# Fair Optimal Transceivers Placement in Foggy Cooperative Dynamic FSO Networks 

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#### Abstract

New optimal transceivers' placement scheme is proposed to improve the performance of fair resource allocation in foggy dynamic FSO networks. The scheme is formulated as bilevel-multi-objective-optimization problem. Results indicate the superior performance over random placements.


OCIS codes: Free space optics (FSO), Foggy link, placement.

## 1. Introduction

FSO is a promising solution for existing last mile connectivity problem. However, FSO channel is affected by various weather conditions. One of these conditions is the foggy weather which could absorb and scatter the transmitted optical signal [1]. To overcome this performance degradation and increase the network reliability, dynamic FSO networks has been proposed [2]. We have proposed in [3] a resource allocation scheme called Lex-Max-Min Fairness (LMMF) scheme, that aims to enhance the performance of cooperative dynamic networks against the atmospheric variation at given number and placement of FSO transceivers. In that work, the number of additional FSO transceivers at each FSO node is assumed to be fixed throughout the performance evaluation, i.e., the effect of changing transceivers' placement has not been investigated. In order to increase the number of alternative paths from/to each node and possibilities of network reconfigurations, the number of additional FSO transceivers must be increased. However, increasing number of additional transceivers without considering their optimal placement could significantly increase the network cost [4]. In this paper, optimal transceivers' placement scheme associated with fair resource allocation is proposed for cooperative dynamic FSO networks.

## 2. FSO Link Model

The FSO channel losses are accumulated from many sources [1]. In the considered networks, both fog and geometrical losses are assumed the dominating ones. Moreover, a homogeneous foggy weather is assumed over the entire network with equal visibility value. When the background radiation level is relatively high (e.g., outdoor), the thermal noise can be ignored and the receiver can be modeled as shot noise limited. On-off keying modulation is used in the transmission over FSO links.

