$

and

$$s_{ij} = N - 1 - \sum_{n=1}^{i} l_{nj}, \qquad i \in \mathcal{X},\ j \in \mathcal{M}.$$

The Decision Rule: Symbol "m" is declared to be transmitted if there exists $m \in \mathcal{M}$ such that

$$(\forall i \in \mathcal{X}) \qquad Y_{im} \geq 1$$
$$\text{and} \qquad (\forall j \in \mathcal{M},\ j \neq m) \qquad Y_{ij} = 0,\ \text{some } i \in \mathcal{X}.$$

Otherwise an incorrect decision is declared.

We now provide an upper bound on the probability of word error P_E. The bit error rate P_b is related to P_E by the well known formula $P_b = \frac{M/2}{M-1} P_E$.

$$P_E = \sum_{j=0}^{M-1} P[E|j] \Pr\{j\},$$

where $\Pr\{j\} = 1/M$ in the case of equally likely data. It is easy to check that $P[E|j]$ is independent of j, Whence

$$\begin{aligned} P_E &= P[E|0] \\ &= \Pr\{Y_{i0} = 0,\ \text{some } i \in \mathcal{X} \\ &\qquad \text{or } Y_{ij} \geq 1\ \forall i \in \mathcal{X},\ \text{some } j \neq 0|0\} \\ &\leq \Pr\{Y_{i0} = 0,\ \text{some } i \in \mathcal{X}|0\} \\ &\qquad + \Pr\{Y_{ij} \geq 1\ \forall i \in \mathcal{X},\ \text{some } j \neq 0|0\}. \end{aligned}$$

The first probability is evaluated as follows:

$$\begin{aligned} &\Pr\{Y_{i0} = 0,\ \text{some } i \in \mathcal{X}|0\} \\ &= -\sum_{i=1}^{w} (-1)^i \binom{w}{i} \Pr\{Y_{1,0} = Y_{2,0} = \dots = Y_{i0} = 0|0\}. \end{aligned}$$

The probabilities under the summation can be evaluated as

$$\begin{aligned}
&\Pr\{Y_{1,0} = Y_{2,0} = \ldots = Y_{i0} = 0|0\} \\
&= \sum_{l_0^i} \Pr\{Y_{1,0} = Y_{2,0} = \ldots = Y_{i0} = 0|0, \kappa_0^i = l_0^i\} \\
&\qquad \times \Pr\{\kappa_0^i = l_0^i\} \\
&= \sum_{l_0^i} \Pr\{\kappa_0^i = l_0^i\} \prod_{j=1}^{i} \Pr\{Y_{j0} = 0|0, \kappa_{j0} = l_{j0}\} \\
&= \sum_{l_0^i} \Pr\{\kappa_0^i = l_0^i\} \prod_{j=1}^{i} \exp[-Q(1 + l_{j0})] \\
&= \exp[-Qi] \cdot E \exp\Big[-Q \sum_{j=1}^{i} \kappa_{j0}\Big] \\
&= \Big[1 - i\frac{\gamma w}{ML} + i\frac{\gamma w}{ML} e^{-Q}\Big]^{N-1} \cdot e^{-Qi} .
\end{aligned}$$

Here Q denotes the average photons per chip pulse. Whence

$$\begin{aligned}
&\Pr\{Y_{i0} = 0, \text{ some } i \in \mathcal{X}|0\} \\
&= -\sum_{i=1}^{w} (-1)^i \binom{w}{i} \Big[1 - i\frac{\gamma w}{ML} + i\frac{\gamma w}{ML} e^{-Q}\Big]^{N-1} \cdot e^{-Qi} .
\end{aligned}$$

