Performance Analysis of Free-Space Optics Systems Adopting Multi-Pulse PPM Techniques in Gamma-Gamma Channels for Thermal Noise Limited Systems

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ABSTRACT

Atmospheric turbulence has a strong effect on the performance of the terrestrial free space optics system and there are many statistical models that are used to describe the atmospheric turbulence but gamma-gamma is shown to be the most suitable model in both weak and strong turbulence. In this paper, we adopt gamma-gamma model and use the characteristics of Meijer G function to get a closed form expression for the average symbol error rate of multi-pulse PPM under gamma-gamma fading channel. Next, we use our derived form to study the effect of atmospheric conditions, operating wavelength and modulation level on the system performance.

Keywords: Atmospheric turbulence, free space optics, gamma-gamma distribution, multi-pulse PPM (MPPM).

1. INTRODUCTION

Free space optics (FSO) is an attractive alternative for the conventional optical fiber in many applications especially when the optical fiber is not available. FSO systems use atmosphere as transmission medium, so their performance is affected by many challenges. Atmospheric turbulence (signal fading) has the most significant effect because it significantly increases the symbol error rate of the system. Atmospheric turbulence was described by many statistical models, e.g., lognormal model, exponential model, and recently gamma-gamma model. Last model is suitable for both weak and strong turbulence [1]. In our analysis we use the gamma-gamma model for its robustness.

Studies that have investigated the performance of MPPM in free-space optical gamma-gamma channels are very rare. Hamkins and Moision have obtained an exact expression for the symbol-error rate (SER) of MPPM schemes for discrete memoryless channels in non-turbulent atmosphere [2]. Nguyen and Lampe have studied coded MPPM FSO transmission using discrete-time Poisson channel model in non-turbulent atmosphere [3]. They have considered two issues, when MPPM would be better than PPM and how MPPM could be used with error control coding. Gappmair and Muhammad have got an exact expression for the SER of PPM under gamma-gamma optical scintillation model [4]. Xu et al. have discussed the use of binary convolutional coding with iterative detection for the case of MPPM [5]. Wilson et al. have studied the use of multiple-input/multiple-output (MIMO) channel for FSO adopting MPPM [6]. They have concluded that the resulting MIMO channel can reduce FSO turbulent effects in both log-normal and Rayleigh-fading channel models. Balsells et al. have calculated the average BER for a rate adaptive transmission technique using MPPM block coding of variable Hamming weight under turbulence conditions [7]. This type of block coding has a variable amount of pulses and has been shown to have a high peak to average optical power ratio. Their analysis was based on a hyperexponential fitting and Monte Carlo simulation.

Up till now, no one has got a closed form expression for the SER of MPPM in the case of FSO under gamma-gamma distribution. In this paper, we derive an expression for the average symbol-error rate of thermal-noise-limited MPPM systems in gamma-gamma channels.

2. MPPM SER ANALYSIS IN ATMOSPHERIC TURBULENCE CHANNEL

In this paper we adopt an intensity modulation/direct detection (IM/DD) FSO system. At the receiver side, the received electrical signal y is given by:

$$y = RPC + n \tag{1}$$

where R is the responsivity of the photodiode, n is a signal-independent zero-mean white Gaussian noise with variance σ_n^2, $C$is one for signal time slot and zero for non-signal time slot, and P is the average optical power received per signal time slot. The average received optical power per symbol $P_R$can be expressed by the link range equation [8]:

$$P_R(h) = P_T\left(\frac{\eta A}{\lambda L}\right)^2 h \tag{2}$$

where P_T is the average transmitted power, η is the efficiency of both the transmitter and receiver optics, A is the transceiver telescopic area, λ is the operating wavelength, L is the distance between the transmitter and receiver, and h is the channel state due to atmospheric turbulence. The channel state h is described by the Gamma-Gamma model as following [1]:

$$f_h(h) = \frac{2(\alpha\beta)^{(\alpha+\beta)/(2)}}{\Gamma(\alpha)\Gamma(\beta)}(h)^{\frac{\alpha+\beta}{2}-1}K_{\alpha-\beta}\left(2\sqrt{\alpha\beta h}\right) \quad (3)$$

where

$$\alpha = \left(\exp\left[\frac{0.49\sigma_R^2}{\left(1+1.11\sigma_R^{\frac{12}{5}}\right)^{\frac{7}{6}}}\right]-1\right)^{-1} \quad (3.a)$$

$$\beta = \left(exp\left[\frac{0.51\sigma_R^2}{\left(1+0.69\sigma_R^{\frac{12}{5}}\right)^{\frac{5}{6}}}\right]-1\right)^{-1} \quad (3.b)$$

where $\sigma_R^2 = 1.23C_n^2(2\pi/\lambda)^{\frac{7}{6}}L^{\frac{11}{6}}$ is unitless Rytov variance, C_n^2 is the refractive-index structure parameter, α and β are the scintillation parameters, $\Gamma(.)$ is the gamma function, and $K_c(.)$ denotes the c_{th} order modified Bessel function of the second kind.