## 3. Placement Vector Parameters

Generally, the cooperative FSO network consists of $N$ nodes ( $v_{1}, \ldots, v_{N}$ ) with arbitrary geographical distribution in addition to the backbone node $v_{0}$. Normally, each FSO node is equipped with one optical transceiver to transmit/receive its own traffic to backbone node, which is equipped with $N$ optical transceivers. In order to take the advantages of cooperative reconfigurable FSO network, additional $w$ optical transceivers are implemented. Given $w$, the total number of optical transceivers at $k^{t h}$ node is $z_{k} \in\{1,2, \ldots, w+1\}$, where $k \in\{1,2, \ldots, N\}$. The number of nodes equipped with $s$ transceivers is given by $n_{s}$ where $s \in\{1,2, \ldots, w+1\}$. The number of feasible placement vectors is $\Lambda$ and each vector is represented as $Z_{a}=\left(z_{1}, z_{2}, \ldots, z_{k}, \ldots, z_{N}\right)$ where $a \in\{1,2, \ldots, \Lambda\}$. Also, all vectors can be summarized in matrix $H$ with dimension $(N \times \Lambda)$ and $\Lambda$ is upper bounded as: $\Lambda<N+\sum_{b=2}^{b=w-1}\binom{N}{b} \times(w-1)^{b}+\binom{N}{w}, w \leq N$. For each transceiver's placement vector $Z_{a}$, there is a number of feasible configurations $\beta_{a}$ that could be realized with this placement, where $\beta_{a}$ is upper bounded by: $\beta_{a}<\left[\sum_{i i=1}^{i i=N}\binom{N}{i i}\right] \times \prod_{s=1}^{s=w+1}\left[\sum_{j j=0}^{j j=s}\binom{N}{j j}\right]^{n_{s}}$. The overall total number of feasible configurations for all placements vectors is $\alpha<\left[\sum_{a=1}^{a=\Lambda} \beta_{a}\right]$. At a given visibility value $V$, the losses of all FSO links are summarized in $\gamma$ matrix, $\gamma=\left(\gamma_{00}, \ldots, \gamma_{0 N} ; \ldots, \gamma_{i j}, \ldots ; \gamma_{N 0}, \ldots, \gamma_{N N}\right)$, where $\gamma_{i j}$ is the loss coefficient
of link between transmitter of $i^{\text {th }}$ node and receiver of $j^{t h}$ node. Clearly, $0 \leq \gamma_{i j} \leq 1, \gamma_{i i}=0$ and $\gamma_{i j}=\gamma_{j i}$ for any $i, j \in\{0,1, \ldots, N\}$. Associated with each placement vector $Z_{a}$ a number of feasible configurations $\beta_{a}$. The feasible $d^{\text {th }}$ configuration, where $d \in\left\{1,2, \ldots, \beta_{a}\right\}$, is identified by binary connection matrix $G_{d}$. Moreover, a matrix $G_{a}$ represents at set of $\beta_{a}$ binary connection matrices that summarize the connections for all feasible configurations associate with placement vector $Z_{a}$. The connection status between network nodes in $d^{\text {th }}$ configuration are summarized in connections matrix $G_{d}=\left(g_{d 00}, \ldots, g_{l 0 N} ; \ldots, g_{d i j}, \ldots ; g_{d N 0}, \ldots, g_{d N N}\right)$, where $g_{d i j}$ is the connection status between $i^{\text {th }}$ and $j^{\text {th }}$ nodes in configuration $d$ and $g_{d i j} \in\{0,1\}$. The connection between nodes $i$ and $j$ is established if $g_{d i j}=1$. Also, bidirectional links are assumed so that $g_{d i j}=g_{d j i}$ and $g_{d i i}=0$. Moreover, all FSO links are assumed to have the same average transmitted power, i.e., the power of optical link between nodes $i$ and $j$ is constant, $P_{d i j}=P_{0}$. However, to increase link capacity and guarantee an error rate less than a specified maximum $B E R_{d i j}<B E R_{\max }$, the link between nodes $i$ and $j$ in configuration $d$ adapts its transmission rate, $T_{d i j}$, to be one of $q+1$ discrete values, where $T_{d i j} \in\left\{0, x_{1}, x_{2} \ldots, x_{q}\right\}$ and $x_{1}<x_{2}<\ldots<x_{q}$. The transmission rate of node $k$ in configuration $d$ is denoted by $T_{d k}$, where $T_{d k}=\sum_{j=0}^{N} T_{d k j}$. The bit rate of node $k$ (its own traffic) through connection to node $j$ is denoted by $R_{d k j}$. The overall bit rate of node $k$ is $R_{d k}=\sum_{j=0}^{N} R_{d k j}$. Obviously, $R_{d k} \leq T_{d k}$ and for practical implementation both $R_{d k}$ and $T_{d k} \in\left\{0, x_{1}, x_{2} \ldots, x_{q}\right\}$. The end-to-end bit error rate of node $k, B E R_{d k}$, is bounded by $B E R_{d k} \leq B E R_{\max }$. The bit rates and bit-error rates associated with all nodes in the feasible configurations could be summarized in $\left(\beta_{a} \times N\right)$ matrices $R_{a}$ and $E_{a}$, respectively. For the $d^{\text {th }}$ configuration, the bit rates for all nodes are represented in vector $r_{d}(1 \times N)$ and $r_{d} \in R_{a}$. Also, The bit-error rates are summarized in vector $e_{d}(1 \times N)$ and $e_{d} \in E_{a}$. The network capacity associated with configuration $d$ is $C_{d}=\sum_{k=1}^{N} R_{d k}$, and all capacities associated with all feasible configuration are summarized in vector $\left(\beta_{a} \times 1\right) C_{a}$ and $C_{d} \in C_{a}$.

## 4. Optimal Transceivers Placement scheme

In the proposed scheme, the optimization problem is formulated in two levels of multi-objective optimization. Each level optimizes different objectives with different priories. The highest priority objective is the reliability (in terms of non-zero bit rate node). The second objective is the fairness $F=\sum_{k=1}^{k=N} R_{d k}^{2} /\left(N \times \sum_{k=1}^{k=N} R_{d k}^{2}\right)$ [5], and the least priority one is the average error rate. In first level of the optimization problem, and for each $Z_{a}$, the scheme obtains the optimal configuration, $\bar{G}_{a}(:,:, m)$, at each visibility interval $\Delta V_{a m}, m \in\left\{1,2, . ., M_{a}\right\}$. This could be achieved by optimizing $r_{d}$ using sequential max-min that reduces number of dropped nodes and hence rises the fairness. If there are more than one solution (configuration) with the same max-min bit rate, the scheme proceeds to optimize $e_{d}$, using sequential min-max that reduces the error rates. The results of this optimization level are two matrices, $\left(M_{a} \times N\right) \bar{R}_{a}$ and $\left(M_{a} \times N\right) \bar{E}_{a}$, which are used in the next level of optimization as shown in (1). In the second level of the optimization problem, the optimal placement vector $Z_{a^{*}}$ along with the associated optimal configuration $G_{a^{*}}(:,:, m)$ are obtained. The optimization aims to get the the optimal placement vector that minimizes number of dropped node at the entire range of visibility $\Omega_{a}^{\Re}$ then increase fairness $\Omega_{a}^{f}$ and finally decrease the average error rate $\Omega_{a}^{e}$.

