Nonlinear Capacity of Few-Mode Fibers using the Gaussian Noise Model

Abdulaziz E. El-Fiqi[1], Abdallah Ismail[1], Ziad A. El-Sahn[2], Hossam M. H. Shalaby[1,2], and Rameash K. Pokharel[3]

abdulaziz.elfiqi@ejust.edu.eg, abdallah.Ali@ejust.edu.eg, ziad.elsahn@ieee.org, shalaby@ieee.org, pokharel@ed.kyushu-u.ac.jp

[1] Egypt-Japan University of Science and Technology (E-JUST), Alexandria 21934, Egypt

[2]Photonics Group, Electrical Engineering Department, Alexandria University, Alexandria 21544, Egypt

[3]E-JUST Center, Kyushu University, Fukuoka, Japan

***Abstract*—A closed-form expression for the nonlinear capacity of a few-mode fiber (FMF) is formulated analytically by extending the Gaussian noise (GN) model. The effects of different nonlinearity penalties on the system capacity are then evaluated via simulations.**

I. Introduction

Space-division multiplexing (SDM) is a promised degree of freedom to increase the transmission capacity, which is rapidly approaching its fundamental limit in single mode fibers [1]. Few-mode fibers (FMFs) are remarkable channels for SDM techniques. However, the nonlinear interaction between different propagation modes in FMFs is a major source of performance limitation which must be addressed for its mitigation. Few analytical efforts have been developed to model the nonlinear propagation in multi-mode fibers [2], [3]. In this paper, we extend the GN-model developed for single mode fibers [4] to address the different nonlinearities impact in FMFs. In [3], a general integral formula for the cross-modal nonlinear interaction has been proposed for multimode fibers. However, in this work, a simple closed-form expression (with less computational complexity) for the nonlinear capacity of FMFs is derived for the case of weak linear coupling regime among the different spatial modes. In addition, the effect of different nonlinearity penalties for various constellation orders are investigated.

II. Proposed GN-Model for Few-Mode Fibers

The signal propagation of mode p in a FMF has been already described in [5]. It is divided into a linear part (dispersion + attenuation) and a nonlinear part, given by $N_p = j\gamma\left(\frac{8}{9} f_{pppp}\left|\bar{\mathbf{A}}_p\right|^2 + \frac{4}{3}\sum_{h\neq p} f_{pphh}\left|\bar{\mathbf{A}}_h\right|^2\right)$. Here $\bar{\mathbf{A}}_p$ is the field envelope of mode p, γ is the fiber nonlinearity coefficient, f_{pppp} is the intra-modal nonlinear coefficient tensor of mode p, and f_{pphh} is the inter-modal nonlinear coefficient tensor between p and h spatial modes. The calculated values of these tensors have been reported in [1].

The GN-model for single mode fibers assumes that the nonlinearity source can be modeled as an additive Gaussian noise which is statistically independent from both the amplifier noise and the transmitted signal [4]. Also, it assumes the transmitted signal as a wavelength-division multiplexed (WDM) comb signal with N_{ch} channels. These assumptions can be extended for FMFs based on the fact that the interaction between any two orthogonal polarization modes is equivalent to that between two spatial modes [6]. Therefore, the performance of a FMF link per mode can be determined by the optical signal-to-noise ratio as $OSNR_p = P_{tx_p}/(P_{ASE} + P_{NL_p})$, where P_{tx_p} is the launch power per mode, P_{ASE} is amplified-spontaneous-emission (ASE) noise power, and P_{NL_p} is the nonlinear interference power per mode.

From Shannon's relation of capacity for the unconstrained additive-white Gaussian noise channel of single-polarization single mode fiber, we can formulate an extended relation for the dual-polarization few-mode fiber capacity per mode as

$$C_p = \frac{2R_s}{B_{ch}}\log_2\left(1 + \frac{R_s}{B_n}OSNR_p\right) \quad \text{bits/symbol/mode,} \tag{1}$$