In MPPM techniques, the symbol duration T is divided into M time slots and optical power is transmitted in w time slots only. The MPPM transmitted signal is thus:

$$x(t) = \frac{P_R\, MR}{w}\sum_{k=0}^{M-1} C_k rect\left(t-\frac{kT}{M}\right) \quad (4)$$

where

$$rect(t) = \begin{cases}1, & 0 \le t < \frac{T}{M} \\ 0, & otherwise\end{cases}$$

where C_k is one for signal time slot and zero for non signal time slot. Next, we use a union bound approximation [9] to get an upper bound on the SER of MPPM:

$$P_{eSym} \approx \frac{w(M-w)}{2}\, erfc\left(\frac{d_{min}\sqrt{R_b}}{2\sqrt{2}\sigma_n}\right) \quad (5)$$

where P_{eSym} is the probability of symbol error, R_b is the bit rate, and d_{min} is the minimum Euclidean distance between any two points in the signal constellation.

Using (2) and (4) we get d_{min} then substituting it into (5) we get:

$$P_{esym}(h) \approx \frac{w(M-w)}{2}\, erfc\left(\frac{RP_T\left(\frac{\eta A}{\lambda L}\right)^2 h}{2w\sigma_n}\sqrt{Mlog_2\binom{M}{w}}\right) \quad (6)$$

Here we focus on a slow fading environment. Thus, we get the average SER ($\overline{P_{esymb}}$) by averaging (6) over $f_h(h)$:

$$\overline{P_{esymb}} = \int_0^\infty P_{esym}(h)\, f_h(h)\, dh \quad (7)$$

Next, we substitute (3) and (6) into (7) and express $K_c(.)$ and erfc(.) in terms of the Meijer G function [10,11] we get:

$$\overline{P_{esymb}} \approx \frac{w(M-w)(\alpha\beta)^{\frac{(\alpha+\beta)}{2}}}{2\sqrt{\pi}\Gamma(\alpha)\Gamma(\beta)}\int_0^\infty h^{\frac{(\alpha+\beta)}{2}-1} G_{0,2}^{2,0}\left(\alpha\beta h\middle|\begin{matrix}-\\ \frac{\alpha-\beta}{2},\frac{-\alpha+\beta}{2}\end{matrix}\right) G_{1,2}^{2,0}\left(\left(\frac{RP_T\left(\frac{\eta A}{\lambda L}\right)^2\sqrt{Mlog_2\binom{M}{w}}}{2w\sigma_n}h\right)^2\middle|\begin{matrix}1\\ 0,\frac{1}{2}\end{matrix}\right)dh \quad (8)$$

Finally, using the integral form for the Meijer G function defined in [10,11], we get the average SER for MPPM system, as in (9).

$$\overline{P_{esymb}} \approx \frac{2^{\alpha+\beta-3} w(M-w)}{\pi^{\frac{3}{2}}\Gamma(\alpha)\Gamma(\beta)} \times G_{5,2}^{2,4}\left(\left(\frac{2RP_T\left(\frac{\eta A}{\lambda L}\right)^2\sqrt{Mlog_2\binom{M}{w}}}{w\sigma_n\alpha\beta}\right)^2 \middle| \begin{matrix} \frac{1-\beta}{2},\frac{2-\beta}{2},\frac{1-\alpha}{2},\frac{2-\alpha}{2},1 \\ 0,\frac{1}{2} \end{matrix}\right) \tag{9}$$

3. NUMERICAL RESULTS

In this section, we use the closed form of the average SER for MPPM that we got in (9) to study the performance of the MPPM system under gamma-gamma channel. In all our calculations, we use the same system parameters as in [10], listed in Table 1.

Table 1. System parameters.