We now proceed to find an upper bound on the second probability:

$$\begin{aligned}
&\Pr\{Y_{ij} \geq 1 \ \forall i \in \mathcal{X}, \text{ some } j \neq 0|0\} \\
&\qquad \leq \sum_{j=1}^{M-1} \Pr\{Y_{ij} \geq 1 \ \forall i \in \mathcal{X}|0\} \\
&\qquad = (M - \gamma) \Pr\{Y_{i1} \geq 1 \ \forall i \in \mathcal{X}|0, \nu_1 = 0\} \\
&\qquad\quad + \sum_{j=1}^{\gamma-1} \Pr\{Y_{ij} \geq 1 \ \forall i \in \mathcal{X}|0\} ,
\end{aligned}$$

where, for any $j \in \{1, 2, \cdots, M-1\}$, $\nu_j \in \{0, 1\}$ denotes the number of pulses that cause a hit (self-interference) in slot j due to the signature code pulses sent in slot 0 by the desired user. The first term in the right hand side of the last inequality is due to the $M-1-(\gamma-1)$ slots that do not have self-interference with slot 0, i.e., $\nu_j = 0$ with probability 1 for these slots. The second term, however, is due to the remaining $\gamma - 1$ slots. These slots interfere with slot 0 at a positive probability, i.e., $\Pr\{\nu_j = 1\} > 0$. Assuming uniformly distributed marks in the code sequences it is easy to see that $\Pr\{\nu_1 = 1\} = \frac{w(w-1)}{L-1}$. Denote $\Pr\{\nu_1 = 1\}$ by q, hence

$$\begin{aligned}
&\Pr\{Y_{ij} \geq 1 \ \forall i \in \mathcal{X}, \text{ some } j \neq 0|0\} \\
&\quad \leq (M - \gamma) \Pr\{Y_{i1} \geq 1 \ \forall i \in \mathcal{X}|0, \nu_1 = 0\} \\
&\qquad + (1 - q)(\gamma - 1) \Pr\{Y_{i1} \geq 1 \ \forall i \in \mathcal{X}|0, \nu_1 = 0\} \\
&\qquad + q(\gamma - 1) \Pr\{Y_{i1} \geq 1 \ \forall i \in \mathcal{X}|0, \nu_1 = 1\} \\
&\quad \leq (M - \gamma) \Pr\{\kappa_{i1} \geq 1 \ \forall i \in \mathcal{X}\} \\
&\qquad + (1 - q)(\gamma - 1) \Pr\{\kappa_{i1} \geq 1 \ \forall i \in \mathcal{X}\} \\
&\qquad + q(\gamma - 1) \Pr\{\kappa_{i1} \geq 1 \ \forall i \in \mathcal{X}^-\} ,
\end{aligned}$$

where the last inequality is justified due to the fact that given a "0" was sent, if $\kappa_{i1} = 0$ and $\nu_1 = 0$ then Y_{i1} should be zero as well. Moreover, if $\nu_1 = 1$ than $\kappa_{i_0 1} \geq 1$ with probability one for some $i_0 \in \mathcal{X}$. Here $\mathcal{X}^- = \mathcal{X} - \{i_0\}$. We proceed by estimating the last probabilities:

$$\begin{aligned}
&\Pr\{\kappa_{i1} \geq 1 \ \forall i \in \mathcal{X}\} \\
&\quad = 1 - \Pr\{\kappa_{i1} = 0, \text{ some } i \in \mathcal{X}\} \\
&\quad = 1 + \sum_{i=1}^{w} (-1)^i \binom{w}{i} \Pr\{\kappa_{1,1} = \kappa_{2,1} = \ldots = \kappa_{i1} = 0\} \\
&\quad = 1 + \sum_{i=1}^{w} (-1)^i \binom{w}{i} \Big(1 - i\frac{\gamma w}{ML}\Big)^{N-1} .
\end{aligned}$$