where B_n is the noise bandwidth of 12.48 GHz (equivalent to the reference 0.1 nm resolution for OSNR calculation), R_s is the signaling rate, and B_{ch} is the WDM channel bandwidth. After a rigorous mathematical analysis, we derive the nonlinear interference power formula through integrating its power spectral density (PSD) over the WDM bandwidth $B_w = N_{ch}B_{ch}$. Furthermore, this PSD is obtained by statistically averaging the square absolute-value of the nonlinear optical field. Next, by assuming a rectangular shaped WDM channel spectrum with bandwidth B_{ch} with has the same value as signalling rate R_s at Nyquist case, a closed-form expression for the nonlinear interference power per mode can be obtained at the center channel frequency [7]. Finally, the overall capacity for the few-mode fiber is formulated by the summation of all mode capacities. The final expression of the overall capacity is shown in (2) at the top of next page, where M is the number of spatial modes, $L_{eff_p} = (1 - e^{-\alpha_p L})/\alpha_p$ is the span effective length of a fiber with length L_s, and a mode fiber loss coefficient α_p, P_{tx} is the total lunch power, β_{2_p} is the mode group-velocity dispersion (GVD), N_s is the number of fiber spans, F is the amplifier noise factor, h is Plank's constant, ν is the center channel frequency, and G is the amplifier gain.

III. Model Results

In this section, we apply the proposed model for a system with the following FMF parameters [1] ($\alpha_p \approx 0.22$ dB/km, $\beta_{2_p} \approx -21.2$ ps^2/km, and $\gamma_p \approx 1.3$ W^{-1}km^{-1}) for three modes (LP_{01}, $LP_{11a,b}$). For the WDM system, the specifications are assumed as: $R_s = 32$ GBaud (that is, a net sampling rate of 25 GBaud + 20% for forward error correction (FEC) and network protocols overheads [4]) and $N_{ch} = 5$. The used

$$C = \frac{2R_s}{B_{ch}} \sum_P \log_2 \left(1 + \frac{1}{B_n N_s} \frac{P_{tx_p}}{(G-1)Fh\nu + \frac{4}{3}\frac{\gamma^2}{M^3}\left[\frac{4}{9} f^2_{pppp} + \sum\limits_{h\neq p} f^2_{pphh}\right] L_{eff_p} \frac{\log(\pi^2 B_w^2 |\beta_{2p}| L_{eff_p})}{\pi^2 B_w^3 |\beta_{2p}|} P^3_{tx.}} \right) \tag{2}$$

amplifier is an erbium-doped fiber amplifier (EDFA) with a noise figure of 6 dB and a gain that compensates the fiber span loss: $G = e^{\alpha_p L_s}$.

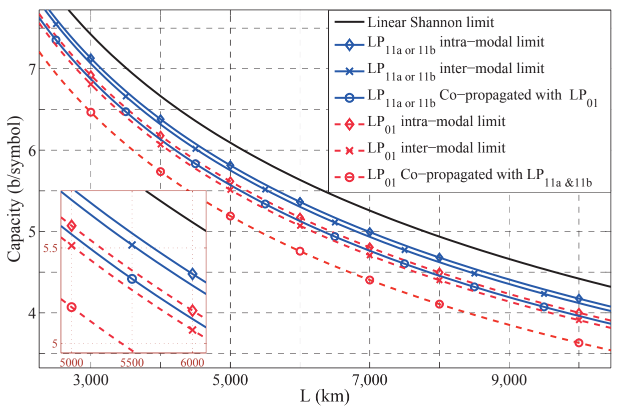

Fig. 1: Capacity versus fiber maximum reach at different non-linear penalties for a FMF of $L_s = 100$ km.

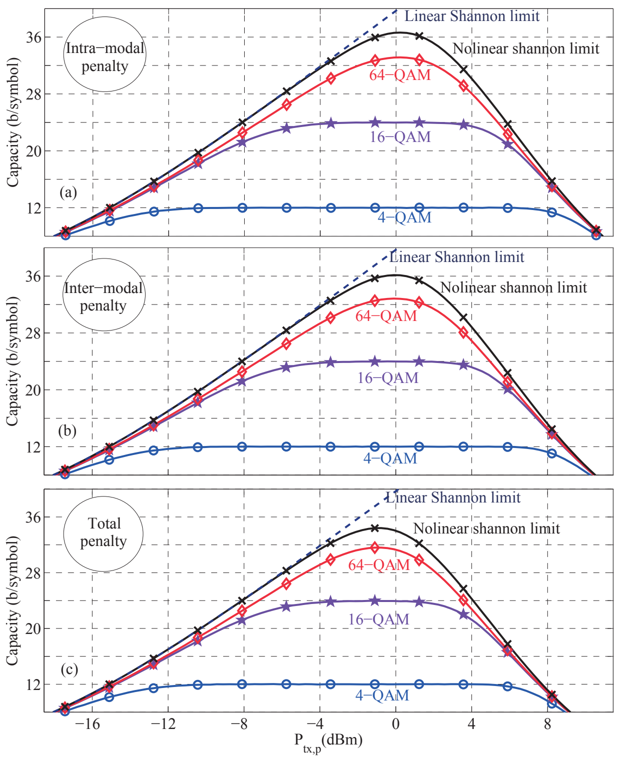

Fig. 2: Capacity versus channel launch power per mode at different nonlinear penalties for a FMF of $L_s = 100$ km and $N_s = 5$.