## Step One : Optimal configurations

Lex-Max-Min : $\left\{r_{d}=\left(R_{d 1}, \ldots, R_{d N}\right): r_{d} \in R_{a}\right\}$


## Subject to:

$d \in\left\{1,2,3, \ldots, \beta_{a}\right\}, z_{k} \in\{1,2, \ldots, w+1\}$,
$R_{d k}=\sum_{j=0}^{N} R_{d k j}, R_{d k} \leq T_{d k}, T_{d k}=\sum_{j=0}^{N} T_{d k j}$,
$B E R_{d k} \leq B E R_{\max }, B E R_{d i j}<B E R_{\max }, P_{d i j}=P_{0}$,
$\left\{R_{d k}, R_{d k j}, T_{d k}, T_{d k j}\right\} \in\left\{0, x_{1}, x_{2}, \ldots, x_{q}\right\}$,
$T_{d j j}=R_{d j j}=0,\{i, j\} \in\{0,1,2,3, \ldots, N\}, j \neq i$.

## Step Two: Optimal Placement

$$
\left.\begin{array}{l}
\operatorname{Max}_{a}\left\{\Omega_{a}^{\Re}=\frac{1}{N} \sum_{m=1}^{m=M_{a}} P\left(\Delta V_{a m}\right) \cdot \Delta V_{a m} \cdot \sum_{k=1}^{k=N} \delta_{m k}\right\} \\
\delta_{m k}=\left\{\begin{array}{rr}
1 & \bar{R}_{m k}>0 \\
0 & \bar{R}_{m k}=0
\end{array} \bar{R}_{m k} \in \bar{R}_{a},\right. \\
\operatorname{Max}\left\{\Omega_{a}^{f}=\sum_{m=1}^{m=M_{a}} P\left(\Delta V_{a m}\right) \cdot \Delta V_{a m} \cdot F_{m}: \bar{R}_{m k} \in \bar{R}_{a}\right\}
\end{array}\right\} \begin{aligned}
& \operatorname{Min}_{a}\left\{\Omega_{a}^{e}=\sum_{m=1}^{m=M_{a}} P\left(\Delta V_{a m}\right) \cdot \Delta V_{a m} \cdot \sum_{k=1}^{k=N} \frac{\bar{R}_{a}(m, k) \cdot \bar{E}_{a}(m, k)}{\bar{C}_{a}(m)}\right\} \\
& \text { Subject to: } \\
& \quad a \in\{1,2,3, \ldots, \Lambda\} .
\end{aligned}
$$

Table 1: Simulation Parameters

| Link parameters | Values | Link parameters | Values |
| :---: | :---: | :---: | :---: |
| Signal wavelength | 1550 nm | Divergence angle | $2 \mathrm{mrad} / \mathrm{m}$ |
| Average Transmitted Power | -12 dBm | Bit Rates in Gbps | $1,3 / 4,2 / 3,1 / 2,1 / 3,1 / 4,0$. |
| Diameter of Transmitter | 4 cm | Average ambient noise | -49.6 dBm |
| Diameter of Receiver | 20 cm | BER threshold | $1 e-4$ |
| Network area | $4 \times 4 \mathrm{~km}$ | Cell length | $1 \times 1 \mathrm{~km}$ |



Fig. 1: Reliability and fairness for different networks.

## 5. Simulation and Numerical Results

The problem is solved numerically by using exhaustive search (ES) method to guarantee the optimal solutions. Table 1 shows the simulation parameters of the FSO links. The values are selected to be in the practical range. In these networks, the longest FSO link is 3.6 km and the shortest FSO link is 0.77 km . Also, the maximum bit rate is 1 Gbps and maximum allowable error-rate is $10^{-4}$. At $N=9$, two networks with $w=3$ and $w=4$ are evaluated. Figure 1a shows reliability and fairness performance of many placement vectors, $Z_{1}, Z_{2}$, and $Z_{3}$ at the entire range of visibility. Depending on the probability distribution $P\left(\Delta V_{a m}\right)$ of the visibility, one of $Z_{2}$ or $Z_{3}$ is the optimal placement. Figure 1 b shows the superior performance of the optimal placement scheme with $w=3$ over the performance of a misplacement network with $w=4$.

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