$$\begin{aligned}
&\Pr\{\kappa_{i1} \geq 1 \ \forall i \in \mathcal{X}^-\} - \Pr\{\kappa_{i1} \geq 1 \ \forall i \in \mathcal{X}\} \\
&\quad = \sum_{i=1}^{w-1} (-1)^i \binom{w-1}{i} \Big(1 - i\frac{\gamma w}{ML}\Big)^{N-1} \\
&\qquad - \sum_{i=1}^{w} (-1)^i \binom{w}{i} \Big(1 - i\frac{\gamma w}{ML}\Big)^{N-1} \\
&\quad = -(-1)^w \Big(1 - \frac{\gamma w^2}{ML}\Big)^{N-1} \\
&\qquad + \sum_{i=1}^{w-1} (-1)^i \Big[\binom{w-1}{i} - \binom{w}{i}\Big] \Big(1 - i\frac{\gamma w}{ML}\Big)^{N-1} \\
&\quad = -(-1)^w \Big(1 - \frac{\gamma w^2}{ML}\Big)^{N-1} \\
&\qquad - \sum_{i=1}^{w-1} (-1)^i \binom{w-1}{i-1} \Big(1 - i\frac{\gamma w}{ML}\Big)^{N-1} \\
&\quad = \sum_{i=1}^{w} (-1)^{i-1} \binom{w-1}{i-1} \Big(1 - i\frac{\gamma w}{ML}\Big)^{N-1} .
\end{aligned}$$

The required upper bound can now be written as

$$\begin{aligned}
P_E \leq M - 1 &+ \sum_{i=1}^{w} (-1)^i \binom{w}{i} \Big(1 - i\frac{\gamma w}{ML}\Big)^{N-1} \\
&\qquad \times \Big[M - 1 - (\gamma - 1)\frac{i(w-1)}{L-1}\Big] \\
&- \sum_{i=1}^{w} (-1)^i \binom{w}{i} \Big(1 - i\frac{\gamma w}{ML} + i\frac{\gamma w}{ML} e^{-Q}\Big)^{N-1} e^{-Qi} .
\end{aligned}$$

B. Users-Throughput Product

One important parameter of performance evaluation in practice is the channel throughput. In OPPM-CDMA, the throughput (in nats/s) for each user can be obtained by

$$R_T \stackrel{\text{def}}{=} \frac{\log M}{T} = \frac{\gamma \log M}{M\tau} = \frac{\gamma \log M}{MLT_c} \qquad \text{nats/s} .$$

The throughput-pulsewidth product (in nats/chip) is thus:

$$R_0 \stackrel{\text{def}}{=} R_T T_c = \frac{\gamma \log M}{ML} \qquad \text{nats/chip} .$$

In [5] we have defined the users-throughput product (denoted by NR) as the product of the number of users times R_0:

$$\mathrm{NR} \stackrel{\text{def}}{=} N \cdot R_0 = N \cdot \frac{\gamma \log M}{ML} \qquad \text{nats/chip} .$$

It has been shown that $\mathrm{NR}_{\max}$ (defined below) is almost a constant quantity for optical CDMA channels and can be used as a figure of merit. We define $R_{0,\max}$ and $\mathrm{NR}_{\max}$ as follows:

$$R_{0,\max} \stackrel{\text{def}}{=} \max_{\substack{\gamma,M,L:\\ P_b^* \le \epsilon}} R_0, \qquad \mathrm{NR}_{\max} \stackrel{\text{def}}{=} \max_{\substack{\gamma,M,L:\\ P_b^* \le \epsilon}} \mathrm{NR} ,$$

where $P_b^* = \frac{M/2}{M-1} P_E^*$ and $P_E^* = \lim_{Q\to\infty} P_E$:

$$P_E^* \le M - 1 + \sum_{i=1}^{w} (-1)^i \binom{w}{i} \left(1 - i\frac{\gamma w}{ML}\right)^{N-1} \times \left[M - 1 - (\gamma - 1)\frac{i(w-1)}{L-1}\right] .$$