Fig. 1 shows the degradation of the FMF capacity with the fiber maximum reach for different nonlinearity penalty limits at the optimal launch power. It is shown that the nonlinear penalty effect is greater in the fundamental LP_{01} than the degenerated modes (LP_{11a}, LP_{11b}) in both the inter-modal and the intra-modal limits. Also, the inter-modal penalty is more significant than those for intra-modal ones in all spatial modes. This penalty variation is related to the different spatial interactions and the fiber effective areas of different modes.

The capacity for different constellation QAM levels (4, 16, 64) is compared to both the nonlinear and linear Shannon limits in Fig. 2. The impact of the nonlinearities does not appear at low constellation levels (4-QAM). However, at moderate levels (16-QAM), the different nonlinearity penalties become significant and limit the FMF capacity from reaching its maximum value (which is 24 b/symbol for a 3-mode dual-polarized signal). In addition, both the inter- and intra-modal impacts are approximately equal as shown in Figs. 2-b and 2-c. At high constellation levels, the impact of the nonlinearities becomes more significant for different penalties. Furthermore, the inter-modal impact becomes greater than the intra-modal one by $\approx 1.5\%$ at nonlinear Shannon limit. These nonlinearity penalties are clear in the nonlinear Shannon capacity curves for different nonlinearity limits. The optimal launched power (top points on curves in Fig. 2) only depends on the penalty limit (nonlinear tensor's values) not on the constellation order.

IV. Conclusions

The GN-model has been extended for FMFs in order to estimate the effects of different nonlinearity penalties. A closed-form formula for the nonlinear FMF capacity has been obtained. Using this formula, it has been verified that the performance degradation due to the inter-modal penalty is greater than those for the intra-modal ones. In addition, the nonlinear impact on the fundamental mode is greater than that for the degenerated modes.

References

[1] I. Kaminow, T. Li, and A. E. Willner, Eds., *Optical Fiber Telecommunications Volume VIB, Sixth Edition: Systems and Networks*, 6th ed. Amsterdam ; Boston: Academic Press, May 2013.

[2] F. Ferreira, S. Jansen, P. Monteiro, and H. Silva, "Nonlinear semi-analytical model for simulation of few-mode fiber transmission," *Photonics Technology Letters, IEEE*, vol. 24, no. 4, pp. 240–242, 2012.

[3] G. Rademacher, S. Warm, and K. Petermann, "Analytical description of cross-modal nonlinear interaction in mode multiplexed multimode fibers," *Photonics Technology Letters, IEEE*, vol. 24, no. 21, pp. 1929–1932, 2012.

[4] A. Carena, V. Curri, G. Bosco, P. Poggiolini, and F. Forghieri, "Modeling of the impact of nonlinear propagation effects in uncompensated optical coherent transmission links," *Journal of Lightwave Technology*, vol. 30, no. 10, pp. 1524–1539, 2012.

[5] S. Mumtaz, R. Essiambre, and G. P. Agrawal, "Nonlinear propagation in multimode and multicore fibers: Generalization of the manakov equations," *Lightwave Technology, Journal of*, vol. 31, no. 3, pp. 398–406, 2013.

[6] A. Mecozzi, C. Antonelli, and M. Shtaif, "Nonlinearities in space-division multiplexed transmission," in *Optical Fiber Communication Conference and Exposition and the National Fiber Optic Engineers Conference (OFC/NFOEC), 2013*, Mar. 2013, pp. 1–3.

[7] A. E. El-Fiqi, A. Ismail, Z. A. El-Sahn, H. M. H. Shalaby, and R. K. Pokharel, "Evaluation of nonlinear interference in few-mode fiber using the gaussian noise model," *submitted in CLEO-2015 (unpublished)*.