Parameter	Symbol	Value	Unit
Tx/Rx optics efficiency	η	80	%
Responsivity	R	0.5	A/W
Noise standard deviation (at 1 Gbps)	σ_n	$5\text{x}10^{-7}$	A
Transmitter and Receiver diameter	D	8	cm
Distance	L	5	km

Figure 1 shows the average SER versus average transmitted power of MPPM at different atmospheric turbulence strength for two operating wavelengths (λ = 850 nm and λ = 1550 nm). As shown in the figure, the performance of short wavelengths is better than that of long one at low power, when the power increases above a threshold level, the performance of long wavelengths is better than that of short wavelengths. This is because in the case of low power, the noise has a dominant effect and since short wavelengths introduce higher gain than long wavelengths [8], the performance of short-wavelength systems will be better. On the other hand, as the power increases above a certain threshold, the turbulence effect dominates and since σ_R^2 is inversely proportional to $\lambda^{(7/6)}$ (for fixed C_n^2) the turbulence would have more effect on short-wavelength systems.

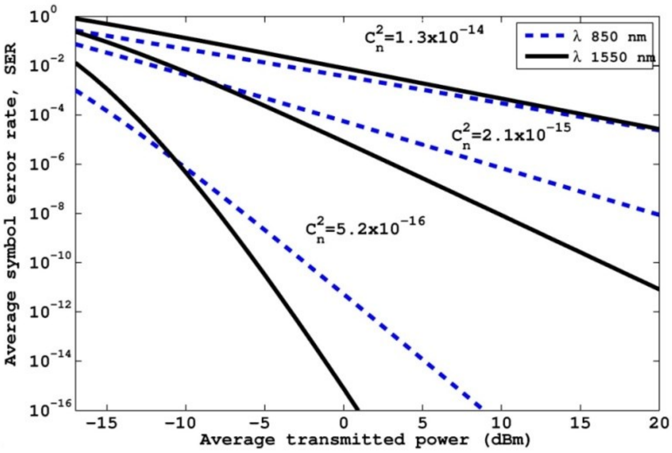

Figure 1. Average SER versus average transmitted power in dBm for MPPM with M=8 and w=4 under different turbulence strength and for λ = 850 nm and λ = 1550 nm.

Figure 2 shows the effect of increasing modulation level on the performance of the FSO. We choose M and w to have a constant data rate. In weak and moderated turbulence the noise has the dominant effect on the performance of the system and since higher modulation level has higher peak power so the average SER of the higher modulation level will be better than it for lower level as shown in Fig. 2. While the system performance cannot be improved in strong turbulent conditions by increasing the modulation level because in that case turbulence has dominant effect on the SER performance.

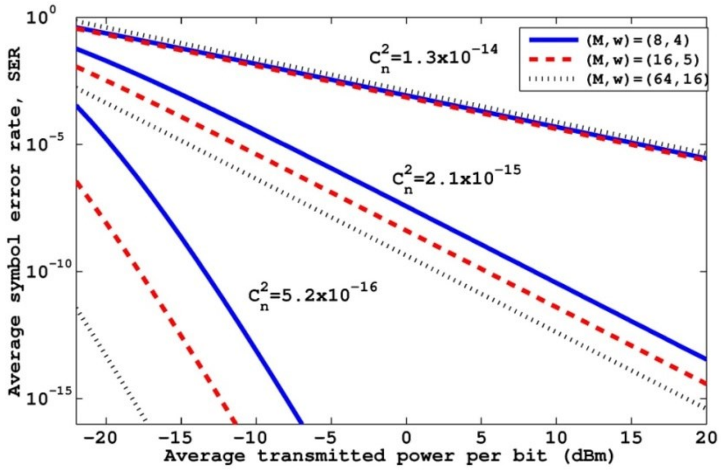

Figure 2. Average SER versus average transmitted power per bit in dBm for MPPM with (M; w) = (8; 4) ; (16; 5) and (64; 16) at different atmospheric turbulence strength for λ = 1550 nm.

4. CONCLUSIONS

A closed form of the upper bound on the average symbol error rate of multi-pulse PPM under gamma-gamma fading channel has been derived for free space optics. Using that expression we get that the performance of short wavelengths is better than that of long wavelengths at low power, whereas as the power increases more than a threshold level, the performance of long wavelengths is better than that of short wavelengths. Furthermore, for both low and moderate turbulence, increasing the number of time slots per symbol improves the system performance, while for high turbulence the system performance cannot be improved by increasing the number of time slots per symbol.

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