V. Numerical Results

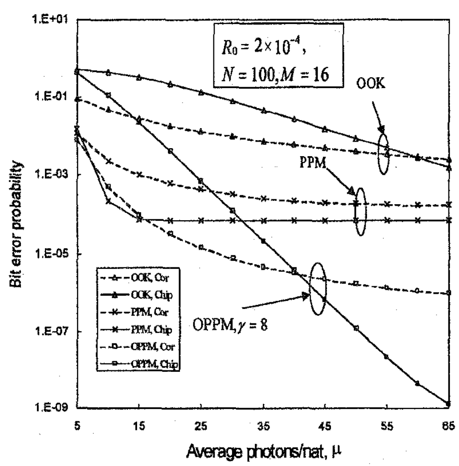

Fig. 4. A comparison between the bit error rate of both correlation and chip-level receivers for OOK-, PPM-, and OPPM-CDMA systems.

In our numerical calculations we hold both the rate of data transmission R_T and pulsewidth T_c fixed. We choose the code length L so as to satisfy the constraint on throughput-pulsewidth product $R_0 = R_T \cdot T_c$. Given a number of users N we choose the code weight w to be the maximum weight that satisfies the code constraint [1]: $N \le \frac{L-1}{w(w-1)}$. Fig. 4 shows the bit error rate for various modulation schemes versus the average photons per nat for a fixed number of users ($N = 100$) and a constraint on the throughput-pulsewidth product ($R_0 = 2 \times 10^{-4}$.) The

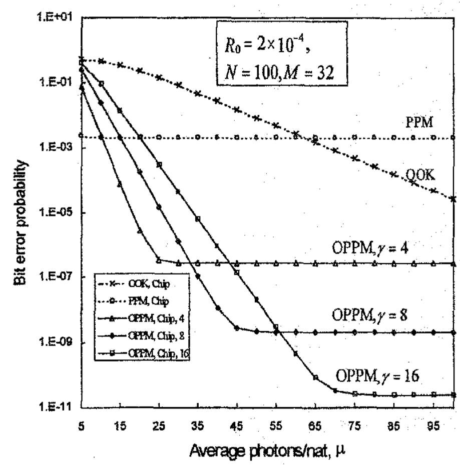

Fig. 5. A comparison between the bit error rate of chip-level receivers for OOK-, PPM-, and OPPM-CDMA systems with different values of γ.

relation between the average transmitted photons per chip pulse Q and the average transmitted photons per nat μ is given by

$$Q = \begin{cases} \frac{\mu \log 2}{w} ; & \text{for OOK,} \\ \frac{\mu \log M}{w} ; & \text{for OPPM.} \end{cases}$$

From this figure we notice that for moderate and high average energy OPPM-CDMA chip-level receiver gives the lowest error rate among all compared receivers. For low average energy, however, OPPM-CDMA correlation receiver is the best. A comparison between chip-level receivers with different values of γ is shown in Fig. 5. For low values of μ the performance of OPPM-CDMA improves as γ decreases but soon the error rate saturates and the performance improves as γ increases. This shows that the larger the value of γ the smaller the error floor. The floor of OOK-CDMA is not seen in Fig. 5 since it saturates at a slower rate. It is obvious that OOK-CDMA floor is less than that of PPM-CDMA. However it is not better than OPPM-CDMA as we can see from next figure. In Fig. 6 we plot the limits of the error probabilities versus the number of users as the average energy increases to infinity. R_0 is still constrained as above. This figure demonstrates that even if there is no constraint on the average energy OPPM-CDMA is still better than OOK-CDMA. Moreover since we have shown in [6] that OOK-CDMA chip-level receiver is asymptotically ($\mu \to \infty$) optimal, thus OPPM-CDMA chip-level receiver performs better than OOK-CDMA optimum receiver.

Maximum achievable throughput-pulsewidth and users-throughput products are plotted in Figs. 7 and 8, respectively, versus the number of users. Both correlation and chip-level receivers for both OOK- and OPPM-CDMA systems are considered. It is obvious from the figures that OPPM-CDMA always gives a better throughput than OOK-CDMA and the best throughput is achieved when using OPPM-CDMA chip-level receivers. For small

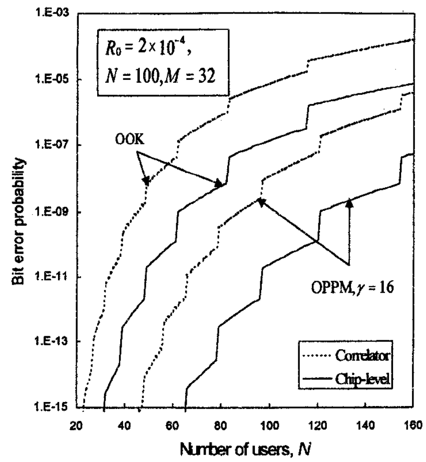

Fig. 6. A comparison between the bit error rate of both correlation and chip-level receivers for OOK- and OPPM-CDMA systems with unlimited energy and R_0=2×10^{-4}.

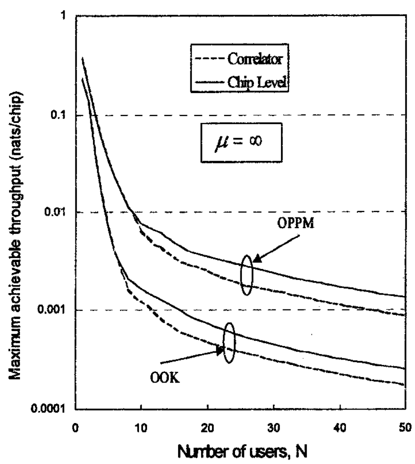

Fig. 7. A comparison between the maximum achievable throughput of both correlation and chip-level receivers for OOK- and OPPM-CDMA systems versus the number of users when there is no limit on the average transmitted energy and the bit error rate does not exceed 10^{-9}.

number of users the throughput limitations of both the correlation and chip-level receivers coincide with each other. This is because for small number of users the main degradation is due to the shot noise process of the photodetector rather than the multiple-user interference.

The users-throughput products for all receivers are almost constant for most of the time and can still be considered as a figure of merit. The users-throughput products for OOK-CDMA chip-level receiver, OPPM-CDMA correlation receiver, and OPPM-CDMA chip-level receiver are 1.4, 5, and 7.5 times greater than that of the OOK-CDMA correlation receiver, respectively.

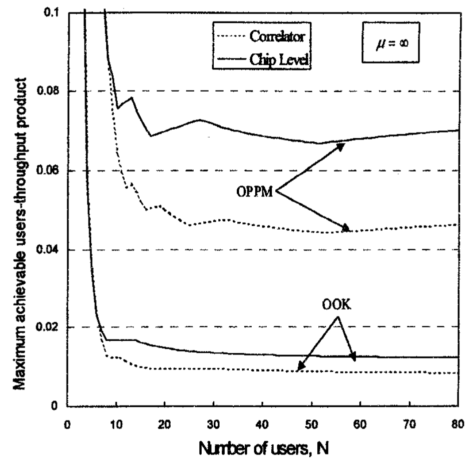

Fig. 8. Same as Fig. 7 but for the maximum achievable users-throughput products versus the number of users.

VI. Concluding Remarks

OPPM-CDMA chip-level detector has been proposed for direct-detection optical CDMA communication systems. The performance in terms of the bit error rate and the throughput limitation has been compared to traditional correlation detectors and previously developed chip-level detectors. In our derivation of the bit error rate we have assumed a Poisson shot noise model for the receiver photodetectors. We can extract the following concluding remarks.

i) For moderate and high average energy per bit, OPPM-CDMA chip-level receiver performs much more better than both OOK- and PPM-CDMA correlation and chip-level receivers, and better than OPPM-CDMA correlation receiver.

ii) OPPM-CDMA chip-level receiver has a throughput limitation that is 7.5 times greater than that of OOK-CDMA correlation receiver if $P_b \leq 10^{-9}$.

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